Convective Heat Transfer in a Vertical Channel Filled with a Nanofluid

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Abstract— Nanofluids are engineered colloids made of a base fluid and nanoparticles (1-100 nm). This article presents the numerical study of a natural convection flow and heat transfer characteristics in a vertical channel filled with a nanofluid. The transport equations along with the boundary conditions are first cast into a dimensionless form and then the resulting equations are solved numerically by finite difference method. Also, we compared the numerical results using semi-numerical-analytical method known as Differential Transformation Method (DTM), and regular perturbation method (PM). The influence of pertinent parameters such as Grashof number, Brinkman number and the nanoparticle volume fraction on the velocity and temperature fields are shown graphically. Results for the skin friction and heat transfer rate for various types of nanoparticles such as Ag-water, Cu-water, SiO_2-water, Diamond-water and TiO_2-water are also tabulated. The results obtained for the flow and heat transfer characteristics reveal many interesting behaviors that warrant further study on nanofluids in a vertical channel.

Keywords: Natural convection, nanofluid, Differential transformation method, Finite difference method, perturbation method.

I. INTRODUCTION

Natural convection heat transfer has been considered by many designers in the past as an important phenomenon in the cooling mechanism of engineering systems due to its simplicity, minimum cost, low noise, smaller size, and reliability. Ostrach [1] was the first among others to reviewed various industrial and engineering applications of natural convection such as thermal insulators for buildings, the electronics industry, solar collectors, and cooling systems for nuclear reactors. Recently, using nanofluids as an advanced technology has a tendency to increase the heat transfer compared to other base fluids. This technology lies under the concepts of using suspended nanoplarticles in order to enhance the rate of heat transfer.

Enhancement of heat transfer is essential in improving performances and compactness of electronic devices. Usual cooling agents (water, oil, etc.) have relatively small thermal conductivities and therefore heat transfer is not very efficient. Therefore numerous methods were proposed to improve the thermal conductivity of these fluids by suspending nano/micro (larger-size) particle materials in liquids. An innovative technique, which uses a mixture of nanoparticles and the base fluid, was first introduced by Choi [2] in order to develop advanced heat transfer fluids with substantially higher conductivities. The resulting mixture of the base fluid and nanoparticles having unique physical and chemical properties is referred to as a nanofluid. Nanofluid describes a liquid suspension containing ultra-fine particles (diameter less than 50 nm). Experimental studies show that even with small volumetric fraction of nanoparticles (usually less than 5%) the thermal conductivity of the base fluid is enhanced by 10-50% with a remarkable improvement in the convective heat transfer coefficient. The characteristic feature of nanofluid is the thermal conductivity enhancement, a phenomenon observed by Masuda et al. [3]. This suggested the possible use of nanofluids in advanced nuclear systems (Buongiorno and Hu [4]). Eastman et al. [5], Xie et al. [6] and Jana et al. [7] showed that higher thermal conductivity can be achieved in thermal systems utilizing nanofluids. Few researchers considered the fluid and solid phase role in the heat transfer process as two-phase model while the others considered both the fluid phase and the solid particles in a thermal equilibrium state and flow with the same local velocity (single phase). Various literatures have mentioned that the two-phase model is not applicable for analyzing nanofluids [8-9]. Owing to their superior characteristic, nanofluids behave like a fluid rather than an unconventional solid/fluid mixture fluid containing micron and large particles. For this reason the application of the modified single phase method is more appropriate for the heat transfer process. Several investigations have been reported in the literature on the convective heat transfer in nanofluids; see, for example, Daungthongsuk and Wongwises [10], Trisaksri and Wongwises [11], Wang and Mujumdar [12], Kumar et al. [13] and the references cited therein. Recently Cimpean and Pop [14] studied analytically the fully developed mixed convection flow of a nanofluid through an inclined channel filled with a porous medium

The differential transformation method (DTM) was first applied in the engineering domain by Zhou [15]. The differential transform method is based on Taylor expansion. It constructs an analytical solution in the form of a polynomial. It is different from the traditional high order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method is computationally taken long time for large orders. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. DTM has been successfully applied to solve many nonlinear problems arising in engineering, physics, mechanics, biology, etc. The differential transform method can overcome the restrictions and limitations of perturbation techniques so that it provides us with a possibility to analyze strongly nonlinear problems. Jang et al. [16] applied the two-dimensional differential transform method to the solution of partial differential equations. Kurnaz and Oturanç [17] applied DTM for solution of system of ordinary differential equations. Arikoglu and Ozkol [18] employed DTM on differential-difference equations. Ravi Kanth and Aruna [19] found the solution of singular two-point boundary value problems using differential transformation method. The method was successfully applied to various application problems [20–24]. Very recently Rashidi et al. [25] applied the DTM to obtain approximate analytical solutions of combined free and forced (mixed) convection about inclined surfaces in a saturated porous medium.

Despite a number of experimental and numerical studies on convection flow in nanofluids (reported in the literature), there is still a lack of information on the problem of heat transfer enhancement in a vertical channel containing water based nanofluids. In fact to the best knowledge of the authors, no studies have been reported in the literature.

The focus of the present study is to analyze the effects of several pertinent parameters such as the free convection parameter, nanoparticle-size volume fraction; and Brinkman number on the flow and heat transfer characteristics with different types of nanofluid particles. The coupled non-linear ordinary differential equations are solved analytically using DTM valid for all values of free convection parameter and Brinkman number. To validate the DTM solutions, the basic equations are solved by numerical method using finite difference method and also analytically using regular perturbation method valid for small values of free convection parameter and Brinkman number.

II. PROBLEM DESCRIPTION AND GOVERNING EQUATIONS

Consider an incompressible water-based nanofluid, which steadily flows between two infinite vertical and parallel plate walls maintained at different constant temperatures extending in the X and Y directions, as shown in Figure 1. The differentially heated walls are filled with a water based nanofluid containing different types of nano-solidparticles namely copper (Cu), silver (Ag), Titanium (TiO_2), silicon (SiO_2) and Diamond. The nano-solidparticles is Newtonian, incompressible, and laminar. The base fluid and the spherical nanoparticles are in thermal equilibrium. The nanofluid is a two component mixture with the following assumptions:

i. steady, laminar and fully developed;
ii. incompressible;
iii. no-chemical reaction;
iv. negligible radiative heat transfer; and
v. nano-solid-particles and the base fluid which is chosen as water are in thermal equilibrium and no slip occurs between them.

The thermophysical properties of the base fluid and the solid particles are given in Table 1(Oztop and Abu-Nada [26]). The thermophysical properties of the nanofluid are assumed to be constant except for the density variation in the buoyancy force term which is determined by the Boussinesq approximation. The fluid rises in the channel driven by buoyancy forces. The flow is assumed to be steady, unidirectional and fully developed. Under these assumptions, the equations governing the convective flow and heat transfer are:

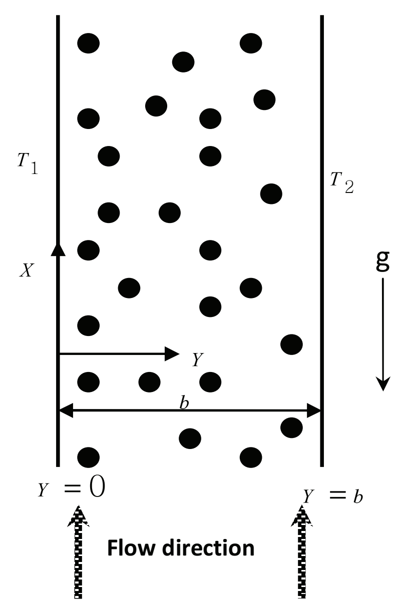

Figure 1. Physical configuration.

$$\mu_{nf}\frac{d^2U}{dY^2}+(\rho\beta)_{nf}\,g\,(T-T_0)=0 \quad (1)$$

$$\frac{d^2T}{dY^2}+\left(\frac{\mu}{K}\right)_{nf}\left(\frac{dU}{dY}\right)^2=0 \quad (2)$$

along with boundary conditions

$U=0$ at $Y=0,b$

$T=T_1$ at $Y=0$, $T=T_2$ at $Y=b$ (3)

where U is the velocity component along Y-axis, g is the acceleration due to gravity, T is the temperature of the fluid, μ is the viscosity and K is the thermal conductivity of the fluid. The effective density of the nanofluid is given as

$$(\rho)_{nf}=(1-\phi)\rho_f+\phi\rho_s, \quad (4)$$

where ϕ is the solid volume fraction of nanoparticles. The thermal expansion coefficient of the nanofluid can be determined by

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s. \quad (5)$$

The effective dynamic viscosity of the nanofluid given by Brinkman [27] is

$$\mu_{nf}=\mu_f\big/(1-\phi)^{2.5} \quad (6)$$

In equation (2), K_{nf} is the thermal conductivity of the nanofluid: For spherical nanoparticles, according to Maxwell [28], this can be written as

$$K_{nf}=K_f\left(\frac{K_s+2K_f-2\phi(K_f-K_s)}{K_s+2K_f+\phi(K_f-K_s)}\right). \quad (7)$$

Here the subscripts nf, f and s respectively are the thermo-physical properties of the nanofluids, base fluid and the nano-solid-particles. The equations (1) to (3) are made dimensionless using the following

$$x=\frac{X}{b},\ y=\frac{Y}{b},\ u=U\frac{\rho_f}{\mu_f}b,\qquad \theta=\frac{T-T_0}{T_2-T_1},$$

$$T_0=\frac{T_1+T_2}{2},\qquad Gr=\frac{g\beta_f\Delta T\,b^3\rho_f^2}{\mu_f^2},$$

$$Br=\frac{\mu_f^3}{K_f\Delta T\,\rho_f^2 b^2} \quad (8)$$

Equations (1) and (2) become

$$\frac{d^2u}{dy^2}+A\,Gr\,\theta=0 \quad (9)$$

$$\frac{d^2\theta}{dy^2}+C\,Br\left(\frac{du}{dy}\right)^2=0 \quad (10)$$

and related boundary conditions (3) become

$u=0$ at $y=0,1$

$\theta=-\frac{1}{2}$ at $y=0$, $\theta=\frac{1}{2}$ at $y=1$ (11)

where $A=(1-\phi)^{2.5}\left(1-\phi+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)$,

$$C=\frac{1}{(1-\phi)^{2.5}}\left(\frac{K_s+2K_f+\phi(K_f-K_s)}{K_s+2K_f-2\phi(K_f-K_s)}\right)$$

We shall solve this coupled nonlinear differential equation using the DTM, and compared the results numerically by finite difference method and the perturbation method. Such solutions are useful and serve as a baseline for comparison with the solutions obtained via numerical schemes.

Basic Idea of Differential Transformation Method (DTM)

The differential transform of function *u*(*y*) is defined as

$$U(k)=\frac{1}{k!}\left[\frac{d^k u(y)}{dy^k}\right]_{y=0} \quad (12)$$

where $u(y)$ is the original function and $U(k)$ is the transformed function which is called the T-function. The differential inverse transform of U(k) is defined as follows:

$$u(y)=\sum_{k=0}^{\infty}U(k)y^{k} \quad (13)$$

In real applications, the function u(y) be a finite series, (13) can be written as

$$u(y)=\sum_{k=0}^{n}U(k)y^{k} \quad (14)$$

and (13) implies that $u(y)=\sum_{k=n+1}^{\infty}U(k)y^{k}$ is neglected as it is small. Usually, the values of n are decided by a convergence of the series coefficients. In order to assess the accuracy of DTM for solving nonlinear equations the solutions obtained from DTM are compared with numerical solutions (FDM) and with analytical solutions (PM). The fundamental mathematical operations performed by differential transform method are listed in Table 2. Taking differential transform of Eqs. (9) and (10), one can obtain

$$U[k]=-\frac{AGr}{(1+k)(2+k)}\Theta[k] \quad (15)$$

$$\Theta[k]=-\frac{C\,Br}{(1+k)(2+k)}\sum_{r=0}^{k}(k+1-r)(r+1)U[k+1-r]U[r+1] \quad (16)$$

The differential transform of the boundary conditions are as follows

$$U[0]=0,\ U[1]=\alpha\ \Theta[0]=-\frac{1}{2},\Theta[1]=\beta \quad (17)$$

Using the conditions as given in Eq. (11), one can evaluate the unknowns α and β. By using the DTM and the transformed boundary conditions, above equations that finally leads to the solution of a system of algebraic equations.

Finite Difference Method (FDM)

The governing Eqs. (9) and (10) together with the boundary conditions (11) are solved using finite difference method. In numerical iterations, computational domain is divided into a uniform grid system. Both the second-derivative and the squared first-derivative terms are discretized with central difference of second-order accuracy and the results are presented in Tables.

Perturbation Method (PM)

Equations (9) and (10) are coupled non-linear equations because of viscous dissipations and it is difficult, in general, to solve them analytically. However for vanishing Br, equations become linear and can be solved exactly. Small values of Br facilitate finding analytical solutions of Eqs. (9) and (10) in the form:

$$u=u_0+Br\,u_1+Br^2u_2........ \quad (18)$$

$$\theta=\theta_0+Br\,\theta_1+Br^2\theta_2........ \quad (19)$$

Substituting (18) and (19) into (9) and (10) and equating the like powers of Br to zero, we obtain zeroth and first order equations. Zeroth order equations take the form

$$\frac{d^2u_0}{dy^2}+AGr\,\theta_0=0 \quad (20)$$

$$\frac{d^2\theta_0}{dy^2}=0 \quad (21)$$

subject to boundary conditions

$u_0=0$ at $y=0,1;$

$\theta_0=-\frac{1}{2}$ at $y=0;$ $\theta_0=\frac{1}{2}$ at $y=1$ (22)

First order equations

$$\frac{d^2 u_1}{dy^2} + A\,Gr\,\theta_1 = 0 \qquad (23)$$

$$\frac{d^2 \theta_1}{dy^2} + c\left(\frac{du_0}{dy}\right)^2 = 0 \qquad (24)$$

subject to boundary conditions

$$u_1 = \theta_1 = 0 \quad \text{at} \quad y = 0, 1 \qquad (25)$$

Solutions of Eqs. (20) and (21) using (22) are

$$\theta_0 = y - 0.5;$$

$$u_0 = \frac{-A\,Gr\left(1 - 3y + 2y^2\right)y}{12} \qquad (26)$$

Solutions of Eqs. (23) and (24) are not presented due to brevity. The physical quantity of interest in this problem is the skin friction and Nusselt number which for the heated wall and cold wall. The dimensionless form of skin friction coefficient is

$$\tau_0 = \frac{1}{(1-\phi)^{2.5}}\left.\frac{du}{dy}\right|_{y=0} \quad \text{and} \quad \tau_1 = \frac{1}{(1-\phi)^{2.5}}\left.\frac{du}{dy}\right|_{y=1}$$

and the dimensionless form of Nusselt number is

$$Nu_0 = \left(\frac{K_{nf}}{K_f}\right)\left.\frac{d\theta}{dy}\right|_{y=0} \quad \text{and} \quad Nu_1 = \left(\frac{K_{nf}}{K_f}\right)\left.\frac{d\theta}{dy}\right|_{y=1}$$

III. Results and discussion

The problem of free convection in a vertical channel filled with nanofluid including th effects of viscous dissipation is analysed. The governing equations are highly nonlinear and coupled for which the closed form solutions are not available. However the approximate solutions are obtained using three different methods i.e. by DTM, FDM and PM. Error analysis is also done between these methods. In order to justify the results obtained by DTM, the problem is linearised and compared with DTM solutions. The solutions obtained by direct method and DTM agree very well. Further finding the solutions of nonlinear equations using FDM also justify the solutions obtained using DTM. To further justify the solutions of DTM, the nonlinear governing equations are solved analytically using PM valid for small values of free convection parameter and Brinkman number.

The velocity and temperature fields are computed for various values of governing parameters such as Grashof number Gr (free convection parameter), solid volume fraction ϕ and Brinkman number Br for different nano-particles and are shown in Figs. 2 to 6. The values of all the graphs are obtained by DTM.

Figure 2 display the velocity profiles for Ag, Cu, Diamond, TiO2 and SiO2 nano-particles. It is observed that the values for silver and copper show closer values and Diamond, SiO2 and TiO2 are also close to each other. The optimal velocity is observed using silver as a nano-particle at the hot wall and minimal value at the cold wall.

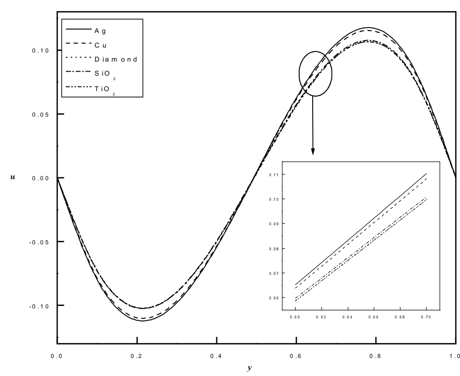

Fig. 2 Velocity profiles for different nanoparticles with $Gr = 20$, $\phi = 0.1$, $Br = 0.1$.

The flow field for different values of free convection parameter *Gr* and solid volume fraction ϕ is shown in Figs. 3 and 4 respectively. As Grashof number increases, velocity increases in the domain $y = 0.5$ to 1 decreases from $y = 0$ to 0.5. It is also observed from Fig. 3 that as the solid volume fraction ϕ increases velocity decreases. ($\phi = 0$ are the profiles for purely viscous fluid). As the Grashof number increases temperature increases slightly in the middle of the channel. Also as solid volume fraction ϕ increases temperature decreases. However the effects of Grashof number *Gr* and solid volume fraction ϕ on the temperature do not show significant effects on the temperature field.

Figures 5 and 6 are the plots of velocity and temperature for variations of Brinkman number *Br* and solid volume fraction ϕ. The nature of graphs is similar to the effect of *Gr* and ϕ as seen in Figs. 3 and 4 respectively. That is as Brinkman number increases velocity decreases near the cold wall and increases near the hot wall for both viscous and nanofluids. The effect of *Br* and ϕ on temperature is not very effective as seen in Fig. 6. The velocity and temperature profiles for viscous fluid are more when compared to nanofluid. This is due to the fact that adding nano-particles to viscous fluid will result in suspensions, similar to that of non-Newtonian fluids, hence adding additives to viscous fluid will result in the reduction of flow nature.

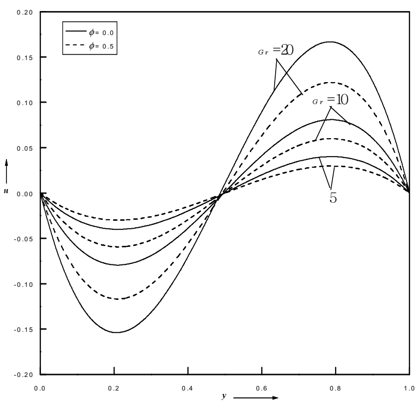

Fig. 3 Velocity profiles for different values of Gr **and** ϕ **with** $Br = 0.1$ **for silver-water nonoparticles.**

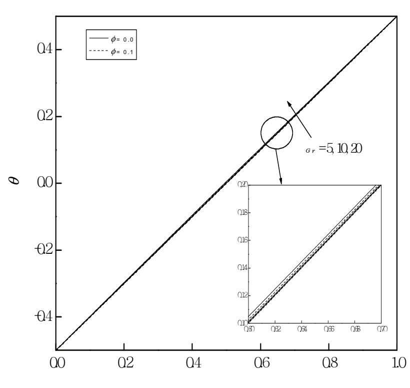

Fig. 4 Temperature profiles for different values of Gr **and** ϕ **with** $Br = 0.1$ **for silver-water nonoparticles.**

To understand the frictional force and rate of heat transfer, at the walls, plots of skin friction and Nusselt number are shown in Figs. 7 and 8 respectively. It is seen from Fig. 7 that skin friction at both the walls decreases as Grashof number increases. The Brinkman number increases the skin friction at the cold wall and decreases at the hot wall. However its effect is not significant for small values of Grashof number at both the walls. Figure 7 also shows that profiles of skin friction for nanofluid lies above viscous fluid at both the walls. The rate of heat transfer for variations of Grashof number, Brinkman number and solid volume fraction is shown in Fig. 8. The effect of Grashof number on heat is not very effective at both the walls. As Brinkman number increases skin friction at the cold wall increases and decreases at the hot wall for large values of Grashof number. The plots of heat transfer at both the walls for nanofluid lies below the viscous fluid.

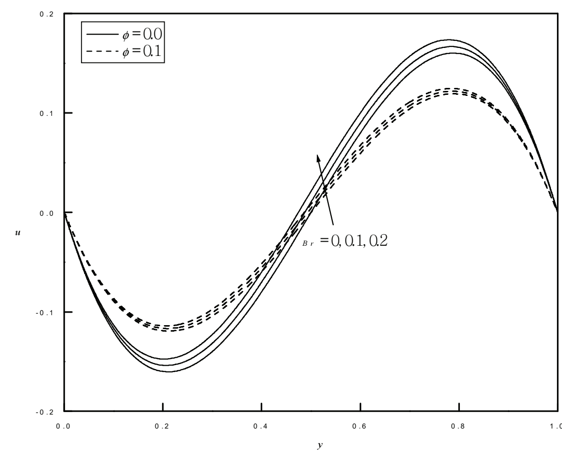

Fig. 5 Velocity profiles for different values of Br **and** ϕ **with** $Gr = 20$ **for copper-water nanoparticles.**

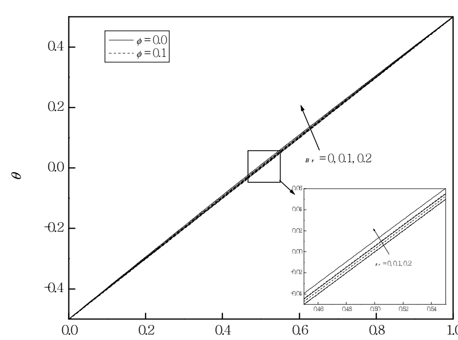

Fig. 6 Temperature profiles for different values of Br and ϕ with $Gr = 20$ for copper-water nanoparticles.

Table 3 and 4 shows the results obtained by DTM, FDM and PM. In the absence of viscous dissipation Eq. (9) to (10) reduces to linear ODEs. Table 3 shows the results of exact solution with the results of DTM solution is agree very well. Table 4 shows the results of temperature for different nano-particles. We observe that convection for SiO_2 is more then followed by Silver, Copper, TiO_2 and less for Diamond. In the presence of viscous dissipations the results are shown in Table 5. It is obvious that present method provides more acceptable results compare with FDM and PM even for $Br = 1$.

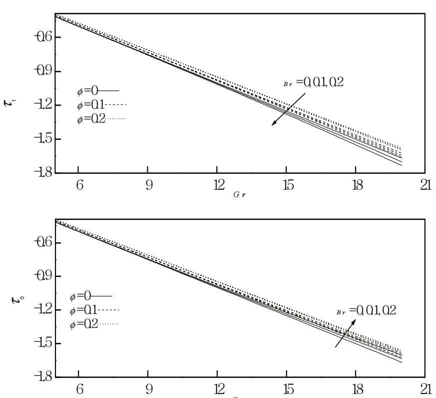

Fig. 7 skin friction profiles for different values of Gr, Br and ϕ for copper-water nanoparticles.

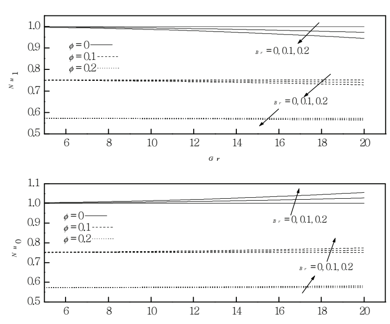

Fig. 8 Nusselt number profiles for different values of Gr, Br and ϕ for copper-water nanoparticles.

Table.1: Thermo-physical properties for pure water and various types of nanoparticles.

Property	Pure water	Ag	Cu	Diamond	SiO_2	TiO_2
$\rho\left(\mathrm{kg/m^2}\right)$	997.1	10500	8933	3510	2200	**4250**
$\mu(\mathrm{N\,m/s})$	0.001	-	-	-	-	**-**
$k\ (\mathrm{W/m\,K})$	0.613	429	400	1000	1.2	**8.9538**
$\beta(1/\mathrm{K})$ **x 10^{-6}**	**207**	**18**	**17**	**1.0**	**5.5**	**0.17**

Table 2: The operations for the one-dimensional differential transform method.

Original function	Transformed function
$y(x) = g(x) \pm h(x)$	$Y(k) = G(k) \pm H(k)$
$y(x) = \alpha g(x)$	$Y(k) = \alpha G(k)$
$y(x) = \dfrac{dg(x)}{dx}$	$Y(k) = (k+1)G(k+1)$
$y(x) = \dfrac{d^2 g(x)}{dx^2}$	$Y(k) = (k+1)(k+2)G(k+2)$
$y(x) = g(x)h(x)$	$Y(k) = \sum_{l=0}^{k} G(l)H(k-l)$
$y(x) = x^m$	$Y(k) = \delta(k-m) = \begin{cases} 1, \text{ if } k = m \\ 0, \text{ if } k \neq m \end{cases}$

Table 3. Comparison of the values of velocity and temperature with exact solution in the absence of viscous dissipation $Gr = 10$ for copper-water nano-particles.

	Velocity				Temperature			
	$\phi = 0.0$		$\phi = 0.5$		$\phi = 0.0$		$\phi = 0.5$	
	Exact	DTM	Exact	DTM	Exact	DTM	Exact	**DTM**
0	0	0	0	0	-0.5	-0.5	-0.5	**-0.5**
0.1	-0.06	-0.06	-0.00920	-0.00920	-0.4	-0.4	-0.4	**-0.4**
0.2	-0.08	-0.08	-0.01227	-0.01227	-0.3	-0.3	-0.3	**-0.3**
0.3	-0.07	-0.07	-0.01073	-0.01073	-0.2	-0.2	-0.2	**-0.2**
0.4	-0.04	-0.04	-0.00613	-0.00613	-0.1	-0.1	-0.1	**-0.1**
0.5	0	0	0	0	0	0	0	**0**
0.6	0.04	0.04	0.00613	0.00613	0.1	0.1	0.1	**0.1**
0.7	0.07	0.07	0.01073	0.01073	0.2	0.2	0.2	**0.2**
0.8	0.08	0.08	0.01227	0.01227	0.3	0.3	0.3	**0.3**
0.9	0.06	0.06	0.00920	0.00920	0.4	0.4	0.4	**0.4**
1	**0**	**0**	**0**	**0**	**0.5**	**0.5**	**0.5**	**0.5**

Table 4. Values of temperature field for different nano-metals in a channel with $Br = 0.1$ and $Gr = 20$.

y	Silver (Au)	Copper (Cu)	Diamond	SiO_2	TiO_2
0	-0.500000	-0.500000	-0.500000	-0.500000	**-0.500000**
0.1	-0.398965	-0.399002	-0.399143	-0.398930	**-0.399102**
0.2	-0.298323	-0.298383	-0.298614	-0.298265	**-0.298547**
0.3	-0.197718	-0.197800	-0.198114	-0.197639	**-0.198024**
0.4	-0.097231	-0.097331	-0.097712	-0.097135	**-0.097602**
0.5	0.002956	0.002849	0.002442	0.003058	**0.002560**
0.6	0.102764	0.102665	0.102284	0.102859	**0.102394**
0.7	0.202281	0.202199	0.201885	0.202360	**0.201975**
0.8	0.301689	0.301628	0.301395	0.301748	**0.301462**
0.9	0.401057	0.401019	0.400872	0.401094	**0.400914**
1	**0.500000**	**0.500000**	**0.500000**	**0.500000**	**0.500000**

Table 5: Comparison of the values of velocity and temperature for copper-water nano-particles ($Gr = 10$, $\phi = 0.2$).

		Velocity			Temperature		
Br		**DTM**	**FDM**	**PM**	**DTM**	**FDM**	**PM**
0.5	0	0.00000000	0.00000000	0.00000000	-0.50000000	-0.50000000	-0.50000000
	0.1	-0.03220928	-0.03220867	-.03220887	-0.39932782	-0.39932231	-0.39932679
	0.2	-0.04276671	-0.04276575	-0.04276594	-0.29891368	-0.29890962	-0.29891198
	0.3	-0.03711756	-0.03711645	-0.03711650	-0.19852332	-0.19852169	-0.19852123
	0.4	-0.02070450	-0.02070335	-0.02070328	-0.09820857	-0.09820755	-0.09820627
	0.5	0.00103412	0.00103527	0.00103540	0.00191299	0.00191515	0.00191536
	0.6	0.02267014	0.02267118	0.02267136	0.10178930	0.10179245	0.10179160
	0.7	0.03878819	0.03878898	0.03878925	0.20147616	0.20147831	0.20147825
	0.8	0.04398289	0.04398331	0.04398366	0.30109139	0.30109038	0.30109309
	0.9	0.03285301	0.03285313	0.03285342	0.40068135	0.40067769	0.40068239
	1	0.00000000	0.00000000	0.00000000	0.50000000	0.50000000	0.50000000
1.0	0	0.00000000	0.00000000	0.00000000	-0.50000000	-0.50000000	-0.50000000
	0.1	-0.03188789	-0.03188644	-0.03188706	-0.39866414	-0.39864463	-0.39866208
	0.2	-0.04215895	-0.04215697	-0.04215737	-0.29783137	-0.29781925	-0.29782796
	0.3	-0.03628207	-0.03628019	-0.03627992	-0.19704489	-0.19704338	-0.19704066
	0.4	-0.01972110	-0.01971943	-0.01971860	-0.09641374	-0.09641509	-0.09640907
	0.5	0.00206902	0.00207053	0.00207165	0.00382726	0.00383029	0.00383208
	0.6	0.02365388	0.02365510	0.02365639	0.10357774	0.10358491	0.10358244
	0.7	0.03962465	0.03962524	0.03962681	0.20295300	0.20295662	0.20295732
	0.8	0.04459227	0.04459210	0.04459387	0.30218890	0.30218075	0.30219245
	0.9	0.03317587	0.03317536	0.03317671	0.40137255	0.40135537	0.40137480
	1	0.00000000	0.00000000	0.00000000	0.50000000	0.50000000	0.50000000

IV. Conclusion

In this study, we presented the definition and operation of differential transformation method (DTM). The DTM has been utilized to derive approximate explicit analytical solutions for nonlinear free convection problem with a small parameter. This new method accelerated the convergence to the solutions. Moreover, it was shown that for this kind of problems, DTM is better than PM and FDM. The figures and tables clearly show this method provides excellent approximations to the solution of these nonlinear equations with high accuracy.

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References

[1] Ostrach,S. Natural convection in enclosures, J. Heat Transfer, 110, pp. 1175–1190, 1988.
[2] S. Choi, Enhancing thermal conductivity of fluids with nanoparticle. in: D.A. Siginer, H.P. Wang (Eds.), Developments and Applications of Non- Newtonian Flows, MD vol. 231 and FED , 66. ASME, pp. 99-105,1995.
[3] H. Masuda, A. Ebata, K. Teramae, N. Hishinuma, Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles, Netsu Bussei, 7 ,227-233,1993.
[4] J. Buongiorno, W. Hu, Nanofluid coolants for advanced nuclear power plants, in: Proceedings of ICAPP'05, Seoul, Paper no. 5705 (May 15-19, 2005).
[5] J.A. Eastman, S.U.S. Choi, S. Li, W. Yu, L.J. Thompson, Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles, Appl. Phys. Lett., 78, pp. 718–720, 2001.
[6] H.Q. Xie, H. Lee, W. Youn, M. Choi, Nanofluids containing multiwalled carbon nanotubes and their enhanced thermal conductivities, J. Appl. Phys., 94,8, pp. 4967–4971, 2003.
[7] S. Jana, A. Salehi-Khojin, W.H. Zhong, Enhancement of fluid thermal conductivity by the addition of single and hybrid nano-additives, Thermo chemica Acta, 462, pp. 45–55, 2007.
[8] R-Y. Jou, S-C. Tzerg, Numerical research on natural convective heat transfer enhancement filled with nanofluids in rectangular enclosures, Int. Commu. Heat Mass Transfer, 33, pp. 727-736, 2006.
[9] K. Khanafer, K. Vafai, M. Lightstone, Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids, Int. J. Heat Mass Transfer, 46, pp. 3639-3653, 2003.
[10] W. Daungthongsuk, S. Wongwises, A critical review of convective heat transfer nanofluids, Renew Sust Energy Rev, 11, pp. 797–817, 2007.
[11] V. Trisaksri, S. Wongwises, Critical review of heat transfer characteristics of nanofluids, Renew Sust Energy Rev, 11, pp. 512–523, 2007.
[12] X. Wang, A.S. Mujumdar, Heat transfer characteristics of nanofluids: A review. Int. J. Therm. Sciences, 46, pp. 1–19, 2007.
[13] Kumar, S., Prasad, S.K., Banerjee, J. Analysis of flow and thermal field in nanofluid using a single phase thermal dispersion model. Applied Mathematical Modelling, 34, pp. 573-592, 2010.
[14] Dalia Sabina Cimpean, Ioan Pop, Fully developed mixed convection flow of a nanofluid through an inclined channel filled with a porous medium Int. J. Heat and Mass Transfer, 55 ,pp.907–914, 2012
[15] J.K. Zhou, Differential transformation and its applications for electrical circuits. Huarjung University Press; 1986. (in Chinese)
[16] M.J. Jang, C.L. Chen, Y.C. Liu, Two-dimensional differential transform for partial differential equations, Appl. Math. Comput, 121, pp. 261–270, 2001.
[17] Kurnaz A, Oturanç G. The differential transform approximation for the system of ordinary differential equations. Int J Comput Math, 82,pp.709–19, 2005.
[18] Arikoglu A, Ozkol I. Solution of differential-difference equations by using differential transform method. Appl Math Comput,181, pp.153–62, 2006.
[19] A.S.V. Ravi Kanth and K. Aruna, Solution of singular two-point boundary value problems using differential transformation method. Physics Letters A , 372, pp. 4671–4673, 2008.
[20] M.M. Rashidi, The modified differential transform method for solving MHD boundary-layer equations. Computer Physics Communications, 180, pp. 2210–2217, 2009.
[21] A.S.V. Ravi Kanth, K. Aruna, Differential transform method for solving the linear and nonlinear Klein–Gordon equation, Computer Physics Commun, 180,pp.708–711, 2009.
[22] M.J. Jang, Y.L. Yeh, C.L. Chen, W.C. Yeh, Differential transformation approach to thermal conductive problems with discontinuous

boundary condition. Appl. Mathematics and Computation, 216, pp. 2339–2350, 2010.

[23] M.M. Rashidi, N. Laraqi, S.M. Sadri, A novel analytical solution of mixed convection about an inclined flat plate embedded in a porous medium using the DTM-Padé, Int. J. Thermal Sciences, 49, pp. 2405–2412, 2010.

[24] Hessameddin Yaghoobi, Mohsen Torabi, The application of differential transformation method to nonlinear equations arising in heat transfer. Int. Commu. Heat and Mass Transfer, 38, pp. 815–820, 2011.

[25] M. M. Rashidi, O. Anwar Beg, N. Rahimzadeh, A generalized differential transform method for combined free and forced convection flow about inclined surfaces in porous media, Chem. Eng. Comm., 199, pp. 257–282, 2012

[26] Abu-Nada, E. Application of nanofluids for heat transfer enhancement of separated flows encountered in a backward facing step. Int. J. Heat and Fluid Flow, 29,pp. 242-249, 2008.

[27] H.C. Brinkman, The viscosity of concentrated suspensions and solution, J. Chem. Phys. 20, pp. 571-581, 1952.

[28] J. Maxwell, A treatise on electricity and magnetism, 2nd ed. Oxford University Press, Cambridge, UK, 1904.