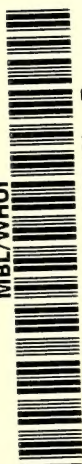


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Theory of

SEAKEEPING

Theory of
SEAKEEPING

by

Prof. B. V. KORVIN-KROUKOVSKY

SPONSORED JOINTLY BY

Ship Structure Committee

The Society of Naval Architects and Marine Engineers

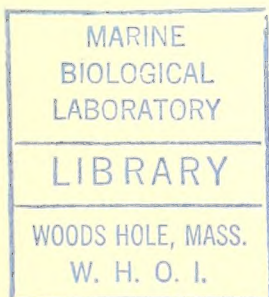


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This monograph has been sponsored by The Society of Naval Architects and Marine Engineers and the Ship Structure Committee. It was prepared under the advisory guidance of a committee appointed jointly by the H-7 Panel of the Hydrodynamics Committee of The Society of Naval Architects and Marine Engineers, and the Committee on Ship Structural Design of the National Academy of Sciences-National Research Council, which advises the design research program of the interagency Ship Structure Committee.

Both sponsoring groups have needed a monograph to undertake a critical survey of the state of the art so as to form a basis for the planning of research in this field. M. St. Denis, Chairman of the H-7 Panel, and E. M. MacCutcheon, then a member of the Ship Structure Subcommittee, were instrumental in arranging a meeting of representatives of the H-7 Panel and of the Committee on Ship Structural Design, which led to initiation of this work.

B. V. Korvin-Kroukovsky had performed fundamental research in the field and was invited by the sponsoring groups to undertake the work of preparing the monograph. He has been assisted by W. Marks of the David Taylor Model Basin in the joint preparation of Section 8 of Chapter 1 and Section 3 of Chapter 3.

As a result of the lack of complete quantitative data in many areas considered by this monograph, and, indeed, basic disagreement as to exactly what the monograph should cover, some controversy has been raised by preliminary drafts distributed to a few individuals working in these fields. The Advisory Committee believes that controversy should not be suppressed and that the publication of this work will encourage research upon which subsequent projects can depend for resolution of these disputed points.

L. TROOST	Massachusetts Institute of Technology, <i>Chairman</i>
D. K. FELBECK	National Academy of Sciences- National Research Council
O. H. OAKLEY	Bureau of Ships, Department of the Navy
M. ST. DENIS	Institute for Defense Analyses; Formerly with the David Taylor Model Basin, Department of the Navy
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Preface

Two methods of preparing this monograph were originally discussed by Panel H-7 (Seakeeping Characteristics) of the Hydrodynamics Committee of SNAME: a composite work in which various sections would be written by experts in the respective fields, and a general survey conducted by one person. The latter method was decided upon as more likely to produce a logically coherent and compact presentation.¹

It was realized that ship behavior in a complex seaway is not yet sufficiently understood to permit writing a definitive textbook. The objective of the monograph was then defined as the *preparation of the critical summary* of the existing state of the art, in order to assist in *planning of further research*. The scope of the monograph was to be the entire field of ship responses to seas, starting with the definition of the seaway and ending with the loads imposed by the seaway on a ship's structure. Consultation of the author with the Monograph Advisory Committee resulted in the following division of the subject matter into chapters:

- 1 Seaway
- 2 Hydrodynamic Forces
- 3 Ship Motions
- 4 Resistance and Loss of Speed in Waves
- 5 Dynamic and Hydrodynamic Loads on a Ship's Structure.

Broadly speaking, the interests of the two sponsoring bodies have been different. The Seakeeping Characteristics Panel has been interested in ship motions (amplitudes, accelerations), wetness, slamming, and steering, inasmuch as these affect the safe speed of a ship in adverse weather conditions. The Committee on Ship Structural Design has been interested primarily in the loads affecting the structural integrity, including the transient loads with which a ship's elastic response is involved. Both interests were to be kept in mind throughout the development of the monograph. The exposition was limited, however, to the definition of external hydrodynamic loads and of accelerations causing dynamic or inertial loads. It has been expected that

¹ This policy was subsequently modified in regard to section 8 of the first chapter and Section 3 of the third one. These sections are concerned with the application of mathematical statistics to the description of sea waves and ship motions caused by them. The Monograph Advisory Committee decided that the chapters written by the original author should be rewritten by a specialist in this field. Mr. Wilbur Marks of the David Taylor Model Basin undertook this task. While he broadly followed the original author's plan, he modified the exposition so that it no longer conforms to the policy and objectives of the other parts of the monograph; the critical attitude and emphasis on the shortcomings of the existing knowledge were replaced by a smooth informative description.

additional projects would be organized for the examination of ship stresses or, more generally, of ship structural responses to these loads. It was intended, nevertheless, to include in Chapter 5 a brief outline of the methods available for computing the transient response to the time-dependent load application occurring in a complex seaway.

The present work is aimed solely at the behavior of surface-displacement ships. Although a large amount of material can be applied to submarines, planing-type craft require a different approach and are definitely excluded.

In research planning, either by the investigators themselves or by sponsoring bodies, the first requirement is to establish the *existing state of the art*, and the second (equally important) is to understand the *shortcomings* of the existing knowledge. These two requirements determined the style of presentation in the present monograph. An effort was made to present the basic principles and the current state of the art as factually, quantitatively and completely as possible within the limitations set by time available and a reasonable size of the monograph. However, the monograph was to be only a summary, and it has not been possible to treat all subjects exhaustively. The treatment of each subject was intended to be in the nature of an introduction and a guide to existing literature. Preparation of an extensive bibliography, therefore, formed an important part of the work. Certain reviewers of the early draft criticized this plan because the references listed have been difficult to obtain. The author solved this difficulty in his own work by relying on microfilm copies. The New York Public Library² has an amazingly complete collection of technical literature and furnishes excellent microfilming and photostatic service. Microfilming service is also available from the Library of Congress, Washington, D.C.

The references have been made as precise and complete as possible. The introductory outlines to each subject and the bibliography are intended to serve as a basis for further development by readers. In order to stimulate this development, great emphasis has been placed upon the shortcomings of the existing knowledge.³ In this connection it should be said that the stated opinions are the author's own, and the sponsoring bodies bear no

² Fifth Avenue at 42nd Street, New York, N. Y. Attention: Photographic Service.

³ In view of this policy, the author wishes to apologize in advance to the many authors whose works are reviewed here in what may appear as an unfavorable light. Exposition of shortcomings is the basic thread of this work.

responsibility for them. Although it is expected that the emphasis on shortcomings alone will inspire the reader to further research, each chapter is concluded with a direct discussion and listing of desired research problems.

Lists of references are attached at the end of each chapter and are divided into three groups: First, a listing of basic texts, which have attained the standing of textbook or are close to it. These are designated by the author's name and a capital reference letter (A, B, C, and so on). This group also includes complete chapters in symposium and encyclopedic book types. The second group is composed of references to the most important research papers and articles available, most of which were actively used in the preparation of the monograph. These are listed by author and year of publication, with the addition of lower-case letters (*a*, *b*, *c*, and so on) if the author published more than one paper in a given year. References cited thus in the text will be found at the end of the chapter in which they are mentioned. References are preceded by chapter number if located in another chapter. For instance, a reference to "Neumann (1952*b*)" cited in the first chapter will be found at the end of that chapter. If cited in another chapter, it will appear there as "Neumann (1-1952*b*)."

The third group is composed of miscellaneous references taken from other sources, but not examined. Responsibility for the completeness and accuracy of these references lies with the sources. The detailed listings are often omitted in this group and references made to a bibliography available elsewhere. No effort has been made to eliminate repetition of references in the various bibliographies used or listed.

The author adopted the practice of quoting original sources, whenever these were sufficiently brief and clear for the condensed exposition of the monograph. No changes were made in quotations from the original text, except of symbols and figure numbers. It was the author's intention at the beginning to retain the original symbols, but the reviewers of the early draft were unanimous in recommending a uniform nomenclature. The author assumes no responsibility for the mathematical work of the sources used. Also, the author does not undertake to reconcile the differences of results and opinions of various investigators. The material published by various researchers is presented and often critically reviewed, but the ultimate deductions are considered to be a part of research activity undertaken in the future.

A large number of copies of the seakeeping monograph draft were distributed to various persons engaged in oceanography and naval architecture, as well as to the members of sponsoring bodies, and critical comments were invited. These comments were considered by the author in the preparation of the final monograph copy. It is, however, unavoidable that differences of opinion will exist among scientists and engineers engaged in various aspects of the broad field of activity covered by the monograph. These are particularly aggravated by the fact that the rational treatment of ship motions and stresses caused by sea waves is a relatively new subject, many aspects of which are still controversial. The author has to accept, therefore, the sole responsibility for statements and opinions given in the seakeeping monograph. The sponsoring bodies are not responsible for these opinions, and indeed, individual members of these bodies may not agree with all of them.

B. V. KORVIN—KROUKOVSKY

New York, N.Y.

January, 1960

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Preface to Chapter 1

It has been the author's experience that the solution of any practical problem connected with wave motion is often accompanied by a large amount of fumbling. The reason for this is that information on waves is widely scattered in various textbooks, written by mathematical physicists, and is often presented in a form which makes direct application to engineering problems difficult. Furthermore, the author's early reading in oceanography has shown him that some of the basic concepts of hydrodynamics have often been disregarded, possibly because of want of easy reference. Appendix A was prepared, therefore, as a ready reference to give a concise summary of the theory of wave motion on the basis of classical hydrodynamics.

Rapid progress in the development of the theory of ship motions in a complex seaway has taken place following the appearance of the paper by St. Denis and Pierson (*Transactions SNAME*, 1953). This activity has been based on the description of an irregular sea surface by an energy spectrum. In this connection the term "Neumann's spectrum" has so often been used that the basis of its development appears to be forgotten. An effort has been made, therefore, to expound Neumann's work rather completely in Appendix B, while in the text only a relatively brief summary appears.

The author has found that at present oceanographic knowledge is rather disorganized in so far as the transmission of energy from wind to water, the growth of waves under the action of wind, and the wave size indicated by the spectrum of a complex seaway are concerned. Very few experts appear to agree on solutions to these problems. Yet it is futile to expect a satis-

factory quantitative description of ship motions or ship stresses at sea without a good knowledge of the complex seaway which causes them. As a result of this state of affairs, Chapter 1 has become more detailed and voluminous than was originally intended.

During the preparation of this monograph, two extensive summaries of the existing knowledge on ocean waves appeared in the press, the first by F. Ursell (Reference M), and the second by H. U. Röll (Reference L).

The general plan of exposition in these two summaries is similar to the one presented here by the author. None of these three summaries eliminates the need for the others, since each author has emphasized different aspects of this vast and complex subject. The present exposition, intended especially for stimulation of research, is much more critical than the others and stresses shortcomings no less than achievements.

The exposition of sea-wave properties given in this monograph is oriented to its ultimate use in naval architecture and in ship navigation. It is, therefore, incomplete in regard to many other fields of endeavor. The ultimate development which is the interest of the present monograph would consist of a specific formulation of a directional wave spectrum as a function of meteorological conditions. Merely empirical attempts to secure such knowledge of the complex sea have not yielded a satisfactory solution. It is the author's opinion that a general theoretical development is also needed for achieving this objective. A comprehensive review of the theoretical work and of the laboratory and open-air observations connected with it was therefore included in the monograph.

B. V. KORVIN—KROUKOVSKY

New York, N.Y.

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1 Introduction

THE wind-swept surface of an ocean usually presents a very complex irregular appearance. A certain regularity exists only in the broadest sense that wave hollows and wave crests progress in some general direction and succeed each other alternately. The time between passages of crests, the heights of crests, and the distances between crests vary widely and erratically. It is impossible to evaluate quantitatively by direct observation the effect of such an irregular surface on the behavior of a ship. It is necessary to undertake a systematic analysis or breakdown of a complex physical event into simple components, to study the effect of these simple components, and to add them together again in order to reach an understanding of the whole. The technique of such a process developed over many years in the fields of heat transfer and electrical network analyses in recent years has been applied to complex ocean waves.

It has been shown that a function of any complexity can be represented as the sum of a large or infinite number of simple harmonic functions (waves) of varying periods and amplitudes. The word "simple" is applied here to a uniform regular wave characterized by a single direction of propagation, a definite frequency, a definite wave length, and a definite amplitude, which are repeated uniformly in each passing wave. The description of a seaway must start then with a definition of the properties of such a simple wave. This simple wave does not normally exist in nature, but can be produced in a laboratory, either for investigating the properties of the wave itself, or for studying its effects on ship models or on harbor installations. A summary of the classical theory of simple waves will be found in Appendix A.

The classical wave theory considers only the propagation of free waves under the action of gravitational and inertial forces. Uniform distribution of air pressure is assumed, and neither the transfer of energy from wind to water nor the mechanism of wave generation has been considered. The following four sections of Chapter 1 will be devoted to discussing these phenomena. Theoretical reasoning, laboratory experiments and specially designed methods for observing natural waves are involved in this problem which so far has eluded a complete solution. Later sections will take up the organization of empirical observations in a form suitable for wave forecasting and for determination of a ship's behavior at sea. Finally, the mathematical sea-surface representation needed for the latter purpose will be outlined.

¹ Throughout, References will be given by author and year, and may be found in the alphabetical list at the end of each chapter.

2 Generation of Waves by Wind—Elementary Rational Approach

The problem of the generation of waves by wind was first attacked by Lord Kelvin (Thomson, 1871),¹ by considering the free-water surface as a dividing boundary between two fluids, water and air. The motions of both were considered separately; i.e., the velocity potentials of both water and air were evaluated. Furthermore, it was postulated that the pressure of the air and that of the adjacent water differ owing to the surface tension of water. The derivation is also given in Lamb (D, art. 267 and 268, pp. 458–462) and Motzfeld (1937, pp. 205 and 206). Kelvin's solution for waves propagating in one direction in deep water was extended by Jeffreys (1925) to waves propagating in two directions. Using the solution of Lord Kelvin, Jeffreys indicated that at an air velocity of 640 cm/sec (about 21 fps) the water surface becomes unstable, and waves develop spontaneously. These first waves have a length of 1.8 cm (0.71 in.) and a celerity of 23.2 cm/sec (0.76 fps) which was shown to be the least possible wave celerity. This wave represents a borderline between longer waves due to gravity, and the shorter (but faster traveling) waves due to the surface tension of water; i.e., the capillary waves.

The foregoing conclusion does not agree with observations of nature, which indicate that waves begin to develop at much lower wind velocities. Jeffreys (1925, 1926) by observation of waves due to light winds on a pond arrived at the conclusion that waves begin to form at a wind velocity of about 110 cm/sec (3.6 fps), have an initial length of 6 to 8 cm (2.4 to 3.1 in.) and a celerity of about 30 cm/sec (about 1 fps). Stanton (1932) found by experiments in a wind flume that gravity waves first form at an air speed of 250 cm/sec (8.2 fps) and are 6 cm (2.4 in.) long. He emphasized the uncertainty of such observations and wrote: "The minimum velocity of the wind required to cause the formation of waves has been observed, although as has been pointed out, the value obtained will depend to a great extent on the conditions of the experiment."

The general picture is made much clearer by Roll (1951) who observed and measured wind and waves on a large shallow pond left on a low sea shore by the receding tide. Fig. 1 shows the frequency distribution of the various waves plotted versus the wind velocity. It is impossible to name with certainty the wind velocity connected with the first formation of waves. By taking the 50 per cent probability of occurrence of gravity waves Roll estimated the minimum necessary wind velocity at 60 to 80 cm/sec (2 to 2.6 fps). He made the very important observation that from the beginning there are

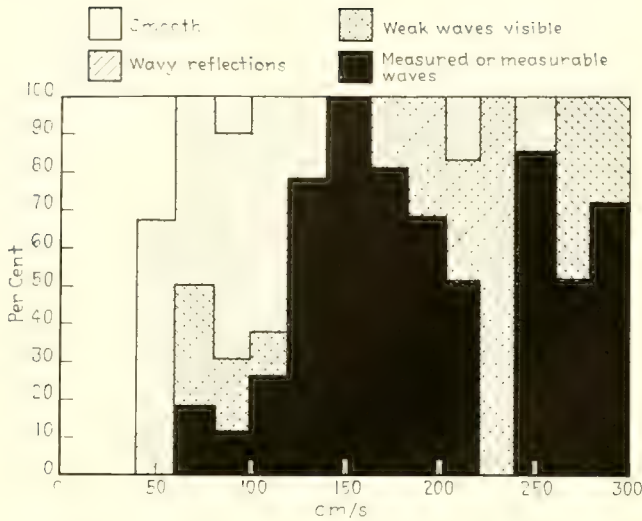


Fig. 1 Frequency distribution of initial waves as function of wind velocity (from Roll, 1951)

wave components traveling at angles to the wind direction. The existence of these wave components he attributed to air turbulence which produces local traveling pressure areas and causes oblique waves (like ship bow waves) to form according to the well-known theory of the traveling disturbance (Lamb, D, art. 256, pp. 433-437; Davidson, 1942).

Because of the glaring discrepancy between the results of Kelvin's analysis and observed data, Jeffreys (1925, 1926) undertook a new analysis in which both dissipation of the energy in water due to viscosity and modification of the air pressures due to air viscosity were taken into account. In the potential flow of air, maximum air velocity and maximum drop of air pressure are found over wave crests, and minimum air velocity and an increase of air pressure occur in troughs. Thus wave-exciting forces exist (causing the instability indicated by Kelvin), but because of the symmetry of the flow on both sides of each wave crest, there is no mechanism for transmitting the energy from air to water. Jeffreys assumed that each element of distorted water surface acts essentially as a flat plate in a fluid stream; i.e., that the pressure acting on it is proportional to its inclination, which can be expressed as $d\eta/dx$. With the wave profile expressed as²

$$\eta = a \sin k(x - ct) \quad (1)$$

the slope of the water surface is

$$\partial\eta/\partial x = ak \cos k(x - ct) \quad (2)$$

Jeffreys assumed an element of the pressure exerted by wind on an element of the wave surface to be

$$p = \beta\rho'(V - c)^2\partial\eta/\partial x \quad (3)$$

² In this case and in the following section the wave formulas differ from those given in Table 1 of Appendix A in that the origin of co-ordinates is taken at the nodal point instead of the crest.

here p is the pressure, β a nondimensional coefficient, ρ' the density of air, and V the air velocity.

Were the air flow potential, the pressure distribution would be symmetrical about the wave crest, and the mean value of the coefficient β would be nil. Jeffreys assumed, however, that in a viscous flow the leeward slopes of the waves are shielded, so that asymmetry of pressure distribution exists and the coefficient β has a value (to be determined empirically) greater than zero.

2.1 Transmission of Energy from Wind to Harmonic Waves. Without making assumptions as to the nature of the air pressure, Lamb (D, p. 625) wrote: "It is not likely that the action of the wind, even on a simple harmonic wave-profile, could be represented by any simple formula. But, neglecting the tangential action, which seems to be of secondary importance, we may imagine the normal pressure to be expressed by a Fourier series of sines and cosines of multiples of $k(x - ct)$, and it is evident that the only constituent which does a net amount of work in a complete period has the form

$$\Delta p = C \cos k(x - ct) \quad (4)''$$

In view of the importance of the foregoing quotation, a more detailed development of this idea will now be presented. The air pressure acts normally to the water surface. For the purpose of analysis it is convenient to consider the normal pressure vector as decomposed into vertical and horizontal components, $p \cos \theta$ and $p \sin \theta$, where θ is the inclination of the water surface. This inclination in waves is generally small and these pressure components can be evaluated in two ways.

(a) Attention can be concentrated on an element of water area at a certain fixed location x moving up and down with vertical velocity v . The mean work done by the pressure p per second is

$$E = \frac{1}{T} \int_0^T p v dt = \frac{ack}{T} \int_0^T p \cos k(x - ct) dt \quad (5)$$

A positive sign is assigned to the work of the downward pressure acting on the surface element with downward velocity.

(b) The alternate method is to consider an element of the wave form moving horizontally with celerity c . The mean work per second is

$$E = \frac{1}{T} \int_0^T p(\partial\eta/\partial x) c dt = \frac{ack}{T} \int_0^T p \cos k(x - ct) dt \quad (6)$$

Let the unknown pressure distribution over the wavy water surface be represented by a summation of sine and cosine harmonics, i.e.,

$$p = \sum_1^n A_n \sin nk(x - ct) + \sum_1^n B_n \cos nk(x - ct) \quad (7)$$

For the wave form represented by equation (1)

$$\partial\eta/\partial x = a k \cos k(x - ct) \quad (8)$$

Substituting equations (7) and (8) into equation (5) leads to integrals of the following forms.

For the first harmonic

$$\int_0^T \sin k(x - ct) \cos k(x - ct) dt$$

and

$$\int_0^T \cos^2 k(x - ct) dt$$

and for higher harmonics

$$\int_0^T \sin n k(x - ct) \cos k(x - ct) dt$$

and

$$\int_0^T \cos n k(x - ct) \cos k(x - ct) dt \quad (10)$$

All of the foregoing integrals vanish upon evaluation, except the one for the first harmonic containing cosine squared in the integrand. This proves equation (4) given by Lamb. The pressure components in phase with the wave, the sine harmonics, transmit no energy to water, and the entire energy transmission is due to the first harmonic of the cosine, i.e., to the out-of-phase component. Retaining only the first harmonics, equation (7) can be rewritten as

$$p = A \sin k(x - ct) + B \cos k(x - ct) \\ = C \sin k(x - ct + \epsilon) \quad (11)$$

This shows that the transmission of energy from wind to water occurs because a sinusoidal pressure distribution differs in phase from the wave form. The distortion of the pressure-distribution curve by higher harmonics apparently has no effect on the energy transmission. In a potential flow the pressure is in phase with the wave, $\epsilon = 0$, and there is no energy transmission. In a fluid of very low viscosity, as is the case with air, the flow is generally potential, but the boundary condition at the water surface is modified by the presence of the boundary layer. This latter thickens locally in the adverse pressure gradient on the lee sides of waves, and thus brings about the phase shift ϵ and the resultant energy transmission. This will be discussed further in connection with experimental wind-tunnel data.

Evaluation of equation (5) after substituting the $\cos k(x - ct)$ pressure term from equation (11) gives the total energy transmitted by wind to waves, per second per unit of sea surface,

$$E = B a c k/2 \quad (12)$$

As can be seen from equations (7) and (11) the coefficient B is the amplitude of the out-of-phase component of the air pressure. Taking it as proportional to the square of the relative velocity between air and wave propagation ($V - c$), B can be expressed in terms of a nondimensional pressure coefficient C_p as

$$B = \Delta C_p^{1/2} \rho' (V - c)^2 \quad (13)$$

where the coefficient C_p depends on the wave form and height. Following Lamb's example of equation (4), the symbol ΔC_p is used to emphasize that only part of the total air pressure, the out-of-phase component, is considered.

In practice it is more convenient to measure the horizontal drag force acting on the water surface. Expressing it in terms of the drag coefficient C_d , as defined customarily in aerodynamics:

$$\text{Drag} = C_d (\rho'/2) (V - c)^2 \quad (14)$$

The total energy per second transmitted by wind to water is

$$E = (\text{Drag}) c = C_d \frac{\rho'}{2} (V - c)^2 c \quad (15)$$

2.2 Effect of Skin Friction. Before proceeding further, it is important to comment on the remark: "... neglecting the tangential action which seems to be of secondary importance..." in the previous quotation from Lamb. In the case of slightly viscous fluids, such as air and water, the flows are generally well described by the potential theory, and a slight dissipation of energy by viscosity does not sensibly modify the results of this theory.³ The primary effect of viscosity on the general flow is indirect; it appears as a modification of the boundary conditions because of formation of the boundary layer. At the boundary of a fluid and a solid, or at the air-water interface considered here, there is a strong velocity gradient, and the viscous effect becomes important in the form of "skin friction." The essentially potential flow can be affected directly only by normal pressures, and not by the tangential skin friction. This latter represents the dissipation of kinetic energy in the form of eddy-making first and finally of molecular motion, or heat, so that this part of the energy is no longer available to the potential motion of waves. It is therefore quite proper to neglect the tangential frictional force in the theory of wave formation just outlined, and to consider only the effect of normal pressure.⁴

Confirmation of the foregoing statement can be found in the theory of wave resistance of ships, as developed primarily by Havelock, Weinblum, and Guilloton, and in experiments conducted in connection with this theory. The particularly relevant references are Wigley (1937), Havelock (1948, 1951), Shearer (1951), Birkhoff, Korvin-Kroukovsky and Kotik (1954) and Korvin-Kroukovsky and Jacobs (1954). The entire treatment is based on the potential theory, yet it gives results in good agreement with towing-tank test data (in the case of fine boats formed of parabolic arcs). Despite the fact that possibly $\frac{2}{3}$ of the total towing energy is dissipated in skin friction, the effect of this on waves (i.e., on the

³ Lamb (D, art. 346, p. 623) also Appendix A art. 4.4; the tremendous success of aerodynamics has been based on this postulate.

⁴ It is interesting to note that frictional drag is also neglected in the recent advanced work on energy transmission from wind to waves by Miles (1958) and Phillips (1958). The air viscosity is considered only in the sense of defining the effective wind velocity.

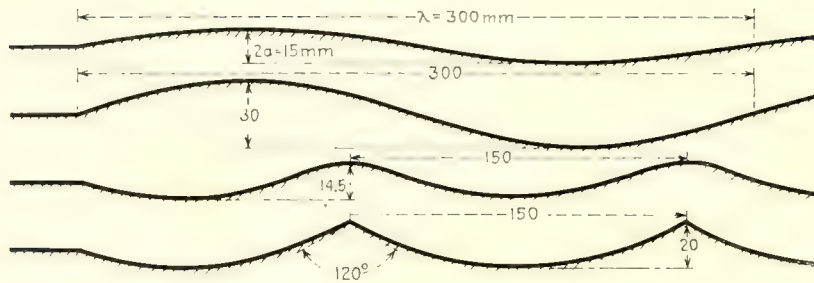


Fig. 2 Wave profiles of models tested in a wind tunnel (from Motzfeld, 1937)

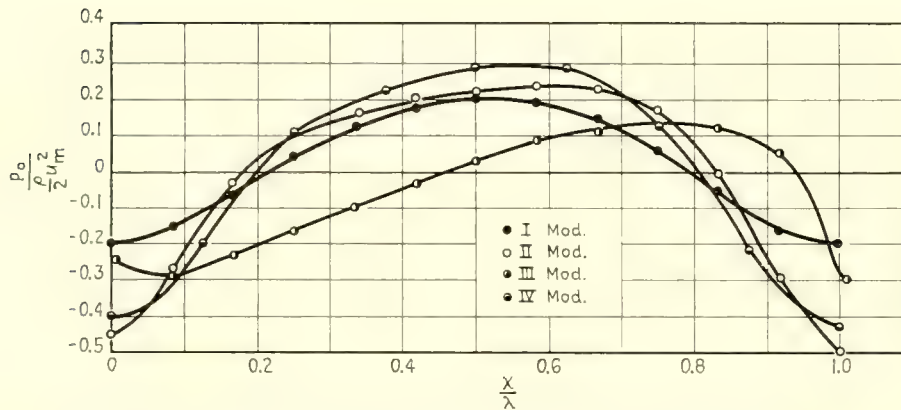


Fig. 3 Pressure distribution along wave profiles (from Motzfeld, 1937)

potential flow) resulting from the action of normal pressures is found to be small.

The fundamental distinction just stated between the kinetic energy of nearly potential flow, and the eddy-making or thermal energy of skin friction has often been disregarded in the literature on wave generation and propagation. A large amount of discussion and the various attempts at evaluation of the skin-friction energy in this connection appear to the author to be in basic contradiction to the laws of hydromechanics. The disorganized eddy-making and thermal energies are no longer available to the potential wave flow.

One of the reasons for considering skin friction heretofore was the necessity of explaining the experimentally observed long waves which travel faster than wind. Since the skin friction depends not on the wave celerity but on the wind velocity with respect to slowly moving water particles, energy transfer appears to exist even for $c > V$. It will be shown later, however, that the energy transfer appears to depend on small steep waves which cover the surface of larger waves rather than on the large waves themselves. Since the celerity of these small waves is low, $(V - c)$ has a large value for them, even when $c > V$ for the predominating large waves. The energy transmitted by normal pressures on small waves provides, therefore, the necessary explanation without involving skin friction.

2.3 Evaluation of the Drag Coefficient C_d : Wind-Tunnel and Flume Experiments. The drag coefficient C_d has been evaluated for different wave forms by three methods: (a) by wind-tunnel or flume measurements on rigid-wave models, (b) by measuring the water-surface inclination caused by the wind-drag force, and (c) by measuring the velocity gradient of the air flow above waves.

(a) Measurements of the pressure distribution over the surface of wave models were made in a wind tunnel by Stanton, Marshall and Houghton (1932) and by Motzfeld (1937), and in a flume by Thijsse (1952). Stanton, et al used a wave model with the excessively small length-to-height ratio of 5, therefore only the more realistic models of Motzfeld and Thijsse will be discussed here.

Motzfeld had four models of waves inserted in the floor of a wind tunnel, and measured the pressure distribution on each model. By integration, he obtained the mean horizontal force and the drag coefficient C_d . He also measured pressures in the air above the model and from these constructed the shapes of streamlines. A high degree of initial turbulence was secured by the length of tunnel in front of the model and by the use of various turbulence-stimulating devices at the entrance and in this leading section of the tunnel. Four models were used:

- 1 A sinusoidal wave of length/height ratio = 20
- 2 A sinusoidal wave of length/height ratio = 10

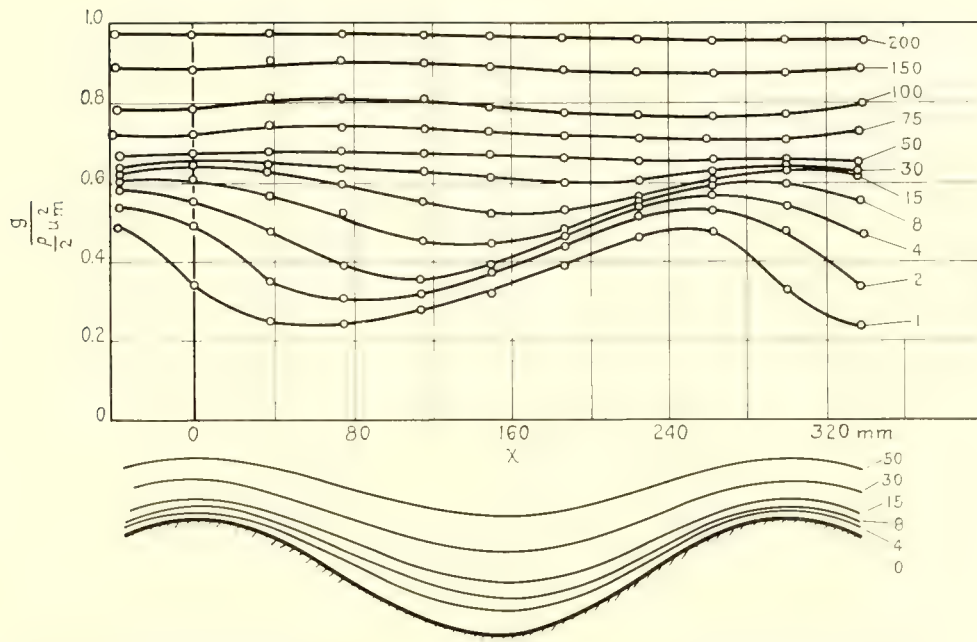


Fig. 4 Streamlines of air flow and total pressure variation at second model (from Mätzfeld, 1937)

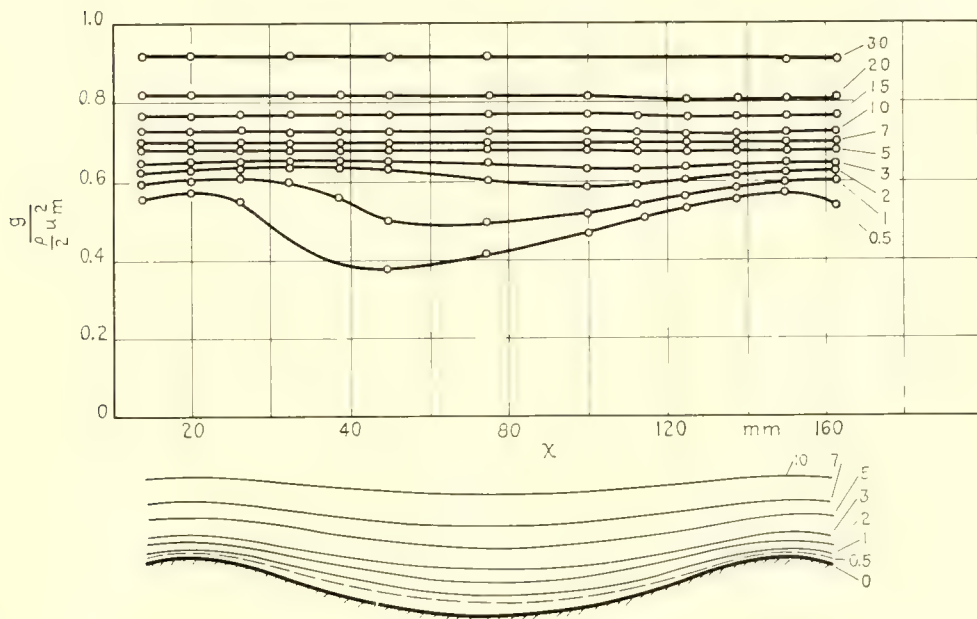


Fig. 5 Same as Fig. 4 but for third model

3 A trochoidal wave of length/height ratio about 10

4 A wave of circular arcs with sharp 120 deg angles at crests and length/height ratio of 7.5, which closely resembles the theoretical wave of limiting height.

The wave profiles and dimensions are shown in Fig. 2, the pressure distribution along the models in Fig. 3, and

the pressure distribution in the air and streamlines in Figs. 4, 5 and 6. Thickening of the boundary layer and the phase lag of the pressure distribution is noticeable on all models, but only on the sharp-crested model No. 4 is separation of the flow observed. The horizontal dynamic (i.e., due to pressure distribution) drag W_a per wave length is obtained by integration

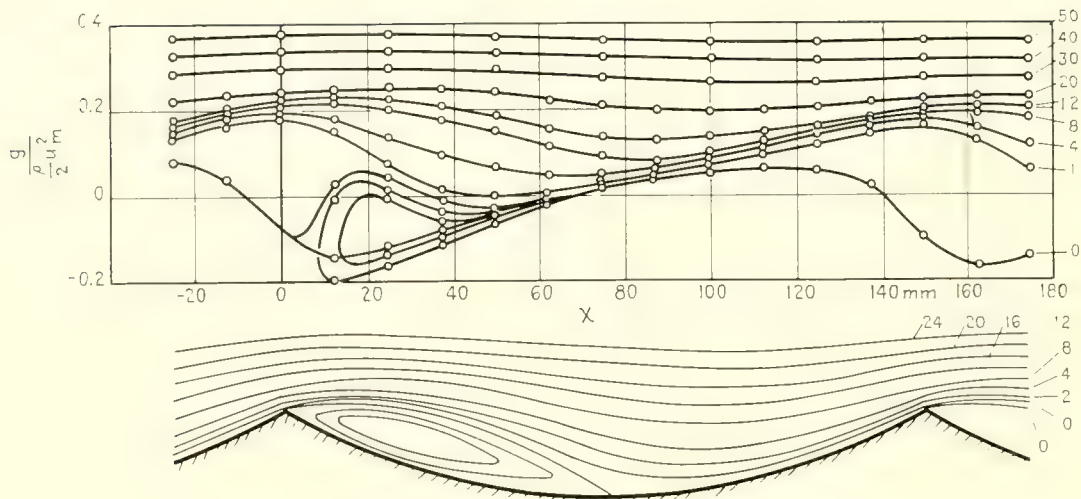


Fig. 6 Same as Fig. 4 but for fourth model

Table 1 Values of Coefficients

Model No.	C_d	C_r^*	$C_d + C_r$
1	0.00085	0.00375	0.0046
2	0.0024	0.0038	0.0062
3	0.0028	0.0035	0.0063
4	0.0195	0.0028	0.0222

* $C_r = (\text{frictional drag})/1/2 \rho u_m^2 \lambda$.

$$W_d = \int_0^S p \sin \theta ds \quad (16)$$

where S is the developed length of the surface of the wave of length λ , and ds an element of the length. The integration is most conveniently performed by noting that $\sin \theta ds = d\eta$, and by plotting pressure p versus surface elevation η . These plots are shown in Fig. 7, and the dynamic drag is represented by the area enclosed by each curve. The nondimensional drag coefficient was defined as

$$C_d = \frac{W_d}{1/2 \rho' u_m^2 \lambda} \quad (17)$$

where u_m is the maximum wind speed in the section of the wind tunnel above the wave nodal point. The values of the coefficients obtained in this way are given in Table 1. The frictional-drag coefficient is listed for the discussion in the following section.

Thijsse (1952) photographed the waves generated in a wind flume, made a model of one of these waves, inserted it at the bottom of a flume and measured the pressure distribution in smooth water flow. The wave profile, air-pressure distribution, and water-velocity distribution are shown in Fig. 8. The dynamic drag coefficient C_d resulting from these measurements is 0.0052, or about twice Motzfeld's drag for the trochoidal model No. 3. Although in this case the wave is slightly lower in height, $\lambda/H = 12$ instead of 10, it is unsymmetrical with a steeper

front (i.e., lee side). Wind-driven waves usually have this unsymmetrical form, so that an increase of resistance over the symmetrical Motzfeld models can be expected.

Lack of a precise and significant definition of air velocity is the outstanding drawback of these experiments. This appears to be a weak point in all experiments on wind-wave energy transmission. A theoretical formulation of a similarity law and its observance in planning observations and tests are sorely needed at this time.

2.4 Determination of C_d from Mean Inclination of Water Surface. As examples of determination of the drag coefficient by measuring the mean slope of the water surface in a wind flume, the work of Francis (1951) and Johnson and Rice (1952) can be cited. Francis used a wind flume about 6 m (19.6 ft) long in which wind velocity up to 12 meters per sec (mps) (39 fps) could be produced. Two alternate forms of experiment were used. In one the waves were generated by the wind and increased in height from zero at the windward end to maximum at the lee end. In another, the waves were artificially generated at the windward end, and diminished or increased towards the lee end depending on the wind strength. The mean elevation of the water surface was measured at several points along the flume, and

Table 2 Results of Measurements

Wind speed, mps	λ/H	$C_d^* = C_d + C_r$
Wind only		
5	10	0.0040
10	10	0.0150
12	12	0.0200
Wind and wave generator		
5	17.5	0.0052
10	9.5	0.0182
12	10.6	0.0242

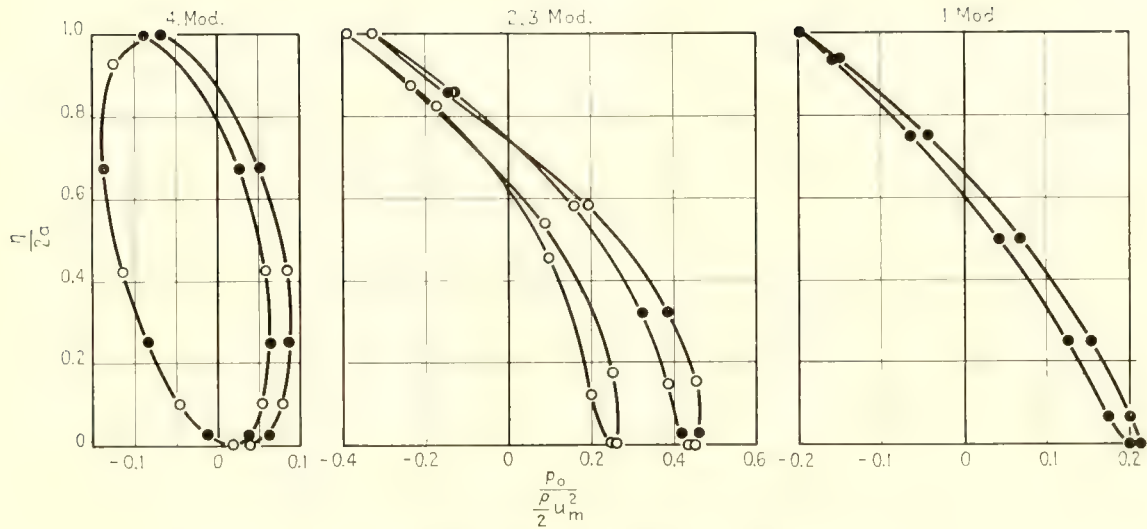


Fig. 7 Plot of air pressure versus wave elevation (from Motzfeld, 1937)

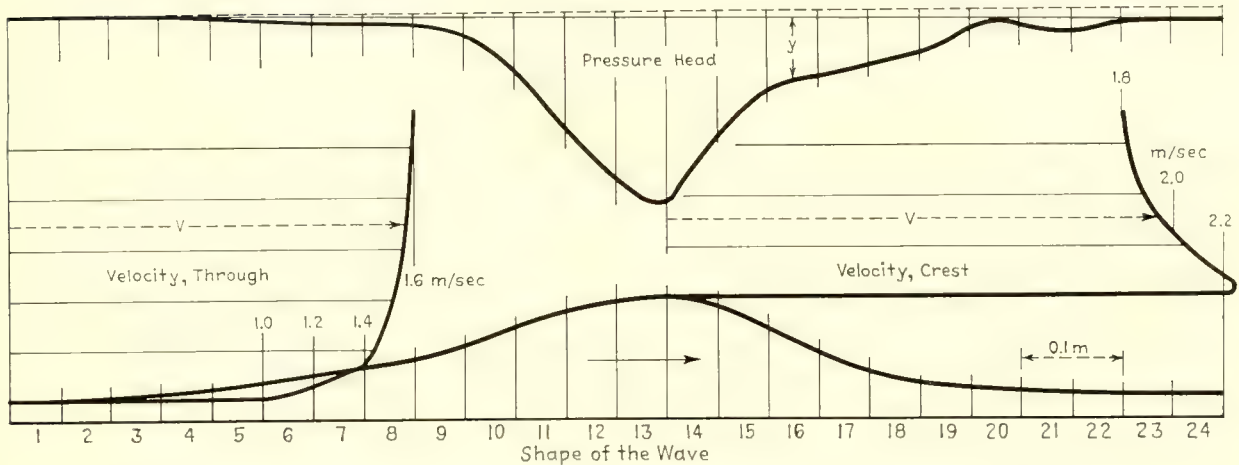


Fig. 8 Measurements on paraffin model of wind-generated wave (from Thijsse, 1952)

from these the slope of the water surface was obtained. The horizontal drag force in this case includes both the dynamic and frictional drag. The mean wave height and length also were measured. The results of these measurements are summarized in Table 2.

The data in Table 2 are approximate. The values of λ/H were read from the mean curves given on the graphs since no tabulated data are given in the reference. In calculating the drag coefficient the assumption was made that $V^2 \cong (V - c)^2$; i.e., the wave celerity of small waves was neglected in comparison with wind velocity. The maximum and minimum values of the total drag coefficient C_d^* agree well with those for the idealized models of Motzfeld. It is clear, however, that the C_d of wind-driven waves is not a unique function of the mean value of the λ/H ratio. These waves are very irregular and while the mean wave is tabulated, lower and higher

waves are found in reality. In particular, sharp and breaking crests occur fairly frequently. The suddenness with which the C_d increases for the sharp-crested profile is probably the most significant result of Motzfeld's tests. It appears that the mean drag of the irregular-wave system may be primarily governed not by the mean λ/H ratio, but by the frequency of occurrence of sharp crests or of steep wave slopes, since the drag of the sharp-crested waves is so much higher than that of the steep trochoidal waves of $\lambda/H = 10$ ratio. At the 5 m/sec wind velocity in Francis' tests, sharp crests probably were absent and the drag was very low. With increasing wind strength the sharp crests must have occurred more often and the C_d increased correspondingly.

The measurements of Johnson and Rice (1952) were also made in a wind flume. Because of the high wind velocity used, the mean waves are exceptionally steep.

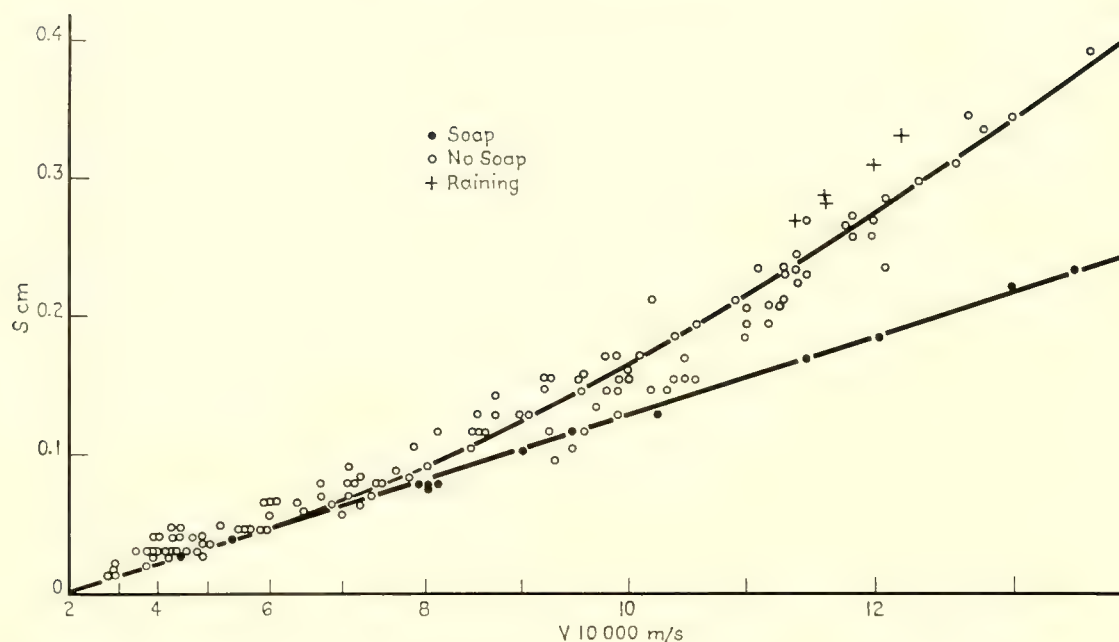


Fig. 9 Original data for setup as a function of wind speed at 10 m elevation. Smooth curves were drawn by inspection. Each point is a 20-min time average for both wind and setup (from Van Dorn, 1953)

Table 3 Results of Wind-Flume Measurements of Johnson and Rice (1952)

Wind velocity, fps	λ/H	$C_d^* = C_d + C_r$
40.8	6.2	0.0282
32.4	7.7	0.0330
35.7	6.7	0.0372
40.1	7.2	0.0202
42.3	7.2 ^a	0.0190

^a At the end of the fetch.

Only the mean drag coefficients $C_d^* = C_d + C_r$ over the flume length were determined. These and the λ/H -values of the waves at about $\frac{2}{3}$ of the available fetch are listed in Table 3.

The drag coefficients are exceptionally high and correspond to the extreme mean steepness of the waves. It should be noted that while the included angle of 120 deg at wave crests, used in Motzfeld's wave model No. 4 (with $C_d^* = 0.0222$) is the minimum for a stable wave, in the breaking of a wind-driven wave sharper angles and, more significantly, steeper slopes of the lee faces may occur instantaneously causing a further increase of air drag. Much sharper crests with the included angle down to 90 deg may also occur (Taylor, 1953) if a standing wave system is present.

2.5 Observations of Keuligan and Van Dorn. The observations of Keuligan (1951) and Van Dorn (1953) on the water surface inclination in a wind require special discussion in view of (i) the form of analysis used, and (ii) the alternate use of natural water surface and surface covered with a detergent. The Keuligan experiments

were made in a wind flume, and Van Dorn's observations in a large outdoor pool. In both cases the primary objectives were to determine the water-surface drag and the surface current generated by it. The wave properties were not measured; nevertheless the type of analysis used permits certain interesting deductions to be made in regard to waves. The method of analysis originated with Keuligan, but it will be used here in connection with the data of Van Dorn.

Van Dorn measured the difference of water level at two points spaced 220 m (722 ft) apart in a pool about 790 ft long. The depth of the pool over most of its length was from 2 to 2.4 m, but owing to the sloping bottom at the ends the mean effective depth was 1.85 m (6.07 ft). The wind velocity was measured at heights of 25 cm, 1 m and 10 m (0.8, 2.3 and 33 ft) above the water surface. Wind velocity always decreases with decreasing height above the surface because of the loss of momentum caused by the drag force. The setup, i.e., the difference in the mean water level at the two ends of the measured stretch of 722 ft, is shown in Fig. 9 plotted against the square of the wind velocity measured at the height of 10 m. Fig. 10 shows similar plots for the wind velocity measured at the heights of 1 m and 25 cm. It was found that by spreading a detergent on the water surface the formation of waves is prevented, and the setup can then be taken as caused entirely by the smooth surface skin friction. This is essentially proportional to the square of the wind velocity, and the plot is a straight line. Using the symbol U for the wind velocity measured at a certain small elevation, the setup S is expressed therefore as

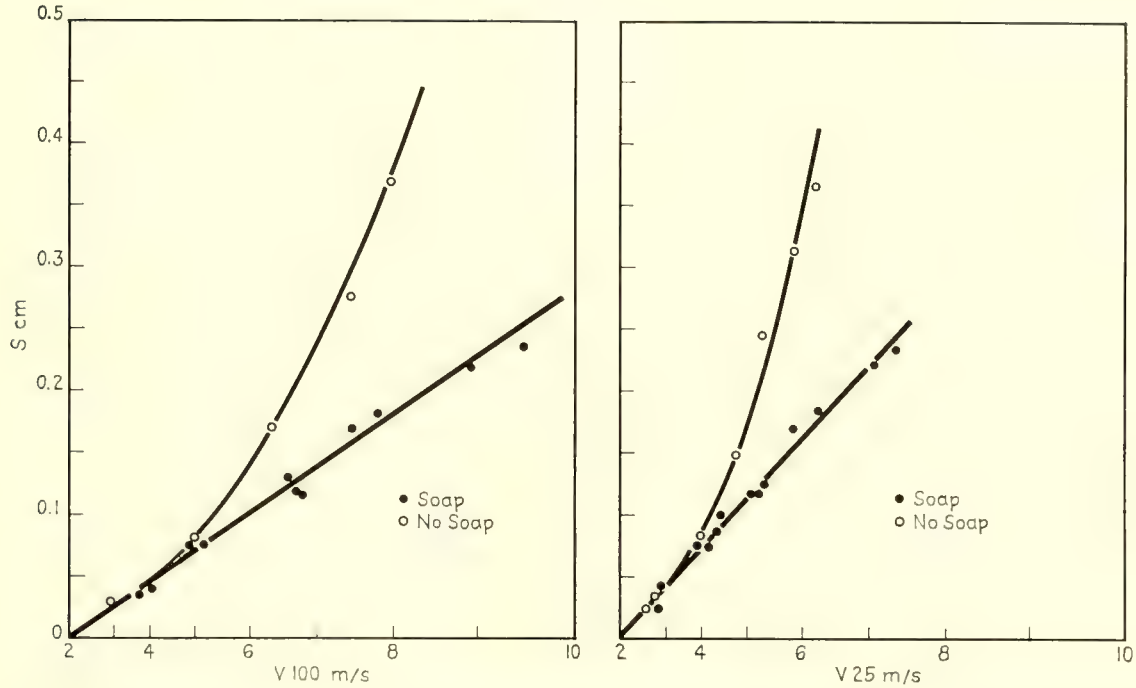


Fig. 10 (Left) Setup as a function of the square of the wind speed at 100 cm elevation; (right) setup as a function of the square of the wind speed at 25 cm elevation (from Van Dorn, 1953)

$$S_1 = aU^2 \quad (18)$$

From Fig. 10 at 100 cm elevation it follows then that, at the wind velocity of 10 mps = 32.8 fps, $S_1 = 0.28$ cm = 0.0092 ft and $a = 0.00000855$ sec²/ft.

For the conditions of Van Dorn's observations the tangential force per unit area is expressed as

$$\tau = \rho g h S_1/L = \rho g h a U^2/L \quad (19)$$

where ρ is water density, h the effective depth and L the distance at the ends of which S_1 is measured. The tangential force or drag coefficient is then

$$C_r = \frac{\tau}{1/2 \rho' U^2} = \frac{2\rho g h a}{\rho' L} = 0.0039 \quad (20)$$

where ρ' is the air density. This is in agreement with Motzfeld's wind-tunnel data given in Table 1.

When waves are formed, there is an added drag due to wave form, and the total setup S becomes

$$S = S_1 + S_2 \quad (21)$$

The curve of S coincides with the straight-line plot of S_1 up to a certain speed V_c , i.e., $S_2 = 0$ for $U \leq V_c$. Above this speed S_2 is expressed by Keuligan, and following him by Van Dorn, as

$$S_2 = b(U - V_c)^2 \quad (22)$$

The foregoing expression contains two unknowns b and V_c . By assuming both of these to be constants, by reading from Fig. 10 the values of $S_2 = S - S_1$ at 6 and 8

mps as $(0.140 - 0.100) = 0.040$ and $(0.390 - 0.180) = 0.210$ cm, and converting to feet, the unknowns are evaluated as

$$b = 0.0000518 \text{ sec}^2/\text{ft} \\ V_c = 14.65 \text{ fps}$$

By the same procedure as was used in connection with the coefficient a , the nondimensional dynamic or form-drag coefficient is then evaluated on the basis of the effective velocity $U - V_c$ as

$$C_d = 0.0236$$

The foregoing analysis was made for comparison with the other previously described wave data. Neither Keuligan nor Van Dorn discussed the details of wave form. They called V_c the "formula velocity," treating it just as an empirical constant, and discussed the coefficient b in equation (22) in terms of the length and depth of the test stretch without any reference to the wave form. However, their distinction between S_1 and S_2 gives an idea as to the distribution of the wave-form drag and the friction drag which is not obtained from other observations of water-surface inclination.

Observing that the tangential force is directly proportional to the setup, the analogy between equations (14) and (22) is of interest. This analogy permits physical interpretation of the "formula velocity" V_c as the celerity of the wave components most significant in producing the wave-form drag.

More accurately it is connected with the horizontal

velocity of translation of the facets of the wave form on which the predominating air pressures act. Both definitions would be identical if the waves were simple harmonic. The true wave structure consists, however, of smaller waves carried on the surface of the bigger ones, and the velocity of translation of small facets of wave form is therefore a complicated function of celerities of waves of all sizes.⁵

The value of the drag coefficient, $C_d = 0.0236$, agrees well with Motzfeld's model No. 4, and with the appearance of the waves shown on Fig. 11. It was derived on the basis of the mean wind speed of 23 fps at which a small c/V -ratio and large percentage of sharp-crested small waves can be expected.

2.6 Summary of C_d Data from Previous Sections.

It appears from the foregoing data that there is generally good agreement among the C_d -values obtained from pressure measurements in wind tunnel and flumes, and derived from the slope of the water surface. In a very mild case (Francis at 5 mps) and in very severe cases (Francis 12 mps and Johnson and Rice) the wave form can be shown to be analogous to Motzfeld's trochoidal and sharp-crested wind-tunnel models. However, for the cases of intermediate severity, the problem of defining the sea surface in a form significant for the drag has not been solved. Clearly λ/H -ratio is not the desired parameter. Its effect is practically discontinuous; a very slowly rising value of C_d with decreasing λ/H , and then a quick and drastic jump to a high value as sharp wave crests are developed. It appears that the C_d values are governed by a statistical parameter depending on the frequency of occurrence of sharp crests or possibly on the frequency of occurrence of steep wave slopes. The problem has to be treated therefore by statistical methods. These will be discussed later in Section 8 of this chapter.

It can be added here that in the wind-flume experiments rather strong wind velocities and short fetches were used, so that the waves were short (of the order of 1 ft) and the ratio c/V of wave celerity to wind speed was very small (less than $1/10$). The predominating waves in the actual sea have a c/V ratio of the order of 0.86 (Neumann 1953, p. 21). The wind-flume data, therefore, although valuable material for the study of various relationships, do not represent ocean conditions directly. The method of analyzing such data thus becomes particularly important. Analyses in the past were generally inadequate because of limitation to overall drag of the water surface, neglect of wave irregularity, and the indeterminateness of the wind velocity to which the data are referred. An important observation in connection with wind-flume tests is that the wave system is very irregular from the outset. The irregularity was commented upon by Francis (1951), and is demonstrated quantitatively by Johnson and Rice (1952), who present a number of graphs of the statistical distribution of wave heights and periods. These are reproduced here in Fig. 12.

⁵ It was derived statistically by Longuet-Higgins (1955, 1957).

2.7 Estimation of Tangential Drag from Wind-Velocity Gradient.⁶

The method of estimating the drag coefficient of a sea surface by measuring the velocity gradient in wind is based on the theory of turbulent boundary layer at a rough surface. The tangential resistance of such a surface causes momentum loss in the immediately adjacent layers of air. The turbulent movements of air particles cause a momentum transfer from one layer of air to another, and, as a result of this, the air velocity diminishes gradually from the velocity of the undisturbed flow V at a large distance from the rough surface to a velocity $u < V$ as the surface is approached. The air velocity u is therefore a function of the distance z from the plate; i.e., $u = u(z)$. The plot of velocity u versus height z has the general shape shown in Fig. 13. The tangential drag of a surface is equal to the shear stress in the air layers in close proximity to the surface and is expressed in the general form as

$$\tau = A(du/dz)_{z=0} \quad (23)$$

where τ is the shearing (or frictional) force per unit area and the coefficient A is yet to be defined. In the most common usage a coefficient of hydrodynamic force is defined in terms of $\rho V^2/2$, as for example in equation (14). In aerodynamic usage in Great Britain, however, it became customary to express it in terms of ρV^2 . This usage has been generally adopted in the field of oceanography, and the tangential force coefficient is written as

$$\gamma = \tau / \rho V^2$$

or preferably

$$\gamma = \tau / \rho u_z^2 \quad (24)$$

In the foregoing expression u denotes the air velocity as measured at a certain specific height z' . It follows then that coefficients C_d^* and γ are related by

$$C_d^* = 2\gamma$$

Attention should be called to the fact that C_d^* represents the total drag coefficient; i.e., $C_d + C_r$ in the notation used in preceding paragraphs.

In the fields of aerodynamics and of hydrodynamics (as applied to ships) the distance z over which $u(z)$ is variable is generally small and it is easy to measure the fluid velocity at a distance from the body where $u(z) = V$. In meteorology and oceanography it is necessary to consider the wind which has blown over a vast distance, and the height z over which $u(z)$ is appreciably variable is so large that it is impossible to measure velocities in the region where $u(z) = \text{const}$, except by means of pilot balloons. It becomes necessary, therefore, to establish the form of function $u(z)$, as well as certain conventions as to the height at which u should be measured.

These conventions have been only very loosely defined. G. I. Taylor (1915, 1916) used the data of pilot-balloon observations over Salisbury Plain in England,

⁶ For a more complete treatment of this subject the reader is referred to Ursell (M).



Fig. 11 Two views of downward end of pond taken at a wind speed of 17 m/sec before and after addition of detergent to the surface. The marker pole is graduated at 1-ft levels (from Van Dorn, 1953)

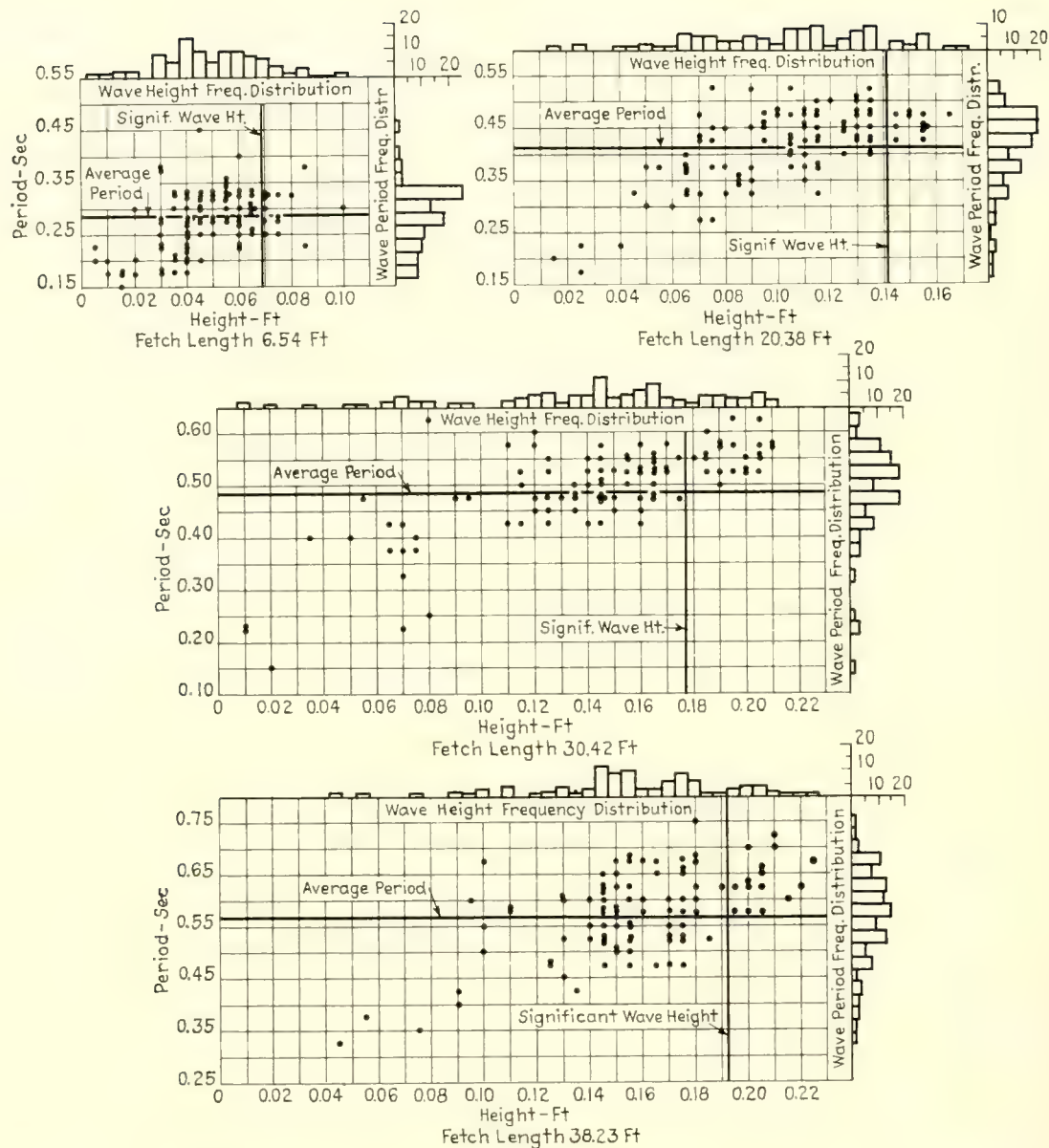


Fig. 12 Joint frequency distribution of wave period and wave height ($U = 42.7$ fps) (from Johnson and Rice, 1952)

and took for U the measured wind velocity at the height of 30 m (about 100 ft). In the published works on oceanography heights of 6 and 10 m (19.6 and 33 ft) are most often used. For measurements made on small waves with small fetches and light winds, Roll (1949) used U as measured at the height of 1 or 2 m. For small waves in a wind flume Francis (1951) measured U at $z = 10$ cm (0.33 ft), while Johnson and Rice (1952) took the mean air velocity in the wind flume. It should be noted that in a wind flume the air velocity first increases with height over the rough-water surface, and then decreases again in approaching the upper wall of the flume. The lack of an established similarity law and a definite convention is evident.

The relationship between wind velocity near a rough surface and the tangential drag τ per unit of surface area can be stated by quoting from Francis (1951):

"The velocity distribution within a boundary layer gives an indirect method of finding the shear stress on the boundary, and this has been used before to find the stress coefficient from field tests. The case of a turbulent flow over a solid rough surface is well known, and it has been shown that at a height z

$$u = 5.75 (\tau/\rho)^{1/2} \log_{10}(z/z_0) \quad (25)$$

where z_0 is a length expressing the effective roughness of the surface . . . By plotting u against $\log z$, a straight line results, of which the slope gives the value of

$5.75 (\tau/\rho)^{1/2}$, and the intercept at $u = 0$ gives $\log z_0$."

Under conditions of a fluid flow along a rough stationary surface, and in particular in the case of wind over land area analyzed by G. I. Taylor (1916), the surface does not absorb the energy. The loss of momentum in the air is accompanied by dissipation of the kinetic energy in eddies, turbulence, and finally in the form of heat. The organized kinetic energy of the potential air flow is in part disorganized, lost in the form of heat, and thus is no longer available. An entirely different situation exists in very mobile sea waves in which the energy is transmitted from air to water. To a large extent the air does work on the moving water surface by normal pressures, so that kinetic energy given up by the air reappears as the kinetic energy of the potential wave motion. Only a part of the kinetic energy of wind is dissipated in friction in the form of air and water turbulence. The theory of a boundary layer at a mobile or oscillating surface has not yet been developed, and so in practice it becomes necessary to assume that the turbulent boundary-layer relationships developed for fixed surfaces remain valid for the mobile water surface. The empirically derived coefficients, however, may not be the same in the two cases in view of the fundamental distinction of the two phenomena. Clearly realizing this distinction Neumann (1948, 1949a) speaks of the "effective tangential force coefficient," which is composed of contributions of both the dynamic, i.e., pressure, drag and the frictional drag, for which alone the turbulent-boundary-layer expressions are truly valid. This represents an assumption that the pressure drag of a moving, wavy surface affects the air-velocity distribution $u(z)$ in the same functional form as the frictional drag. Practical application of the method appears to confirm this assumption, but apparently no deeper investigation of this question and no crucial experiments were made. In connection with the foregoing Neumann (1948, 1949a) emphasized that the roughness parameter z_0 in equation (25) is a purely nominal quantity, characteristic of the sea surface but bearing no direct relationship to the apparent roughness of the sea. In fact, as will be shown later, the roughness parameter z_0 is often shown to decrease with increasing wind and apparent sea roughness.

In the computation of the drag of the earth's surface by G. I. Taylor (1916) the dimensions of the roughness (ground undulations, trees and so on) were small as compared to the height z used, and z was therefore obtained by measurements over the ground without ambiguity. In oceanography the usual measurements of the wind velocity are made at a relatively small height over large waves. A more specific definition of the height z is therefore needed. Roll (1948) expressed z as $z' + H/2$, where z' is the height above wave crests, and H the wave height. This definition is close to but not identical with measuring z over the undisturbed water surface. However, there was a kink in his plotted curve of $\log z$ versus u at a low value of z . Neumann (1949a) suggested that $z = z' + H$ be used; i.e., the lowest level of wave troughs be used as the reference level. In the

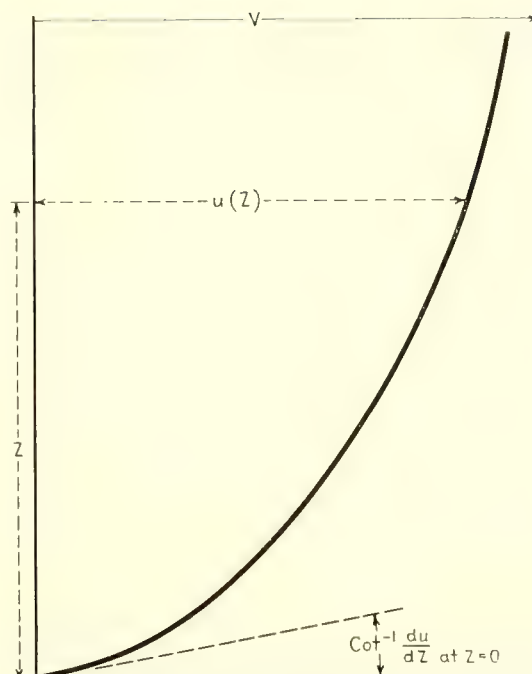


Fig. 13 Variation of mean air velocity versus height in vicinity of ground

plots made by him on this basis the kinks disappeared and the desired straight-line plot resulted. At a wind of 4 mps (about 13 fps) and wave height $H = 50$ cm (about 1.6 ft) shown by Roll's (1948) measurements, Neumann estimates $z_0 = 2$ cm (0.79 in.).⁷

Francis (1951) applied the method described in the quotation from his work given before to his measurements in a wind flume. At low z in proximity to the waves, however, the plot of u versus $\log z$ exhibited violent kinks, and only for a short range of the larger z heights measured was the plot linear. Although the measurements showed wide scatter, Francis obtained a mean value of 5.0 for the coefficient in equation (25), which is close to the theoretically expected 5.75. It was apparently impossible to evaluate z_0 (and therefore τ) from these plots.

3 Energy Balance in Waves and Energy Dissipation

The energy of a wave system grows with distance by the amount of the energy received from the wind less the amount dissipated by internal friction. An elementary analysis of this process will be given.

Consider a stretch of sea of unit width, traversed by imaginary control planes located at fetches F_1 and F_2 . The mean rate of energy gain E per square foot over the distance $F_2 - F_1$ is expressed as

$$dE/dx = \frac{E_2 - E_1}{F_2 - F_1} \quad (26)$$

⁷ For a recent discussion on properties of the boundary layer at the sea surface the reader is referred to Ellison (1956) (see p. 105).

where E_1 and E_2 are the energies per square foot per second carried over through the planes at F_1 and F_2 . From equation (60) of Appendix A

$$E_1 = (1/4) \rho g c_1 a_1^2 \quad (27)$$

in any consistent set of units, or

$$E_1 = 16 c_1 a_1^2 \quad (28)$$

in foot-pound units for sea water at $\rho g = 64$ pcf. Here c_1 is the wave celerity and a_1 the wave amplitude at F_1 . An identical expression with subscripts 2 will hold at F_2 .

The foregoing rate of change is equal to the difference between energy E_p , received from wind, and E_{ds} , dissipated in internal friction; i.e.,

$$dE/dx = E_p - E_{ds} \quad (29)$$

E_p is evaluated on the basis of normal pressures acting on water as

$$E_p = C_d \frac{\rho'}{2} (U - c)^2 c \quad (30)$$

where for "standard air" $\rho' = 0.00237$ pounds per cu ft.

The energy E_{ds} dissipated in internal friction is given for classical gravity waves by equation (66) of Appendix A in terms of the molecular coefficient of viscosity μ . G. I. Taylor (1915), in his study of atmospheric turbulence, introduced a coefficient of turbulent viscosity which is larger than μ . This coefficient will be designated by μ^* . Neumann (1949b) also shows that the effective turbulent coefficient of viscosity μ^* in wave motion is many times larger than μ . The turbulence responsible for this increase results partly from the energy transmitted from wind by skin friction and partly from the kinetic energy of wave motion dissipated in the process of the breaking of wave crests. Assuming for the present that the classical expression is valid with a new coefficient μ^* ,

$$E_{ds} = 2 \mu^* k^3 c^2 a^2 \quad (31)$$

where k is the wave number, $2\pi/\lambda$. Expression (31) is valid in any consistent set of units. In the foot-pound system, and taking $\mu = 2.557 \times 10^{-5}$ for sea water of 59 F (15 C), it becomes

$$E_{ds} = 0.065 n (a/\lambda)^2 \quad (32)$$

where n denotes the ratio μ^*/μ .

Since the wave height, $H = 2a$, and wave length λ are reported in all wave observations, the energies E_1 and E_2 can be computed readily. The energy received from wind, E_p , also can be computed, provided the drag coefficient C_d is known. C_d can be reasonably estimated from the data of the foregoing section. The only quantity completely unknown is the ratio n . It can be computed on the basis of equations (26) and (29) as

$$n = \frac{1}{0.065(a/\lambda)^2} \times \left[C_d \frac{\rho'}{2} (U - c)^2 c - (E_2 - E_1)/(F_2 - F_1) \right] \quad (33)$$

Data and computations for six cases found in the literature are shown in Table 4. This is but a small sample of data of varying reliability (for the present purpose) but nevertheless a few conclusions can be drawn:

a) There is little purpose in analyzing in this manner more of the data found in the current literature. Data must be obtained specifically with this type of analysis in mind for the results to be reliable.

b) The ratio n is not a constant but varies with (a/λ) ratio and with wave height. Expression (31) must therefore be modified by including a proper functional relationship for μ^* .

c) In the minute and mild waves (case 3) n is about 6. For essentially the same wave height but greater steepness and hence larger C_d in case 5, n increases to 41.

d) The n -values of 207 and 439 in cases 1 and 6 are apparently exaggerated by the excessive influence of the wave celerity c in the factor $(U - c)^2 c$ entering into E_p . In Table 4, the c -values are based on the reported mean or significant waves. It appears more probable that the energy-transfer calculations should be based on the smaller waves or ripples by which the significant waves are overlaid. This would call for smaller λ and c in the foregoing calculations.

e) If the same reasoning were applied to wave dissipation, the calculations also would have indicated greater energy dissipation, since smaller waves of lower c/U -ratio usually have higher a/λ -ratio.

f) Case 2 should be omitted. A comparison with case 1 shows too low a/λ for a similar c/U -ratio. Apparently, the waves preformed by the wave generator were too long for the ambient wind conditions.

3.1 Energy Dissipation (by Bowden, 1950). The development indicated by paragraph b) of the foregoing section was attempted by Groen and Dorrestein (1950) who, on the basis of the previous work of Richardson (1926) and Weizsäcker (1948), assumed μ^* to be proportional to $\lambda^{4/3}$. Bowden (1950) showed that Weizsäcker's reasoning is not applicable to waves and, on the basis of dimensional reasoning, derived a new relationship. If μ^* depends on wave proportions, it should be a function of wave length, amplitude and period, so that

$$\frac{\mu^*}{\rho} = K \lambda^\alpha a^\beta T^\gamma \quad (34)$$

It follows that $\alpha + \beta = 2$ and $\gamma = -1$. Bowden took the simplest assumption that $\alpha = \beta = 1$ and wrote

$$\frac{\mu^*}{\rho} = K c a \quad (34a)$$

where K is a nondimensional coefficient. The rate of energy dissipation is then

$$E_{ds} = 2 \rho K k^3 c^3 a^3 \quad (35)$$

Bowden confirmed the foregoing results by a derivation based on von Kármán's (1930a and b) similarity hypothesis for shearing flows.

The application of the foregoing equation to cases 1 to

Table 4 Estimated Energy Balance in Observed Waves

Case	$F_{2,1}$, ft	λ , ft	c , fps	a , ft	$E_{2,1}$	$(E_2 - E_1)/$ $(F_2 - F_1)$	U , fps	$(U - c)^2 c$ mean	C_d	E_p	a/λ mean	$0.065(a/\lambda)^2$ mean	$n =$ μ^*/μ
1	30.42	1.22	2.49	0.0935	0.348	0.0134	42.7	3251	0.0258	0.100	0.080	0.00042	207
	6.54	0.416	1.46	0.0345	0.028		42.7						
2	16.4	1.44	2.71	0.0935	0.379	0.0462	39.3	3463	0.0214	0.088	0.047	0.00014	300
	9.2	1.15	2.42	0.0345	0.046		39.3						
3	16.4	0.26	1.15	0.013	0.0031	0.0004	16.4	219	0.0052	0.0014	0.050	0.00016	6.2
	9.2	0.10	0.70	0.005	0.0003		16.4						
4	67.5	1.33	2.61	0.0637	0.169	0.0030	19.5	603	0.0052	0.0038	0.051	0.00017	4.7
	18.0	0.50	1.60	0.0272	0.019		18.6						
5	2.62	0.17	0.93	0.0112	0.00187	0.00085	25.5	482	0.0235	0.0136	0.069	0.00031	41.3
	0.98	0.09	0.68	0.0066	0.00047		25.0						
6	...	106	23.28	3.25	...	0	37	4370	0.0052	0.0272	0.031	0.000062	439
		(Steady Conditions)					(Est.)						

Legend:

Case 1. Johnson and Rice (1952)—Run 1a. C_d estimated as mean of first four lines of Table 3, assuming $C_r = 0.0028$ on basis of Table 1, Model 4.

Case 2. Francis (1951)—Severe Condition: Generator and Fan—12 mps. Data estimated from curves; $C_d + C_r = 0.0242$ from Table 2, C_r estimated as 0.0028.

Case 3. Francis (1951)—Gentle Condition: Fan only, 5 mps; $C_d + C_r$ listed in Table 2 would leave improbably small value for C_d . $C_d = 0.0052$ is assumed from Thijssse's (1952) experiments.

Case 4. Roll (1951)—Open Air Observations, Nos. 24 and 26. Conditions at two fetches and wind are means of several observations and do not represent a simultaneous set of conditions. C_d estimated as for Case 3. Roll's (1948a, Table 1) wind-profile measurements give the unreasonably low value of $C_d + C_r = 0.0025$.

Case 5. Roll (1951)—Observations 29 and 32. In shorter fetch and stronger wind than Case 4. In view of mean steepness of waves, C_d is assumed as the mean of Cases 1 and 2.

Case 6. Neumann (1950, page 41)—A typical sea condition in North-East trade winds quoted from Larisch-Wind Beaufort 5 to 6. C_d assumed from Thijssse's (1952) experiments.

6 of Table 4 yields values of K of 0.022, 0.022, 0.010, 0.0008, 0.073 and 0.000069, respectively. Bowden, on the basis of swell attenuation, gave the order of magnitude of K as 5×10^{-5} , which checks well with the figure computed for Neumann's case 6 in trade winds. The variability of the coefficient K indicates that the problem is not yet solved. It appears to the author that Bowden's dimensional reasoning was faulty in that T and λ are not independent, and λ and a are not dimensionally distinguishable. It is suggested that the derivation based on von Karman be investigated further.

3.2 Summary of the Energy-Balance Problem. Summarizing the foregoing, neither the problem of energy transmission from wind to waves nor the problem of energy dissipation in waves has been solved. The solution of the first hinges primarily on evaluating the celerity of the wavelets overlaying larger waves.⁸ The solution of the second requires theoretical and experimental determination of the function form of the turbulent viscosity μ^* . Flume and open-air wave observations have not been reported in a form useful for evaluating the wave-energy balance. New observations and flume tests are needed. These must be planned specifically with a suitable analytical procedure in mind. It cannot be too strongly emphasized that wind-flume and small-scale natural waves differ radically from natural ocean waves of significant size in the ratios c/U and a/λ . Small values of the former and large values of the latter predominate in small waves. Small-scale tests and observations, therefore, cannot be used as direct representation of open-sea conditions. The objective of the small-

scale observations should be, therefore, the development of rational relationships which then can be applied to any wave system.

The decrease of wave steepness a/λ with increase of wave length λ is one of the most reliable relationships observed in ocean waves. The quantitative relationship between these quantities was first noted by Sverdrup and Munk (1946, 1947) and later confirmed by Roll (1951) and Neumann (1950a and b, 1953a and b). Bowden appears to be the first to offer a rational explanation of this observed fact. His formula, equation (35), indicates a rapid increase of the energy-dissipation rate with wave height and length. The formula for the transfer of the energy from wind, on the other hand, depends on C_d which is a function of the wave form, but is independent of wave height. Thus the balance between the energy received and dissipated can only be achieved in higher waves at a lower a/λ value. While formula (35) appears to have exaggerated the influence of the wave size, the significance of this factor can scarcely be doubted.

The exposition of the energy balance in the growth of sea waves has been presented simply, in order to call attention to the basic elements of the problem. Both the transfer of the energy from the wind and the dissipation of the energy by turbulence and internal friction must be considered in defining the growth of waves. A procedure analogous to the foregoing, but much more elaborate, was employed by Neumann (1952a, b, c). It is reviewed in Appendix B. Although it is listed under "Practical Approach" because of the many empirical and intuitive steps involved, it is the most complete discussion of the fundamental principles involved in the energy balance in waves available to date. It served as a basis for the wave-forecasting method of Pierson, Neu-

⁸ A more exact formulation of this statement was given on page 9 (last paragraph) and it will be discussed further in Section 4.4.

mann, and James (H). On the other hand, the broadness of the subject is overlooked in the recent "advanced" work discussed in Section 4, and attention is concentrated entirely on the transfer of the energy from the wind.

4 Generation of Waves by Wind—Advanced Rational Approach

Under this heading several recent papers can be reviewed briefly. These are elaborate mathematical developments and the reader is referred to the readily obtainable original papers for the complete exposition of the subject. Only a general outline of the principles used will be given here, primarily in order to indicate the apparent shortcomings and desirable directions of further development.⁹

The advanced approach to the subject of wave formation by wind has taken two broad directions. Eckart (1953*a, b, c*) and Phillips (1957) took air-pressure fluctuations in a gusty wind as the primary cause of wave formation. The pressure fluctuations in this case are uncorrelated with the wave form. In another approach, Munk (1955) and Miles (1957), following in principle the elementary method of Jeffreys (Section 2), considered the air pressures as caused by the air flow about the wave profile. In this case the air-pressure variations are completely correlated with the waves. Two such different concepts can be entertained only in the early stages of development of a subject. It is probable that at a later stage of the theoretical development a concept of partial correlation will be introduced. In these early stages of development, also, the energy dissipation in waves has not been considered.

4.1 Eckart's Theory. The wind pressure is assumed to be everywhere normal to the undisturbed surface of water and caused by an ensemble of "gusts." A gust is defined as an area of high or low pressure, which moves with the mean wind speed, and has a radius $D/2$ and a duration or life T . At the end of the time period T a gust "blows itself out" leaving its wake to dissipate in the form of free gravity waves. First, the theory of this phenomenon is developed for a single gust. Next the effect of the ensemble of gusts is treated on the basis of the time average, as commonly used in the theory of turbulence. A very large number of gusts of uniform diameter and intensity is assumed to be distributed at random through space and time. This ensemble of gusts makes a storm.

Quoting from Eckart (1953*c*): "In the generating area the waves may be many meters high, and thus represent a large surface density of energy. This energy cannot be supposed to have been obtained from the air instantaneously and locally. Much of it will have been obtained from the air earlier and at a considerable distance from the point of observation (though still in the storm area). It will have been transported by the water essentially according to the laws of free wave motion.

This has long been recognized by the use of the concept of fetch . . ." The effect of the few gusts near to a point of interest on the sea surface is, therefore, negligible compared to the many distant gusts the effect of which has accumulated with fetch.

The waves caused by the ensemble of randomly distributed and fluctuating pressure areas represent the summation of many component waves of different wave lengths, heights, phases and directions of propagation. Such waves are described by a spectrum. As will be discussed in greater detail later, the resultant appearance of the sea is that of groups of waves separated by calmer regions, each group consisting of a few waves of varying heights. Quoting further from Eckart (1953*c*): "Since the surface disturbance has a random character, no unique value of wave number and frequency can be assigned to it; these dominant values correspond roughly to a maximum in the spectrum." From a consideration of the empirically observed number of 5 to 10 waves in such groups, Eckart concluded that in a wind of 20 mps (about 39 knots) for instance, the life T of the gust is 15 to 30 sec, and the typical gust radius $D/2$ is 40 m (130 ft).

Eckart's solution covers regions inside and outside of the storm area; the latter case is simpler and the solution is more precise. Outside the storm area the spectrum of wave directions is symmetrically disposed with respect to the radial direction from the storm center, i.e., the velocity of propagation of the dominant wave is in the radial direction. The existence of a spectrum of wave directions causes wave short-crestedness, and at the radial distance r of 10 storm diameters D , for instance, the average length of wave crests is 2.2 of wave length λ (between succeeding crests).

Inside the storm, the wave components have not yet separated, and there is no similarly dominant direction. Each point is traversed by waves travelling in many directions. The conditions are particularly confusing in the center of the storm area. The predominating direction of wave propagation becomes more clearly defined as a point under consideration moves from the center to the periphery of the storm area. Generally, the sector of wave directions inside the storm is not symmetric with respect to the wind direction, the asymmetry decreasing toward the edges of the storm.

Outside of the storm area the wave remains constant, since it is no longer influenced by atmospheric disturbances. Inside the storm area the formulas derived by Eckart imply that the spectrum is a function of position in the area, and in particular, that the effect of a given fetch depends on its position in the storm area.

While the theory developed by Eckart explains many of the observed characteristics of storm-generated waves, it fails to predict correctly the wave height. The air pressure needed to generate the observed waves according to the theory is shown to be possibly ten times greater than its probable value.

One of the possible reasons for this discrepancy is contained in the initial formulation of the problem by

⁹ Only in the realm of basic ideas. Mathematical techniques will not be discussed.

Eckart. The storm is modeled mathematically by the succession of *traveling pressure areas*, and no consideration is given to the effect of the *wind velocity*. The wind velocity U enters into Eckart's theory only in that the pressure areas are assumed to travel with the wind velocity. Neglect of the wind velocity itself is consistent here with the initial assumption of "infinitesimal" waves. Quoting again from Eckart: "The term infinitesimal in this connection, refers to the neglect of nonlinear terms in the hydrodynamic equations; it seems certain that this must be ultimately remedied if all phenomena connected with the interaction of wind and water are to be treated theoretically . . ." Particularly important in this definition are the infinitely small slopes of the water surface. Since the wind action on water appears to depend on the square or higher power of these slopes, the mechanism of the kinetic-energy transfer from wind to water is essentially absent in Eckart's case; only changes of static pressure of the air are considered. It should be clear from the preceding sections that the transfer of energy from wind depends specifically on the finite height of waves, and particularly on the existence of sharp slopes which are not considered in this case.

While the foregoing paragraph represents a plea for consideration of wave-correlated pressure areas, it should be noted that Eckart's conclusions (except for wave height) probably would not be greatly affected by such considerations. Since the waves themselves are randomly distributed, and in fact have the appearance of randomly distributed groups, it can be assumed that consideration of correlated pressures would have taken a form similar to the one used by Eckart. An apparent major difference would be the use of wave-group velocity instead of wind velocity for the propagation of pressure areas. Most of Eckart's conclusions, except the mean value of wave heights, can therefore be assumed to be valid for the actual sea surface, at least for the time being. Unfortunately, observational data on the angular dispersion of wave propagation and on the lengths of wave crests are very meager. In particular, data on the waves within the storm area appear to be almost completely absent. Such data as for instance Weinblum's (3-1936) stereophotographs on the *San Francisco* in a severe storm have not yet been analyzed in a form suitable for comparison with the theory discussed here. Furthermore they usually cover too small an area to be valid statistically.

The work of Eckart (1953*a, b, c*) can be considered as extremely important not only for its results, but for the method of attack as well. Further work based on this method, but considering the action of the wind on the irregular sea surface with large wave slopes, should be encouraged and sponsored.

4.2 Phillips' Theory. Apart from the mathematical methods, Phillips' theory differs from Eckart's by the adoption of a more general randomness. While Eckart postulated random distribution of gusts in space and time, he, by independent reasoning, specified the diam-

eters and durations of gusts. He also assumed that gusts move with the wind velocity U . Phillips made the statement of the problem more general by assuming that the dimensions and lifetime of gusts are also random. This included the smaller gusts moving near the sea surface in the air stream of reduced velocity $u(z) < U$. In stating the problem Phillips, therefore, postulated an (as yet) unknown velocity U_c .

A random distribution of fluctuating pressures is described by a spectrum;¹⁰ i.e., it is thought of as composed of a superposition of sinusoidally varying pressure fluctuations of different amplitudes, frequencies and phase relationships. The term "spectrum" or, more exactly, "spectral density" is applied to the mean amplitudes of fluctuations within a narrow frequency band. Waves excited by such pressure fluctuations also are described by a spectrum; i.e., by the summation of sinusoidal wave components of various amplitudes and frequencies. The end result of Phillips' solution is an expression defining the wave amplitudes in terms of the amplitudes of pressure fluctuations at all frequencies. This relationship is time dependent and the wave amplitude is shown to increase in proportion to the elapsed time. In the process of solution, the effective gust travel velocity U_c was defined.

Quoting Phillips, "It is found that waves develop most rapidly by means of a resonance mechanism which occurs when a component of the surface pressure distribution moves at the same speed as the free surface wave with the same wave number.

"The development of the waves is conveniently considered in two stages, in which the time elapsed [from the onset of a turbulent wind] is respectively less or greater than the time of development of the pressure fluctuations. An expression is given for the wave spectrum in the initial stage of development, and it is shown that the most prominent waves are ripples of wavelength $\lambda_{cr} = 1.7$ cm, corresponding to the minimum phase velocity $c = (4gT/\rho)^{1/4}$ ¹¹ and moving in directions $\cos^{-1}(c/U_c)$ to that of the mean wind, where U_c is the 'convection velocity' of the surface pressure fluctuations of length scale λ_{cr} or approximately the mean wind speed at a height λ_{cr} above the surface. Observations by Roll (1951) have shown the existence, under appropriate conditions, of waves qualitatively similar to those predicted by the theory.

"Most of the growth of gravity waves occurs in the second, or principal stage of development, which continues until the waves grow so high that nonlinear effects become important. An expression for wave spectrum is derived, from which the following result is obtained:

¹⁰ An outline of the notation and mathematics used in connection with random processes (particularly sea waves) will be found in Section 8. The reader is asked to accept the brief and incomplete statements on the subject in Sections 4 and 5, and thus a temporarily incomplete understanding of these sections, with the hope that he will return to them after perusing Section 8, "Mathematical Representation of the Sea Surface."

¹¹ T here is the surface tension.

$$\bar{\eta}^2 = \frac{\bar{p}^2 t}{2 (2 \rho^2 U_c g)^{1/2}} \quad (36)$$

where $\bar{\eta}^2$ is the mean square surface displacement, \bar{p}^2 the mean square turbulent pressure on water surface, t the elapsed time, U_c the convection speed of the surface pressure fluctuations, and ρ the water density. . . ."

"We are now in a position to see rather more clearly the probable reason for the failure of Eckart's theory to predict the magnitude of the wave height generated by the wind. His less precise specification of the pressure distribution has 'smoothed off' the resonance peak of the response of the water surface, and it is the wave numbers near the peak that can contribute largely to the wave spectrum at large durations."

Application of equation (36) requires knowledge of the mean pressure fluctuation \bar{p}^2 . Quantitative data on the turbulence in the boundary layer of the wind at the sea surface are meager and uncertain. However, Phillips used certain plausible data and evaluated \bar{p}^2 and $\bar{\eta}^2$ as functions of the elapsed time. He was thus able to demonstrate excellent agreement of wave-height growth versus time with the data of Sverdrup and Munk (see Section 5.1). The author believes, however, that this comparison is premature and has little meaning, since dissipation of the energy in waves has not been considered. It is evident that Phillips made a major contribution to the subject of wave generation by wind. He has ably treated, however, only one facet of the problem. This must be combined with other aspects (wave-correlated pressures, energy dissipation) before a comparison with observed waves can be meaningful. Phillips' results may be directly applicable to the initial formation of small ripples, at which time the energy dissipation depends on the molecular viscosity and is small, and the drag coefficient C_d and therefore the wave-correlated pressures are also small. The application of Groen and Dorrestein's (1950) and Bowden's (1950) results showing that energy dissipation grows with wave height and length may limit the indicated wave growth and eliminate the need of uncertain reference to nonlinearities.

In his 1958 work, Phillips defined by dimensional reasoning the theoretical shape of the high-frequency end of a wave spectrum. This definition was based on the observed occurrence of sharp-crested waves, the physical definition of a sharp crest by the vertical water acceleration $\ddot{\eta} = -g$, and the mathematical-statistical definition of a discontinuous function expressing the water surface elevation. Phillips found the spectrum¹² to be

$$E(\omega) = \alpha g^2 \omega^{-5} \quad (36a)$$

where α is a constant, g the acceleration of gravity and ω the circular frequency.

4.3 Statistics of the Sea Surface Derived from Sun Glitter (Cox and Munk 1954a, b). This work, describ-

ing the method and the results of observations at sea based on the statistical theory and outlining certain important relationships of this theory, serves as one of two basic constituent parts of the work of Munk (1955), to be discussed in the next section.

The following resume is abstracted from Cox and Munk (1954a): If the sea surface were absolutely calm, a single mirror-like reflection of the sun would be seen at the horizontal specular point. In the usual case there are thousands of "dancing" highlights. At each highlight there must be a water facet, possibly quite small, which is so inclined as to reflect an incoming ray from the sun towards the observer. The farther the facet is from the horizontal specular point, the larger must be its slope in order to reflect the sun's rays back to the observer. The distribution of the glitter pattern is therefore closely related to the distribution of surface slopes.

In order to exploit this relationship plans were laid in 1951 for co-ordination of aerial photographs of glitter from a B-17G plane with meteorological measurements from a 58-ft schooner, the *Reverie*. One of the objects of this investigation was a study of the effect of surface slicks. In the methods adopted oil was pumped on the water, . . . With 200 gal of this mixture, a coherent slick 2000 by 2000 ft could be laid in 25 min, provided the wind did not exceed 20 mph. Two pairs of aerial cameras, mounted in the plane, were wired for synchronous exposure. Each pair consisted of one vertical and one tilted camera with some overlap in their fields of view. One pair gave ordinary image photographs for the purpose of locating cloud shadows, slicks, and vessels; this pair also gave the position of the horizon and the plane's shadow (to correct for the roll, pitch, and yaw of the plane). The other pair of cameras, with lenses removed, provided photogrammetric photographs.

The method consists essentially of two phases. The first identifies, from geometric considerations, a point on the sea surface (as it appears on the photograph) with the particular slope required at this point for the reflection of sunlight into the camera. This is done by suitable grid overlays. Lines of constant α (radial) give the azimuth of ascent to the right of the sun; lines of constant β (closed or circumferential curves) give the tilt in degrees.

The second phase interprets the average brightness of the sea surface (darkening on the photometric negative) at various α - β intersections in terms of the frequency with which this particular slope occurs. On the density photographs the glitter pattern appears as a round blob with a bright core (on the positive print) and a gradually diminishing intensity to the outside. The density of the blob on the negative is then measured with a densitometer at points which correspond to the intersection of appropriate grid lines.

The results are expressed as the mean of squares of wave slopes in up-down wind direction σ_u^2 , and in cross wind direction σ_c^2 . The data on the observed waves and on measured mean squares of slopes are given in Table 5. This table represents an abstract of data from Cox and Munk (1954a), Table 1, with columns of λ/H and

¹² The reader is referred to Sections 6 and 8 for the discussion of wave spectra.

Table 5 Wave Data Obtained in Sun-Glitter Observations of Cox and Munk (1954)

Photograph designation	Wind V , ^a fps	Significant Waves ^b					Mean square slope components		
		Period T , sec	Celerity, c , fps	Length, λ , ft	Height, H , ft	Ratios		Transverse σ_c^2	Wind direction σ_u^2
						λ/H	c/V		
Clean Water Surface									
28 Aug. b	33.3	4	20.5	81.9	3.5	23.4	0.62	0.0211	0.0390
28 Aug. p	40.0	5	25.6	128	6	21.4	0.64	0.0294	0.0484
28 Aug. u	41.5	5	25.6	128	6	21.4	0.62	0.0287	0.0452
28 Aug. v	41.3	5	25.6	128	6	21.4	0.62	0.0276	0.0404
3 Sept. j	1.2	3	15.4	46.1	1.5	30.7	12.9	0.00337	0.00489
3 Sept. q	25.8	3	15.4	46.1	2	23.0	0.60	0.0224	0.0230
4 Sept. k	12.3	2	10.2	20.5	1	20.5	0.83	0.00694	0.00977
4 Sept. n	24.0	3	15.4	46.1	2	23.0	0.64	0.0136	0.0191
4 Sept. r	19.0	3	15.4	46.1	4	11.5	0.81	0.0134	0.0170
4 Sept. v	19.5	3	15.4	46.1	4	11.5	0.79	0.0136	0.0186
4 Sept. y	14.8	3	15.4	46.1	4	11.5	1.04	0.0172	0.0174
5 Sept. b	5.4	4	20.5	81.9	3	27.3	3.80	0.00534	0.00906
5 Sept. g	6.5	4	20.5	81.9	3	27.3	3.16	0.00609	0.00875
5 Sept. j	10.2	4	20.5	81.9	5	16.4	2.00	0.0102	0.0125
6 Sept. c	32.5	4	20.5	81.9	4	20.5	0.63	0.0252	0.0265
6 Sept. k	31.2	4	20.5	81.9	4	20.5	0.66	0.0254	0.0357
6 Sept. q	35.5	4	20.5	81.9	5	16.4	0.58	0.0254	0.0374
10 Sept. r	16.0	3	15.4	46.1	3	11.5	0.96	0.0137	0.0179
11 Sept. l	15.1	3	15.4	46.1	2	23.0	1.02	0.0136	0.0137
17 Sept. e	29.5	3	15.4	46.1	4	11.5	0.52	0.0209	0.0264
17 Sept. c, h, k, n, q	29.3	3	15.4	46.1	4	11.5	0.52	0.0230	0.0322
17 Sept. A	31.0	3	15.4	46.1	5	9.2	0.50	0.0224	0.0365
Water Covered by an Oil Film									
6 Sept. c	32.5	4	20.5	81.9	4	20.5	0.66	0.0111	0.0108
10 Sept. k	25.7	3	15.4	46.1	3	15.4	0.60	0.0102	0.0117
10 Sept. m	21.6	3	15.4	46.1	3	15.4	0.71	0.00860	0.0100
10 Sept. r	16.0	3	15.4	46.1	3	15.4	0.96	0.00967	0.00985
11 Sept. f	15.2	3	15.4	46.1	2	15.4	1.01	0.0107	0.0109
13 Sept. e	7.1	5	25.6	128	4	32	3.60	0.00391	0.00467
13 Sept. f	7.1	5	25.6	128	4	32	3.60	0.00724	0.00957
17 Sept. e	29.5	3	15.4	46.1	4	11.5	0.52	0.0106	0.0126

^a Wind was measured at the heights of 9 and 41 ft. The mean value, corresponding approximately to the often-used "anemometer height" of 24 ft is listed here.

^b The wave periods and heights were recorded. The listed values of wave length λ and celerity c are obtained from Table 3 of Appendix A for the corresponding periods.

c/V added. The σ^2 are shown plotted against wind speed in Fig. 14 and the probability distribution of σ^2 is shown in Fig. 15. This latter will be discussed later in connection with the statistics of the irregular sea.

The major conclusions can be given by a partial quotation from the summary of Cox and Munk (1954):

a) As a first approximation the slope distribution is found to be Gaussian; this can be accounted for by an arbitrarily wide continuous spectrum of ocean waves, but not by a spectrum consisting of a few discrete frequencies.

b) The ratio of the up/down wind to crosswind components in the mean square slope is less than two; this indicates the directional "beam width" in excess of 130 deg for the relatively short waves that constitute the slope spectrum.

c) The mean square slope, regardless of direction, increases linearly with wind speed and reaches a value of $(\tan 16^\circ)^2$ for a wind speed of 14 mps (about 45 fps or 27 knots); this empirical relation follows in form and to an order of magnitude from a spectrum proposed by Neumann on the basis of wave-amplitude observations. A

spectrum proposed by Darbyshire cannot be reconciled with our observations.

d) Oil slicks laid by the vessel over an area of $1/4$ sq mile reduce the mean square slope by a factor of 2 or 3.

Several conclusions of lesser importance for the present monograph pertain to statistical properties of records and are omitted in the foregoing quotations. The discussion of statistics and spectra will be deferred until later chapters on irregular seas, and only the features bearing on the next section, on Munk (1955), will be discussed here.

This work is extremely valuable in introducing an ingenious technique based on statistics, in providing very valuable data on the sea surface, and in discussing statistical relationships for the sea surface. The conclusions appear, however, to be too broadly stated in some respects, without regard to the limitations of the relatively small scope of observations. An examination of the wave data in Table 5, particularly of the columns of λ/H and c/V shows that the observed significant waves are mostly either too small or too large in relation to the observed wind. The former case indicates that either

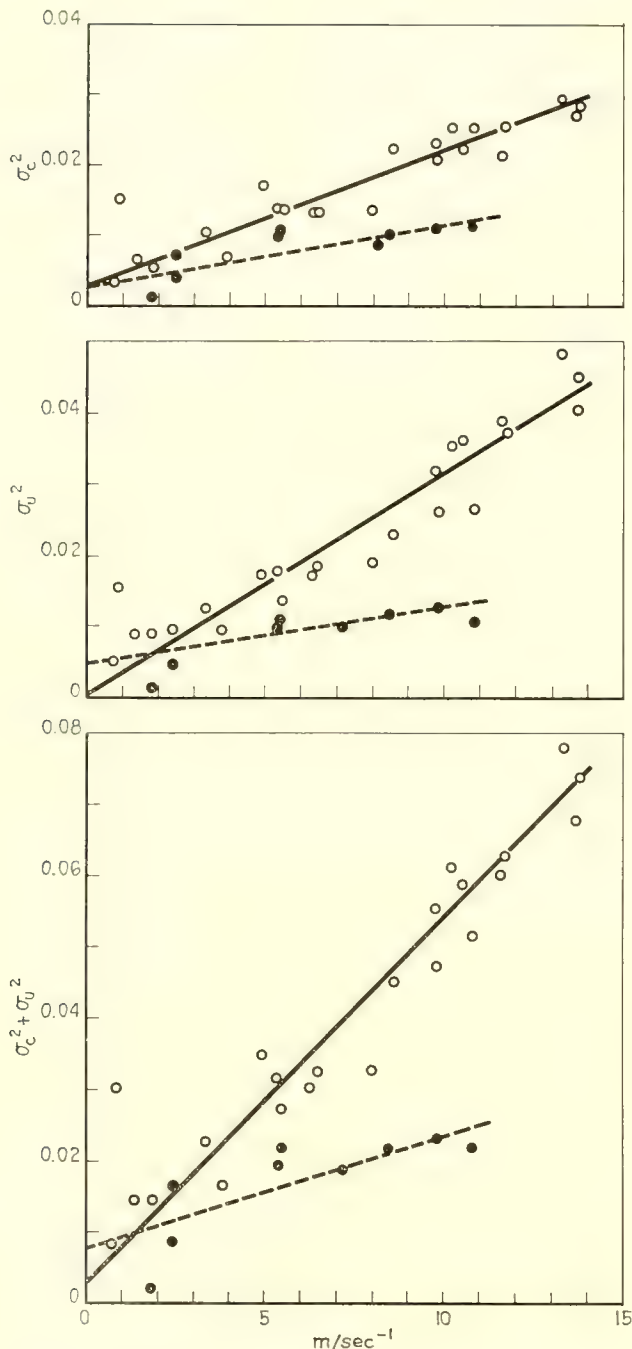


Fig. 14 Mean-square, wave-slope components and their sum as functions of wind speed. Open circles and solid lines for clean sea surface, solid dots and dotted lines for detergent covered (slick) surface (from Cox and Munk, 1954)

the fetch or the wind duration were not large enough to give a fully developed sea. The latter case indicates that the observed significant waves were to a large extent due to the presence of a swell and not due to the local wind. In any study pertaining to waves it is very important to make a clear distinction between the swell

and the wind sea, but such a distinction is missing in the work under consideration.¹³

A ratio of squared slopes σ_u^2/σ_c^2 of about 2.5, indicating a directional spread of waves of about 130 deg, applies essentially to the small waves by which the larger observed waves are overlaid. Spreading of an oil film eliminated these small waves, leaving the larger waves unaffected. It is surprising to find that the ratio σ_u^2/σ_c^2 in this case is reduced to nearly unity, indicating an increased degree of short-crestedness. A possible explanation is that several swells of different directions (independent of the wind) were present, while the small and steep waves were caused by the local wind.

Examination of Table 5 and of Fig. 14 shows clearly that steep wave slopes are connected with the small waves by which the larger observed waves are overlaid. The recorded slopes are drastically reduced when these small waves are eliminated by the oil film. The mean square values of slopes σ^2 are seen to have little relationship to the observed wave dimensions, since these small waves are neglected in the definition of the "significant wave" as the mean of the $\frac{1}{3}$ highest waves. On the other hand σ^2 is seen to depend directly on the wind strength.

The conclusion that σ^2 is proportional to the wind velocity, as shown by Fig. 14 and as stated by Cox and Munk (1954a) may, however, be misleading. This relationship is shown to exist within the scope of observations, but the wave slopes cannot increase indefinitely, and the statistical observations should not be extrapolated without regard to the physical properties of waves.

4.4 Horizontal Drag Force Exerted by Wind—W. H. Munk's Hypothesis. The objective of this work is stated in the following quotation from Munk (1955a): "The problem of wind stress on water plays an essential part in studies of ocean circulation and storm tides, and of the momentum balance of atmospheric circulation. The present work is an attempt to connect results from recent experimental determinations of wind stress with the results from measurements of wave statistics . . ."

The starting point is the expression by Jeffreys (1925) for the pressure exerted by wind of velocity U on an element of the wave surface

$$p = s\rho'(U - c)^2 \partial\eta/\partial x \quad (37)$$

where s is a coefficient called by Jeffreys "sheltering coefficient" and assumed to be constant. The horizontal component of this pressure (i.e., drag force or wind stress) is

$$\tau = s\rho' \langle (U - c)^2 (\partial\eta/\partial x)^2 \rangle \quad (38)$$

where the symbol $\langle \rangle$ indicates that the mean value is taken.

The foregoing formulas were written for a simple harmonic wave, the celerity c of which is known. When the

¹³ A more complete description of the environmental conditions of these observations was published by Darbyshire (1956a).

water surface is formed by superposition of many waves, the shorter ones riding the surface of the longer ones, the definition of c is not immediately evident. Munk has introduced the concept of "facet velocity," i.e., he defined c for the present purpose as the horizontal velocity of translation of a small element of water surface; this velocity will be designed here as C . The vertical velocity of water surface and the horizontal facet velocity C are connected with the wave slope by the relationship

$$\partial\eta/\partial t = -C \partial\eta/\partial x \quad (39)$$

Equation (37) can be simplified by assuming C to be much smaller than U , and neglecting the square of it. Then

$$p \cong s\rho' U(U - 2C) \partial\eta/\partial x \quad (40)$$

Munk omits the factor of 2 in equation (40), and by using equation (39) writes

$$p = s\rho' U(U - C) \partial\eta/\partial x = s\rho' U \left(U \frac{\partial\eta}{\partial x} + \frac{\partial\eta}{\partial t} \right) \quad (41)$$

The form drag force per unit area becomes then

$$\tau = s\rho' U \left\langle U \frac{\partial\eta}{\partial x} \frac{\partial\eta}{\partial x} + \frac{\partial\eta}{\partial x} \frac{\partial\eta}{\partial t} \right\rangle \quad (42)$$

For a complex sea formed by superposition of waves of all amplitudes, directions, and lengths, the differential expressions in the foregoing are averaged by statistical methods (Eckart, 1953b) and the drag force is expressed as

$$\tau = s\rho' U \times \int_0^\infty \int_{-\pi}^\pi [U \cos \Theta - c(k)] k^2 \cos \Theta S(k, \Theta) k dk d\Theta \quad (43)$$

where $k = 2\pi/\lambda$ is the wave number and Θ is the direction of wave propagation. The expression $S(k, \Theta) k dk d\Theta$ designates the contribution to the total mean-square elevation by waves with wave numbers in the interval $k - (1/2)\delta k$ to $k + (1/2)\delta k$, which travel in the direction $\Theta - (1/2)\delta\Theta$ to $\Theta + (1/2)\delta\Theta$ relative to wind direction.

The wind stress is then evaluated for different assumptions as to the form of the function $S(k, \Theta)$. By considering waves propagating in one direction, i.e., $\Theta = 0$, and assuming c to be negligibly small, Munk arrives at the expression

$$\tau = s\rho' U^3 \sigma^2 \quad (44)$$

where σ^2 is the mean square slope. From Cox and Munk (see Section 4.3) σ^2 is taken as proportional to the wind velocity U . Neglect of the small celerity c leads finally to

$$\tau = s\rho' (\text{const}) U^3 \quad (45)$$

For a range of wave directions Θ , within the sector $-\Theta_0/2 < \Theta < \Theta_0/2$, equation (42) yields

$$\tau = s\rho' (\text{const}) f(\Theta_0) U^3$$

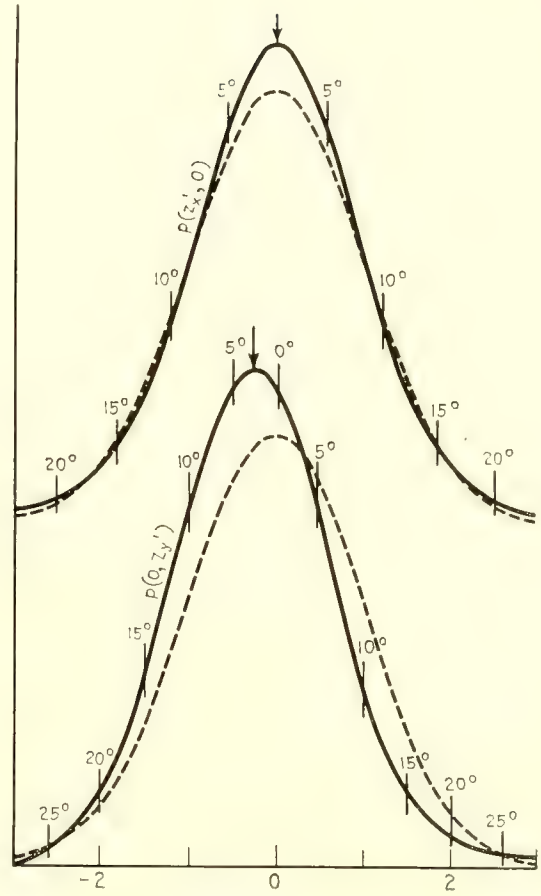


Fig. 15 Probability distributions of wave slopes along crosswind axis (upper) and up/downwind axis (lower curves, positive upwind). Solid curves refer to observed distribution; dashed to a Gaussian distribution (from Cox and Munk, 1954)

where

$$f(\Theta_0) = \frac{1}{2} \left(1 + \frac{\sin \Theta_0}{\Theta_0} \right) \quad (46)$$

The values of $f(\Theta_0)$ are plotted versus Θ_0 in Fig. 16. The dotted vertical line indicates the value of Θ_0 derived by Cox and Munk (1954) from the sun glitter measurements.

A third form of $S(k, \Theta)$ considered by Munk corresponds to the Neumann spectrum. A special section will be devoted to this spectrum later on; suffice it to say here that it includes waves of a wide range of wave numbers k . For the present purpose Munk (1955a) replaces the spectrum in terms of k by one in terms of β where $\beta = c/U$. The spectra $S(k, \Theta)$ or $S(\beta, \Theta)$ define the contribution of all waves to the mean-square amplitude. Similarly spectra can be expressed for the wave slopes. To the mean-square elevation spectrum $S(k, \Theta)$ corresponds the mean-square slope spectrum $k^2 S(k, \Theta)$. Fig. 17 taken from Munk (1955a) shows these two spectra in terms of $\beta = c/U$, as well as the integrand of the corresponding expression for the tangential force

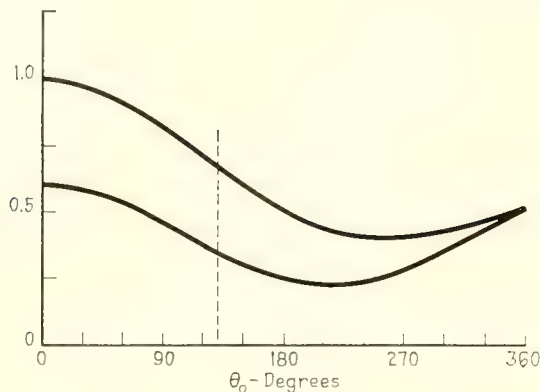


Fig. 16 Beam factor of $f(\theta_0)$ relating beam width θ_0 to form drag. Upper curve: high-frequency spectrum; lower curve: Neumann spectrum (Section 6.2). Beam width of a tradewind sea is 130 (from Munk, 1955)

$$\begin{aligned} \tau &= s\rho'g^2U^{-2} \\ &\times \int_0^\infty \int_{-\pi}^\pi \cos \Theta (\cos \Theta - \beta)\beta^{-4} S(\beta, \Theta) d\beta d\Theta \quad (47) \\ &= Sb f(\Theta_0) (\text{const}) U^3 \end{aligned}$$

where

$$f(\Theta_0) = \frac{1}{2} \left(1 + \frac{\sin \Theta_0}{\Theta_0} \right) - \left(\frac{2}{\pi} \right)^{1/2} \frac{\sin^{1/2} \Theta_0}{\Theta_0}$$

Quoting from Munk (1955a): "In Section (above) we have dealt with the case of a 'fully-arisen sea' (Neumann 1953) where it is assumed that all spectral components have attained their equilibrium value. If the wind fetch or duration is small, the effect will be that only the high frequencies (small β) have attained their equilibrium values. For prediction purposes Neumann introduces what is essentially an abrupt high-pass filter. This is equivalent to specifying some critical wave age, say β_m , which depends on wind speed, fetch, and duration. The spectrum is presumed to be fully developed for $\beta < \beta_m$ and zero for $\beta > \beta_m$.

"Suppose now that $\beta_m = 0.5$. It will be seen from Fig. 17 that the mean-square elevation is a very small fraction of the equilibrium value. The mean-square slope has more than half its equilibrium value. The form drag, however, has nearly its equilibrium value. The form drag is therefore much less affected by limitations in fetch and duration than the wave amplitude . . .

"The opposite example, that of a low-pass filter, is provided by the action of oil or detergent spread on the water surface. Suppose for the moment that the spectrum, for β_m less than 0.5, is dissipated by the surface-active agent. This would leave the mean-square elevation virtually unchanged, reduce the mean-square slope by a large factor and essentially eliminate the form drag. In general this corresponds to what is observed . . ."

Figs. 16, 17 and the foregoing quotation probably

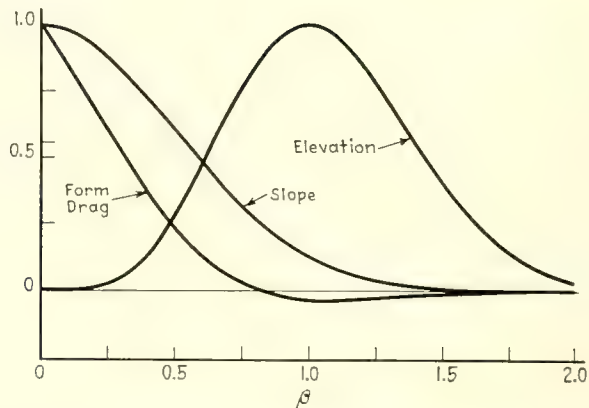


Fig. 17 Contributions to mean-square elevation, mean-square slope, and form drag by waves of different wave age β (from Munk, 1955)

represent the most important results of Munk's (1955) work. Together with the work of Cox and Munk (1954) and Van Dorn (1953) it demonstrates the predominating importance of the small high-frequency waves or ripples in the transmission of energy from wind to water.

An equally important conclusion is that wave slopes are much more significant for the wind stress than wave elevations.

The conclusion, from equations (45), (46) and (47), that form drag is proportional to U^3 is in direct contradiction to Neumann (1948, 1949a, b) who shows the force to be proportional to $U^{1.5}$. In either case the basic relationship is that the force is proportional to U^2 times a certain drag coefficient. Neumann finds this coefficient to be proportional to $U^{-0.5}$, and Munk to σ^2 or U . Conceivably this discrepancy may have been caused by environmental conditions. Neumann's observations were made in open seas. Cox and Munk's observations, on the other hand, apparently were made in an area surrounded by islands. The fetch may have been too short for a fully arisen sea to develop at higher wind velocities, and the waves therefore may have been steeper than they would be in an open ocean. This situation appears to be indicated by the c/U and λ/H ratios listed in Table 5. It is possible, therefore, that the proportionality of the drag τ to U^3 is valid for a "young sea," but will not necessarily apply to the more developed sea in an open ocean.

Theoretically, the proportionality of the drag coefficient, or C_d , to the wind velocity U was indicated in equations (45), (46) and (47) on the assumption of a constant value for the pressure coefficient s . However Jeffreys' expression (37) setting the air pressure per unit area proportional to the wave surface slope $\partial\eta/\partial x$ does not correspond to physical facts. The coefficient s is in reality a variable dependent on the wave form, which in turn depends on the wind velocity. The equations may therefore be misleading in showing U^3 conspicuously; the product sU^3 may be a different function of the wind velocity.

Jeffreys' sheltering coefficient s is related to the form-drag coefficient C_d by the relationship

$$C_d = 2sa^2k^2 \quad (48)$$

The drag coefficient C_d has been defined by (14) in Section 2.1, following aeronautical practice, merely as an empirical coefficient to be obtained experimentally for each body (or wave) form. The coefficient s is also found at the end to be purely empirical, but is defined in terms of an erroneously assumed relationship between air pressure p and wave slope $\partial\eta/\partial x$. The important result of Motzfeld's experiments (Section 2.3 and Table 1) is that C_d is very low and can be assumed nearly constant for smooth waves with rounded crests, but suddenly increases several times in magnitude with the occurrence of sharp wave crests. It appears, therefore, that the drag coefficient C_d should be defined in terms of the frequency of occurrence of sharp crests. It is possible that with this frequency of occurrence increasing with wind velocity, C_d will be found a smooth function of U , but not necessarily proportional to σ^2 or to U . Neumann pointed out that small waves are damped out by turbulence after breaking of the crests of larger ones. The percentage of the sea surface covered by small sharp-crested waves may therefore be reduced in a strong wind, bringing about a reduction of C_d .

While Munk (1955) has introduced the important concept of "facet velocity" he uses only one symbol, C , in his work and does not distinguish clearly between the celerity c of a harmonic wave, C of the facet velocity and \bar{c} of the celerity of significant waves.¹⁴ In deriving expression (45), for instance, the celerity c of a small ripple, assumed to be harmonic, may be negligibly small. The facet velocity C , however, may be much larger than the small ripple celerity c ; it appears to be identifiable with the "formula velocity" V_c of Keuligan and Van Dorn. The analysis of Van Dorn's data in Section 2.5 has indicated that in his case V_c was of the order of $0.6U$.

The factor $(U - c)^2$ in the expression for air pressure in the case of irregular waves should clearly depend on the facet velocity C . In expression (43) Munk used the one symbol C not distinguishing between c and C . In reproducing the expression here it was interpreted as $c(k)$, since the entire formula represents a summation of the effects of simple harmonic components; C itself is a function of the superposition of many harmonic wave profiles and therefore the expression $C(k)$ would have no meaning. On the other hand, the factor $[U \cos \Theta - c(k)]$ also cannot be accepted as valid without further discussion. $(U - C)$ has the well-defined physical meaning of relative velocity of the wind with respect to the moving water-surface facet. The expression $[U - c(k)]$ has, however, no such meaning, since $c(k)$ is merely the result of a mathematical analysis, and not a physically visible entity. With $c(k)$ negligibly small in comparison to U for small ripples and $C = 0.6U$, use of the expression $U[U - c(k)]$ in place of $(U - C)^2$ appears to

have introduced something like a six-fold exaggeration of τ . It would appear that statistical work should be directed primarily to the evaluation of the corresponding pairs of values of the facet velocity C and the wave slope $\partial\eta/\partial x$, for subsequent use with the relationship $(U - C)^2$.

Munk's (1955a) work has been directed to evaluating wind stress or form drag of waves. In application to energy transfer from wind to waves, the wind stress has to be multiplied by the facet velocity; i.e., in a simple form,

$$E = \langle K(V - C)^2 C \rangle \quad (49)$$

This indicates the extreme importance of developing methods of evaluating the facet velocity C for the realistic irregular sea.¹⁵ The coefficient K in (49) also depends on the wave form, and K and C are therefore interrelated.

4.5 Miles' Theory. Following the earlier work of Jeffreys (1925, 1926) and Munk (1955a), Miles (1957) considered the transfer of energy from wind to water waves as caused by the air pressures resulting from the wave form. In defining these pressures, however, he considered the properties of the thick turbulent boundary layer in proximity to the sea surface. A slightly viscous air was assumed so that the velocity gradient $u(z)$ had formed,¹⁶ while the analysis based on the velocity potential and stream function could, nevertheless, be applied.

Under the assumption of a two-dimensional sinusoidal wave, the pressures acting on the water surface were defined in terms of the wave slope $k\eta$ as

$$p = (\alpha + i\beta)\rho'U_1^2k\eta \quad (50)$$

where $k\eta$ is the local slope of the waves, $\eta = ae^{ik(x-ct)}$ and the real parts of complex quantities are implied in the final interpretation.

The vectorial form of the nondimensional pressure coefficient $(\alpha + i\beta)$ represents a sum of pressure components in and out of phase with the wave. As was shown in Section 2.1, only the pressure component out of phase with η contributes to the energy transfer. Nothing more need be said, therefore, about the coefficient α and the primary problem is a rational evaluation of the coefficient β .

In the elementary discussions in previous sections of this monograph, the pressure was related to $(U - c)^2$.¹⁷ Miles, however, like Phillips, treats the "reference velocity" U_1 as unknown at first. His paper, therefore, has two objectives; evaluation of both the velocity U_1 and the coefficient β . The coefficient β , while nondimensional, is not a constant but is a function of the wave celerity c and the wave number $k = 2\pi/\lambda = g/c^2$.

¹⁵ Valuable material in this connection can now be found in two papers by Longuet-Higgins (1956, 1957).

¹⁶ See Fig. 13; also refer to Section 2.7 for the elements of boundary-layer properties and the definition of the roughness parameter z_0 .

¹⁷ The symbol U refers to the air velocity $u(z)$ at an arbitrary elevation z at which an anemometer is located.

¹⁴ The use of the symbol \bar{c} is limited to this section only.

A crucial step in the analysis leads to definition of the critical elevation z_c at which the air velocity $u(z)$ is equal to the wave celerity c ; i.e. $u(z) - c = 0$. This step is glossed over quickly with a definition of an appropriate Reynolds number and with reference to Lin.¹⁸ The principal result is that the *energy transfer* (from wind to waves) is *proportional to the curvature of the velocity profile at that point in the profile where the mean air speed is equal to the wave speed*. It follows that (quoting Miles):

“(1) Only those waves having speeds in that range of the wind profile for which $-d^2u/dz^2$ is large may be expected to grow; the lower limit for (the celerity) c may be imposed by the existence of a sub-layer of linear profile or by the interaction of the waves with the wind profile, while the upper limit will be rather less than the wind speed outside of the boundary layer;

“(2) in the *initial* phases of wave formation, those waves having speeds well down in the profile (large $-d^2u/dz^2$) may be expected to predominate;

“(3) experimental measurements of aerodynamic forces on stationary wave models (Stanton, et alia 1932; Motzfeld 1937; Thijsse 1951) may not yield significant values of such parameters as Jeffreys’ sheltering coefficient, since the point at which $u(z) = c$ then occurs right at the boundary.”

The reference speed U_1 is defined by the relationship

$$c = U_1 \log_e (z/z_0) \quad (51)$$

In application to Roll’s observations with the “anemometer height” $z = 2$ m (6.5 ft), Miles indicates that

$$U_1 = U/10$$

The pressure coefficient β is expressed as an integral which is evaluated by a serial expansion in terms of the ratio z_c/λ . The maximum value of β occurs at $z_c = 0.006 \lambda$. At $z_c = 0.05 \lambda$, β drops to about $1/5$ of its maximum value, and it practically vanishes at $z_c = 0.16 \lambda$.

The pressure coefficient β and the energy transmission from the wind drop to nil at c/U_1 between 9 and 11, depending on the values of z_0 and U_1 . Thus, in the absence of energy dissipation the waves are expected to cease growing when the wave celerity approaches the wind velocity.

Miles made two attempts at verification of his theory. In the first, he treated the case of wave initiation with dissipation of energy by molecular viscosity. Taking the probable value of z_0 , he obtained a minimum wind velocity of 80 to 100 cm/sec with a corresponding wave celerity of 40–50 cm/sec. These figures are in agreement with Jeffreys’ (1925, 1926) observations.

In another attempt, by superposition of his simple wave results he calculated the drag coefficient of a complex sea based on Neumann’s spectrum in the form used by Munk (1955). The computed coefficients were approximately double those of Munk. Miles strongly emphasized that mathematical approximations on one hand and uncertainty in estimating the roughness parameter

z_0 on the other might make the results off by a factor of 2 either way. It is interesting to recollect, however, that Jeffreys (1925, 1926) has shown that the drag coefficient for a short-crested sea is half that of a regular long-crested sea. The drag coefficient of bodies in three-dimensional flow is generally lower than that in two-dimensional flow, the simplest illustrative case being that of a sphere and a cylinder. This may well explain the factor of 2 in comparing Miles’ and Munk’s results.

Miles’ theory is based on the action of pressure; i.e., of the normal force, on the wave surface. It considers, therefore, the transfer of the energy of potential air flow to potential wave energy. The observed data on the roughness parameters z_0 , on the other hand, include the total tangential drag; i.e., the pressure drag plus frictional drag. In mild waves these two drag components may be of equal magnitude (see Motzfeld’s Table 1). The computed coefficient β and the rate of energy transfer in such a case should be approximately double the true ones.

Another important aspect to mention is that Miles sought to avoid the limitation to small wave heights by measuring elevations z not from a level surface but from a wavy streamline. This procedure is equivalent to assuming that the entire velocity profile $u(z)$ shifts up and down with the wave elevation without change of form. This may be an admissible assumption in the case of long and low waves (small H/λ ratio), to which Miles’ results should, therefore, be limited.

In the case of steeper waves, the $u(z)$ -curve appears to follow the logarithmic law at sufficient elevation above the wave but at lower z it varies with position along the wave profile. In particular, it is drastically changed at wave crests. This should have a large effect on the pressure distribution (and therefore on the drag and energy transfer), since Miles demonstrated how small the z_c is in comparison with the wave length when the pressure coefficient is significant. An extension of Miles’ work to take into account this effect would make an interesting and important project.

5 Growth of Waves in Wind—Practical Approach

The words “rational” and “practical” are used in this work conditionally for convenience of reference. The difference is in the degree to which empirical information and hydrodynamic theory are used. In the “rational” approach empirical data are limited to form-drag coefficients in simple waves and to the observed statistical distribution of some quantity, say wave directions, in the actual sea. The effect or the mutual interrelation of all other variables is then obtained by observing the laws of mechanics and hydromechanics. It is necessary to pursue this approach in order to gain complete understanding of the nature of the seaway in all its details. However, this development progresses slowly, and for immediate practical use many short cuts become necessary. In these, theoretical reasoning and empirically obtained data are intermixed in various forms and proportions

¹⁸ Lin, C. C., “The Theory of Hydrodynamic Stability,” Cambridge University Press, 1955, chapter 5.

chosen intuitively by the individual researcher. In particular, the effects of several separate parameters are usually combined in groups; it has been found that functional relationships between judiciously selected groups of parameters can be evaluated empirically with much greater reliability than relationships between individual parameters. In this connection, the shortcomings of the partially developed theoretical reasoning are implicitly compensated for in establishing empirical relationships. The bulk of this kind of information has been accumulated since 1943, and had its incentive in the need for predicting surf conditions at European invasion beaches in World War II. The problems of wave formation in a storm area and of wave decay or dispersion outside of the storm area had therefore to be treated. The objective was to arrive at the conditions existing at a beach at a given time due to a storm or several storms which had occurred some time earlier, often at a distance many hundreds of miles away. In the present text only the wave conditions in or near the storm area will be discussed. Only a brief outline of the most important methods used to obtain information on these conditions will be given. Details will be found in the readily available references.

5.1 Method of Sverdrup and Munk (1946, 1947). Quoting from Sverdrup and Munk (1947): "Within the generating area there always exist a large number of such trains of waves of different length, traveling with the wind or at small angles with the wind direction. From interference and criss-crossing there results an extremely irregular appearance of the sea surface, but the larger waves can be recognized and the theoretical relationships between period, length, and velocity apply to these (Krümmel 1911, Sverdrup, et al, 1942).

"Because of the simultaneous presence of many trains, the wave characteristics have to be described by some statistical terms. For that purpose it has been found convenient to introduce the average height and period of the one-third highest waves. The waves defined in this manner are called 'the significant waves,' but the definition requires further refinement because the composition of the 'one-third highest waves' depends on the extent to which the lower waves have been considered. Experience so far indicates that a careful observer who attempts to establish the character of the higher waves will record values which approximately fit the definition. . . ."¹⁹

The "significant waves" are assumed to have all the properties of simple waves of finite height (Stokes' waves) except that their length, height and energy content do not remain constant as is the case with simple waves. These quantities increase with the time and the distance over which the energy of the wind is transmitted to waves. The details of this process of wave growth are not known, but certain energy relationships can be established. In this connection, Sverdrup and Munk

make the following observation: ". . . When a wind of constant velocity has blown for a long time over a limited stretch of water, such as a lake, a steady state is established. At any fixed locality the significant waves do not change with time, but on the downwind side of the lake they are higher and longer than on the upwind side. If on the other hand a uniform wind blows over a wide ocean, waves grow just as fast in one region as in any other region and the significant waves change with time but do not vary in a horizontal direction."

The energy balance is then written for these conditions by equating energy growth to energy received from wind. The dissipation of energy by molecular or turbulent viscosity is neglected. The energy growth is represented by the growth of wave height, length, and celerity all of which are taken as functions of time t and distance x along the fetch. The energy is transmitted from wind to waves by the work of the normal pressure times the vertical water surface velocity

$$E_p = \frac{1}{\lambda} \int_0^\lambda pw \, dx \quad (52)$$

where λ is the wave length and w is the vertical velocity of the water surface. Following Jeffreys, the out-of-phase component of pressure is written as

$$p = \pm s\rho' (U - c)^2 \partial\eta/\partial x \quad (53)$$

This equation indicates wave growth for $U > c$, and decay for $U < c$. To reconcile this with the fact that waves are often observed to travel faster than wind, the energy transfer by the frictional drag force is introduced

$$E_\tau = \frac{1}{\lambda} \int_0^\lambda \tau U \, dx \quad (54)$$

where u denotes the horizontal component of water-particle velocity (orbital and drift velocity) at the sea surface. τ is the stress which wind exerts on the sea surface, and is evaluated as

$$\tau = \gamma\rho'U^2 \quad (55)$$

where for U measured at a height of 8 to 10 m, the coefficient γ is taken as 0.0026. Owing to the symmetry of an harmonic wave the integral in (54) vanishes, but for Stokes' waves of finite height, a characteristic of which is mass transport (see Appendix A, Section 3.3), the integral has a finite value. Since the water-particle velocity is low in comparison with wind velocity, energy is shown to be transmitted to water, and waves can grow even if $U < c$.²⁰

The energy balance equations are now written:

For the transient state,

$$\frac{dE}{dt} + \frac{Edc}{cdt} = E_\tau \pm E_p \quad (56)$$

and for steady state,

¹⁹ For a comparison of instrumental and visual observations see Roll (1955).

²⁰ It has been found that small wave components or ripples are instrumental in absorbing the energy from the wind, making the assumption of skin friction unnecessary as has been discussed in previous sections.

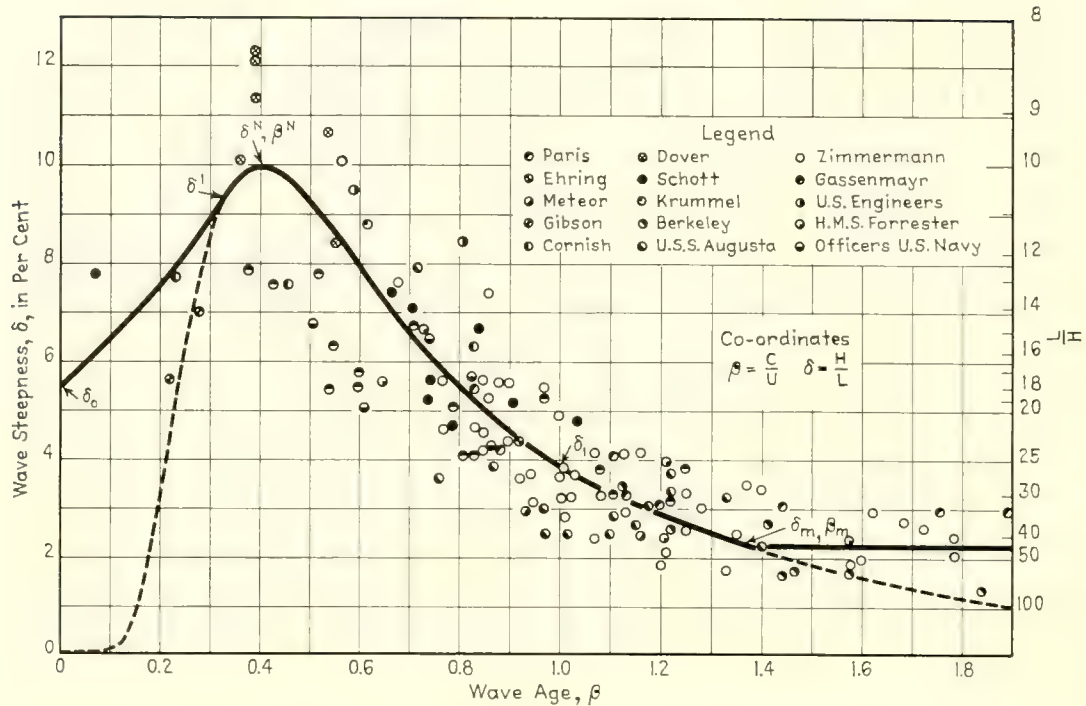


Fig. 18 Relation between wave steepness and wave age. Observed values shown by symbols, assumed relationship shown by full drawn line (from Sverdrup and Munk, 1947)

$$\frac{c}{2} \frac{dE}{dx} + \frac{Edc}{2dv} = E_T \pm E_P \quad (57)$$

Each of these independent equations contains two unknowns, E and c . The solution is made possible by introducing an assumed functional relationship

$$H/\lambda = f(c/U) \quad (58)$$

Sverdrup and Munk (1947) collected many observations and plotted $\delta = H/\lambda$ versus $\beta = c/U$. The first quantity was called "wave steepness" and the second "wave age," and these terms have since been adopted for common usage. The resultant plot is shown in Fig. 18. By making certain assumptions as to subdivision of energy in the growth of wave height and wave length, a curve was obtained, which fits the observed data very well. At the time the original plot was made, very few data for $\beta < 0.4$ were available. These were obtained later mostly by Roll (1951) and Francis (1951). Fig. 19 shows a plot taken from Neumann (1952b). For many years in the past, attempts to relate wave height directly to wind velocity were unsuccessful; a wide variance existed in the published literature. The parameters H/λ and c/U on the other hand, fell readily into a well-defined pattern for $\beta > 0.4$. The relationship below this value remains uncertain.

Having established empirically the relationship $\delta = f(\beta)$, equations (56) and (57) were solved and the results were plotted in the form of nondimensional ratios c/U and gH/U^2 versus gF/U^2 for the steady state, and versus

gt/U for the transient state. Fig. 20 taken from Johnson and Rice (1952) shows the steady-state relationships as developed further by Bretschneider from data which became available subsequent to the original work of Sverdrup and Munk. The second plot taken from the latter is shown in Fig. 21. These plots are of very practical form and have been widely used for prediction of wave conditions on the basis of meteorological data. For a discussion of wave decay outside the generating area the reader is referred to the original papers of Sverdrup and Munk (1947) and Bretschneider (1952).

5.2 Neumann's Work on Wave Generation. In this section the work of Neumann on wave generation (1948, 1949a, b, 1950, 1952a, b) will be outlined. He has treated the matter in two different ways, considering the significant wave as did Sverdrup and Munk (1946, 1947), or taking the sea as composed of three predominating wave systems. Neumann's formulation of the continuous sea spectrum (1953, 1954, and Pierson, Neumann and James, H) will be the subject of Section 6.2.

Neumann's work on wave generation is outstanding because:

a) The experimental material, taken from previous observations of other oceanographers and obtained by Neumann himself, covers a very wide range of wind and sea conditions and geographic locations. (All observations were visual.)

b) A determined attempt was made in the analysis to adhere as closely as possible to rational procedure.

c) The mathematical and statistical formulations are

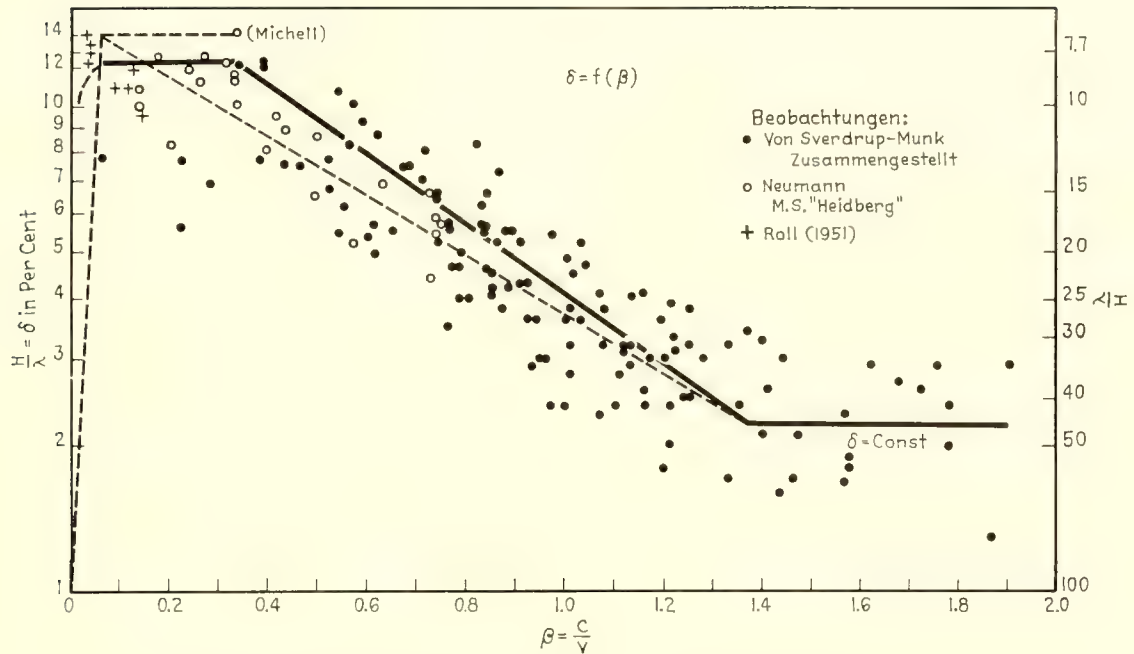


Fig. 19 Wave steepness δ as function of $\beta = c/v$ (from Neumann, 1952b)

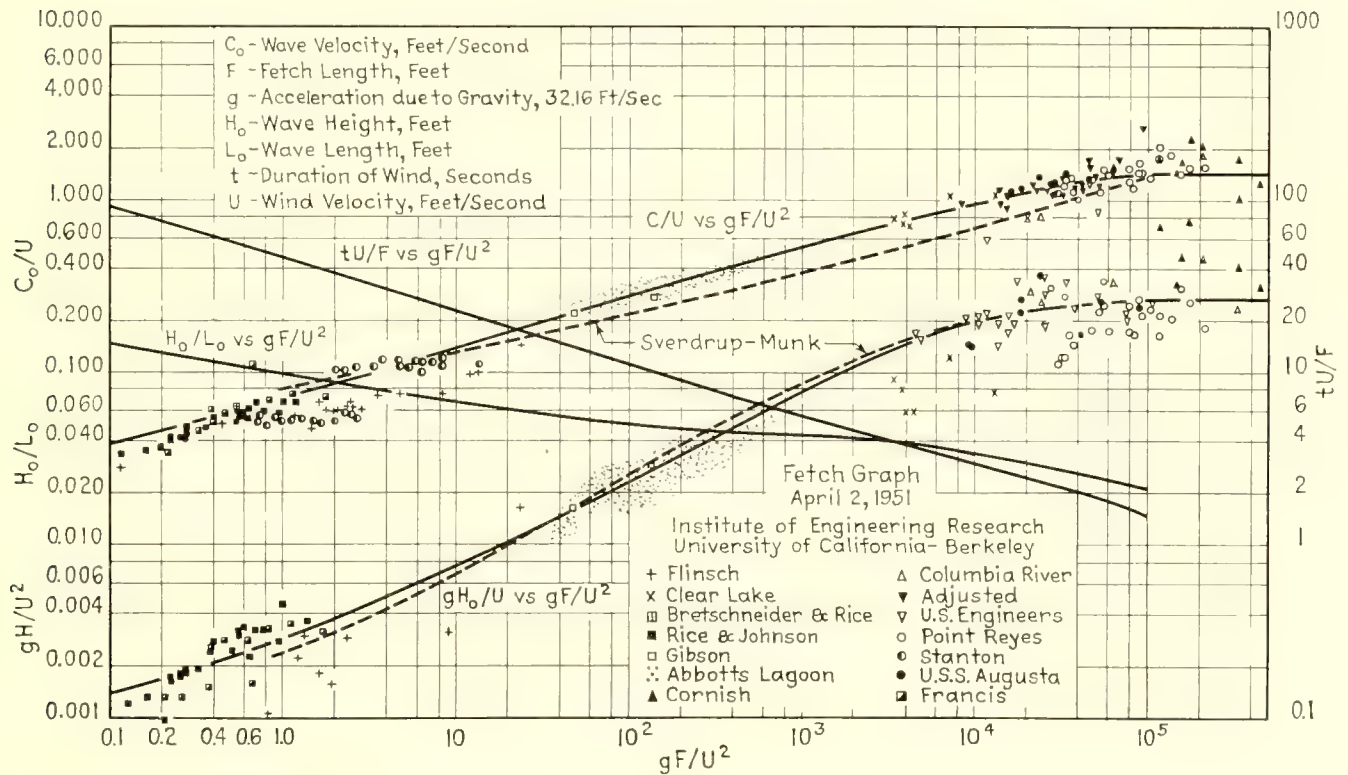


Fig. 20 Relationship between wind speed and duration, wave height and period, and fetch; see Bretschneider (1952) for references (from Johnson and Rice, 1952)

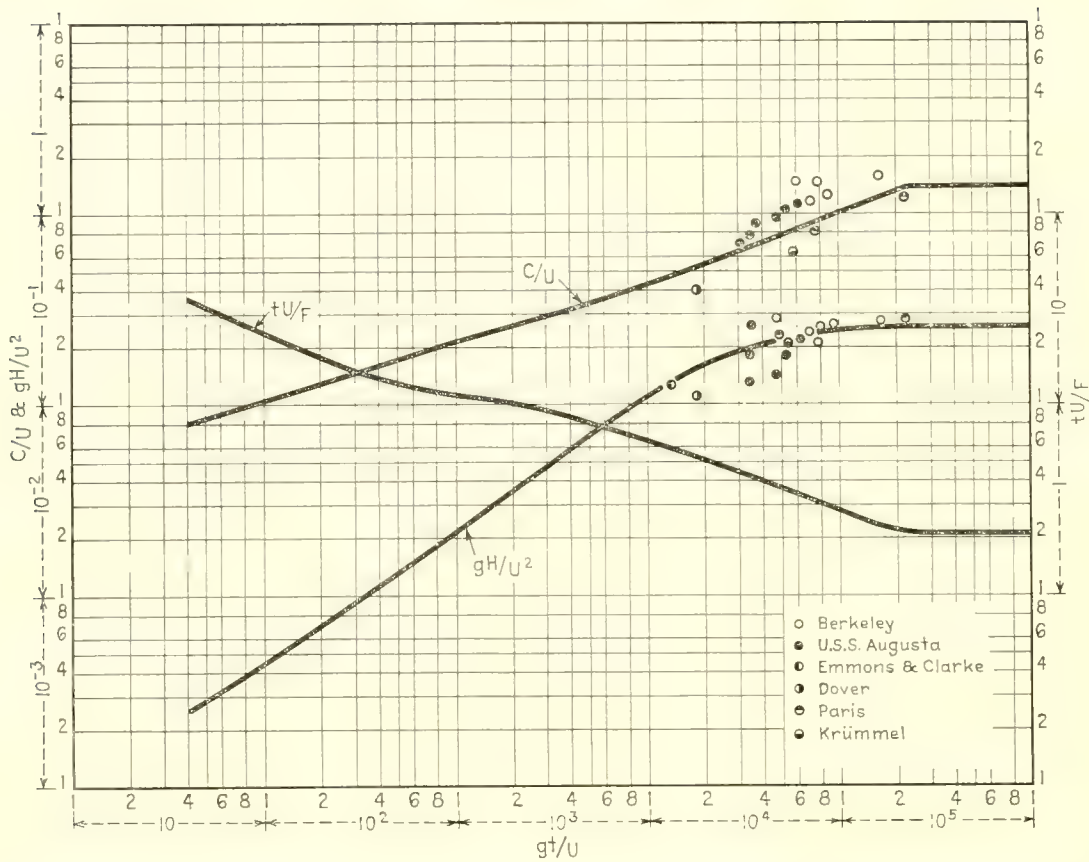


Fig. 21 Wave height and velocity as functions of duration, using nondimensional parameters and relation between minimum duration, fetch, and wind velocity. Theoretical relationships shown by curves; observations by symbols (from Sverdrup and Munk, 1947)

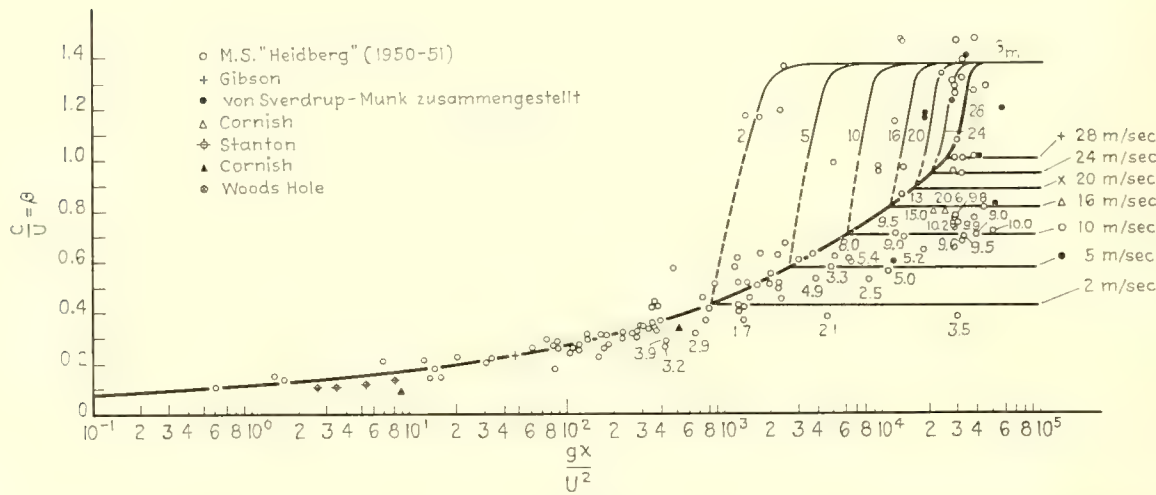


Fig. 22 Wave celerity c of characteristic waves in complex seas as function of fetch x and wind velocity U in nondimensional presentation (from Neumann, 1952b)

accompanied by extensive descriptions of the physical sea.

nificant wave or of a composite of three distinct waves is certainly obsolete. In spite of this, the results of this development are currently used in the wave-forecasting

However a development based on the idea of a sig-

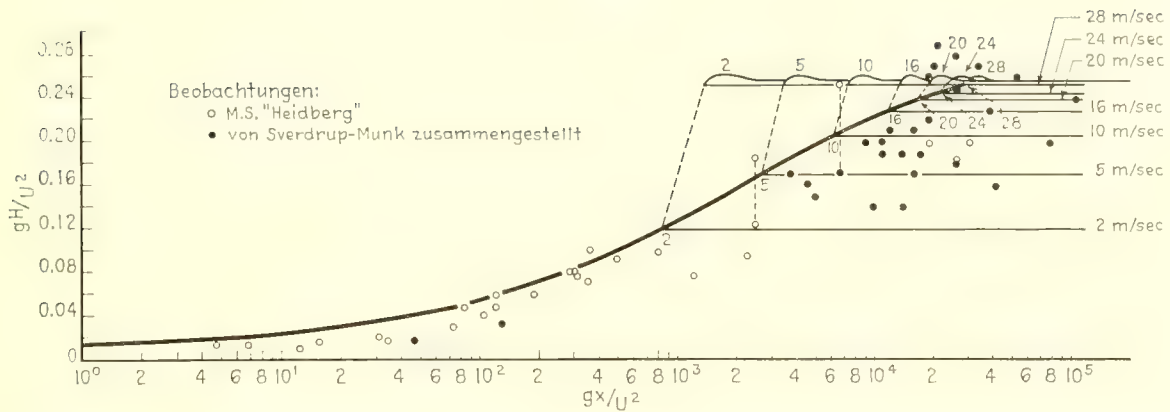


Fig. 23 Height H of characteristic waves in complex seas as function of fetch x and the wind velocity U in non-dimensional presentation (from Neumann, 1952b)

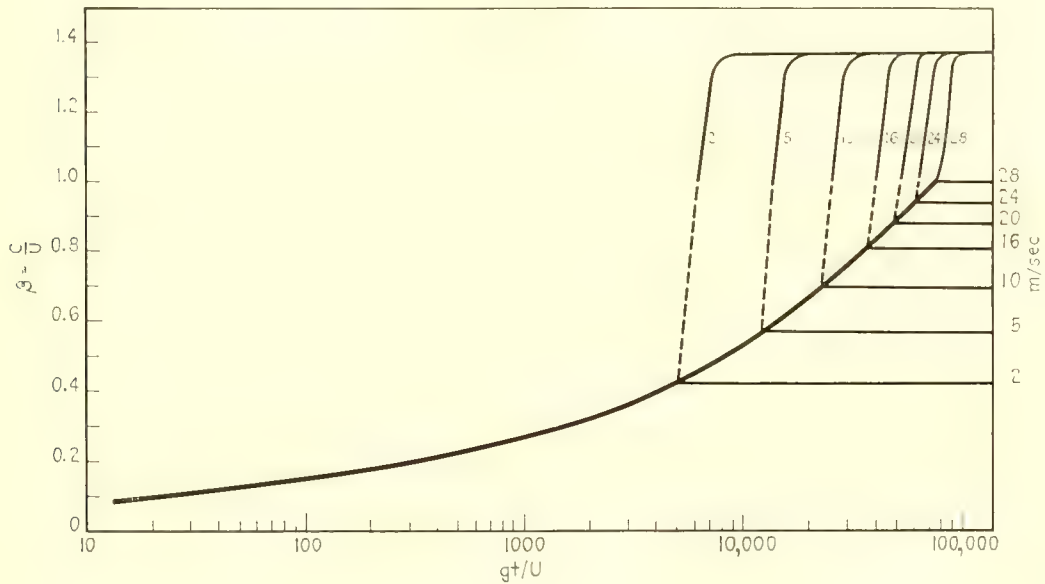


Fig. 24 Wave celerity c of characteristic waves as function of wind duration t (from Neumann, 1952b)

method of Pierson, Neumann and James (H). The large amount of supporting material and the thorough discussion of the tangential wind force and of energy dissipation will be of importance to any new investigator. Neumann's vivid descriptions of the physical sea characteristics also will be of value to investigators who did not have the opportunity to gather this experience themselves. To this author, the broad point of view and the thoroughness in considering the entire process of wave growth under the influence of both the energy transmitted from the wind and the energy dissipated in waves makes Neumann's work unique.

The bulkiness and complexity of Neumann's (1948–1952) work do not permit inclusion in the monograph text. A comprehensive summary has been made in Ap-

pendix B. Only the final results will be stated here. These are shown in Figs. 22 to 25. Fig. 22 shows the wave celerity c (and therefore length λ) versus the fetch F for any wind velocity in terms of nondimensional coefficients c/U and gF/U^2 . Fig. 23 shows the wave height H versus the fetch, in terms of nondimensional parameters gH/U^2 and gF/U^2 . Figs. 24 and 25 give the wave celerity and height as a function of the wind duration t . The curves were computed by Neumann, and the points show various observed data.

6 Wave Spectra

Efforts to predict surf conditions during World War II showed the inadequacy of visual wave observations. A

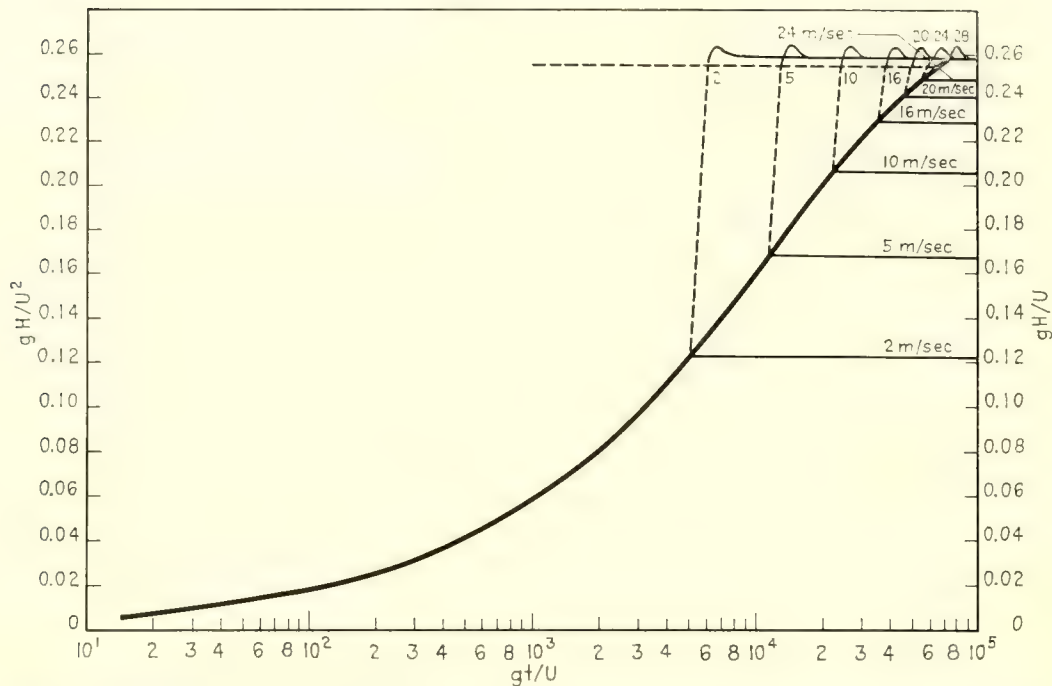


Fig. 25 Height H of characteristic waves as function of wind duration t (from Neumann, 1952*b*)

number of pressure gages were installed on the sea bottom near the shores of England and continuous recordings of the wave-caused pressure fluctuations were obtained. Visual examinations of these did not yield satisfactory results, and, therefore, a frequency analyzer was constructed (Barber, Ursell, Darbyshire and Tucker, 1946). Barber and Ursell (1948) presented a theory for tracing various Fourier components of the recorded swell to their origin, often many hundreds of miles away. In 1952 Darbyshire used such recordings and analyses in formulating the shape of the spectrum and in estimating the wave heights caused by the wind. The mathematical details of the spectral analysis will be presented in Section 8. At present, quotations from Darbyshire and Neumann in the next two sections will suffice. Several formulations of the spectra have been developed since 1952, and these will be described briefly in the following sections.

6.1 Darbyshire's Wave Spectra. Quoting from Darbyshire (1952): "This paper describes an investigation of the height and length of ocean waves and swell in relation to the strength, extent and duration of the wind in the generating area, and the subsequent travel of the swell through calm and disturbed water. The investigation is based on records of waves made on the North Coast of Cornwall, in the Irish Sea, and in Lough Neagh. It is a practical continuation of the work of Barber and Ursell (1948), who showed that the waves leaving the generating area behave as a continuous spectrum of component wave trains which travel inde-

pendently with the group velocities appropriate to their periods. The spectral distribution of energy in the storm area is considered, and the relative amplitudes of the different components are deduced empirically under various wind conditions . . ." "A method of deriving the wave spectrum from a wave record is described by Barber, Ursell, Darbyshire and Tucker (1946)." . . . "The method of analysis . . . gives a Fourier analysis of a 20 to 30-minute record; the analysis appears in the form of a series of peaks, each corresponding to a harmonic component which is an exact submultiple of the total length of the record. An example of such an analysis containing waves due to a local storm and a band of swell from a distant storm is shown in Fig. 26. While it cannot be implied that these discrete periodicities are actually present in the sea waves, it is possible, for the duration of the record, to represent the pressure variations at the point of measurement by a combination of independent sine waves with periods which are submultiples of the duration of the record and with amplitudes proportional to the heights of the peaks on the spectrum.

"The state of the sea can best be described in terms of the wave energy. Assuming that for the duration of the record, the wave pattern consists of a combination of independent sine waves with periods and amplitudes corresponding to those of the peaks on the spectrum, it is possible to evaluate the wave energy. Since the energy per unit area of a single sine wave of height H is $\rho g H^2/8$, the total energy for all the waves in the spectrum $= g \rho \Sigma H_n^2/8$ where H_n corresponds to the height of the

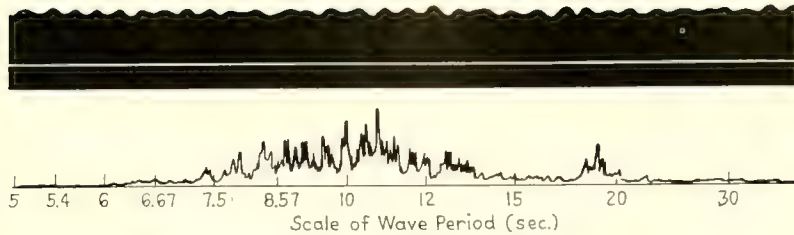


Fig. 26 Typical wave record and spectrum (from Darbyshire, 1952)

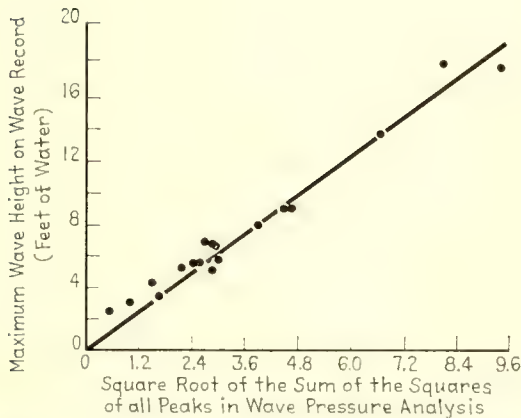


Fig. 27 Comparison between maximum wave height and equivalent wave height (from Darbyshire, 1952)

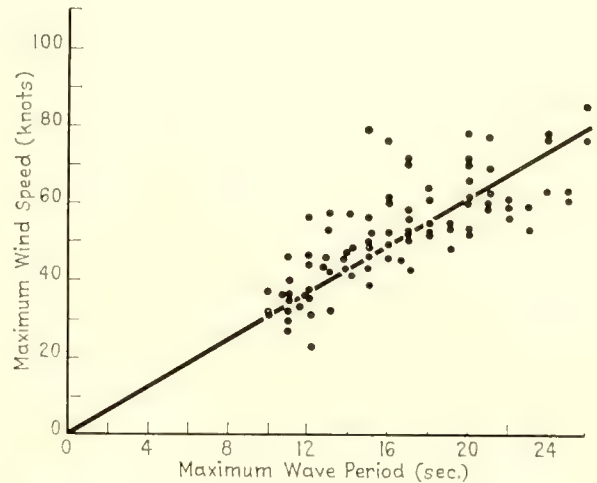


Fig. 28 Correlation of maximum wave period with maximum wind strength for all storms (from Darbyshire, 1952)

n th peak in the spectrum. If H is the height of a hypothetical single sine-wave train which has the same energy per unit area as the complicated wave pattern, then $g\rho H^2/8 = g\rho \sum H_n^2/8$ and H can be defined as the equivalent height of waves. To compare such equivalent height with the maximum heights measured on wave records, corresponding values of H and maximum height H_{\max} are plotted in Fig. 27. The graph shows that $H_{\max} = 2H$.

"The idea of an equivalent height can be extended to parts of a wave spectrum as well as the whole. The energy E_T in a unit wave-period interval $T - (1/2)$ to $T + (1/2)$ is given by

$$E_T = \frac{1}{8} g\rho \sum_{T-1/2}^{T+1/2} H_n^2 = \frac{1}{8} g\rho H_T^2 \quad (59)$$

"The information available consists mostly of synoptic meteorological charts and records of wave motion usually in the form of records of the pressure fluctuations produced by the waves on the sea bed at a depth of about 50 feet at one point on the coast, . . ." "There are not usually sufficient observations of wind strength in the wave generating area to allow detailed comparisons between the wave and wind characteristics, and it was made a general practice to compute the wind from isobaric charts. Six hourly charts provided by the Naval Weather Service were found most convenient, and gra-

dient wind speeds were calculated according to the instructions in the Admiralty Weather Manual. The relation between surface wind and gradient wind varies with the atmospheric stability and other factors, but Gordon (1950), using observations over the sea, found that the ratio between the two varied between 0.60 and 0.80, the mean value being 0.66. The best that can be done at present is to assume a constant ratio of 0.66 and to compare wave characteristics with the gradient wind speeds. There is some advantage in doing this since in any application of this work it is more likely that weather charts will be available than wind observations."

From examination of the wave record, Darbyshire came to the conclusion that wave characteristics become practically independent of fetch after 200 to 300 miles.²¹ As the characteristic information to be abstracted from the records, Darbyshire chose maximum wind speed vs. maximum component wave period (see Fig. 28) and mean wind speed versus the wave component period cor-

²¹ Very much longer fetches are needed to attain steady wave conditions according to Sverdrup and Munk (1947) and Neumann (1952, 1953). The use of the chart in Fig. 20 indicates that in the gradient wind of 60 knots, corresponding to anemometer height wind of 40 knots, a fetch of about 700 nautical miles is required.

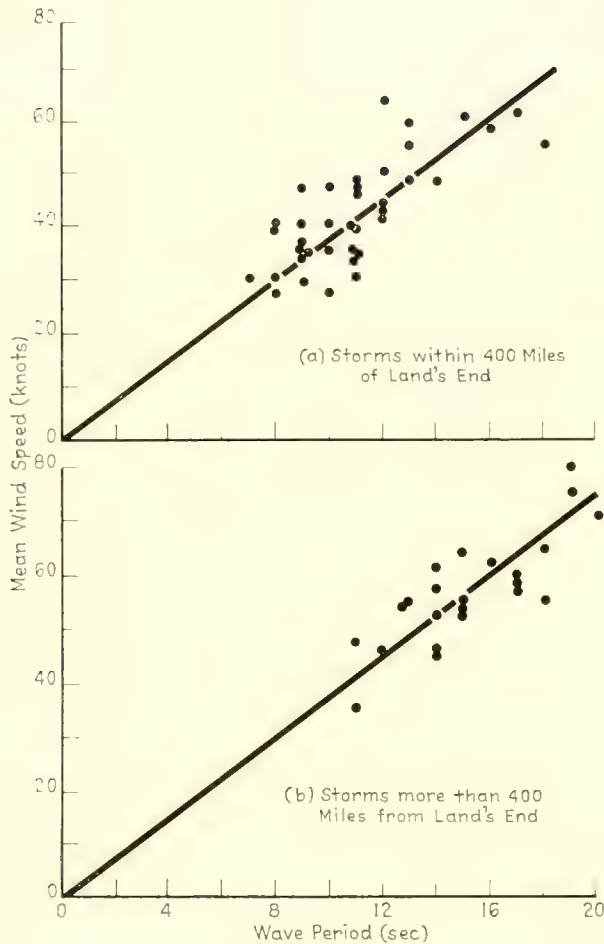


Fig. 29 Correlation of period of maximum amplitude with mean gradient wind speed in strongest part of storm (from Darbyshire, 1952)

responding to the maximum wave component amplitude (see Fig. 29). Implicit in this choice of characteristics is the assumption (quoting from Darbyshire, 1952): "... that in the development of a wave spectrum, the wave components all grow independently. The empirical formulae obtained for the spectral distribution of energy in the storm area imply that waves with periods covering a wide range grow together under the action of the wind..."²²

From Figs. 28 and 29 it follows that

$$\begin{aligned} (\max T)/V_{\max} &= 0.33 \\ (T \text{ for } \max H)/V_{\max} &= 0.25 \end{aligned} \quad (60)$$

The symbol V is used in (60) in the notation of the present monograph for gradient wind, which corresponds to undisturbed velocity in aerodynamics. The symbol

²² It does not appear reasonable that the waves of longest period would occur immediately at the onset of a wind. Neumann assumed that high-frequency waves appear first, and that the spectrum will develop from the high-frequency end. This appears to be confirmed by Ijima's observations (see section 6.4).

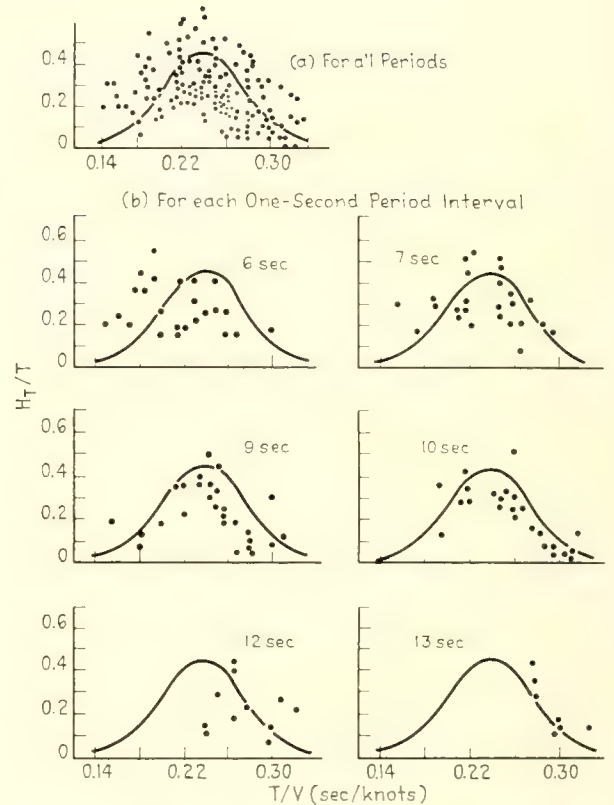


Fig. 30 Graphs of H_T/T against T/V (from Darbyshire, 1952)

U_1 was used in the previous sections for the wind velocity at "anemometer" or "mast-head" height. The gradient wind velocity V is in knots throughout Darbyshire's work.

Having obtained relationships (60) Darbyshire writes "these two relations . . . suggest that if H_T is plotted against T/V to give the envelope of the wave spectrum, the envelope should always have the same shape and an increase in wind speed should change only the vertical scale of the curve. This implies that the envelope can be expressed in the form

$$H_T = f(T/V) \quad (61)$$

where V is the mean wind speed and $f(T/V)$ is a function which is maximum at T/V (T sec, V knots) = 0.25 and nearly zero when $T/V = 0.33$. Assuming $H_T = V^n f(T/V)$, $H_T/T^n = (V/T)^n f(T/V)$ and the values obtained by dividing H_T by some power of the corresponding value of T should lie on a curve which is a function T/V . . . " "The values obtained by letting $n = 1$ gave the most consistent results. These values of H_T/T are shown plotted in Fig. 30." Despite a very large scatter, Darbyshire fits a curve of the form

$$y = Ke^x$$

where $K = 0.44$

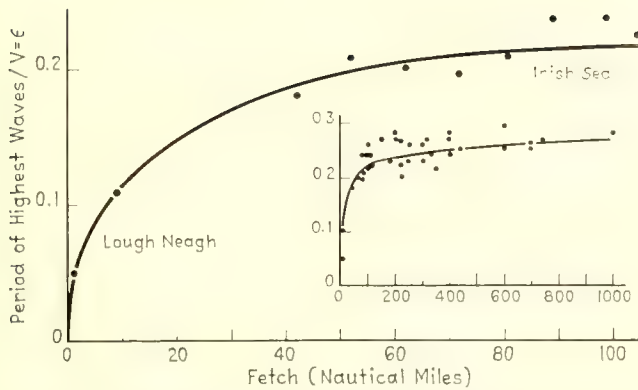


Fig. 31 Graph of ϵ against fetch (from Darbyshire, 1952)

$$y = H_T/T$$

$$x = - \frac{(T/V - 0.24)^2}{0.0027} \quad (62)$$

The resultant curve is shown by the solid lines in Fig. 30. This relationship and the plots of Fig. 30 were obtained by considering wave components of the same period under the action of winds of different speeds. The maximum value of H_T/T is 0.44 and it occurs at $T/V = 0.24$. At $T/V = 0.33$ the value of H_T is very small, only one fourteenth of the maximum value of H_T .

The equivalent wave height H is then found by inte-

Table 6

H_{mean}	Mean wave height (average of all waves)	1.0
H	Equivalent wave height (height of simple sine wave having same energy content as complicated wave pattern)	$1.2H_{\text{mean}}$
H_s	Significant wave height (average of one-third highest waves)	$1.6H_{\text{mean}}$
H_{max}	Maximum wave height (highest in 100 waves)	$2.4H_{\text{mean}}$ or $1.5H_s$

grating $H_T^2 dT$ from 0 to ∞ and taking the square root of the result, which gives

$$H = 0.027 V^{3/2} \quad (63)$$

where H is in feet, V in knots.

For a fetch shorter than 100 miles the correction of the foregoing is obtained from observations made on a lake and on the Irish Sea, yielding

$$H = 0.027 V^{3/2} (1 - e^{-0.23x^{1/2}}) \quad (64)$$

where x is the fetch in nautical miles. In addition a somewhat more complicated if more accurate expression is given. The variation of wave period with fetch is shown in Fig. 31.

On the basis of the previous work of Barber (1950), the relative values of H and of other forms of sea definition are given in Table 6.

According to Barber, approximately 13 per cent of the waves have heights greater than the significant height. Examination of wave records has shown that the signifi-

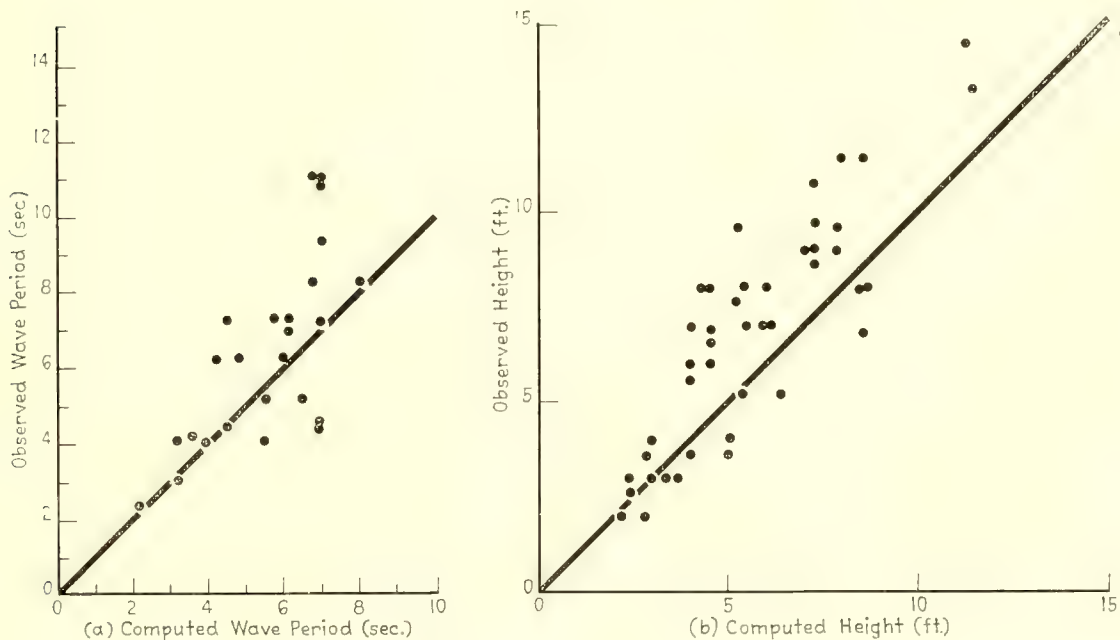


Fig. 32 (a) Comparison of computed and observed wave periods (Sverdrup and Munk's data). (b) Comparison of computed and observed wave heights (Sverdrup and Munk's data)

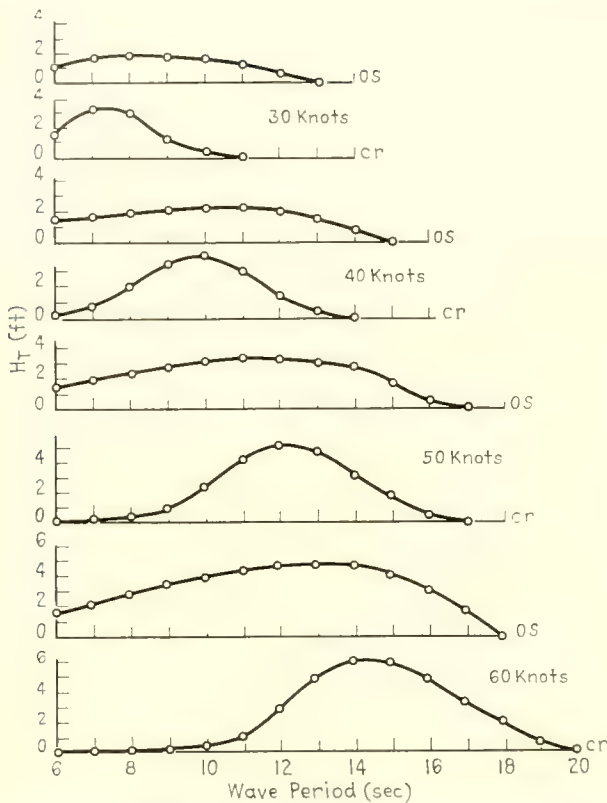


Fig. 33 Predicted wave spectra for various wind speeds for open sea and western coastal region (from Darbyshire, 1955b)

cant period corresponds very closely with the period of maximum amplitude of the analysis.

To obtain an independent check, the expressions for significant height and period have been applied to the wind and wave data listed by Sverdrup and Munk (1947) and the predicted and observed wave heights and periods are compared in Fig. 32. While the mean order of magnitude is correct, the excessive scatter of data makes it questionable whether the spectrum is correctly expressed.

Cox and Munk (1954) discuss the validity of Darbyshire's spectrum on the basis of the spectrum of wave slopes which they measured by means of the sun's glitter (see Section 4.3). Quoting from their work: "The spectrum was obtained by frequency analysis of pressure records of waves generated by local storms at Lands End, England. Only those storms were analyzed for which the fetch was sufficient to generate equilibrium waves. The bottom-pressure transducer was usually located at depth of 50 ft. This requires that for wave periods less than 14 sec the usual hydrodynamic relationship must be used to convert bottom pressure to surface elevation, as has been done by Darbyshire. But for periods less than 6 sec the bottom-pressure oscillations are reduced to such a low level that this is no longer practicable. Apparently these short waves have been

ignored by Darbyshire. But it will be shown that these waves contribute about five sixths of the total wave energy and that consequently Darbyshire's spectrum is too narrow and the total energy too small. A pressure recorder at 50 ft is evidently not a suitable instrument for obtaining the energy spectrum." Indeed the sample record in Fig. 26 shows no periods below 6 sec, in Fig. 28 below 9 sec, and in Fig. 29 (a) and (b) below 7 and 11 sec, respectively.

6.11 The Darbyshire (1955) spectrum. The work described in the previous section was based on the records of a pressure gage installed at a depth of about 50 ft off Land's End, England. In the subsequent period 1953–1955, Darbyshire collected a number of records from a shipborne wave recorder (Tucker, 1952b, 1956) which consisted of a pressure gage at a depth of about 10 ft and an integrating accelerometer for recording the heaving motions of the ship.

Quoting Darbyshire: "It is now possible to investigate wave generation in deep water, for since February 1953 waves have been recorded by the ocean weather ship *Weather Explorer* using shipborne wave recorder described by Tucker (1952). Records were taken eight times daily while the ship was at sea. Most of them when she was stationary. The ship occupied, in turn, the positions 'India' (61° 00' N, 15° 20' W) and 'Juliet' (52° 20' N, 20° 00' W). The sensitivity of the recorder is nearly constant for wave periods from 8 to 24s."

"The records taken by the *Weather Explorer* are of 7 to 10 min duration. . . . To avoid uncertainties about wave attenuation, the study was confined to waves generated by winds acting within about 100 miles of the ship's position, and a large number of records were analyzed to find forty-five which showed no evidence of extraneous swell from distant storms . . . The examples chosen covered a fairly wide range, in which the fetch (the distance the wind acts on waves) varied from 50 to 400 miles. The previous paper showed that the effect of fetch was not very marked after 100 miles, and this appears to be true for deep-sea storms also."

The same analytical method was used as before. A much larger content of high-frequency waves was found than before. This difference cannot be completely explained by the lesser depth of the pressure pick-up. It was concluded that the waves in the previous investigation were modified by the presence of the continental shelf.

The spectrum is now expressed as

$$H_T = 0.0036(V^{1/2} - 0.43T)^{5/2} \quad (65)$$

The corresponding period of the highest energy content is

$$T_{\max} = 1.64 V^{1/2} \quad (66)$$

"Equivalent" wave height is given as

$$H = 0.0038 V^2 \quad (67)$$

Fig. 33 shows the spectra for four wind speeds. The upper spectrum in each case is the one given above for the open ocean, and the second for the coastal region (Darbyshire, 1952). The wind speeds are for "gradient wind," and the "anemometer height" wind can be taken on an average as $\frac{2}{3}$ of it.

6.2 Neumann's Wave Spectrum. This section covers the spectrum of waves in a complex seaway as it was derived by Neumann (1953, 1954) and described by Pierson, Neumann, and James (H). The material is clearly presented in these readily available publications. Only a brief outline will be given here in order to make clear subsequent developments.

Quoting from Neumann (1953): "The wind never raises well defined wave trains with uniform heights, periods or wave lengths. When waves are being generated by wind, the energy supplied from wind to waves is distributed more and more over a certain range of wave lengths with different heights as the sea grows. The spectrum of ocean waves is being formed, with wave components ranging from ripples to large billowing waves in a storm sea. Therefore, it is extremely difficult to describe the wind-generated wave pattern by means of only two single parameters such as 'wave height' and 'wave period' of some kind of a fictitious single wave train. A general method of wave forecasting and describing the sea must embrace more than this . . . At the present stage of our knowledge it seems that further progress is only possible by taking the entire wave spectrum into account with the assumption of a continuous distribution of component wave trains."

"Since the wave energy per unit area of sea surface for a component wave with the period T is proportional to the square of the wave height h_T^2 , the spectral height distribution may be given by H_T^2 as a function of T , where $\Delta h_T^2 = H_T^2 \Delta T$.²³ Fig. 34 shows such a hypothetical wave spectrum by the dashed curve.

"Assume a continuous distribution of an infinite number of wave components in the complex wind generated wave motion, and let all wave periods between 0 and ∞ be possible in the spectrum of fully arisen sea. Like for spectra resulting from temperature radiation, only an energy interval ΔU_T can be specified, corresponding to a prescribed range of periods ΔT around the average period T . That is, only the ratio has a finite value, and a 'wave' can be defined approximately by a *spectral height*, H_T , as related to ΔU_T for the period interval ΔT . (For a 'sharp wave,' that is, for a wave with a 'sharp' period T , where $\Delta T \rightarrow 0$, also $\Delta U_T \rightarrow 0$ and $h_T \rightarrow 0$.)

"In Fig. 34 the spectral band concentrated around the

²³ The reader is referred to Darbyshire (1952, see Section 6.1) for a similar definition of the "equivalent wave." Attention is called to the fact that only wave energies or the squares of wave heights are linearly superposable. Wave heights are nonlinearly connected with energies, and therefore cannot be superposed without violating the principle of conservation of energy. For example, two wave trains of amplitudes a_1 and a_2 have total energy proportional to $a_1^2 + a_2^2$. Direct superposition of amplitudes would correspond to an amount of energy $(a_1 + a_2)^2$ which exceeds the initially available energy by $2a_1a_2$.

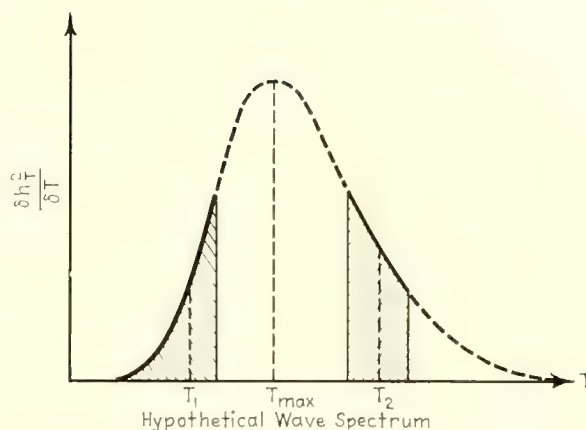


Fig. 34 Hypothetical wave spectrum (from Neumann, 1953)

period T_1 , would correspond to a very young sea, whereas the spectral band around T_2 would correspond to a swell if separated from the original wind generated wave spectrum.

"The spectral wave energy ΔU_T for the average period T in the interval ΔT is proportional to the square of the *spectral wave height* in the same interval. This spectral wave height can be defined for an infinitely small spectral band

$$H_T^2 = \lim_{\Delta T \rightarrow 0} \left(\frac{\Delta h_T^2}{\Delta T} \right) = \frac{\partial h_T^2}{\partial T}, \text{ cm}^2 \text{ sec}^{-1} \quad (68)$$

where h_T^2 is the square of the heights of individual waves in the spectrum. Let the spectral energy density be²⁴

$$W_T = \partial U_T / \partial T, \text{ erg cm}^{-2} \text{ sec}^{-1} \quad (69)$$

With sufficient accuracy for our purposes

$$\Delta U_T = W_T \Delta T = \frac{1}{8} g \frac{\partial h_T^2}{\partial T} dT, \text{ erg cm}^{-2} \quad (70)$$

can be defined as the mean energy per unit area of the sea surface of waves whose periods lie between $T - \Delta T/2$ and $T + \Delta T/2$.²⁵

"The total energy in the wind generated wave pattern will be given by

$$\bar{U} = \frac{1}{8} g \rho \int_0^\infty \frac{\partial h_T^2}{\partial T} dT \quad (71)$$

"The spectral wave height not only will be a function of the wave period (or wave length), but it also will de-

²⁴ The symbol W_T will not be used in other sections of this monograph outside of Section 6.2 on Neumann's spectrum, and therefore is not included in the list of symbols on page 90.

²⁵ The reader can compare this discussion based on the assumption of a continuous spectrum with the one given by Darbyshire (1952, see Section 6.1) based on the summation of Fourier components resulting from an harmonic analysis of a record of finite length.

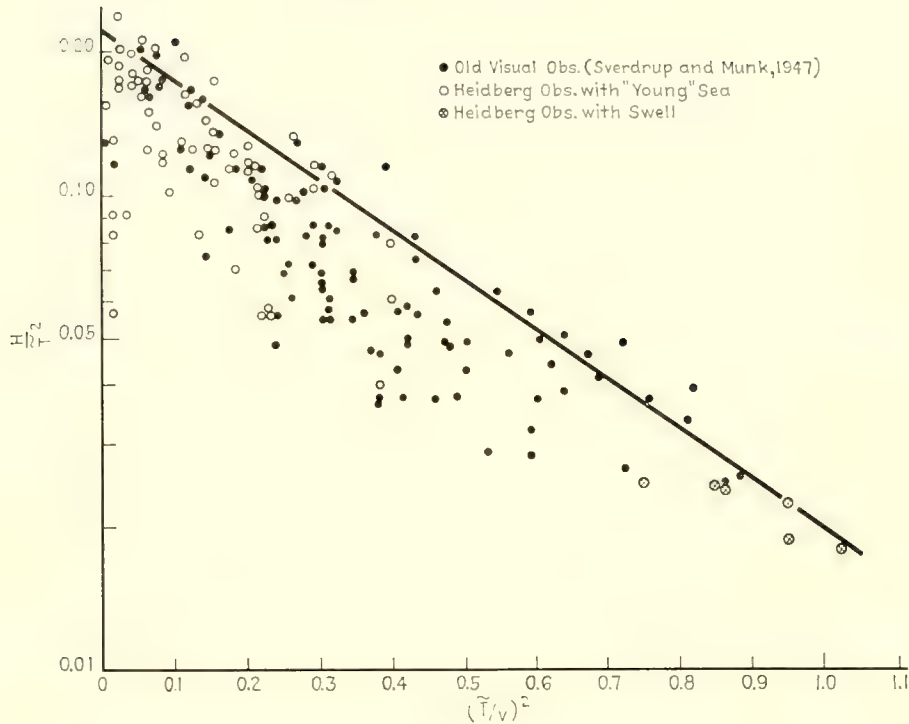


Fig. 35 Ratio of wave heights to square of apparent wave periods H/\tilde{T}^2 , as observed with different ratios of apparent wave period to wind speed v , in generating area (visual observations) (from Neumann, 1953)

pend on the wind velocity. For example, a 6-second component wave train in a wind generated sea at 5 meters per second wind velocity certainly has a spectral wave height which is much different from that with a 6-second period wave at 10 meters per second. . . .

Equation (70) expressing the relation $\Delta U_T = f(T)dT$, is the general expression for a spectrum. The practical solution requires evaluation of the unknown function $f(T) \approx \partial h_T^2 / \partial T$. It has been demonstrated by Sverdrup and Munk, and later by Bretschneider and by Neumann that the relationship $H/\lambda = f(c/U)$ shown by Figs. 18 and 19 represents probably the most reliable observable characteristic of significant waves at sea. The relationship $\lambda = f(c)$, which is well defined for simple waves, is not known sufficiently well for the "apparent" wave lengths, celerities and periods in a complex seaway. Neumann introduces therefore a plot of the relationship H/\tilde{T}^2 versus (\tilde{T}/U) , shown on Figs. 35 and 36. The symbol \sim (tilde) placed above a letter designates an "apparent" (meaning a directly observed) quantity in a complex seaway. The first figure is based on the visual observations (presumably of significant wave heights) collected by Sverdrup and Munk (1947) and made by Neumann (1952a). Here H refers to significant waves. The second figure contains all wave heights, as obtained from the records of a wave gage. In this case individual apparent waves were measured on records as shown in Fig. 37. Neumann finds that the straight line (on semi-log paper)

$$H/\tilde{T}^2 = 0.219 \exp [-2.438 (\tilde{T}/U)^2] \quad (72)$$

well represents the envelope of all observed and recorded data, and accepts it as a basis for evaluation of the spectrum form. Here H is in meters and U in meters per second.²⁶

By putting \tilde{T}^2 proportional to the apparent wave length, $\tilde{\lambda}$, the function (72) can be made dimensionally correct

$$H/\lambda = (\text{const}) \exp \left[-\left(\frac{g}{2\pi U} \tilde{T} \right)^2 \right] \quad (73)$$

or

$$H = (\text{const}) \frac{g \tilde{T}^2}{2\pi} \exp \left[-\left(\frac{g}{2\pi U} \tilde{T} \right)^2 \right] \quad (74)$$

The numerically equivalent expression $(g/2\pi)^2$ is substituted in the foregoing for the (dimensional) empirical factor 2.438 (with $g = 9.81$ mps), U in meters per second (mps).

The assumption is now made that the foregoing relationship derived for the envelope of the observed apparent waves applies to the spectral wave components, and therefore

²⁶ It is rather unfortunate that Neumann has used dimensional ratios so that any derived constant may be presumed a function of an unknown parameter.

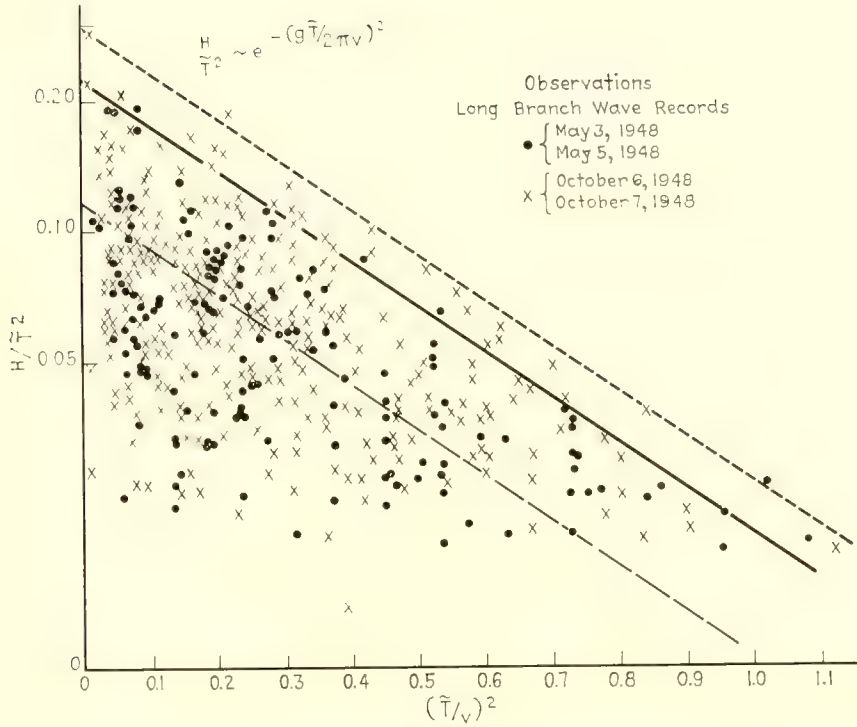


Fig. 36 Ratio of wave heights of square of apparent wave periods, H/\tilde{T}^2 as observed with different ratios of apparent wave period to wind speed v , in generating area (from Neumann, 1953)

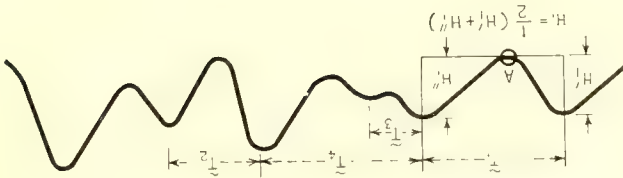


Fig. 37 Apparent wave periods and heights in a wave record (from Neumann, 1953)

$$\frac{\partial h_T^2}{\partial T^2} = H_T^2 = C \frac{g^2 T^4}{4\pi^2} \exp \left[-2 \left(\frac{gT}{2\pi U} \right)^2 \right] \quad (75)$$

(cm² sec⁻¹)

With the substitution of the above value, equation (70) becomes,

$$dU_T = C \frac{1}{32} \rho \frac{g^3}{\pi^2} T^4 \exp \left[-2 \left(\frac{gT}{2\pi U} \right)^2 \right] dT \quad (76)$$

where the constant $C(\text{sec}^{-1})$ has yet to be determined. This is the first basic expression for the spectrum, giving the wave-energy distribution as a function of the *wave period*. The second form of the spectrum, the distribution of the wave energy as a function of the *circular frequency* $\omega = 2\pi/T$ is obtained by substituting into equation (76).

$$d\omega = -\frac{2\pi}{T^2} dT \text{ and } W_\omega d\omega = W_T dT$$

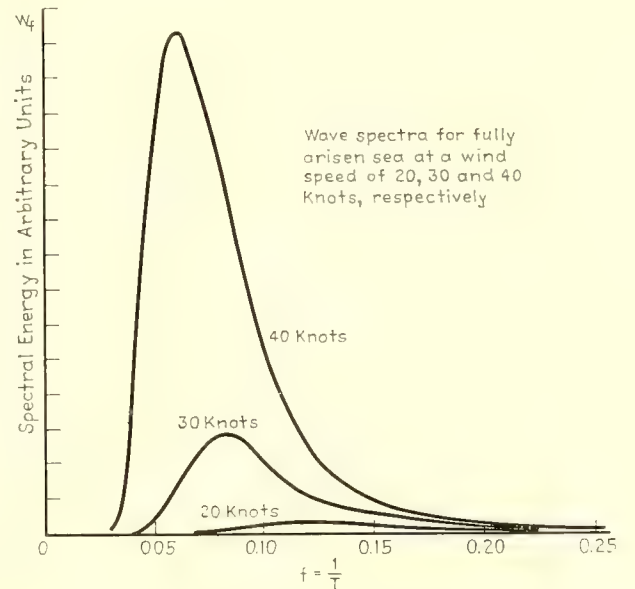


Fig. 38 (from Neumann, 1953)

thus obtaining the second basic expression

$$dU_\omega = -C\rho g^3 \pi^3 \omega^{-6} \exp \left(-\frac{2g^2}{U^2 \omega^2} \right) d\omega \quad (77)$$

The relative energy distribution (wave spectrum)

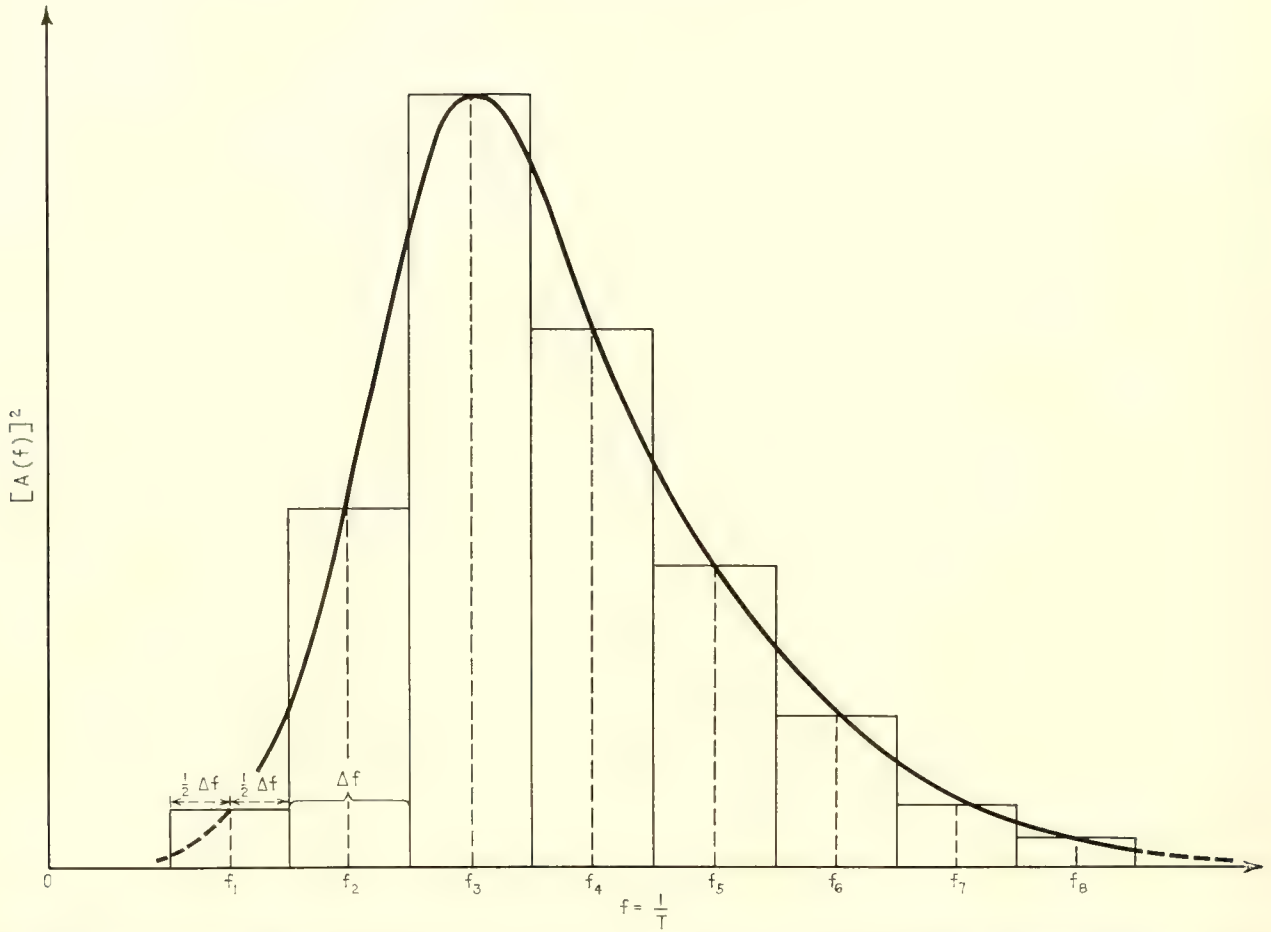


Fig. 39 Stairway approximation to wave spectrum. Area of a given rectangle is proportional to square of height of sine wave with associated frequency (from Neumann, 1953)

$\partial U_\omega / \partial \omega$ for a fully arisen sea at a wind velocity of 20, 30 and 40 knots is shown in Fig. 38 as a function of cycles per second $f = 1/T = \omega/2\pi$. These curves are computed from equation (77). The ordinate,²⁷ $W_f = \partial U_f / \partial f$, is proportional to the spectral wave height. It is given with an arbitrary scale, since it depends on the constant C .

6.21 Total wave energy and evaluation of C . The total wave energy in the spectrum between periods 0 and T or between frequencies ∞ and ω is given by

$$\bar{U} = \int_{\infty}^{\omega} \partial U_{\omega} = -C\rho g^3 \pi^3 \int_{\infty}^{\omega} \omega^{-6} \exp\left(-\frac{2g^2}{U^2 \omega^2}\right) d\omega \quad (78)$$

After substitution of $b = 2g^2/U^2$

$$b/\omega^2 = x^2,$$

and

$$d\omega = -(b^{1/2}/x^2)dx,$$

equation (78) can be evaluated as

²⁷ Subscript f indicates that W and U are functions of frequency f .

$$\bar{U} = C\rho g^3 \pi^3 \left\{ \frac{3}{4\sqrt{b^5}} \int_0^x (e^{-x^2}) dx - \frac{e^{-x^2}}{\sqrt{b^5}} \left[\frac{x^3}{2} + \frac{3}{4}x \right] \right\} \quad (79)$$

where the integral

$$\int_0^x (e^{-x^2})^2 dx = \frac{\sqrt{\pi}}{2} \Phi(x) \quad (80)$$

and $\Phi(x)$ is the error integral which can be taken from tables.

The total wave energy in the case of *fully arisen sea* follows by integration between limits $\omega = \infty$ and $\omega = 0$; i.e., from $x = 0$ to ∞ :

$$\bar{U} = \left(C\rho\pi^3 \sqrt{\frac{\pi}{2}} \right) \left(\frac{3}{32} \frac{1}{g^2} U^5 \right) \quad (81)$$

or

$$\bar{U} = (\text{const}) U^5$$

The constant C can now be evaluated if the relationship between the total energy U_ω and the wave height is established. First, it is found convenient to omit the constant quantities ρ , g and the numerical factors in the foregoing, and to replace the energy \bar{U} by the quantity E

Table 7

	Amplitude	Height
Average wave.....	$0.886\sqrt{E}$	$1.772\sqrt{E}$
Mean of $1/3$ highest waves.....	$1.416\sqrt{E}$	$2.832\sqrt{E}$
Mean of $1/10$ highest waves.....	$1.800\sqrt{E}$	$3.600\sqrt{E}$
Mean of $1/100$ highest waves.....	$2.359\sqrt{E}$	$4.718\sqrt{E}$

equal to the sum of squares of the amplitudes a of the individual component wave trains which go to make up the actual wave motion as it is observed. It is related to the total wave energy by the expression

$$E = \Sigma a^2 = \bar{U}(2/\rho g) \quad (82)$$

The reader's attention is called to the fact that so far in the discussion *wave heights* were used, but now E is defined in terms of *amplitude*. Darbyshire (1952) has defined an "equivalent wave" height H as that of a simple harmonic wave of the same energy content as the complex seaway. If the amplitude of this wave were designated by A , then $A = \sqrt{E}$. It follows that $E = 1/4$ of the area of the spectrum drawn in terms of the *wave heights*, as is shown by Fig. 34. Fig. 39, on the other hand, shows the spectrum drawn in terms of *wave amplitudes*, and in this case $E =$ the area of the spectrum.

The value of the foregoing definition lies in that the area of the spectrum, or the quantity E , is connected with various statistical properties of the observed seaway; i.e., with the distribution of the apparent wave heights. Longuet-Higgins (1952) has shown theoretically that the statistical relationships given in Table 7 exist

It follows from Table 7 that theoretically

$$H_{av}/H_{1/3} = 0.625$$

$$H_{1/10}/H_{1/3} = 1.27$$

According to a summary by Munk (1952), the foregoing ratios computed from wave records are 0.65 and 1.27. The first ratio is also confirmed by the observations of Darlington (1954). Thus, very close agreement is found between the theoretical relationships of Longuet-Higgins and the wave relationships observed at sea.

The foregoing relationships serve as the connecting link between the energy spectrum of waves (in terms of amplitudes) defined by its area E , and the observable properties of the sea defined most frequently by the height of the "significant wave"; i.e., the mean of the $1/3$ highest waves. It is now possible to evaluate E , and from it the constant C on the basis of available sea-wave observations. For this purpose, Neumann uses the results of his own visual observations made during a voyage on the *MS Heidberg*. Fig. 40 shows the plot of wave heights versus wind speed. Vertical lines show the range of variation of the observed waves. In the earlier work—Neumann (1952a)—these were described as "characteristic" waves, and it was mentioned that at 15–16 mps, for instance, the heights of characteristic waves fluctuate between 4 and 9 m. The upper limit of these fluctuations is now interpreted as the mean of $1/10$ highest waves, and the empirical solid line

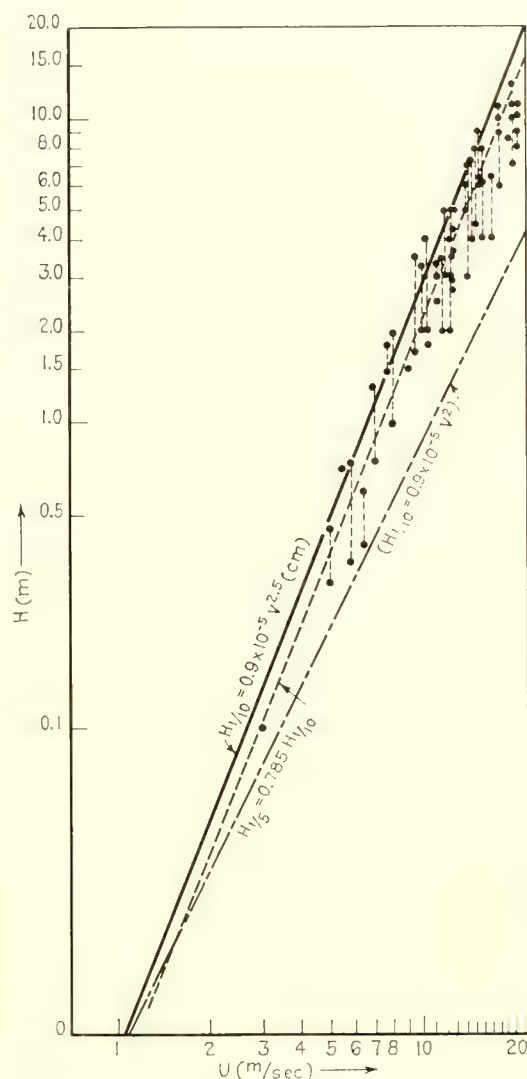


Fig. 40 Wave-height observations at different wind speeds (MS Heidberg) (from Neumann, 1953)

$$H_{1/10} = 0.000009 U^{2.5} \quad (83)$$

is fitted. The broken line, representing the significant wave, is drawn parallel to it with the coefficient reduced in accordance with the Longuet-Higgins relationships. By the use of these relationships also the average wave height is expressed as

$H_{avg} = 0.492 H_{1/10} = (0.443 \times 10^{-5}) U^{2.5}$ (in cgs units) and from this

$$E = \left(\frac{H_{avg}}{1.772} \right)^2 = \left(\frac{0.443 \times 10^{-5}}{1.772} \right)^2 U^5$$

or

$$\bar{U} = \frac{1}{2} \rho g E = 3.125 \times 10^{-9} U^5 \text{ (erg cm}^{-5}\text{)} \quad (84)$$

and by comparison with equation (81), the constant C is evaluated as 8.27×10^{-4} .

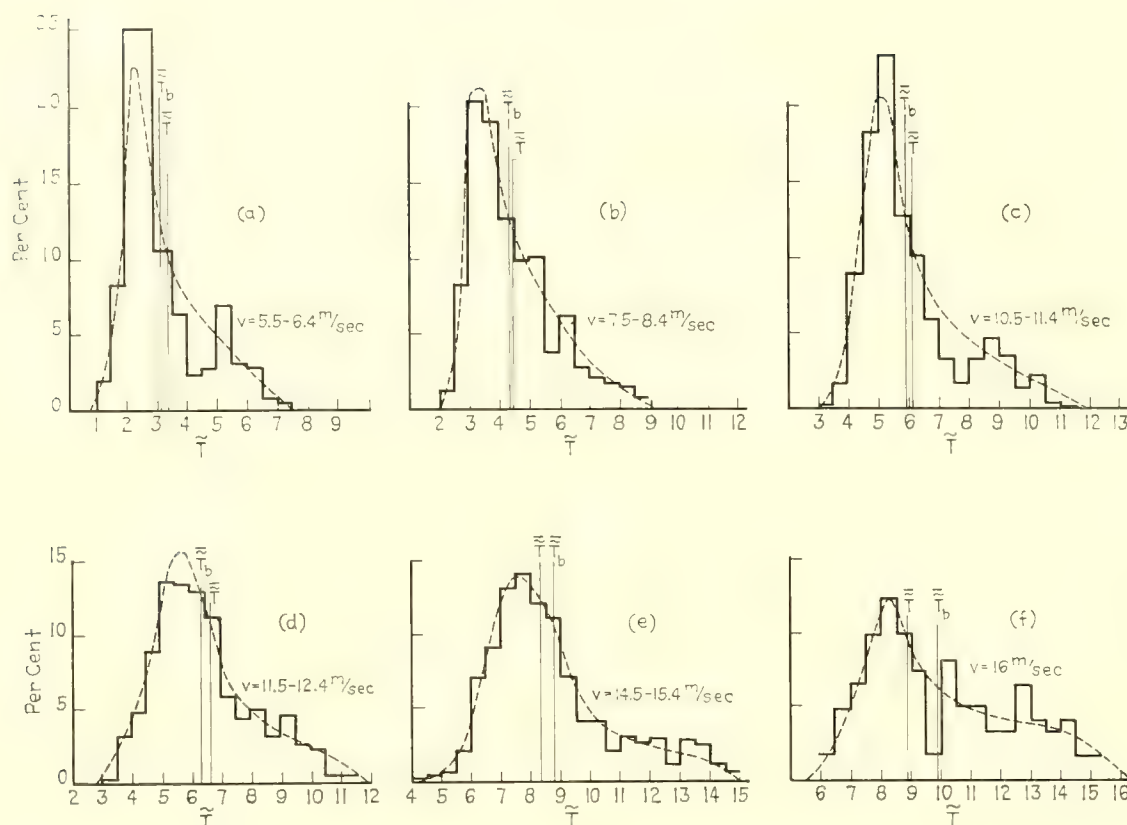


Fig. 41 Observed frequency distribution of apparent periods of waves at different wind speeds. \bar{T} denotes the mean period computed from spectrum, \bar{T}_b the observed mean period (from Neumann, 1954)

Table 8 Characteristics of Fully Arisen Sea

V , knots	\bar{T} , sec	T_{max} , sec	H_{avg} , ft	$H_{1/3}$, ft	$H_{1/10}$, ft
10	2.8	4.0	0.88	1.41	1.79
12	3.4	4.8	1.39	2.22	2.82
14	4.0	5.6	2.04	3.26	4.15
16	4.6	6.5	2.85	4.56	5.8
18	5.1	7.2	3.8	6.1	7.8
20	5.7	8.1	5.0	8.0	10.2
22	6.3	8.9	6.4	10.2	12.8
24	6.8	9.7	7.9	12.5	16.0
26	7.4	10.5	9.6	15.4	19.6
28	8.0	11.3	11.3	18.2	23.1
30	8.5	12.1	13.5	21.6	27.6
32	9.1	12.9	16.1	25.8	32.8
34	9.7	13.6	18.6	29.8	38.0
36	10.2	14.5	21.6	34.5	43.8
38	10.8	15.4	24.7	39.5	50.2
40	11.4	16.1	28.2	45.2	57.5
42	12.0	17.0	31.4	50.1	63.9
44	12.5	17.7	36.0	57.7	73.4
46	13.1	18.6	39.7	63.5	80.6
48	13.7	19.4	44.4	71.0	90.4
50	14.2	20.2	48.9	78.0	99.4
52	14.8	21.0	54	87	110
54	15.4	21.8	59	95	121
56	15.9	22.6	64	103	130

Table 9 Significant Range of Periods in Fully Arisen Sea at Different Wind Velocities, V

(T_L = lower limit, T_u = upper limit of significant periods)

V , knots	T_L , sec	T_u , sec	V , knots	T_L , sec	T_u , sec
10	1.0	6.0	34	5.5	18.5
12	1.0	7.0	36	5.8	19.7
14	1.5	7.8	38	6.2	20.8
16	2.0	8.8	40	6.5	21.7
18	2.5	10.0	42	6.8	23
20	3.0	11.1	44	7.0	24
22	3.4	12.2	46	7.2	25
25	3.7	13.5	48	7.5	26
26	4.1	14.5	50	7.7	27
28	4.5	15.5	52	8.0	28.5
30	4.7	16.7	54	8.2	29.5
32	5.0	17.5	56	8.5	31.0

\bar{T} = average "period" (seconds).
 T_{max} = period of most energetic wave in the spectrum.
 H_{avg} , $H_{1/3}$, $H_{1/10}$ = height of average, $1/3$ highest and $1/10$ highest waves (range of significant periods: see Table 9).

The wave heights of a fully developed sea, resulting from the foregoing relationships for different wind speeds, are shown in Table 8. Attention should be called to the fact that the *form* of the spectrum and its *area*, the quantity E , have been derived quite independently, and therefore should be verified independently. The form was based in Fig. 36, while the quantity E was computed from the visually observed wave heights in Fig. 40. The dependence of energy E on wind speed to the fifth power was obtained in the process of developing

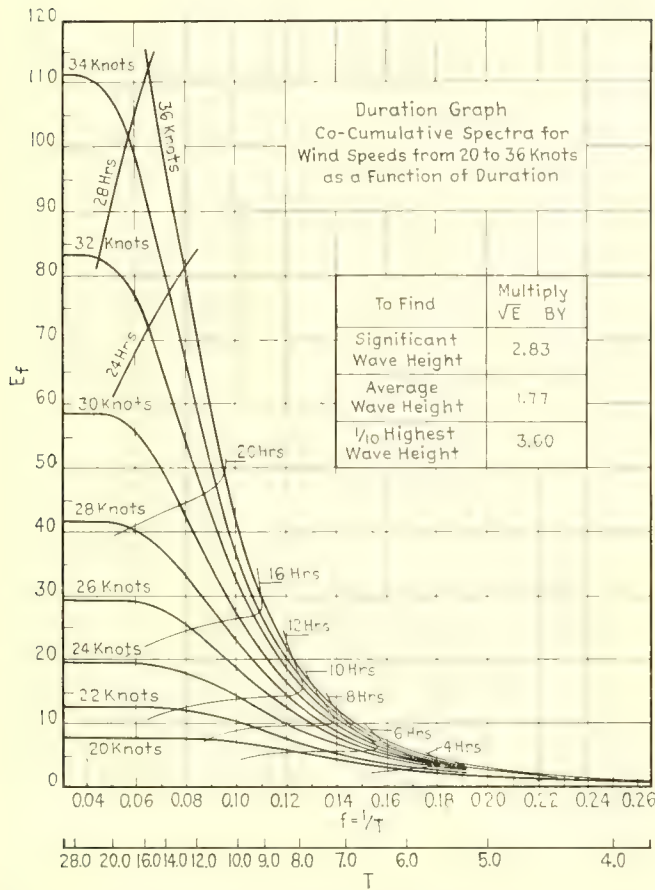


Fig. 42 Duration graph co-cumulative spectra for wind speeds from 20 to 36 knots as a function of duration (from Neumann, 1953)

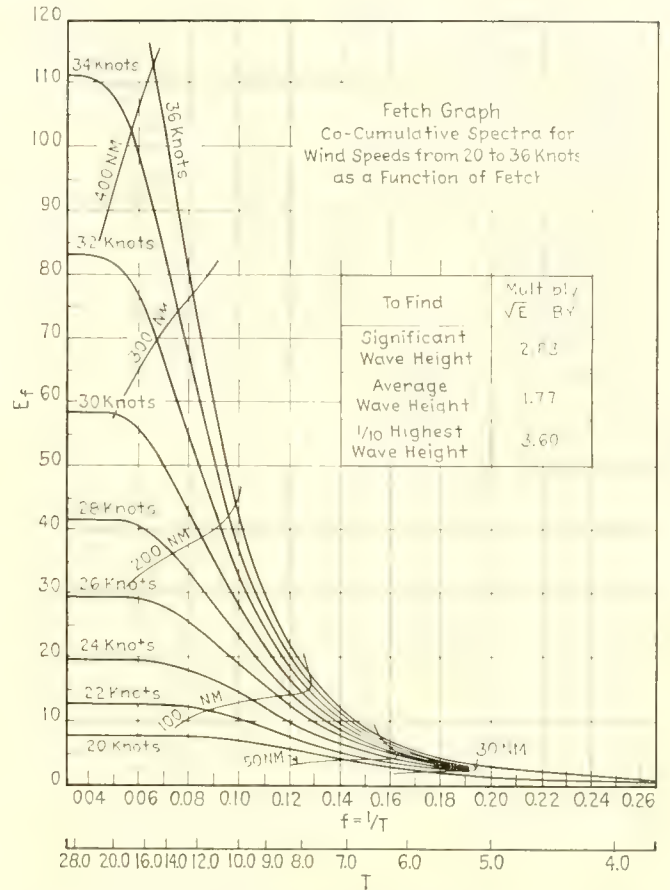


Fig. 43 Fetch graph co-cumulative spectra for wind speeds from 20 to 36 knots as a function of fetch (from Neumann, 1953)

the form; this evidently influenced the fitting of the $H_{1/10}$ curve in Fig. 40.

The form of the spectrum can be characterized broadly by the total range of periods T , by the value of T at the frequency of maximum energy, usually designated T_{max} , and by the mean value of T . In principle, all values of period T or of frequency f are possible. However, examination of Fig. 38 shows that above and below a certain range of frequency, f , the energies contained in waves are negligibly small. Thus, at a 30-knot wind the significant range is between $f = 0.048$ and $f = 0.240$, or between periods of $T = 21$ and 4 sec, respectively. The frequency ranges at different wind speeds are shown in Table 9.

The mean value of the frequency provides a convenient method of checking the form of the spectrum. In terms of the theoretical spectrum it is given by²⁸

$$\bar{T} = 2\pi \left[\frac{\int_0^\infty dE(\omega)}{\int_0^\infty d[\omega^2 E(\omega)]} \right]^{1/2} \quad (85)$$

Here the symbol \bar{T} is used to designate the mean value of "apparent" periods. This period also can be evaluated by direct observation of periods in a seaway. Fig. 41 shows the period distributions observed by Neumann (1954) over six ranges of wind velocity. The average periods calculated from the spectra, T , and resulting from the observed data, \bar{T}_b , are shown by vertical lines. A reasonable agreement is demonstrated. No such agreement is established as yet for the spectrum area E ; i.e., for the significant wave height corresponding to a given wind, and practically all observers indicate wave heights somewhat lower than Neumann's, particularly at higher wind speeds.

6.22 Co-cumulative energy spectrum and seaway development for a limited fetch or wind duration. Since the wave height is proportional to the square root of the spectrum area E , plots were made of the value of $2/\rho g$ times the integral, equation (78), versus frequency f : such curves are called *co-cumulative spectra*, abbreviated as *CCS curves*. Figs. 42 and 43 are reproduced here as

²⁸ Neumann (1953) attributes this formula to Dr. W. J. Pierson, Jr.

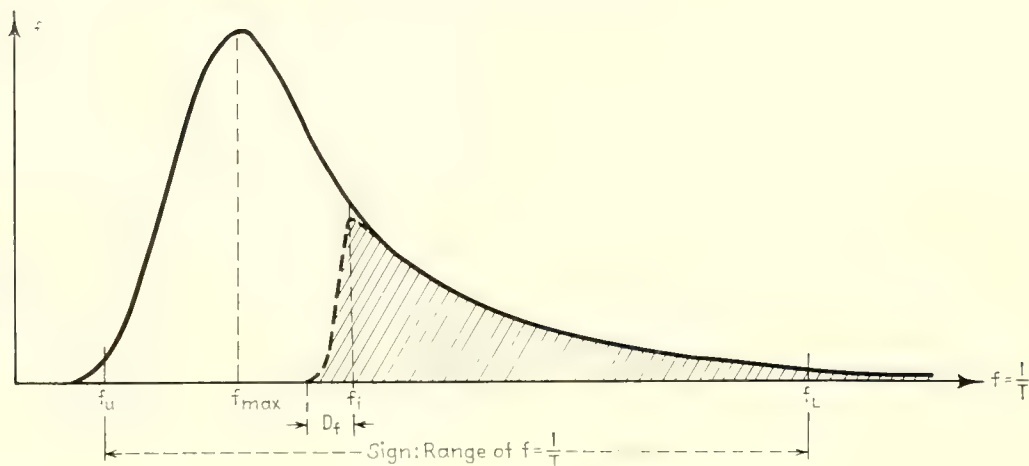


Fig. 44 Wave spectrum in fully arisen sea with a significant range of periods between f_u and f_L . Wave spectrum in non-fully developed state is shown schematically by dashed area, where upper limit of significant periods is given by $f_i - D_f$ (from Neumann, 1953)

examples. Additional figures for different ranges of wind velocity will be found in Neumann (1953) and in Pierson, Neumann and James (H).

The value of E for the total spectrum, i.e., the value indicated by the intercept on the LH vertical scale of these figures, characterizes a fully arisen sea. In practice it was found convenient to neglect the waves of very high and very low frequencies which contribute little to the total energy. Quoting from Neumann (1953): "As a rule, by empirical evidence, about 5% of the total E value in the fully developed state of wind generated sea can be cut off at the upper part of the curves. At the lower part it is 3% of the total E value. For example, the total E value at a wind speed of 30 knots is read off at the ordinate in Fig. 42, $E = 58.5$ (feet)².

$$5\% \text{ of } E = 2.9$$

thus, 55.6 on the E scale intersects the CCS curve at $f = 0.06$ or $T = 16.7$ seconds. This is the upper limit of the significant range of periods in a fully developed sea with a 30 knot wind. If it is wanted to determine a lower limit of the significant range of periods, 3% of $E = 1.76$ is the ordinate value which intersects the CCS curve at $f = 0.213$, or $T = 4.7$ seconds. Therefore, the significant range of periods is between 4.7 and 16.7 seconds . . ." Table 9 shows the range of significant periods in a fully arisen sea at different wind speeds. The co-cumulative curves in Figs. 42 and 43 are intersected by lines of wind duration in hours on the first and lines of fetch in nautical miles on the second. The position of these lines were derived from the data of Figs. 22 to 25.

Quoting Neumann (1953, p. 32): "The intersection points of the CCS curves with the duration or fetch lines, respectively, show the limit of the development of the composite wave motion at the given duration or fetch. Physically, it means that the state of development is limited by a certain maximum amount of total energy

which the wave motion can absorb from the wind with the given conditions. The E value of the ordinate of each intersection point is a practical measure of the total energy accumulated in the wave motion of the no[t]-fully arisen state, limited either by the fetch or duration.

"Under actual conditions, both fetch and duration may be limited, and the E value for any given situation in most cases will be different for the fetch and duration. It is easily seen that in such cases the smaller of the two E values has to be taken.

"From the E value, the wave height characteristics can be computed, as in the case of a fully arisen sea.

"The upper limit of significant periods in not-fully arisen sea is approximately determined by the 'frequency of intersection', f_i , that is, the frequency of the intersection point between the CCS curve of a given wind speed and the given fetch or duration line, respectively. By this, theoretically, the wave spectrum is cut off abruptly at a given maximum period, T_i (or minimum frequency, f_i), without considering possible wave components with periods a little longer than $T_i = 1/f_i$, which are just in the beginning stage of development (Fig. 44). These wave components probably have a small amplitude, and contribute so small amount of energy to the total wave energy, that they may be neglected in most practical cases of wave forecasting."

6.23 Roll, Fischer and Walden's modification of Neumann's spectrum. Neumann's spectrum has often been referred to as theoretical but this description has also often been objected to on the grounds that the spectrum derivation really represented an ingenious treatment of empirical data. In particular, two crucial steps in the derivation appear to be intuitive:

a) The transition from the *apparent* wave steepness distribution to the distribution of *spectral component* waves.

b) Abandonment of the data on which the spectrum's form was based, Fig. 36, in favor of visual observation data, Fig. 40, in evaluating the constant which governs the height of the predicted waves.

The explanation given by Neumann for the first of these is not clear. In two papers, Walden and Piess (1957, 1958) attempted to clarify this step as well as to improve the precision of various details connected with the spectrum. They came to the conclusion that the validity of Neumann's derivation can be neither proved nor disproved by these considerations and rests on the gathering of sufficient empirical data.

The experience of Roll and Fischer²⁹ indicates that, had Neumann consistently used Fig. 36 for the evaluation of the constant as well as the spectral form, he would have arrived at about double the actual wave height. Instead, he substituted the wave heights found by his previous visual observations, Fig. 40, and on this basis arrived at an acceptable constant.

The foregoing remarks, taken with the fact that Neumann's constant C has the dimensions of $1/t$, lead to a plausible hypothesis. Were Neumann's spectral form accepted as invariable for all significant wave heights, the "constant" C could be taken as variable and a function of the mean period \bar{T} . For the light sea conditions on which Fig. 36 was based, the constant may possibly have double its value for the mean sea conditions shown in Fig. 40. This hypothesis appears to have its confirmation in the trend of the observed points in Fig. 40 toward a curved rather than a straight line. The corresponding trend can also be seen in Fig. 36. A more exact curve-fitting would have given a concave curve, rather than a straight line.

Roll and Fischer (1956) called attention to the apparent discrepancy in the mathematical form of the spectrum given by equations (76) and (77). Both equations are meant to describe the same wave form, and on first thought one would expect that the same wave length or period T_{\max} would be indicated in both as corresponding to the wave component of maximum energy. T_{\max} can be found by differentiating equations (76) and (77) with respect to period T and frequency ω , respectively, and by equating the derivative to zero. When this is done, two different values of T_{\max} are found: $0.641 U$ and $0.785 U$ (with T in sec, U in mps). Neumann used the second value only, without discussing the subject.

Such an apparent discrepancy and a method for dealing with it had been known previously in the theory of thermal radiation. It will be recollected that the energy can be defined only for an interval ΔT or $\Delta\omega$, at a certain value of T or ω . The apparent discrepancy arises because of the difference in the corresponding intervals $d\omega$ and dT :

$$d\omega = -\frac{2\pi}{T^2} dT \quad (86)$$

while

$$W_T dT = W_\omega d\omega$$

This apparent confusion can be avoided by defining the periods and frequencies in logarithmic form, $d(\ln T) = dT/T$ and $d(\ln \omega) = d\omega/\omega$. Equation (76) is then written as

$$\Delta U_T = W_T \frac{dT}{T} = C\rho \frac{g^3}{32\pi^2} T^4 \exp \left[-2 \left(\frac{gT}{2\pi U} \right)^2 \right] \frac{dT}{T} \quad (87)$$

and from

$$\frac{dT}{T} = -\frac{d\omega}{\omega} \text{ and } W_\omega \frac{d\omega}{\omega} = W_T \frac{dT}{T}$$

it follows that

$$\Delta U_\omega = W_\omega \frac{d\omega}{\omega} = -C\rho \frac{g^3\pi^2}{2} \omega^{-4} \exp \left(-\frac{2g^2}{U^2\omega^2} \right) \frac{d\omega}{\omega} \quad (88)$$

The powers of T and $1/\omega$ are now the same in both equations, and both yield

$$T_{\max} = 0.641 U \quad (89)$$

The total energy is obtained by integration as

$$\bar{U} = \int_{-\infty}^{\infty} dU_\omega = -C\rho \frac{g^3\pi^2}{2} \int_{-\infty}^{\infty} \omega^{-5} \exp \left(\frac{-2g^2}{U^2\omega^2} \right) d\omega \quad (90)$$

This expression is integrable, and by substitution of

$$b = \frac{2g^2}{U^2}; \quad \frac{b}{\omega^2} = x^2; \quad d\omega = -\frac{\sqrt{b}}{x^2} dx$$

\bar{U} is evaluated as

$$\bar{U} = C\rho \frac{\pi^2}{16g} U^4 \left[1 - \left(1 + \frac{2g^2}{U^2\omega^2} \right) \exp \left(-\frac{2g^2}{U^2\omega^2} \right) \right]_{-\infty}^{\infty} \quad (91)$$

and for the fully developed sea

$$\bar{U} = \frac{C\rho\pi^2}{16g} U^4 \text{ (erg cm}^{-2}\text{)} \quad (92)$$

It is observed that in this case the energy is shown to be proportional to U^4 ; i.e., the wave height to U^2 , as against $U^{2.5}$ originally derived by Neumann in connection with the spectrum. The dependence of the wave height on the square of the speed has been shown previously by Sverdrup and Munk (1947) by the horizontality of the H versus t -curve in Fig. 21. It is likewise indicated by the horizontality of the curves at the RH side of Figs. 22 to 25 taken from Neumann (1952b). Darbyshire (1952, 1955) also shows wave height as proportional to wind velocity squared.

It will be recollected that Neumann used Fig. 36 in evaluating the form of the spectrum, but abandoned it without explanation in evaluating the constant C in favor of a plot of his own visual observations in Fig. 40. Roll and Fischer (1956) now return for evaluation of the constant to the original empirical relationships (67) and (68) established by Neumann on the basis of Fig. 36. They then follow Neumann in assuming that the rela-

²⁹ To be discussed later.

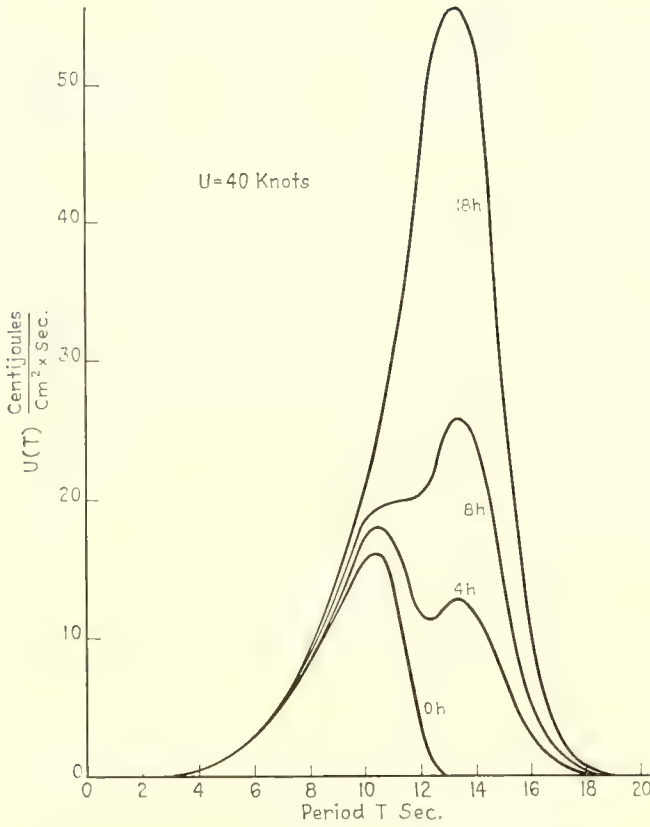


Fig. 45 Energy spectrum of waves $U(T)$ at wind speed of 40 knots and wind duration of 4, 8 and 18 hr. Spectrum marked 0h is attained within 1 hr (from Gelci, Casalé, and Vassal, 1957)

tionship derived for observed apparent wave heights and periods is applicable to the component waves forming the spectrum, but they take the spectrum in the logarithmic form shown earlier. Writing the increment of energy as

$$\Delta U_T = W_T \frac{dT}{T} = \frac{g\rho}{8} H_T^2 \frac{dT}{T} \quad (93)$$

and substituting H_T^2 resulting from equation (74) they obtain

$$\Delta U_T = C^* \rho \frac{g^3}{32\pi^2} T^4 \exp \left[-2 \left(\frac{gT}{2\pi U} \right)^2 \right] \frac{dT}{T} \quad (94)$$

From a comparison of equation (94) with (87) and consideration of equation (72) it follows that

$$C = C^* = 1.96 \times 10^{-2} \quad (95)$$

It is not possible to compare this constant with that of Neumann because of the difference in form of equations (76) and (94). It should be noted, however, that the constant C in equation (94) is nondimensional,³⁰ while in equation (76) it has the dimensions of sec^{-1} . By using (82) to convert the energy into the quantity E and using

³⁰ The use of dT/T and $d\omega/\omega$ in order to avoid dimensionality of the constant has been discussed previously by Darbyshire (1952).

Longuet-Higgins' relationships from Table 6, the average wave height is obtained as

$$\bar{H} = \frac{0.275}{g} U^2 \quad (96)$$

and the significant wave height as

$$\bar{H}_{1/3} = \frac{0.44}{g} U^2 \quad (97)$$

The corresponding relationship found by Sverdrup and Munk (1947) for the fully developed seaway is

$$\bar{H}_{1/3} = \frac{0.26}{g} U^2 \quad (98)$$

Walden (1956) pointed out that in using Neumann's relationship Roll and Fischer evaluated the constant on the basis of the envelope curve of Fig. 36, which (according to Neumann) corresponds to the mean of $1/10$ highest waves. It appears more correct to derive the constant on the basis of the average wave height, which is about half of the former. With this correction, equation (97) is rewritten as

$$\bar{H}_{1/3} = \frac{0.22}{g} U^2 \quad (99)$$

The wave height is now indicated to be somewhat lower than in Sverdrup and Munk's equation (98) but checks well with Walden's (1953/54) data derived from a study of many reports of weather observation ships (see section 6.5).

To summarize, the suggestions of Roll, Fischer and Walden were:

- 1 To express spectra in logarithmic form so that the maximum of the spectral curve would occur at the same wave length whether plotted versus T or versus ω .
- 2 By the foregoing action, to make the constant C dimensionless.
- 3 To use Fig. 36 consistently for both spectrum form and evaluation of the constant.
- 4 To base this derivation on the mean, rather than on the envelope curve of Fig. 36.

6.3 Gelci, Casalé and Vassal's Spectrum. Quoting from Gelci, Casalé and Vassal (1956, 1957) (in a free translation): "The following method of predicting waves is based on the *independence of the various sinusoidal components*. This is the hypothesis adopted by J. Darbyshire (1952) in a classical study. It is in reality difficult to state the limits of validity of this hypothesis. Recently G. Neumann (1953) adopted implicitly a particular hypothesis of *interdependence of various sinusoidal components*: the long components grow only when the lesser ones already exist. We have here reconciled the independence hypothesis of Darbyshire with the results of G. Neumann, relating to the limiting established state of sea under constant wind action (fully arisen sea)."

The foregoing reconciliation consists of the statement that while all wave components exist at the same time,

the long ones grow very slowly at the beginning. They are at first not visible in the same sense that tidal waves are not visible under wind sea.

The reader is referred to Gelci, Casalé and Vassal's 1956 and 1957 papers for explanation of the energy spectrum properties and the appropriate methods of estimating the waves caused by winds of various strengths and directions. The spectrum of the fully arisen sea is finally defined as

$$\frac{\partial U_T}{\partial T} = 209 T^4 \quad (100)$$

if $T \leq 0.315 U$, and

$$\frac{\partial U_T}{\partial T} = 209 T^4 \exp \left[-329 \left(\frac{T}{U} - 0.315 \right)^2 \right]$$

if $0.315 U \leq T$,

where $\partial U_T / \partial T$ is the spectral-energy density of the fully arisen sea in ergs/cm²/sec, U the wind at anemometer height in knots, and T the wave-component period in seconds.

Considerable discussion of the directional variability of component waves is given in the referenced papers. This question is placed in Section 8 of this monograph.

The spectrum for a limited wind duration is defined, in the words of Gelci, et al: "In order to define the ideas, let us consider a generating area with a very large fetch; the sea being originally calm, the wind U (in knots) begins to blow instantaneously. At the end of a relatively short time (of the order of an hour) the spectral energy density is expressed (in ergs per cm² per second) as:

$$\left. \begin{aligned} \frac{\partial U_T}{\partial T} &= 209 T^4 \\ &\left\{ 1 - \exp \left[-329 \left(\frac{T}{U} - 0.315 \right)^2 \right] \right\} f(\theta) \end{aligned} \right\} \quad (101)^{31}$$

if $T \leq 0.315 U$

$\partial U_T / \partial T = 0$ if $0.315 U \leq T$

"We will designate by ρ_0 this function of T and U . . . On the scale of meteorological synoptic charts, it is even possible to assume that this regime establishes itself instantaneously.

"Then, each wave component grows linearly following the law:

$$\frac{\partial U_T}{\partial T} = \rho_0 + \frac{t}{18} 209 T^4 \exp \left[-329 \left(\frac{T}{U} - 0.315 \right)^2 \right] f(\theta) \quad (102)$$

t expressed in hours is below 18. For $18 \leq t$ the regime is stationary (fully arisen sea)."

The successive spectra of the developing wave in a 40-knot wind are shown in Fig. 45.

³¹ Here $f(\theta)$ designates a function of the directional wave distribution. This is not considered in this section of the monograph.

The author has not found a discussion of the fetch in the papers by Gelci, Casalé, and Vassal. It should be emphasized that formulation of the spectra (as by Gelci, et al, and by Darbyshire) is only a part of the problem. Another important part of references cited is the detailed discussion of methods of evaluating effective fetch and wind strength.

6.4 Spectra of Incompletely Developed and Decaying Seas. Most often in nature winds of significant strength do not blow long enough to develop a "fully arisen sea"; i.e. waves in which the energy intake from the wind is equal to the energy dissipation and for which the wave structure remains constant. In wave prediction it is necessary to treat such incompletely developed seas. In this connection, as indeed throughout the spectrum discussions, the form of spectrum and the wave height indicated by its area must be considered separately. The prespectrum methods of wave prediction of Sverdrup and Munk (with Bretschneider's extension) in the U.S.A. and Suthon and Bracelin in England, have been successful in predicting the height and period of significant waves, but give no indication of the distribution of the component waves in a complex sea. It will be shown later, in Section 2 of Chapter 3, that minor variations of the spectral curve have little significance for the analysis of a ship's motion. The position of maximum spectral density on the frequency or period scale is, however, important since it determines the conditions under which a ship falls into synchronism with waves.

Neumann's method (included in Pierson, Neumann, and James' forecasting manual) of predicting the wave conditions for a limited fetch or limited wind duration was discussed in Section 6.22. In this method the shape of the spectral curve at the higher frequency end depends only on the wind strength. A limited fetch or duration provides cut-off points beyond which longer periods (lower frequencies) are assumed not to exist. This method, therefore, determines principally the mean (or the significant) wave height but defines the spectrum shape only crudely, and in particular gives no information on the position of the maximum of the spectral curve.

Gelci, Casalé, and Vassal's method (designated D.S.A. II), described in the foregoing section and illustrated by Fig. 45 defines the entire spectrum curve for any wind duration. Gelci, et al, agree with Neumann that the spectrum grows from the high-frequency (low period) end.

The effect of fetch was evaluated by Darbyshire in connection with the coastal region spectrum (1952) but not with the open-sea spectrum (1955). The author was not able to find in Darbyshire's references any discussion of the wind-duration effect. Gelci, et al, however, discuss Darbyshire's spectra thus: "The behavior of the successive spectra corresponding to duration of action of 4 hours and 20 hours will be found in Fig. (46).³² It will be noted that the spectrum grows first

³² Of the figures cited, one for the wind of 40 knots is reproduced here as Fig. 46.

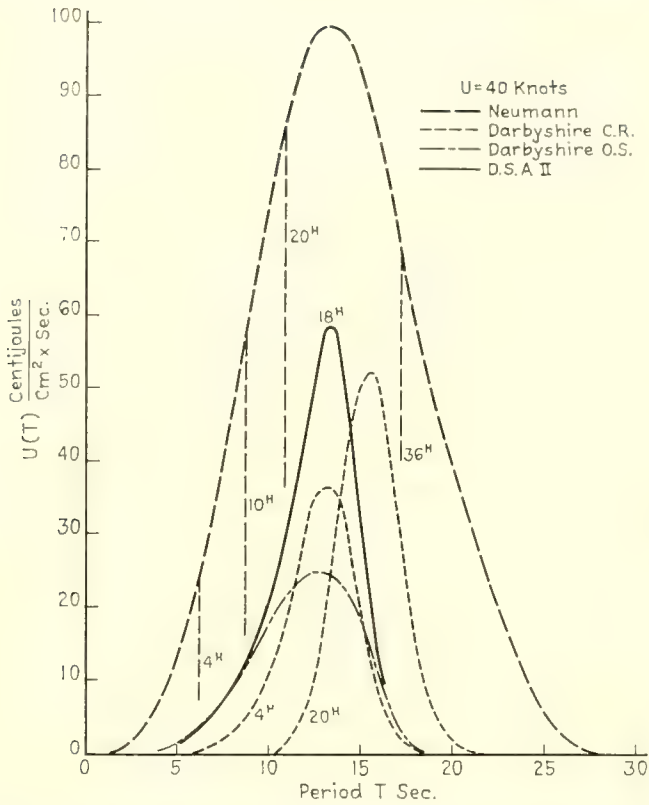


Fig. 46 Energy spectra of fully arisen ocean waves $U(T)$ at a 40-knot wind according to different formulations (from Gelci, Casalé, and Vassal, 1957)

very rapidly, and then is slowly displaced towards the longer periods. Strictly speaking, there is no limiting stationary regime, but rather a slowly variable regime which we will call 'quasi fully arisen sea', after a dozen hours. It is this regime which is represented by the preceding equations.³³

No fetch or duration effects were discussed for Darbyshire's (1955) (open-sea) spectrum but it was assumed to exist after a wind duration of about 10 hrs.

Fig. 47 was prepared by Gelci, et al (1957) to show the mean of $\frac{1}{10}$ highest waves as a function of duration of a 40-knot wind as depicted by various forecast methods.³⁴ The original report by Gelci, et al, contains similar plots for 30 and 50-knot winds in which the relative placement of the curves is in the same order.

Each of the methods discussed so far was based on the investigator's interpretation of the sea data available to him. Apparently the only direct measurements of the growth of waves with wind duration are found in the work of Ijima (1957) from which Figs. 48, 49 and 50 are taken. These figures are based on observations at the Port of Sakata on the west coast of Japan, where appear well-defined and often stationary generating areas of moderately strong wind over the Sea of Japan. Ijima's paper gives data on seven summer and three winter storms.

³³ Darbyshire's wave height spectrum transformed into energy spectrum in ergs/cm²/sec.

³⁴ The author has not examined the reference to Suthon (1945), but believes that it is the basic material used later by Bracelin (1952) who is cited in Walden's (1953-54) discussion in the next section.

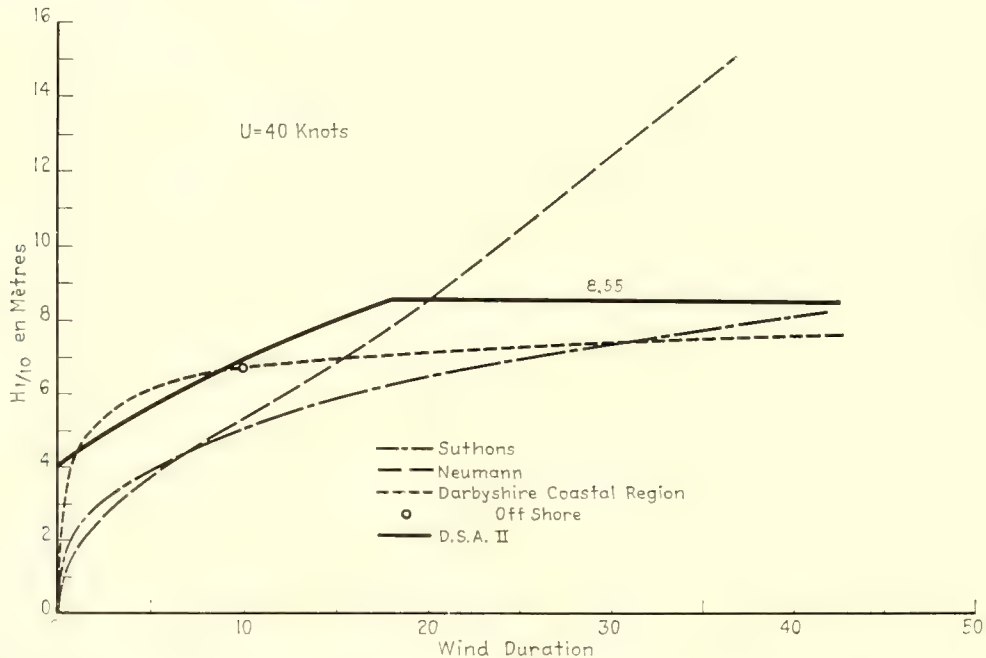


Fig. 47 Growth of waves with duration of 40-knot wind according to different formulations (from Gelci, Casalé, and Vassal, 1957)

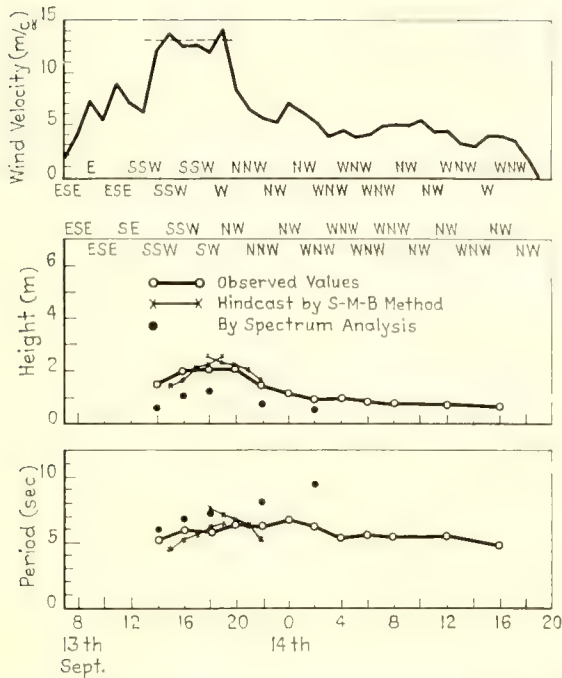


Fig. 48 Wave height and period at Port of Sakata from September 13-14, 1955 (from Ijima, 1957)

Figs. 48 and 49 were chosen from this set primarily because they permitted the best reproduction.

Observations were carried out using an underwater pressure meter installed at about 900 ft from the end of a breakwater, where the depth was 31.8 ft. Recordings were made continuously for 20 min every 2 hr. The solid curves in Fig. 49 show the recorded pressure-wave heights and the dotted curves show the wave components corrected for gage-depth attenuation. The time histories of spectrum growth and decay varied somewhat for different recorded cases. Average conditions are however well represented by Fig. 50. Confirming the conclusions of Neumann and Gelci, et al, the spectrum develops gradually from the high-frequency end. However, contrary to Neumann's assumption, considerable energy is contained in the spectrum at periods longer than T_{max} . No detailed analysis was made to show the degree of compliance of the curves in Fig. 50 with those expected from the various spectrum definitions. By visual examination alone (remembering the opposite directions of the abscissas T and $1/T$), Ijima's spectra are seen to bear a close resemblance to those of Gelci, et al, Fig. 45.

The decay process appears to be the reverse of the growth one. The longer waves disappear first, and the T_{max} is gradually shortened. In addition there is a certain attenuation of short-wave amplitudes.

6.5 Comparison of Various Spectra and Methods of Wave Forecasting. In this section the results of the various investigations will be compared under:

a) Spectra of fully arisen sea.

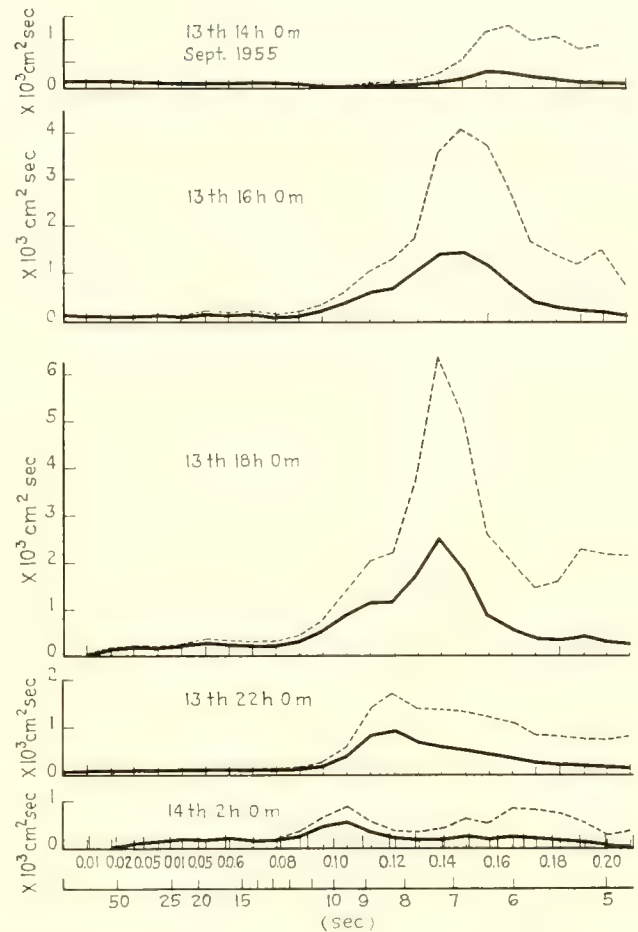


Fig. 49 Wave-energy spectrum of waves shown in Fig. 48 (from Ijima, 1957)

- b) Wave development with time and fetch.
- c) Overall results of wave forecasts.

In all cases the form of the spectral curve and the predicted mean wave characteristics will be distinguished. Since the practical importance of the three subjects grows from a) to c), it is gratifying to find (as will be shown in the following) that the discrepancies between the various methods diminish from a) to c), although they never vanish.

6.51 Spectra of fully arisen sea. A comparison of various spectra formulations are shown in Figs. 46 and 51. The demonstrated differences among these spectra formulations are appalling, and indicate to this author the inadequacy of an empirical approach and the pressing need to pursue development of a rational theory of wave generation by wind.

The observational data for direct verification of the spectra are extremely meager. The ideal conditions of a well-defined single fetch and wind duration are very seldom found in nature. In strong winds incompletely developed seas are usually found, and in lighter winds (of minor practical significance) waves are most often

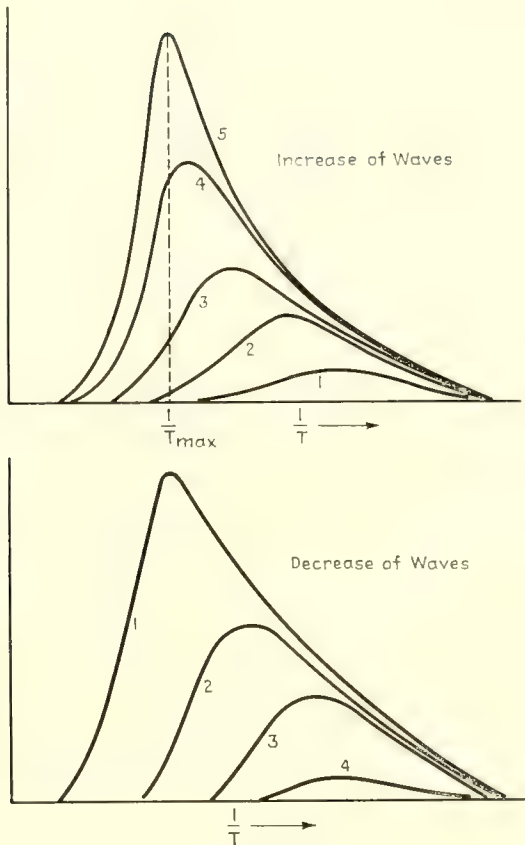


Fig. 50 Typical change of wave energy in build-up and decay of waves (from Ijima, 1957)

compounded of several generating causes. Most important of all, however, is the fact that a sea spectrum is a statistical concept. Waves are expected to conform to the predicted spectrum *on the average*, and it should not be expected that an isolated record will conform exactly. Conversely, an isolated record should be considered of little value in qualifying a spectrum formulation. The rare occurrence of the ideal conditions in nature, however, makes it very difficult to collect sufficient data for a statistical evaluation. Also, until recently, Darbyshire's (1955) data were the only ones obtained by instrumental measurements in the open ocean.

In regard to the spectrum area, i.e., significant wave height prediction, the subject is further complicated by the indeterminateness of the wind specification. Various formulations indicate that spectrum area is proportional to either the fourth or fifth power of wind velocity. Small differences in measured wind velocity will, therefore, strongly affect the relative position of the spectral curve on such plots as Figs. 46 and 51. Yet winds are measured indiscriminately at various "anemometer heights," and no standard for such measurements has been established. Aerodynamic experience also has indicated that interference of a body with an air stream extends for a considerable distance. In calibrating air-

craft speed, a "bomb" carrying the speed pick up has sometimes been suspended on a cable many feet below the aircraft. The conclusion is inevitable that wind-speed measurement at practical locations on a ship is faulty. Conceivably a kite or a small balloon could be used on weather ships.

Apparently the only available data from sea recording and analysis for spectrum verification were obtained in project SWOP (Chase, et al., 1957). This project will be described more completely in Section 8 in connection with directional wave spectrum. For the present it is sufficient to say that two stereophotographs of an open-sea area were taken from two airplanes simultaneously within a 20-min period of wave-pole recording. Two spectra were computed, one from the stereophotographic analysis and the other from the wave-pole analysis. One of these is reproduced in Fig. 51 for comparison with other spectra.

A discussion of the discrepancies among the spectra shown in Fig. 51 will be found in Neumann and Pierson (1957).

6.52 Wave development with time. Some of the appalling differences among the spectra shown in Figs. 46 and 51 are compensated by the differences in their rates of growth. An abnormally high Neumann spectrum is developing so slowly that its full wave height is hardly ever reached in practice in strong winds. The much lower wave height indicated by Darbyshire's (1955) spectrum, on the other hand, is quickly attained. The differences among the different formulations are therefore much reduced for the wind durations most frequently found in practice.

The mean of $1/10$ highest waves in a 40-knot wind is shown in Fig. 47 as a function of wind duration. This figure shows the wave heights *expected* in idealized conditions of uniform wind over unlimited fetch. Walden (1953/54) collected the material to show what was *actually observed*. Weather and visual wave-observation data had been radioed at regular intervals by ten weather-observation ships in the Atlantic Ocean. This information was systematically collected at the sea-weather bureau (Seewetteramt) in Hamburg, Germany. These records, covering the period from February to December 1953, were examined in conjunction with working weather maps.

Out of a very large number of records, 515 suitable for wind-duration effects, and 264 suitable for fetch-effect evaluation were chosen. In these records the wind started suddenly, reached full force in a short time and then blew for a long time with approximately uniform velocity. In addition, in the records chosen, the sea was calm before the wind started. The conditions were sufficiently close to the ideal ones implied in spectrum formulations so that only minor computational corrections were necessary. The results of these investigations (for winds of 26 and 30 knots) are shown in Figs. 52 and 53 for the effect of wind duration and Figs. 54 and 55 for the effect of the fetch. Of the wave-forecasting

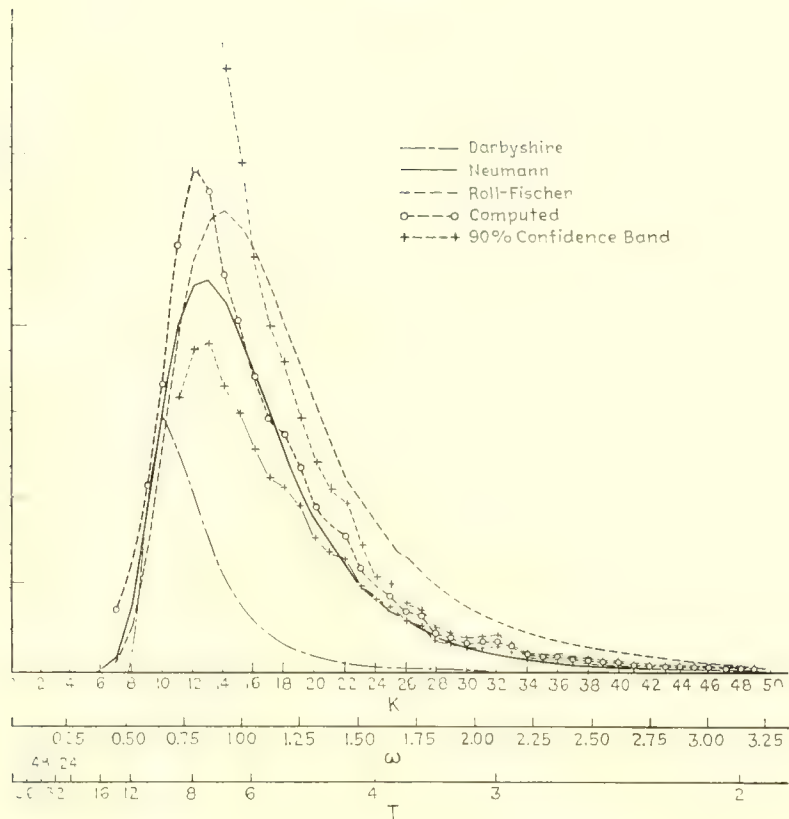


Fig. 51 Computed spectrum and its 90 per cent confidence bands compared with theoretical Darbyshire, Roll-Fischer, and Neumann spectra for an 18.7-knot surface (or 28.05-knot gradient) wind (from Neumann and Pierson, 1957)

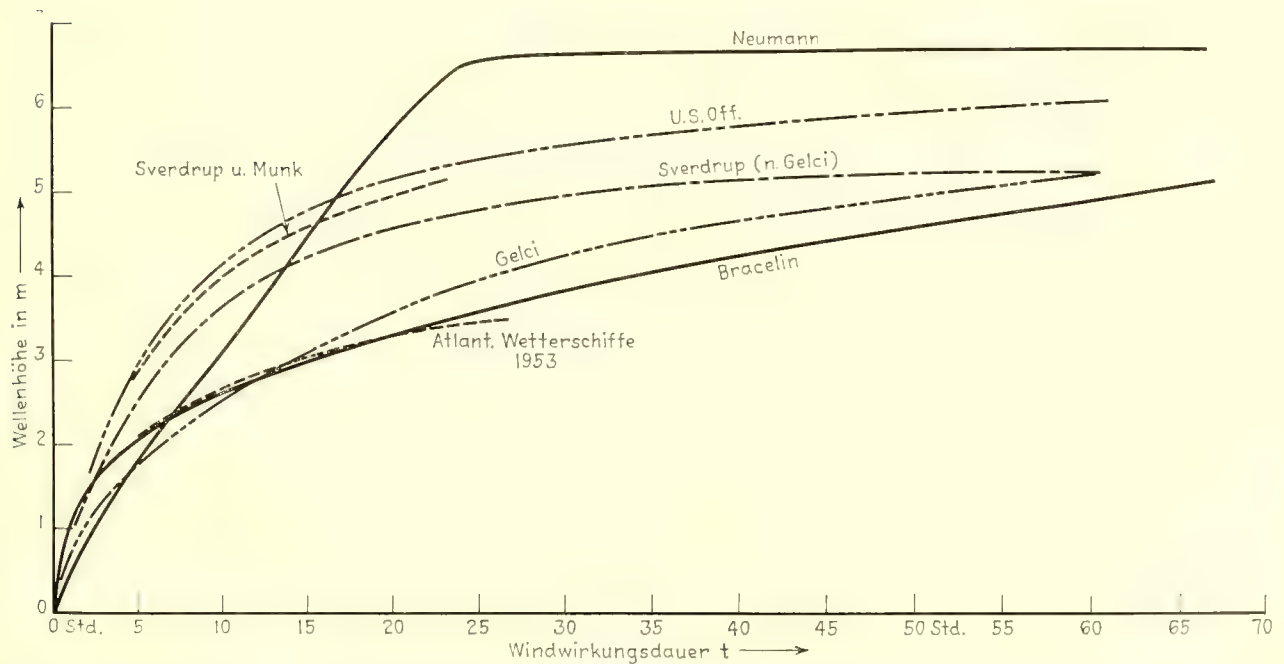


Fig. 52 Relationship between wind duration and wave height at wind speed of 30 knots. Ordinates are heights of significant waves in meters. Abscissas—wind duration in hours. Walden's data, obtained from ten weather-observation ships are shown by dotted line labeled "atlant. Wetterschiffe 1953" (from Walden, 1953/54)

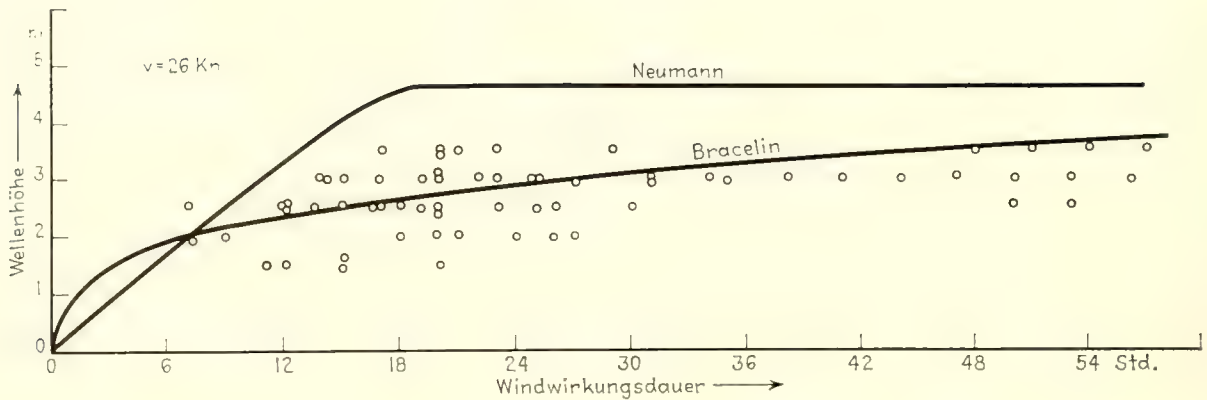


Fig. 53 Relationship between wave heights and wind duration at wind speed of 26 knots. Labeling of co-ordinates is same as in Fig. 52. Dots indicate weather-ships' observations (from Walden, 1953/54)

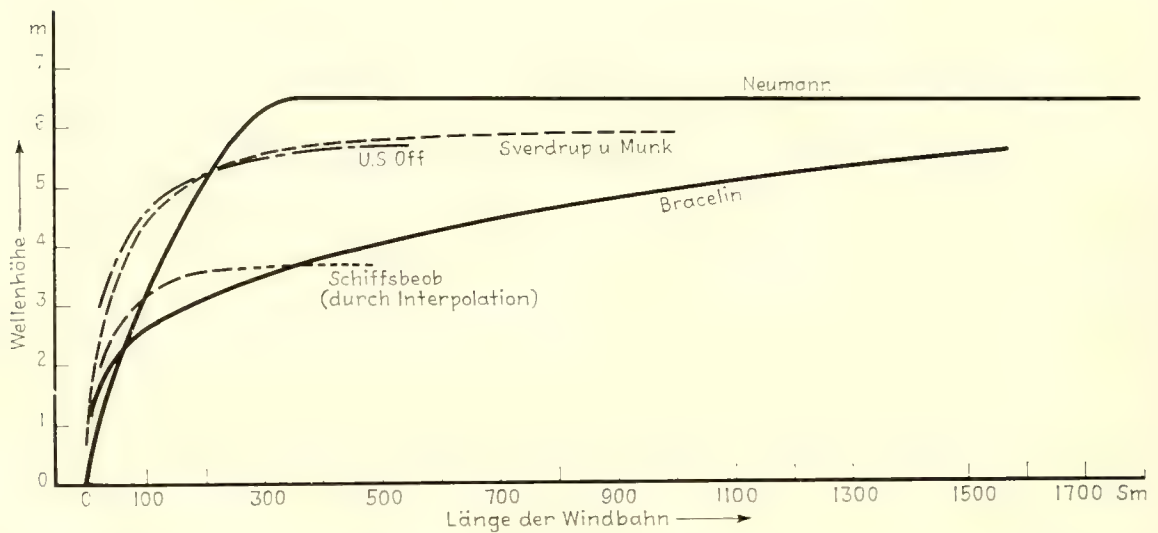


Fig. 54 Relationship between fetch and wave height at wind speed of 30 knots. Ordinates and abscissas are significant wave heights in meters and fetches in nautical miles, respectively. Walden's summary of weather-ships' observations is shown by dotted line labeled "Schiffsbeob" (from Walden, 1953/54)

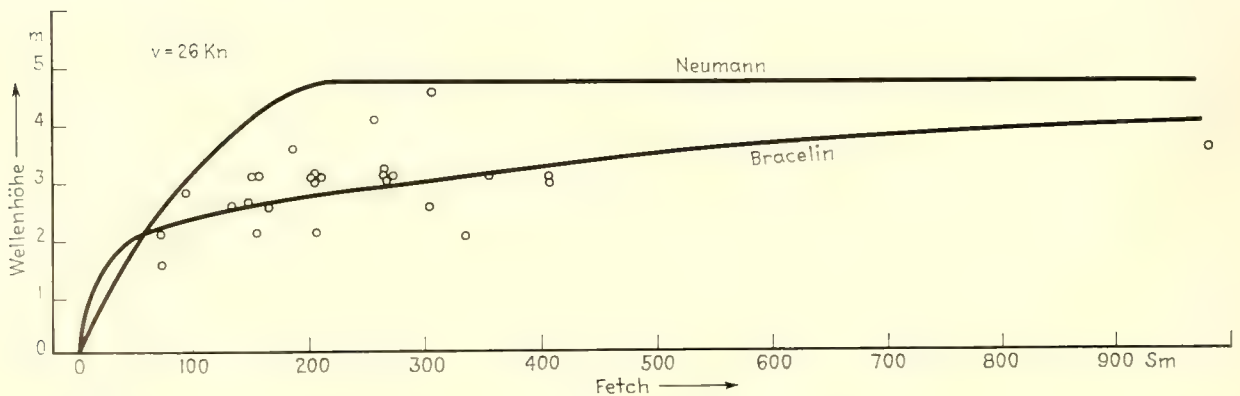


Fig. 55 Relationship between fetch and wave height. Labeling of co-ordinates is same as in Fig. 54. Dots indicate weather-ships' observations (from Walden, 1953/54)

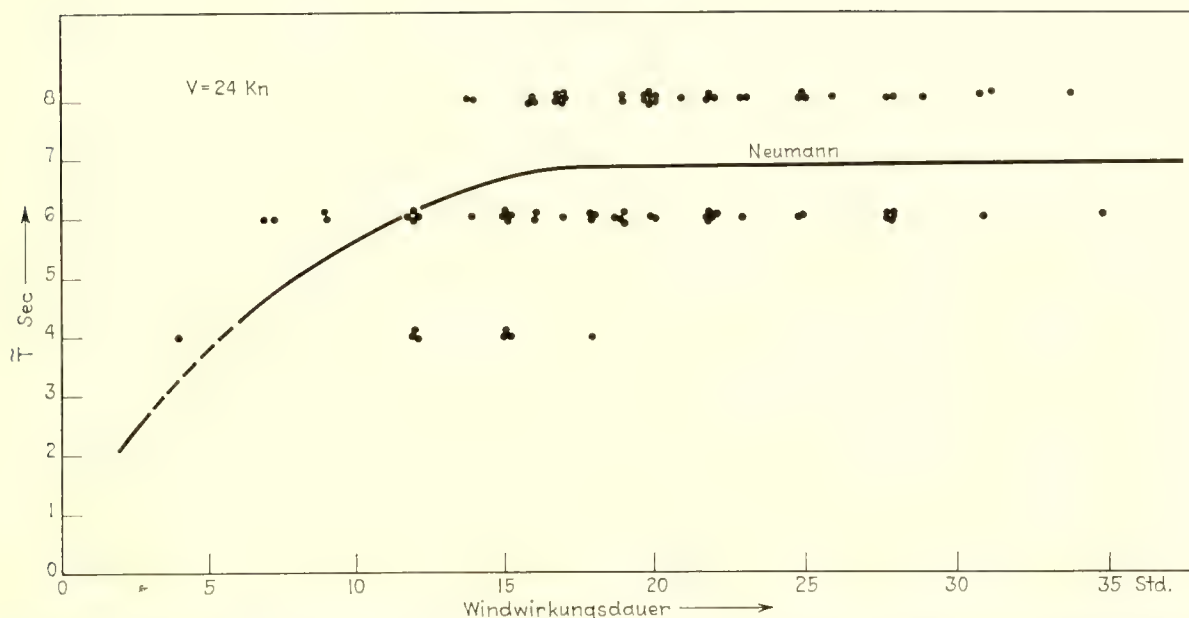


Fig. 56 Apparent wave periods plotted versus 24-knot wind duration (abscissa scale in hours). Dots indicate weather ship reports made in intervals of 2 sec (from Walden, 1953/54)

methods, Bracelin's (prespectrum) one gives results in best agreement with Walden's analysis of weather-ship observations for wind duration up to 30 hr. For longer duration it somewhat exaggerates the wave heights. Neumann's method gives a too slow wave rise during the first 5 hr of wind action, but strongly exaggerates waves after that.

Fig. 56 compares the apparent wave periods computed by Neumann's method with those reported by weather ships. The apparent periods are reported by ships in 2-sec intervals, and the resultant series of points is seen to straddle Neumann's period when wind duration exceeds 15 hr for a 24-knot wind. This finding is in agreement with results demonstrated by Neumann in Fig. 41. It can be considered as partial confirmation of the shape of the Neumann spectrum. The prediction of the wave height, indicated by this spectrum, can be improved following suggestions by Walden. On the basis of his data Walden (1953/54) recommended reduction of the constant in Neumann's spectrum expression. Later, Walden (1956) recommended a reduction of this constant on the basis of a reinterpretation of Neumann's Fig. 36. The apparently too slow initial rise of waves shown by Neumann's method remains uncorrected.

6.53 Practical wave prediction. In the cases chosen by Walden in the preceding section, weather conditions had to correspond to the assumptions made for the spectrum formulations. In normal wave forecasting, however, the effects of fetches with different wind velocities, of moving fetches,³⁵ of several fetches together,

of swells from distant storms, and so on, must be considered. It almost appears that the initial spectrum formulation becomes subordinate to the forecaster's reading and interpretation of weather maps. Nevertheless, the particular formulation is important since a forecaster will get different results following different forecasting methods. Comparisons of the results of different methods by Rattray and Burt (1956) and by Walden and Farmer (1957) will be cited here.

(a) **Rattray and Burt (1956).** Rattray and Burt presented a hindcast of an abnormally severe storm in the North Pacific Ocean. The forecasting methods of Sverdrup and Munk (1947), of Bretschneider (1952) and of Pierson, Neumann and James (H) were used, and the results of each were compared with observations of the weather ship which was in the storm area. Fig. 57 shows the wind condition and the results of the comparison in the form of significant wave heights and significant periods. These are defined as the average height and period of one-third highest waves.

The highest significant waves were apparently correctly computed by all methods. Hindcast time histories of wave development and decay deviate considerably from the observed ones. The wave-period hindcast following Sverdrup and Munk is best.

The word "apparently" was used advisedly in the first sentence of the foregoing paragraph. In formulating the spectra and forecasting methods the air-water temperature differences are averaged out and usually are not considered specifically. In the application of a forecasting method to an individual case, however, it is advisable to consider the temperature difference since it

³⁵ References, Walden (1956b), Wilson (1955).

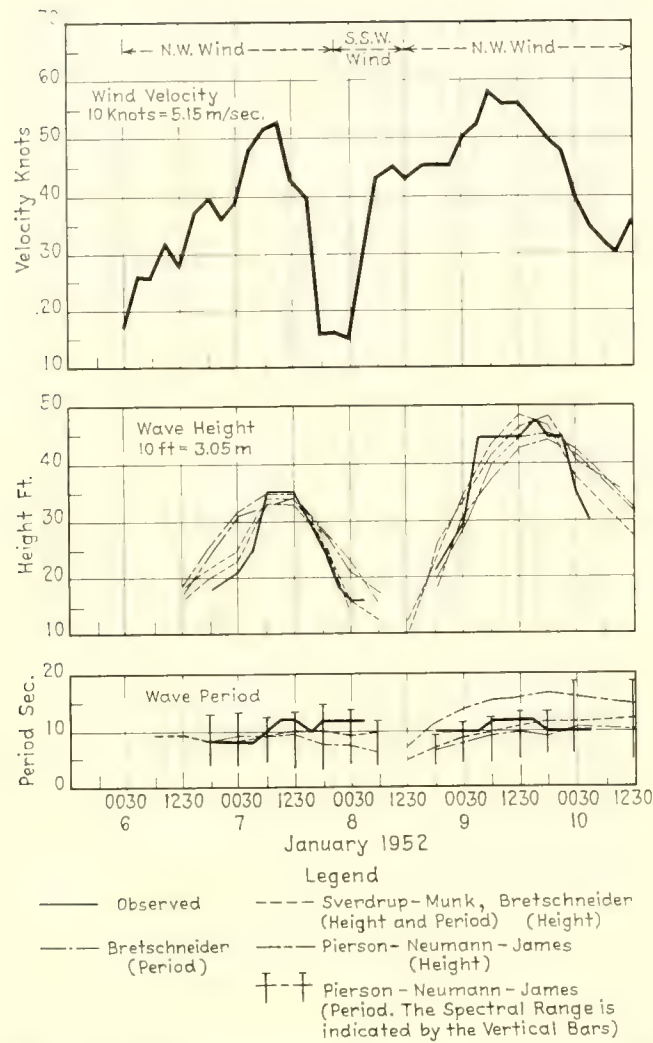


Fig. 57 Comparison of hindcast and observed waves at Ocean Station Papa (50°N, 145°W) (from Ratray and Burt, 1956)

may make considerable difference in wave height. Cold air over warm water is unstable and more turbulent. It has, therefore, greater capacity for exciting waves. There is no generally agreed method of allowing for this effect, and it appears that Bracelin (1952) is the only one to have included it in his forecasting instructions. Fig. 58 shows the relationship between the wave height and sea-air temperature difference according to Roll (1952). In the case illustrated here the air was 12 deg cooler than the water, and (by extrapolation) a factor of possibly 1.3 should have been applied to the average predicted wave heights. Good agreement without this factor indicates that the “averaging” methods of prediction exaggerate the wave height by possibly 30 per cent.

(b) **Walden and Farmer (1957).** Walden and Farmer made 47 wave recordings on the MV *Atlantis* of Woods Hole Oceanographic Institute in November and December 1947, on a cruise along the 50.5° W meridian from Newfoundland Banks to the South American Conti-

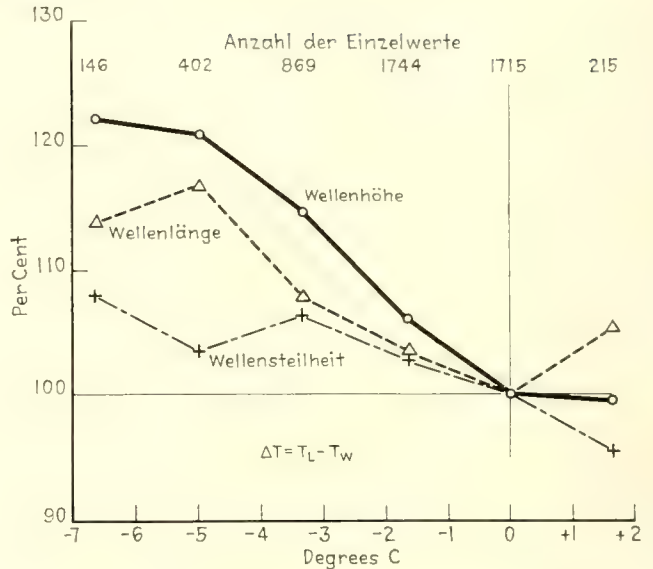


Fig. 58 Increase of wave height and steepness plotted versus air-water temperature difference. Air temperature is designated T_L , water temperature T_W . Numbers above diagram indicate number of individual observations included in each plotted point (from Roll, 1952)

Table 10 Data Corresponding to Walden and Farmer's Fig. 59

Figure Code	Date, November	Hours-Minutes	Latitude ^a N	$H_{1/3}$, ^b ft	\bar{T} , sec
a	14	08-55	40.9	11.5-11.7	7.9-7.5
b	14	11-00	40.8	13.0-13.2	8.3-8.1
c	14	17-30	40.5	9.0-9.3	7.1-6.9
d	16	14-15	38.6	4.6-4.9	6.3-6.1
e	18	12-32	35.6	8.7-8.8	7.7-7.5
f	22	01-37	27.9	17.0-17.0	9.1-8.9

^a Longitude uniformly 50.5° W.
^b Determined from spectra using Longuet-Higgins' (1952) relationships for a narrow spectrum.

Table 11

Pierson, Neumann, James	Bretschneider and Wilson	Walden ^a	Darbyshire
+19.6 (10) ^b	+52.0 (14)	+7.1 (17)	-7.0 (5)

^a Walden (1953/54, 1954, 1955, 1955/56a, 1955/56b).
^b Figures in parentheses indicate the number of analyzed cases.

ental Shelf. Fifteen of these were analyzed by the Hydrographic Office, Washington, D.C., and representative wave spectra for six records are shown in Fig. 59. The wind and wave conditions corresponding to these spectra are given in Table 10.

The wave records were taken with Tucker's (1956) ship-borne wave recorder. The dotted lines in Fig. 59 show the spectra corrected for the pressure-gage depth. Recordings were of 20-min duration, and were taken mostly with the ship drifting and in a few cases at very low speed in head seas. The local wind was measured

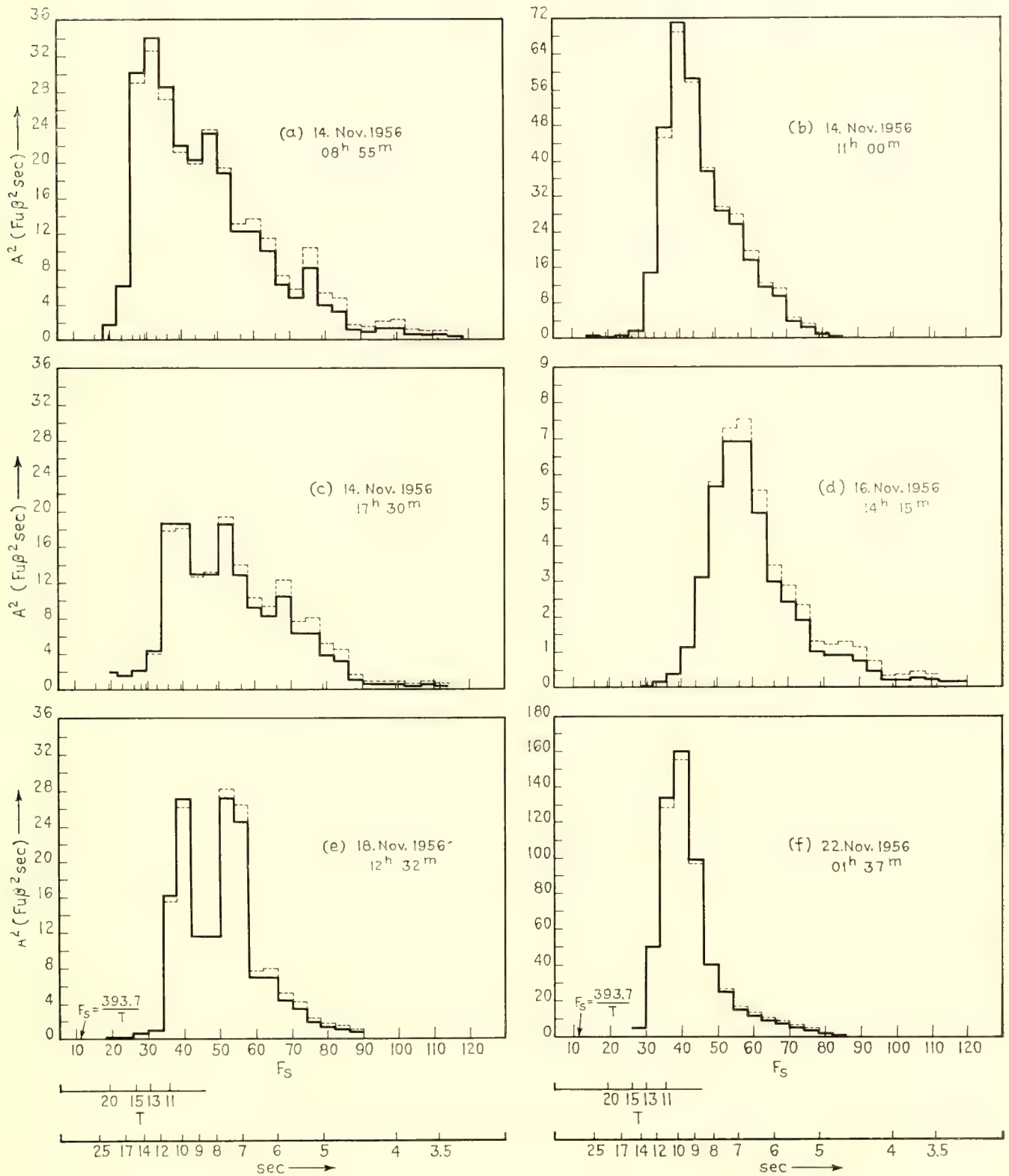


Fig. 59 Energy spectra $E(f)$ of waves listed in Table 10 in ft²/sec (from Walden and Farmer, 1957)

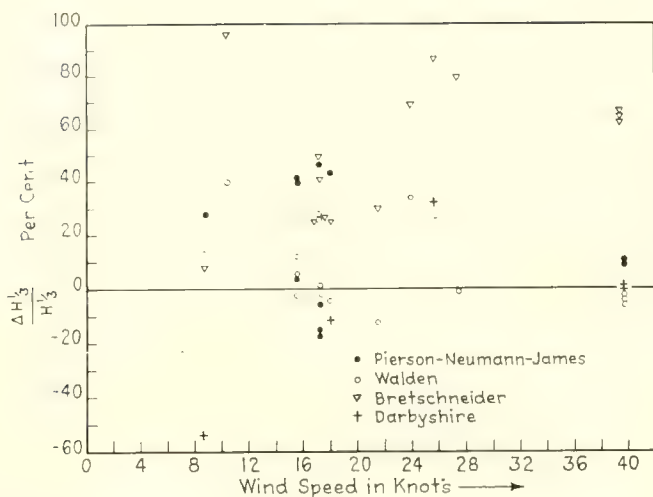


Fig. 60 Deviations of hindcast wave heights from observed ones in per cent for different methods of hindcasting (from Walden and Farmer, 1957)

with an anemometer on the ship's mast at the height of 30 ft. Hindcasts were made on the basis of weather maps. The percentage deviations of the hindcasts from the observed significant waves are shown in Fig. 60, and the average deviations in Table 11.

It will be observed that although there is a hopeless disparity between Neumann's and Darbyshire's spectra shown in Figs. 45 and 51, the disparity in the results of practical applications of their methods to normal sea conditions is much less.

6.6 Descriptive Spectrum of Voznessensky and Firsoff (1957). The spectra outlined in the foregoing section describe an irregular sea as a function of wind speed, duration and fetch. They have been developed for the purpose of forecasting sea conditions and usually require the perusal of weather maps for their evaluation. There is, however, another problem, that of describing an observed sea regardless of the wind and fetch condi-

tions which caused it. Such a description is needed, for instance, when a comparison of a ship's motions and stresses with wave conditions is called for.

The foregoing parts of Section 6 have dealt with prediction of wave spectra. The last section (8) of the monograph will be concerned with the analysis of a wave record which will give a description of a sea state without resort to its previous history. However, everything pertaining to the irregular sea must be handled by statistical methods, and a standardized (but flexible) sea description is desirable in this connection. Voznessensky and Firsoff developed one such form of wave-energy spectrum in connection with their study of a ship's rolling.

It will be shown later in Section 8 that the first result of a digital analysis of a wave record is the "auto-covariance function," the plot of which is called a "correlogram." Fig. 65 shows nine correlograms obtained by the analysis of instrumental wave records. All correlograms are similar in three aspects: the maximum ordinate is at zero on the abscissa scale, there are periodic oscillations and a decay in the amplitudes of oscillation. Such functions are approximately expressible by an equation of the form

$$R(\tau) = R(0)e^{-\alpha\tau} \cos \beta\tau \quad (103)$$

where, for the time being, τ can be considered simply as a number (designating time) on the abscissa scale of a correlogram. It will be shown in Section 8 that at $\tau = 0$, the function $R(0)$ represents the area of a spectrum, and is therefore a measure of the wave height. The symbol E will therefore be substituted for $R(0)$. The parameter α is a measure of wave irregularity. The autocovariance or correlation function of a regular sinusoidal wave is also a sinusoidal wave, and therefore $\alpha = 0$ in this case. The rate of attenuation of oscillations increases with broadness of spectrum; i.e., when the distribution of the spectral energy of the irregular sea is over a wider band of frequencies. This is indicated by increasing values of α . The parameter β is the circular frequency of oscillations. In the limiting case of a sinusoidal wave, β is the fre-

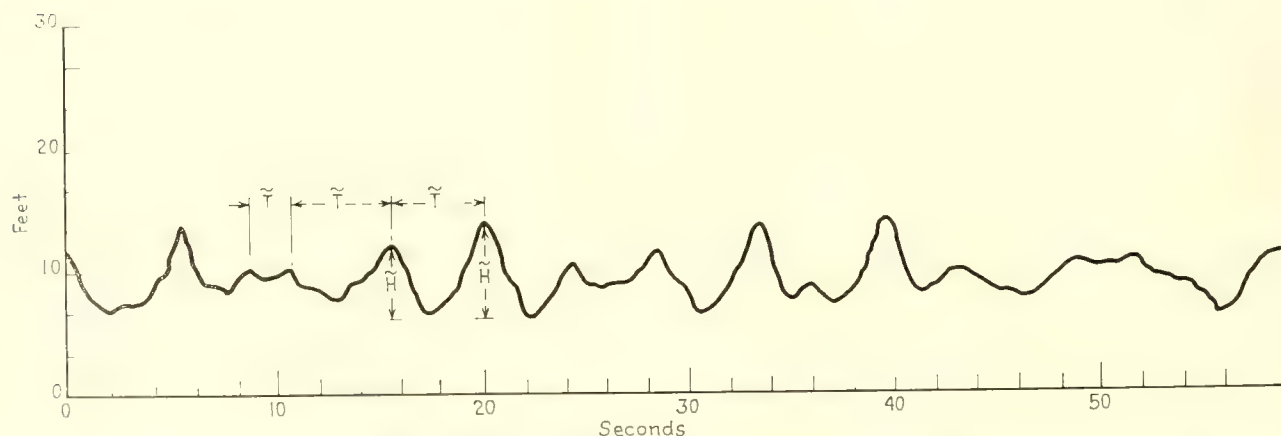


Fig. 61 Definitions of apparent wave heights and periods in measurements from a wave record (from Neumann, 1954)

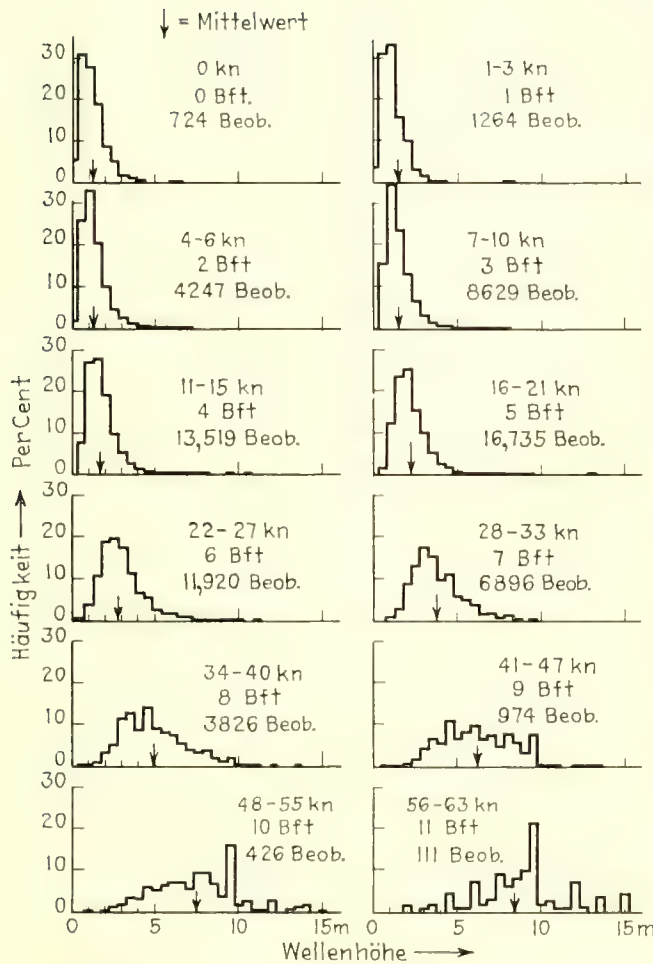


Fig. 62 Waves at different wind speeds. Frequency distribution of ten weather-ships' observations in Atlantic Ocean (from Roll, 1955)

quency ω of this wave. In the case of an irregular sea it appears to bear a relationship to the mean wave frequency.

The spectrum obtained from equation (103) by a Fourier transform (see Section 8) is

$$E(\omega) = \frac{2E}{\pi} \alpha \frac{(\omega^2 + \alpha^2 + \beta^2)}{(\omega^2 - \alpha^2 - \beta^2)^2 + 4\alpha^2\omega^2} \quad (104)$$

The parameters α and β were evaluated on the basis of analyses of numerous records made in 1951–1952 in the Barrents Sea using a wave-measuring buoy. Individual records contained on the average 400 waves. The parameters were found to be functions of average wave height. They are shown in Fig. 66 plotted against average wave height.

Voznessensky and Firsoff have thus defined a sea spectrum in terms of the parameters E , β , and α . These parameters correspond to the spectrum characteristics most important in defining ship motions; namely, wave height, frequency at the maximum of the spectral

density curve, and degree of sea irregularity respectively. The spectrum was derived on the basis of instrumentally measured and analyzed waves but the definitions can be useful in connection with visual observations or (preferably) visual inspection of wave records. The mean

apparent period \tilde{T} of the waves can be estimated either by visual observations or by inspection of wave records. A small amount of additional research may establish the quantitative connection between the observed frequencies and the parameter β . Likewise, a measure of the sea irregularity can be devised on the basis of which the parameter α can be evaluated. Thus a three-parameter description of an observed sea spectrum can be developed for use by ships' personnel, particularly aboard weather ships.

The above remarks should not be construed as recommending visual observations. It is highly desirable that ship-borne wave recorders be used on all weather ships. It is also desirable that shipboard analyzing equipment be provided. However, with Voznessensky and Firsoff's method there is a possibility that, after a small amount of additional research, results of visual wave observations, of visual examination of wave records, and of spectral analysis of wave records can be presented in the same spectral form. The method provides therefore the possibility of collecting spectral sea data on a broader scale than heretofore.

7 Statistics of Directly Observable Sea Waves

Three stages are discernible in the observation and study of sea waves, the first, visual observation with a stop watch as the only instrument, the second, instrumental recording of wave elevations, and the third, the development of spectral analysis techniques. The concept of a wave spectrum and the formulations of various spectra were discussed in Section 6. Methods of evaluating spectra from wave records and various mathematical problems arising in this process will be discussed in Section 8. The present section will be concerned with the handling of data obtained in the first two stages prior to the emergence of the third. The older methods are by no means superseded by the spectral techniques; all are being currently used for different purposes.

The great bulk of past wave data has been obtained by visual observation. Instrumental recordings until recently were obtained from pressure recorders located at appreciable depth near shore. The only instrumental wave recordings in the open sea available at present are those of Darbyshire (1956) and of Walden and Farmer (1957). The same form of simple statistical analysis has been used for the visual observations as for the instrumental recordings. Fig. 61 shows a section of a long record of irregular sea waves. The record is clearly not sinusoidal, but one can nevertheless speak of the time intervals between successive crests as "apparent periods," designated as \tilde{T} in order to distinguish them from the true period T of a sinusoidal wave. Likewise, the vertical distance from a trough to the neighboring peak is

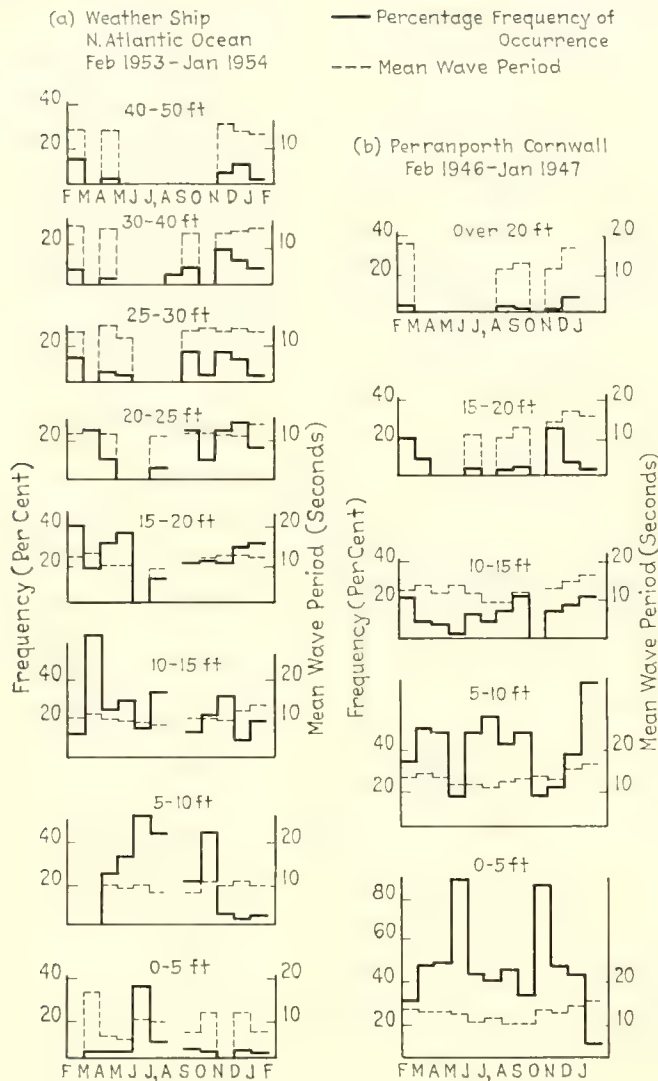


Fig. 63 Monthly distribution of wave heights (from Darbyshire, 1952a)

called "apparent height," or \bar{H} . While described thus with respect to a record, the apparent amplitudes can be judged by direct visual observations, and the periods measured with a stop watch.

Both \bar{T} and \bar{H} are highly variable. A number, say 200, of consecutive apparent periods can be measured and collected in a table. The entire content of this table is then divided into classes of equal intervals, for instance 0.5 sec, so that the classes are from 2 to 2.5 sec, 2.5 to 3 sec, 3 to 3.5 sec, and so on. The number of observations falling into each class is plotted as ordinate versus \bar{T} as abscissa. The actual frequency of occurrence of each class may be plotted, but it is preferable to plot the ratios of these numbers to the total number of readings. These percentages are usually referred to as the "frequency distribution." Examples of such diagrams which are often called "histograms," are shown in Fig.

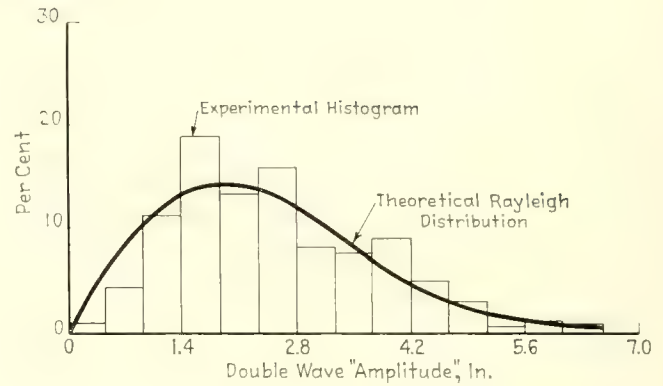


Fig. 64 (From Lewis, 3-1956)

41. Similar diagrams are prepared to show the frequency distribution of wave heights. Fig. 62 shows the wave height distribution in the Atlantic Ocean based on visual observations reported by ten weather ships and subdivided according to wind speed ranges on Beaufort scale. The last number in the legend of each subdivision indicates the number of wave reports used in making the plot. Fig. 63 shows the comparative distribution of wave heights as measured by Darbyshire with a shipborne wave recorder in the open ocean and with a pressure instrument off the coast of England. Frequency distributions of apparent wave periods and heights in a wind flume test are shown in Fig. 12.

Consider a histogram of the wave height and designate by N the total number of heights recorded, by H_i the heights of individual waves, and by $\Delta N/N$ the percentage of points in a class of interval ΔH . If the intervals are sufficiently small and a smooth curve is drawn to replace the stepped histogram, the "probability density" of H_i can be defined as

$$p(H_i) = \lim_{\Delta H \rightarrow 0} (\Delta N/N)/\Delta H \quad (105)$$

The integral of this curve

$$P(H_i) = \int_0^{H_i} p(H_i) dH \quad (106)$$

is known as the probability of H_i . It defines the percentage of the total number of observations N (assumed to be very large) which will have values below a specified wave height H_i .

The histograms shown in the figures have an apparently wide range of shapes and appearances. However, there is cogent reason for believing that the frequency distribution of a large number of individual measurements of periods or amplitudes of waves follows a well-defined law developed in the statistical theory.

The following parameters are first defined:

$$\text{Mean amplitude } \bar{H} = \frac{1}{N} \sum_{i=1}^N H_i \quad (107)$$

$$\text{Variance } \sigma^2 = \frac{1}{N-1} \sum_{i=1}^N (H_i - \bar{H})^2 \quad (108)$$

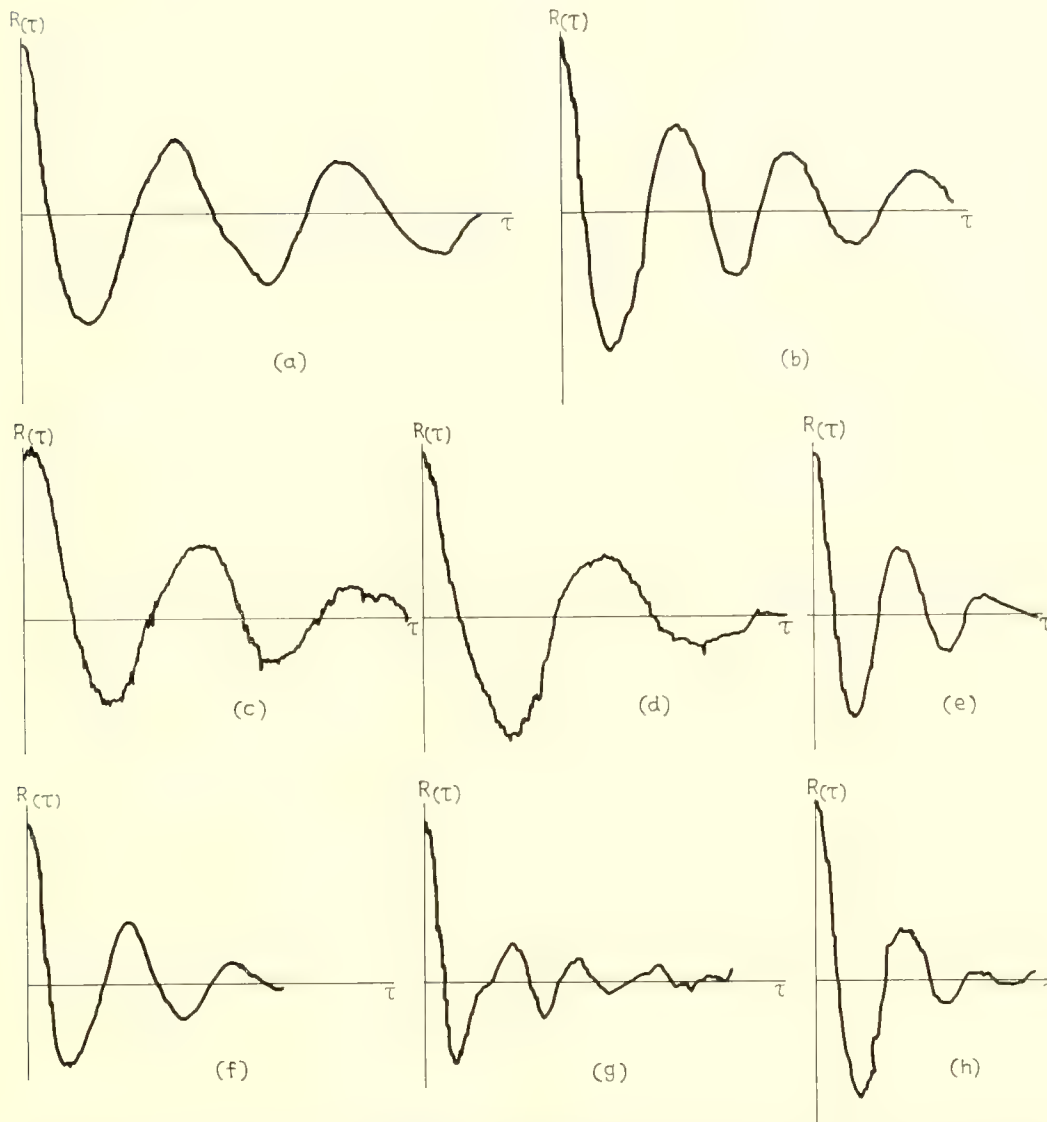


Fig. 65 Sample autocovariance diagrams obtained from an analysis of waves recorded at sea (from Voznessensky and Firsoff, 1957)

where $H_i - \bar{H}$ is the deviation of an individual wave height from the mean value.

$$\text{Mean square } E = \frac{1}{N} \sum H_i^2 \quad (109)$$

Three frequency distribution laws have been formulated:

1 The "normal distribution":

$$p(H_i) = \frac{1}{\sigma \sqrt{2\pi}} \exp [-(H_i - \bar{H})^2 / 2\sigma^2] \quad (110)$$

2 The "log-normal distribution," a normal distribution of the logarithm of H_i :

$$p(H_i) = \frac{1}{\sigma \sqrt{2\pi}} \exp [-(\log H_i - \overline{\log H_i})^2 / 2\sigma^2] \quad (111)$$

where $\overline{\log H_i}$ denotes the mean value of the logarithms of H_i .

3 The "Rayleigh distribution" defined as

$$p(H_i) = \frac{2H_i}{E} \exp (-H_i^2/E) \quad (112)$$

where

$$H_i \geq 0$$

Fig. 64 shows the histograms of the wave heights in an irregular sea test in a towing tank and the corresponding Rayleigh distribution. A simple derivation of this distribution, equation (112), will be found in the Appendix to Williams' (3 1952) paper. General discussion of the foregoing and numerous samples of sea and ship motions

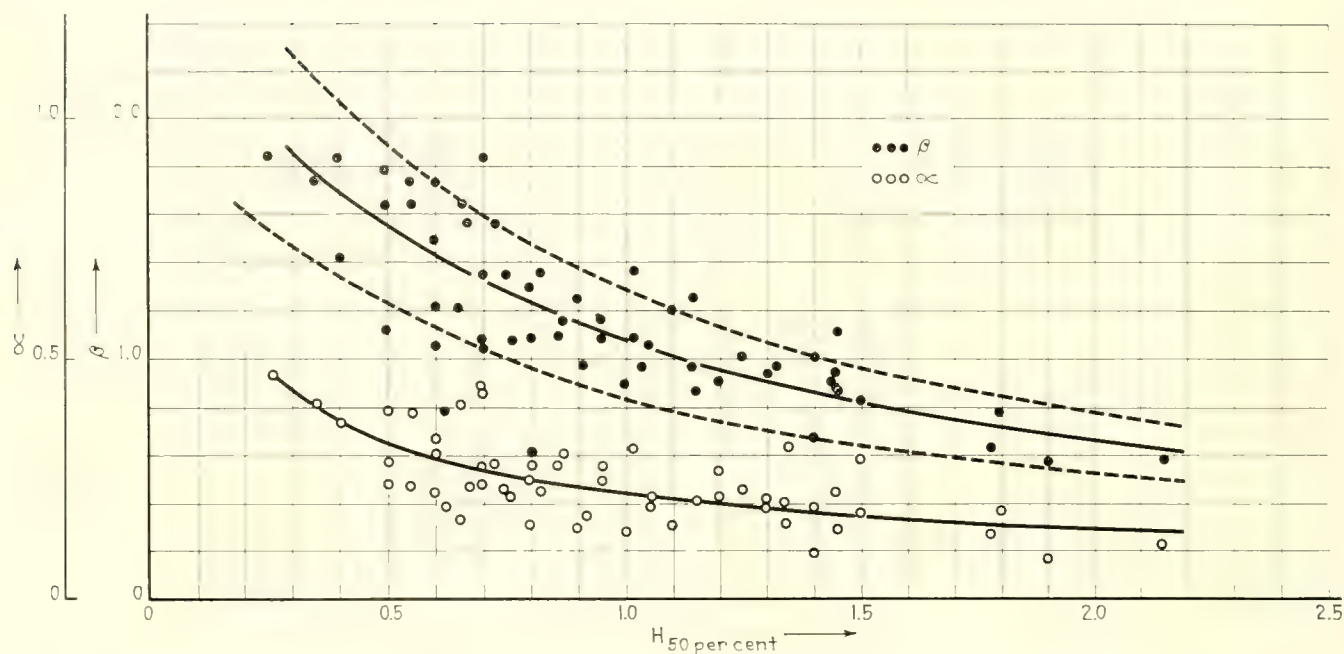


Fig. 66 Values of parameters α and β which define shape of autocovariance function (from Voznessensky and Firsoff, 1957)

were presented by Jasper (3-1956). Reference was made to H. Cramer, "Mathematical Methods of Statistics," Princeton University Press, Princeton, N.J., 1946.

The histograms describe an observed sea without disclosing functional or causal relationships. By plotting a series of histograms (as for instance in Fig. 62 for a series of wind speeds), an idea of the functional relationship may be achieved, but not a mathematically defined quantitative relationship. Since the plan of this monograph is to trace the quantitative effect of waves on ship motions and on ship stresses, the histogrammatic approach will not be discussed further. Attention will be concentrated on spectral analysis, which permits the formulation of functional relationships.

8 Mathematical Representation of the Sea Surface

8.1 Probabilistic Versus Deterministic Representation. In the first five sections of this chapter, the reader is acquainted with the great amount of time and effort that has been expended in the development of the various theories relating the generation and growth of ocean waves to the winds blowing over the sea surface. It is quite natural that the reader should come to expect at least some attempt at a partial unification of all this work with perhaps a view toward formulation of a deterministic representation of the sea surface. Unfortunately, and regrettably, this is yet to be done. Notwithstanding the wealth of ideas and material revealed in these five sections, a champion is yet to appear on the oceanographic scene who can specify the precise behavior of the sea surface for all time and/or space simply on the

basis of a set of initial conditions. The fact is, we just don't know enough about the interaction of the wind with the sea surface to do this.

This is not difficult to understand if one but looks at a time history of the waves passing a fixed point, Fig. 61. Even this simplified description of the seaway illustrates quite clearly the absence of any regularity; high waves follow low waves in no set pattern and even "period" measurements fail to demonstrate any regularity. To make matters worse, the appearance of extra small wiggles in the record defies classification and many high-frequency wavelets are not even sensed by most transducers. Examination of very long records (1 hr duration) reveals that no portion of the record repeats itself throughout. It is natural therefore to assign a property of "randomness" to ocean waves. On the basis of this assumption and fortified with ignorance of any deterministic method for representation of the sea surface, the appeal of a probabilistic definition could hardly be resisted.

The suggestion of probabilistic methods was not immediately embraced because there was a natural suspicion (by those not well grounded in this branch of mathematics) of the implication of qualified statistical averages. However, when the virtues of probability theory are examined, in terms of defining the state of the sea, the matter takes on a positive hue. Consider again, the random (stochastic) process in Fig. 61. One fact becomes strikingly clear. This precise event, recorded in the open sea, may be expected *never* to occur again. Consequently, any description of this event as a particular realization of the seaway is only useful in a very

limited way. The basic problem is to define that particular seaway from which this "sample" record was obtained.

This record is only one of many samples which might have been taken from the population which is that particular seaway. Other records of the same length, taken at other times and other places, will in general yield, if long enough, the same information on the state of the sea, provided the seaway retains its statistical homogeneity throughout the sample space and time; that is, remains stationary.

The sample record is then seen to be one member of a large family of possible sample records. It may be thought of as a single function in an ensemble of functions. If in addition to being stationary and random (stochastic), the sea surface is Gaussian as well (that is, points read at random from the sample record are distributed according to the normal probability law), then the sample record lends itself to computation of certain statistical quantities. An important property of such ensemble functions is given by the Ergodic theorem, for stationary Gaussian processes, which states that time and space analyses yield the same statistical properties as an ensemble analysis. In other words, *if the seaway is stationary and random and Gaussian then analysis of a long enough sample record from this sea surface may define (statistically) the seaway for all time and space where these assumptions apply.* This is what is desired in most expositions relating to the seaway. One sacrifices determinism for the statistical properties of one member (record) of an ensemble of functions which, if it describes the average statistical properties of that ensemble, is a good description of the parent population (seaway) from which the original sample was drawn.

Ocean waves are observed to be random in that they are, in general, unpredictable space and/or time-wise. Analysis of wave elevations chosen at random from many different wave records has shown a high percentage of frequency distributions which conform well to the normal probability (Gaussian) law. This permits the computation of certain statistical quantities which describe the distribution of wave heights in the seaway from which the particular sample (record) was taken. (See Section 8.6). Stationariness is somewhat arbitrary and usually is based on the appearance of the record. If the waves are not obviously growing or decaying during the recording period, the seaway is considered to be stationary for that interval of time.

The treatment of wave data is independent of the conditions just mentioned. However, interpretation of the results depends very much on the assumptions that the data are stationary, Gaussian, and that only a relatively narrow range of wave frequencies is involved. Justification for results of wave analysis is on the user of the results and not on the experimental data or on the methods of analysis. The fact that the complexity of waves forces statistical treatment does not, however, relieve one of the responsibility of exercising good judgment in the experimental design of wave analysis.

The tool most applicable to realization of a probabilistic definition of waves is the energy spectrum of the sea surface. This is essentially an analysis of the variance, or second moment property of the record of the seaway. Some of the seaway statistics derivable from such an analysis were given in Section 7; more will be discussed in this section.

At this stage then, determinism has been abandoned (at least temporarily) for probability methods, and the key to a statistical description of ocean waves is found to be the energy spectrum, because it may yield the kind of information desired of this stationary, random process.

As has been noted, the qualifications pertaining to the process have no bearing on computation of the spectrum but relate only to the statistics derivable from the spectrum. In this connection, the applicability of such terms as stationary and Gaussian are somewhat arbitrary. It is generally agreed that "nearly stationary" and "nearly Gaussian" conditions are acceptable assumptions, but these again are somewhat arbitrary and will bear further discussion later on.

Section 6 has already introduced the reader to various attempts at an analytical formulation of a one-parameter family of spectra that is representative of the sea surface, under any conditions. Which particular formulation is the best is still open to question. The problem however is being investigated vigorously by a number of oceanographers and although some of these are quite adamant in their particular beliefs, at this time, basic differences in their spectrum formulations are not so great that a mutually acceptable and reasonably reliable spectrum may not be realized at some future time.

The "empirical-theoretical" spectra discussed in Section 6 are useful in that they may describe the state of the sea, if once the time-space variation in the wind field is determined. If the point of wave observation and the date and time are specified, then weather maps are consulted for resolution of the wind field and by the carefully documented work of, for example, Pierson, Neumann and James (*H*),³⁶ the state of the sea may be defined. The importance of this work lies in the fact that it enables prediction treatment of partially developed seas, (slowly) changing seas, and swell originating from distant storms. If the spectrum formulation of Pierson, Neumann, and James is unacceptable to an investigator, their work still permits the adaptation of any other spectrum form given in Section 6.

The use of weather maps (spaced 6 hr apart) is still somewhat subjective and it is desirable, for the time being, to be able to estimate the energy spectrum of the seaway from wave records or other suitable measurements. This is especially true for full-scale seakeeping trials where the results always relate to the state of the sea. A mechanically observed, computer-derived state of sea is the only accurate description available. As such, it is also used in wave research in attempts to verify the

³⁶ Other sea-state determinations have been given by Bracelin (1952); Bretschneider (1959)(see ref. p. 105); Gelci, Cazalé, and Vassal (1957), and Walden (1958).

spectrum formulations of Section 6. In fact, it is very likely that a great many wave spectra of different sea conditions will need to be measured and interpreted before a particular equation will be settled upon as adequately descriptive of all possible wave conditions. It should be mentioned that some oceanographers are at present engaged in collection of wave data on a large scale for the primary purpose of settling the wave-spectrum formulation question. A symposium dedicated to the wave spectrum will be held in 1961, and perhaps some unification of wave theories will result.

The intent of this section is to quit the realm of questionable sea spectrum prediction and to acquaint the reader with this alternative formulation of the spectrum of the seaway; namely, that derivable from direct observation of waves and analysis via the concepts of random process theory. Naval architecture is indebted to Prof. W. J. Pierson, Jr. for recognition of the applicability of random-process theory (well known in the field of communications), to ocean waves and ship motions, and for making these methods available to naval architects for practical use (Pierson, 1952*a*; St. Denis and Pierson, 1953).

Several quotations from Press and Tukey (1956) will partially summarize what has been discussed in these introductory remarks and will suggest what is to follow:

"Fourier methods in particular have found wide regions of application in aeronautics and have provided many useful results The introduction of a chance or random element leads naturally to the application of probability and statistical notions which in combination with classical Fourier techniques have in recent years evolved into a general technique for the analysis of problems of this type. These techniques stem from the field of random-process theory and from generalized harmonic analysis. Of particular appropriateness for present purposes are the methods centered around the use of power spectra Fourier techniques are described, as those techniques, which when fused with statistical theory, form the basic mathematical structure for spectral methods."

8.2 Evolution of the Description of the Seaway from Wave Measurements. Most of the discussion in Section 8 will be devoted to the formal reduction of wave data to energy spectra and the problems which arise from the nature of the data and from the various means of data handling. In order to provide the reader with a suitable perspective that will prepare him for exposure to some of the subtleties of random-process theory, it will be worth while to discuss the various paths which may be taken, in proceeding from the observation stage to the final form of sea-state representation.

There are five basic steps involved in the realization of statistical information from the energy spectrum of waves: 1) Observation, 2) recording, 3) preparation, 4) reduction, 5) presentation. It is not necessarily the case that all five steps must apply (for example, one may go directly from the transducer to the analyzer), but these exceptions will be rare occurrences pertaining to

operational use of the energy spectrum rather than to the energy spectrum as a descriptive tool in a particular investigation.

8.21 Observation. Transducers used in wave observation are more varied than those for any other event related to seakeeping. The reason is that waves are harder to measure than ship motions. One can find in the literature such waves sensors as: graduated staffs fixed to piers, pressure transducers on the bottom, floating (capacitance, resistance, inductance) poles, floating accelerometers, stereo-photography, combinations of accelerometer and pressure transducer (shipboard), etc. None of these is without error due to distortion and noise, of some magnitude, but most are fairly reliable and find application in different investigations. It is incumbent upon the user to ascertain that the error in sensing, of his "wave observer," is either sufficiently small to neglect or to qualify his results in terms of that error.

8.22 Recording. Almost always, the raw wave data will be analyzed in the laboratory, rather than at the transducer, hence the necessity for a memory or storage system. The information observed and interpreted by the transducer is delivered to the memory, the recorder; that is, the sensor observes the event and may reproduce the observation as a continuously fluctuating voltage. This voltage is transmitted to a recorder which may reconvert the data to a trace on paper, modulated electromagnetic signal (tape recorder), trace on film (camera recording output of oscilloscope), sequence of numbers, or any one of a number of other memory media. Chart paper and magnetic tape are the two most popular wave-recording methods in use.

Like the wave-sensing equipment, recorders are not without some error owing to distortion and noise; for after all, a recorder is first a transducer and second a memory. The qualification of results due to recording error apply in the same way as to the wave observing transducer.

8.23 Preparation. The data, at this point, are still in their original form and must be prepared for input to an analyzer, either digital or analog. The digital input will usually be in the form of equally spaced measurements of deviations from the mean of the wave record, recorded on punch cards or tape. The analog input is usually the fluctuating voltage presentation on magnetic tape, sped up many times, so that the frequency range of the input signal is amenable to survey by filters of practical bandwidth.

8.24 Reduction. The data are now ready for spectrum analysis. Analog computers are usually required to perform the operations of filtering, squaring and smoothing, while digital computers perform calculations for the autocovariance function, Fourier cosine transform and smoothing. Both techniques are compatible and will produce the same results, if properly treated. In fact, the two techniques may successfully be switched, (that is, digital filters versus electronic autocorrelators), though this is not particularly convenient.

8.25 Presentation. The output of the analyzer is

either a graphical presentation of relative spectral density versus frequency (usual output of analog computers) or a set of numbers representing relative spectral density (usual output of digital computers). Such a spectrum determination provides information on the general distribution of energy. Further statistics are derived from the total energy in the spectrum as well as from the various even moments of the spectral-density function.

This review of data collection and data reduction illustrates that a variety of errors may be introduced into the signal long before it is analyzed. It is generally agreed however, that if the precautions discussed are taken, the net error in the signal, at the time of analysis, may be neglected when compared to the computational errors and the more serious error due to sampling variability. These last two errors will be discussed in detail in subsequent sections.

8.3 Scalar Sea Spectrum—Theoretical Determination.

Wave records, of finite length, can be analyzed by familiar Fourier-series methods. This however is found to be impractical, at least by numerical methods. Complete analysis of a record may require a computation of as many as 300 harmonics with little contribution to the total energy in the spectrum in the first hundred harmonics. Using analog-filter techniques, such an analysis is not nearly so cumbersome. Blackman and Tukey (1958) point out that such elementary (Fourier series) methods frequently fail because indefinitely long records may not be available. The high-precision, high-resolution analysis of such short records results in spectra which may be quite misleading because they represent very well the fluctuations of the particular record being analyzed rather than the ensemble of functions it is desired to represent. A quote from Lee and Wiesner (1950) will illustrate further the shortcomings of Fourier techniques:

"... we wish to know the spectrum of the fluctuating voltage. If only Fourier series and Fourier integral theories for periodic functions and transients are at our disposal we are not sufficiently equipped to solve a problem of this sort. The reason is simply that these theories, as they stand, are not applicable to functions which are specified in terms of statistics and probability and which are not representable by specific analytic expressions giving their precise values for all values of the independent variable. However, the extension of the Fourier theorems to the harmonic analysis of random processes through the medium of correlation functions has enabled us to obtain a solution to our problem with surprising ease."

Since we are dealing with a single function $y(t)$, we shall speak of the autocovariance (autocorrelation) function as the descriptive property of such a record. However, Wiener (1930) showed that the autocovariance function and the energy spectrum are Fourier transforms of each other. This being the case, both must yield the same information, in principle. In practice, confidence criteria derived for the autocovariance function are very complicated and difficult to apply, while

the work of Tukey (1949), on sampling effects, finite length of record and computational procedure, is easily applied to the energy spectrum. In addition, the energy spectrum, being in the frequency domain (rather than the time domain), appeals to the naval architect who deals with transfer functions and ship responses. The energy spectrum has gained wide use and acceptance in such fields as meteorology, oceanography, seismology, and aerodynamics.

The application of harmonic analysis to random stationary processes appears in many places in the literature. A particularly clear development is given by Rice (K), who extended the ideas of Wiener (1930) to obtain derivations of certain important statistical properties of Gaussian random processes. The procedure of Rice in defining the energy spectrum, via the covariance function, will be paraphrased here to apply to ocean-wave records.

Let a sample wave record, T seconds long, such as is shown in Fig. 67, be considered periodic,³⁷ with period T ,

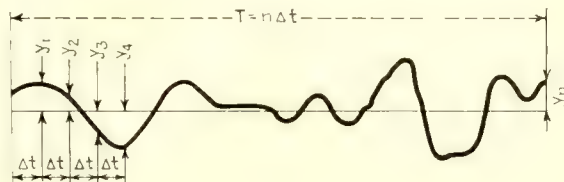


Fig. 67

and absolutely integrable in that period, i.e.,

$$\int_0^T y(t) dt < \infty$$

then $y(t)$ can be represented by a Fourier series

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \omega_n t + b_n \sin \omega_n t) \quad (113)$$

where $\omega_n = 2\pi n/T$ and n is an integer. The coefficients a_n and b_n are the amplitudes of the n th cosine and sine waves. These amplitudes are evaluated in the theory of Fourier series by the integrals

$$a_n = \frac{2}{T} \int_0^T y(t) \cos \omega_n t dt \quad (114)$$

$$b_n = \frac{2}{T} \int_0^T y(t) \sin \omega_n t dt$$

Let the record now be shifted by an amount of time τ . Then for the interval $-\tau \leq t \leq T - \tau$

$$y(t + \tau) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \{a_n \cos \omega_n (t + \tau) + b_n \sin \omega_n (t + \tau)\} \quad (115)$$

³⁷ The condition of periodicity will not disturb us, because we are interested only in the properties of $y(t)$ in any interval exactly T seconds long.

The coefficients a_n and b_n remain unaffected by the shift τ , provided that T is sufficiently long. However, a new relationship now appears—that of the dependence of the measured $y(t + \tau)$ on the record shift τ . In order to find the characteristics of this dependence, series (113) and (115) are multiplied and integrated with respect to t and taken in the limit as T goes to infinity. This defines the autocovariance (autocorrelation) function which by the ergodic theorem completely describes the process.

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T y(t) y(t + \tau) dt \\ = \frac{a_0^2}{4} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \cos \omega_n \tau + C \quad (116)$$

where C stands for correction terms which must be added because series (115) does not represent $y(t + \tau)$ in the interval $(T - \tau, T)$ when $\tau > 0$, or in the interval $(0, -\tau)$ if $\tau < 0$. The integral expression for $R(\tau)$ in equation (116) is commonly known as the "autocovariance function".

On the extreme right-hand side of equation (116), the sum of the amplitudes $a_n^2 + b_n^2$ is the square of the total amplitude of the wave for each harmonic component ω_n . The amplitude squared, multiplied by a suitable constant, represents the energy which is contained in the n th wave component. An interval of frequency $\Delta\omega$ can be visualized as containing a number of harmonics so that the average energy density in the frequency interval $\Delta\omega$ is proportional to

$$\frac{1}{\Delta\omega} \sum_{n=\omega-\Delta\omega}^{n=\omega+\Delta\omega} (a_n^2 + b_n^2) \quad (117)$$

The limit of this ratio as $\Delta\omega \rightarrow 0$ was variously designated as $w(f)$ by Rice (K, p. 167), $\Phi(\omega)$ by Tukey and Press (1956), $0.5 [A(\omega)]^2$ by Pierson (1952), and $E(\omega)$ by Longuet-Higgins (1955) and is seen to be a measure of the total energy in the wave system. In Rice's definition, the frequency $f = \omega/2\pi$ is used instead of the angular frequency ω . On the basis of the foregoing, equation (116) also can be written³⁸

$$R(\tau) = \int_0^{\infty} E(\omega) \cos \omega \tau d\omega \quad (118)$$

The corrective coefficient C of equation (116) vanishes because of the assumption that $T \rightarrow \infty$. The coefficient a_0 is eliminated by measuring the record ordinates, y , from the mean level of the record.

The value of $R(\tau)$ can be estimated from wave-record measurements and the integral equation (118) can be solved for $E(\omega)$ by using a Fourier transform, yielding the spectral density,

$$E(\omega) = \frac{2}{\pi} \int_0^{\infty} R(\tau) \cos \omega \tau d\tau \quad (119)$$

$R(0)$ designates the value of equations (116) and (118) for $\tau = 0$. In this case equations (116) and (118) become

$$R(0) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [y(t)]^2 dt = \int_0^{\infty} E(\omega) d\omega \quad (120)$$

This is a manifestation of the important theorem of Parseval which states that the average energy in the signal is equal to the sum of the average energies in each frequency component.

Equation (120) is proportional to the mean energy of the wave system, or to the variance of $y(t)$. Because of this relationship to the energy, the function $E(\omega)$ is known as the "power spectrum" or "energy spectrum." The first term is universally used in electronics and the second has been recommended for use in oceanography and naval architecture. Both of these refer to the distribution function which is $E(\omega)$ versus frequencies ω or f . $E(\omega)$ is defined more correctly as the spectral density function.

In application to the wave spectrum, Pierson, (J, 1952) introduced the symbol $[A(\omega)]^2$ in place of $2E(\omega)$. In this form, the proportionality of the energy to the square of the component wave amplitude is used to emphasize that the spectrum is everywhere positive. He also introduced the symbol $E = 2R(0)$ as the measure of the total energy contained in the spectrum. Since Pierson's definition of E has been widely used in characterizing the wave heights, the factor 2 will be hereafter incorporated in definitions of covariance functions. The reason will be given in the next section.

Lucid and authoritative expositions of the spectral analysis of stationary random processes are given by Press and Tukey (1956) and by Blackman and Tukey (1958).

8.4 Scalar Sea Spectrum—Calculation by Numerical Method: 8.41 Formulation for computing procedure.

The continuous time history which is the wave recording may, in theory, be operated upon by equations (116) and (119) in order to obtain the wave spectrum. In practical application, the continuous wave record is replaced by a sequence of equally spaced readings of $y(t)$; the approximation can be made as close as desired, by judicious choice of sampling interval. This sequence of numbers is then converted to a new sequence composed of numbers denoting deviation from the mean of the wave record. This new sequence of numbers is then operated upon by a discrete approximation to (116) of the form

$$R_p = \frac{2}{N - p} \sum_{q=1}^{N-p} y_q y_{q+p}, \quad (p = 0, 1, \dots, m) \quad (121)$$

where

- R_p = autocovariance estimate for lag $\tau = p\Delta t$
- Δt = time interval between values of $y(t)$ read from original record
- y_q = value of $y(t)$ at time $q\Delta t$ read from \bar{y} as origin
- n = number of data points available in sample $y(t)$
- p = number of intervals defining lag $\tau = p\Delta t$
- m = maximum number of lags

³⁸ For a more rigorous transition from summation to integration, the reader is referred to Rice (K, p. 167).

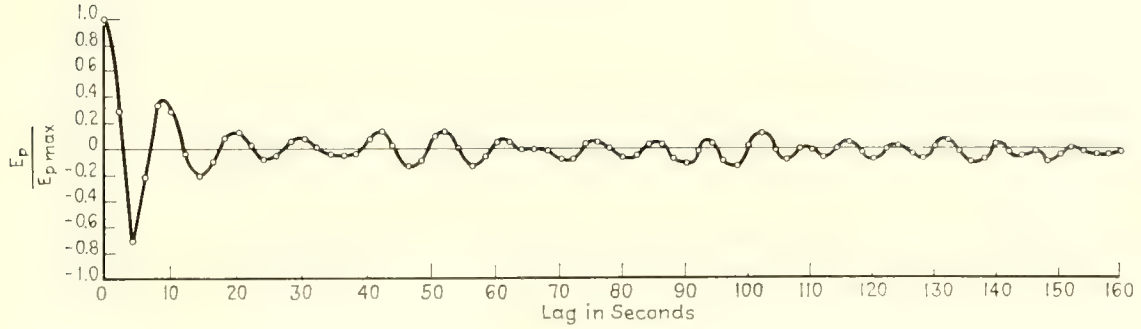


Fig. 68 Non-normalized autocorrelation function reduced to unit amplitude by division by $E_{p \max}$ for pressure record of 10-18-51 from 2258 to 2323 EST at a depth of 32.5 ft MS off Long Branch, N. J. (from Pierson and Marks, 1952)

The number 2 is inserted in (121) to account for the symmetry of the autocovariance function. It accounts for the products $y_q y_{q-p}$ in computing the total variance in the record. The autocovariance function of a wave record appears in Fig. 68. The energy spectrum of the finite sample, $y(t)$, may be estimated from the discrete values of the autocovariance function through a discrete Fourier cosine transform which is the same as using the trapezoidal rule for integration to approximate equation (119). The result is

$$L_h = \frac{1}{m} \left[R_0 + 2 \sum_{p=1}^{m-1} R_p \cos \pi p h / m + R_m \cos \pi h \right], \quad (h = 0, 1, \dots, m) \quad (122)$$

where the L_h are estimates of the spectral densities at frequencies

$$\omega = \frac{\pi h}{m \Delta t} = h \Delta \omega \quad (123)$$

The quantity $\pi/m\Delta t$ in (123) is the interval between spectral estimates computed in (122) and is bound up in that equation as the frequency bandwidth. Therefore, if the L_h are summed, the result is an estimate of the total energy of the spectrum computed from the sample. This important property will be used in Section 8.6 when statistics are obtained from the computed spectrum.

The process of discrete cosine transformation can be thought of as a weighting or filtering process of the form

$$L_h = \int_{-\infty}^{\infty} E(\omega) G(\omega - \omega_h) d\omega \quad (124)$$

The filter operator, $G(\omega - \omega_h)$, on the true spectrum of the sample, $E(\omega)$, has the form

$$G(\omega - \omega_h) = \Delta t \frac{\sin [2\pi m \Delta t (\omega - \omega_h)]}{\tan [\pi \Delta t (\omega - \omega_h)]}$$

In the ideal case, it is desired that $G(\omega - \omega_h)$ be constant in the interval $\omega_h - \Delta\omega/2 < \omega_h + \Delta\omega/2$ and zero elsewhere. Then, L_h would be an accurate estimate of the energy density in each frequency band. Actually, the function $G(\omega - \omega_h)$ has the form shown in Fig. 69 and is a poor approximation to the ideal filter. Because of the high-amplitude side lobes, it is expected that spectral-

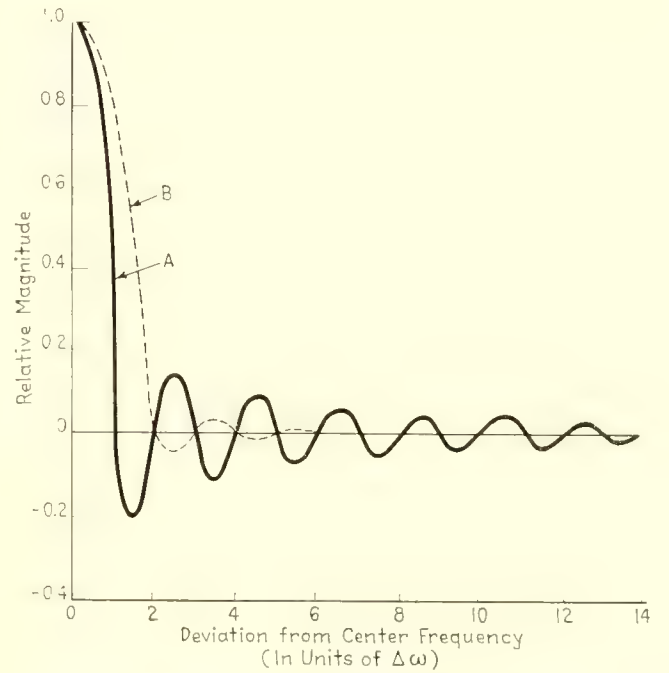


Fig. 69 Filter response ($M = 20$) (from Fleck, 1957)

energy estimates at any frequency will be affected by energy in neighboring frequencies. Also, the negative lobes yield "negative" quantities of energy. The work of Pierson and Marks (1952) and Fig. 70 show the computation of "negative" energy at certain frequencies.

A simple smoothing operation on equation (122) will improve the shape of the filter, Fig. 69, and results in better estimates of the spectral density. The smoothed spectral estimates take the form

$$\begin{aligned} E_0 &= 0.5L_0 + 0.5L_1 \\ E_h &= 0.25L_{h-1} + 0.5L_h + 0.25L_{h+1} \\ E_m &= 0.5L_{m-1} + 0.5L_m \end{aligned} \quad (125)$$

$$h = 1, 2, \dots, m-1$$

This form of smoothing (0.25, 0.5, 0.25) is called Hanning (after van Hann). Another form, used extensively

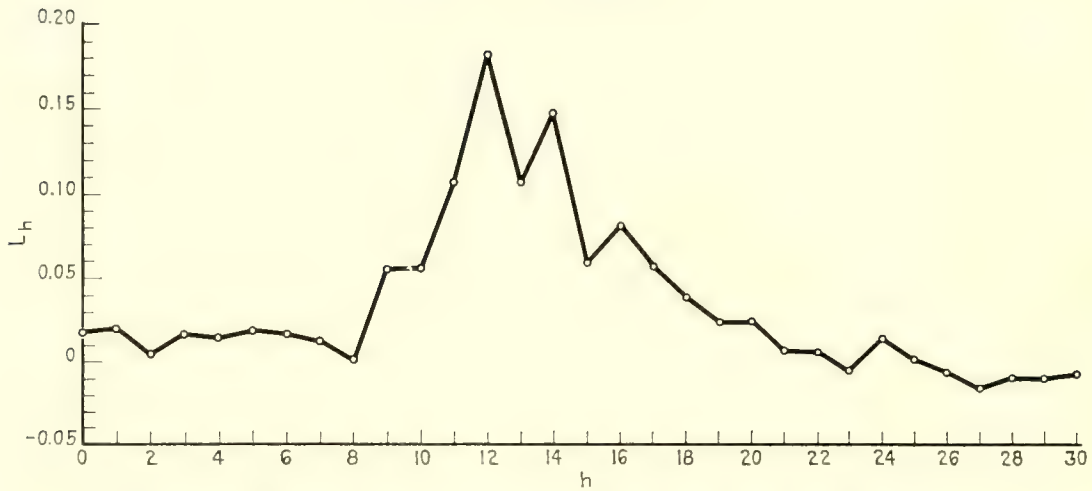


Fig. 70 "Raw" or unfiltered power spectrum (from Pierson and Marks, 1952)

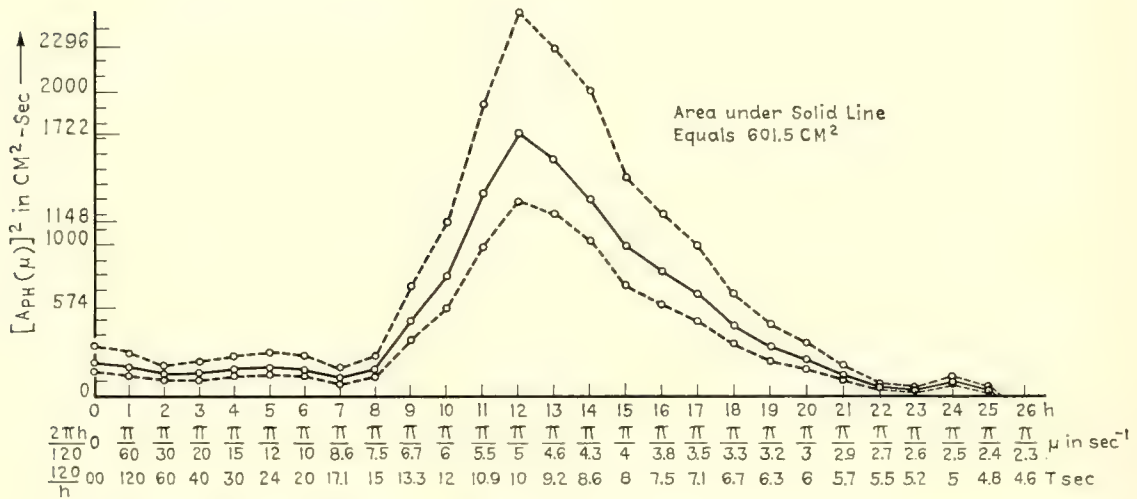


Fig. 71 Best estimate of pressure-power spectrum of autocorrelation function plotted in terms of $[A_{PH}(\mu)]^2$ versus μ . True spectrum lies between dashed boundaries 90 per cent of time (from Pierson and Marks, 1952)

in the literature involves weighting coefficients of 0.23, 0.54, 0.23 and is called Hamming (after Hamming). There are any number of other smoothing functions that can be used and some are listed by Fleck (1957).

The procedure for numerical computation of spectral density given here is thoroughly discussed by Blackman and Tukey (1958) and clearly outlined by Fleck (1957).

Fig. 68 shows a normalized autocovariance function, $R(\tau)/R(0)$ computed from wave records obtained by a pressure transducer. The "raw" spectrum, appropriate to this function is shown in Fig. 70 and the "smoothed" spectrum obtained by equation (125) is shown by the solid line in Fig. 71. These graphs are from Pierson and Marks (1952). This paper also contains an important section dealing with the extrapolation of the subsurface

pressure-wave record to the free surface. These two spectra, superimposed, are shown in Fig. 72.

8.42 Choice of computational parameters—interpretation of spectrum. The discussions on measurement of the seaway so far given in Section 8 would ordinarily suffice for the purposes of this monograph on seaworthiness and we would be tempted to go on with a brief description of the statistics derivable from the sea spectrum plus some qualifying remarks on the statistics and then close the issue. It turns out however (as will be seen in Chapter 3, Section 3) that wave records and ship motion records have the same basic property of randomness. This suggests that the same treatment given wave data, to extract certain information, will apply in the same way to ship motion data obtained at sea or in the

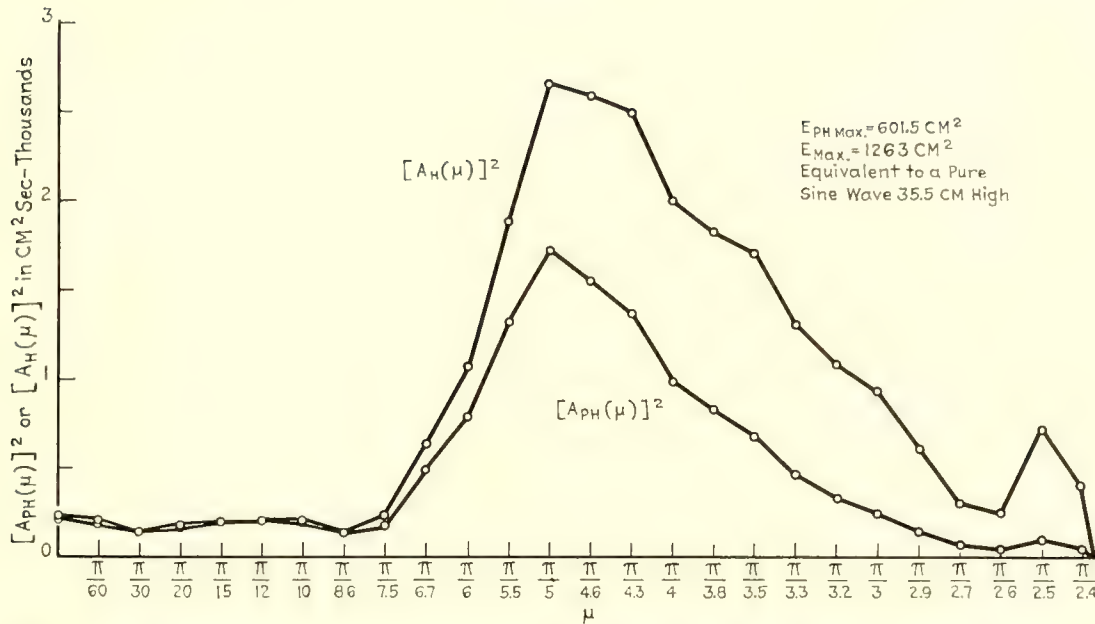


Fig. 72 Comparison of pressure-power spectrum with free-surface power spectrum. Note the much greater amplification of the low-frequency components (from Pierson and Marks, 1952)

towing tank (in irregular-wave tests). Since motion measurement is a vital part of seaworthiness studies, it will be worth while to study in detail the nature of the statistics derivable from a sample record and hence from the parent population from which the sample was drawn. In this connection, all that has preceded in this section and all that will follow applies equally well to ship-motions records as to wave records.

At this stage of discussion, there is available a wave record and a means (theoretical and numerical) of reducing this wave record to an energy spectrum of the seaway. The question arises: What is the best way of utilizing the means available for going from the record to the spectrum, in order to obtain the most reliable results? This is not easy to answer and is, in fact, complicated by our failure to specify a particular problem, which in turn implies something of the nature of the results we are seeking. As will be seen, the specification of the problem plays an important part in the choice of computational parameters. However, it will be most profitable to keep the discussion as general as it will permit.

Three questions may be posed at the outset that will have profound effects on the ultimate estimate of the energy spectrum. They are:

- 1 How long a record shall be taken?
- 2 How large shall the sampling interval (Δt) be?
- 3 How many lags shall be computed?

The last two questions relate directly to the accuracy of the spectral-density estimates; the first question relates to the sampling variability; i.e., it determines whether the sample, from which the spectrum is determined, truly represents the population from which it was drawn.

Table 12 Reliability of Power Spectrum Estimates^a

f	Possible error in observed value—				
	2.5 pct	5 pct	50 pct	95 pct	97.5 pct
1	1000	250	3.1	0.26	0.2
2	40	20	1.5	0.33	0.21
3	14	8.5	1.3	0.36	0.32
4	8.3	5.63	1.2	0.42	0.36
5	6.0	4.37	1.15	0.46	0.39
6	4.8	3.6	1.12	0.48	0.42
8	3.8	2.8	1.10	0.51	0.46
10	3.1	2.6	1.07	0.55	0.49
15	2.4	2.1	1.05	0.60	0.55
20	2.1	1.8	1.02	0.63	0.59
50	1.55	1.45	1.02	0.74	0.69
100	1.35	1.29	1.00	0.79	0.78
Large		$10^{-1/f^{1/2}}$		$10^{-1/f^{1/4}}$	

^a From Tukey (1949).

It turns out however, that these three questions cannot be treated independently; they must be considered together in order to design the analysis procedure properly. As was stated in Section 8.3, the work of Tukey (1949) combined the effects of finite-record length and computational procedure into a compact expression that defines the confidence criteria which may be applied to the estimated spectral densities. A quote from Pierson and Marks (1952) will introduce the expression for the confidence bands on the spectrum: "The final results are estimates. Estimates can be in error, and the important part of this method of analysis is that it also yields a statistical analysis of how much in error the estimates can be. Tukey (1949) and Tukey and Hamming (1949)* have shown that the errors are distributed according to a chi-squared distribution with f degrees of freedom where f is given by

*See reference p. 105.

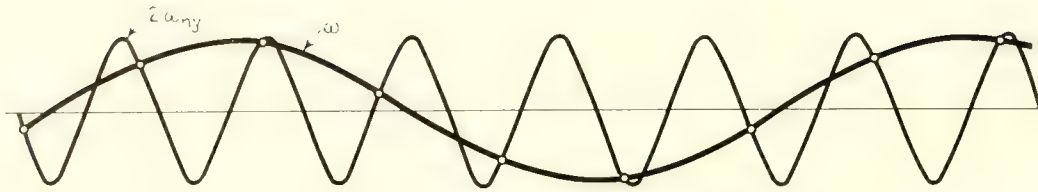


Fig. 73 Aliasing a short wave record (from Press and Tukey, 1956)

$$f = \frac{n - m/4}{m/2} \approx \frac{2n}{m} \quad (126)$$

Given f , then the use of Table 12 yields information on the reliability of the estimate for each frequency band obtained by the analysis of one record.

"If a given analysis has, say, ten degrees of freedom, then the number obtained from (125) can be multiplied by 2.6 and by 0.55. The true value of the power in the band under analysis will lie between these extremes 90 pct. of the time. Stated another way, given a great many analyses of different short samples from a long run of time series, then the set of bands given above will enclose the true power in the band 90 times out of 100 if the sample length is such that ten degrees of freedom are obtained and if a very large number of samples are evaluated.

"In this analysis, $n = 750$ and $m = 30$. The result from (126) is approximately 50 degrees of freedom. Therefore, if each ordinate of the smoothed spectrum is multiplied by 1.45 and by 0.74, it can be said that the true pressure power spectrum lies between these bounds (dotted lines in Fig. 71) 90 pct. of the time."

Equation (126) was first introduced by Pierson (1952a) as a deduction from the work of Tukey (1949), and Tukey and Hamming (1949). A paper by Blackman and Tukey (1958, Section B.9) leads to the expression

$$f = \frac{n - m/6}{m/2}$$

The difference is insignificant. Both equations tend to $2n/m$ as n becomes large and any small differences in f would hardly be noticed in Table 12 where the confidence bands vary slowly above $f = 10$.

It is clear that the confidence limits obtained through equation (126) and Table 12 define the probable ensemble spread and consequently qualify the usefulness of the spectrum as a descriptive tool. The narrower the confidence limits, the more reliable the spectrum. This implies the desirability of a large number of degrees of freedom (f) in (126). We are now in a position to evaluate the three basic questions in terms of the desired properties of the spectrum.

On the question of record length, one extreme rules out very short records. It is clear that one or two wave oscillations can hardly serve our needs for description of the seaway. The other extreme, that of an infinitely long record, is ideal, in principle, since it can completely

and accurately describe the sea surface, yet is equally ridiculous in that such a record cannot be recorded and if it could it cannot be analyzed. Compromise requires the convergence of these two sampling concepts to an optimum record length that embodies sufficient length for an adequate description of the seaway yet not so long that the additional length, beyond a time T , returns little, in minimizing sampling variability, for a great deal of computational effort. Finally, the record, however long, must be stationary (or nearly so) throughout. The seaway is usually stationary for hours at a time and in this respect wave analysis is more fortunate than ship motions analysis where many different samples must be taken in the same interval of time. Nevertheless, it has been shown in the example of Pierson and Marks that a 25-min record yields 50 degrees of freedom, a worthwhile mark at which to aim. Since there is some freedom in the recording time of waves (unless the sea state changes rapidly), one should aim for a minimum of 20 min and maximum of 1 hr of wave-recording time.

Once the record is obtained, its length is fixed, but the number of discrete points in the sample is not determined until the sampling interval Δt is specified, because there are

$$n = (T/\Delta t) + 1$$

sample points, where T is the record length. It is obvious that a Δt which is small, compared with the shortest period that contributes energy to the spectrum, will have misleading effects on equation (126). In fact, f can be made as large as desired, by letting Δt become sufficiently small. This is contrary to the intent of (126) which implies some measure of independence between the n -discrete values taken from the original record. The "small" Δt does, in fact, provide a better approximation to the record compared to an optimum Δt (which will be defined) but adds little to the energy spectrum of that record, for large additional computational effort. The net effect is the computation of more spectral estimates of zero value for the higher frequencies and no improvement in resolution.

On the other hand, if Δt is large, compared with the shortest period that contributes energy to the spectrum, the n -discrete values do not adequately represent the higher frequencies in the record (see Fig. 73) and the energy in harmonic components above f_c , where $f_c = 1/2\Delta t$ is the Nyquist frequency (Shannon, 1949), is aliased (transposed) into the energy content of the lower

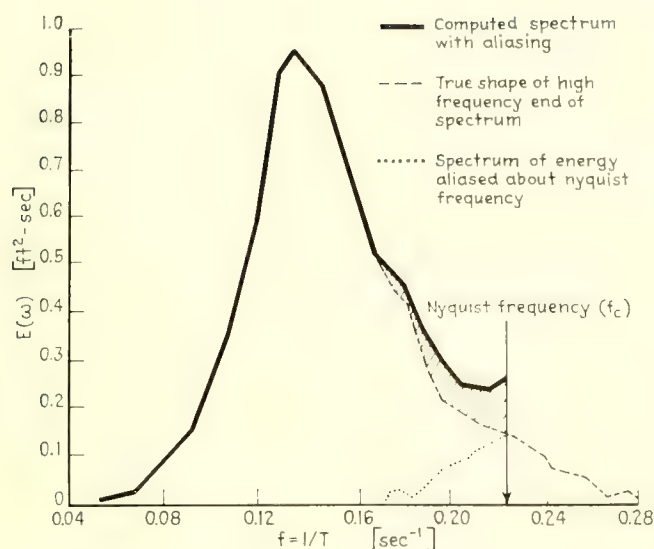


Fig. 74 Example of aliasing a wave spectrum. Hatched area shows erroneous energy contribution

frequencies. Information at $f_c + f$ cannot be resolved because of the discrete sampling and the energy spectrum is “folded” about f_c . The Nyquist frequency is often called the folding frequency or cut-off frequency. Fig. 74 illustrates the erroneous distortion in the wave spectrum due to a large Δt .

“Selection of an optimum Δt depends on the highest frequency in the wave spectrum above which no appreciable wave energy can be aliased back into the spectrum.” This glib statement, which appears often in the literature, is predicated on the assumption that there is *a priori* knowledge of the shape of the spectrum. Most often this is not the case and a simple rule of thumb, commonly used, suggests the selection of that Δt which is one half the smallest “apparent” period in the wave record. This is still somewhat subjective in that it involves a definition of what is a cycle of wave motion. In the illustration of the wave-record analysis of Pierson and Marks, the pressure transducer had a natural cut-off at 0.25 cps because of the attenuation of wave energy with depth. This only obtains with pressure transducers. It is however possible to prefilter either at the transducer or recorder and this is often worth while in eliminating spurious noise. It also affords a natural choice for the Nyquist frequency. An example of electronic prefiltering will be given in Section 8.43.

The number of lags to be computed, equation (126), for the autocovariance function determines the number of estimates of spectral density (actually $m + 1$) that will be made in accordance with equation (122). These spectral densities will be spaced at distances of $f = f_c/m$, [see equation (123), for the equivalent expression]. According to Fleck (1957): “. . . Since the effective width of the filter corresponding to the numerical procedure for computing the spectrum is $2\Delta f$, the value of m must be

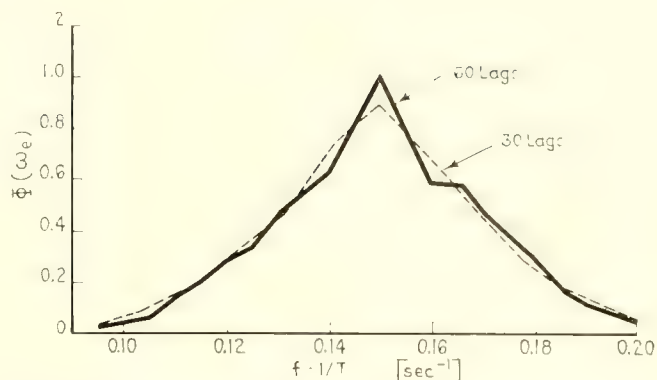


Fig. 75 Spectra of a seakeeping event computed from the same record for 30 and 60 lags (courtesy David Taylor Model Basin)

chosen so that $2\Delta f$ is the minimum resolution interval desired or some smaller interval. Then $m = 1/2\Delta f\Delta t$. It is recommended that m be at least twice this minimum value to insure adequate resolution.”

From equation (126) it is seen that the greater the resolution (greater m) the smaller the number of degrees of freedom (f) and consequently the less confidence. That is, the more nearly we represent the sample record through high resolution of its spectrum the greater is the probability that we deviate from the true spectrum of the parent population. Compromise must be made between resolution and confidence. In most wide-band spectra it has been found that 60 lags is more than adequate for a good description of the record and 30 lags is usually ample. In the case of a swell spectrum, adequate resolution of the steep sides will require somewhat more lags; twice the number needed for a wide band spectrum may be a good rule of thumb. Fig. 75 shows a wide-band spectrum (not waves) computed for 30 and 60 lags. The 90 per cent confidence limits, Table 12, for each of these spectra will contain the other spectrum completely. No significant peaks have been masked in going from 60 to 30 lags and it is seen that it would be worth while to use 30 lags and improved confidence, for this spectrum anyway.

The purpose of the foregoing was to acquaint the reader with some of the subtle aspects of spectrum analysis. One cannot rush headlong into the computations without consideration of what is desired and of the penalties that must be paid to achieve the desired spectrum.

8.43 Additional sources of error. There are some additional sources of difficulty connected with arriving at the correct wave spectrum of the sample. These are involved with the collection and preparation of data discussed in Section 8.2 rather than with the computational procedure discussed in Section 8.42.

Very often, spurious information appears in the spectrum due to d-c drift in the recording electronics. In this case the wave record drifts away from the preset zero of the recorder and the spectrum manifestation of this event is a spike, at or near zero frequency, corresponding

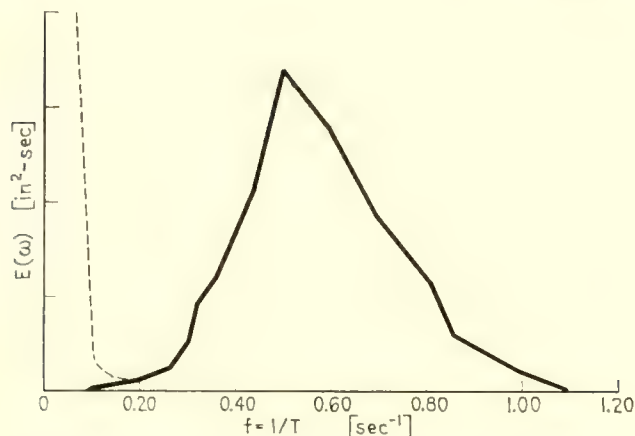


Fig. 76 Wave spectrum of irregular waves measured in model tank. Dashed curve indicates distortion in true wave spectrum due to introduction of artificial linear drift in wave record (courtesy David Taylor Model Basin)

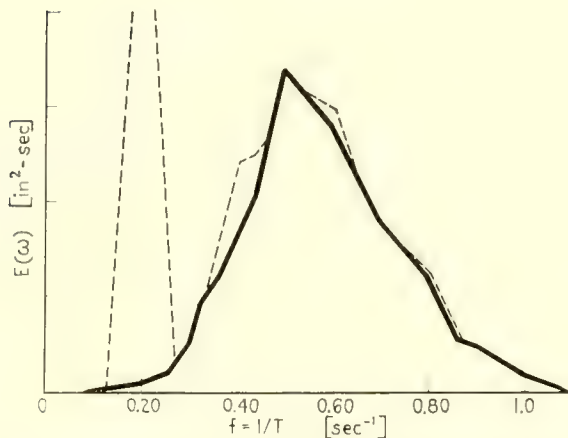


Fig. 77 Wave spectrum of irregular waves measured in model tank. Dashed curve indicates distortion in true wave spectrum due to introduction of artificial sinusoidal drift in wave record (courtesy David Taylor Model Basin)

to the apparent infinite period generated by the drift. In order to determine the nature of this misinformation a record of irregular waves, recorded in the Taylor Model Basin tenth-scale model of their Seakeeping Facility, was distorted to simulate the type of drift just defined. The effects on the computed spectra are shown in Fig. 76. Except for the huge spike near zero frequency, due to linear drift, there is essentially no change in the spectrum derived from the uncontaminated wave record. Therefore, a spectrum with a spike at zero frequency may be considered not to actually have that spike and it should be ignored. There is one exception to this rule. A shipborne wave recorder which acts as a moving-point source of wave observation may record energy at zero frequency of encounter, for certain wave components, whose speed is equal to the component of ship speed in

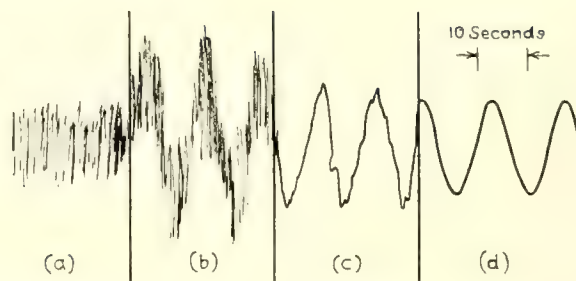


Fig. 78 Filtering a pure tone to eliminate noise: (a) No filter; (b) 10-cps filter; (c) 1-cps filter; (d) 0.1-cps filter (from Campbell, 1959)

that direction when both waves and ship travel in the same general direction. Here, the true wave energy is mixed with the energy due to the linear drift of the electronics and separation is a serious problem.

The linear-drift problem is not serious. A sinusoidal drift is another matter. Such difficulties may arise from use of an airborne radar altimeter or from the depth-keeping problems of a moving submarine or from an unstable power supply. The problem was investigated in the same way as for the linear drift; this time a low-frequency sinusoidal signal was artificially introduced into the same Taylor Model Basin wave record. The resultant spectra, Fig. 77, show a distortion not only at the frequency of the intruding signal (0.2 cps) but at higher harmonics which are integral multiples of the fundamental. It is clear that the additional energy applied to the main portion of the spectrum is not easily separable. In addition, the main spike may easily be mistaken for a band of swell. Consequently, a wave spectrum, with a spike outside the main energy band, but not at zero frequency, should be treated with great caution.

A source of error commonly encountered is the introduction of "noise" into the system as stated in Section 8.2. This may occur either in the transducer or in the recorder or both. It may be an inherent property of the electronics or an artificially induced vibration in the sensing element. Such noise is usually of much higher frequency than is measured in most seakeeping parameters. This noise can be eliminated by passing the signal through a low-pass filter (Campbell, 1959). An example of this appears in Fig. 78 where noise is introduced into the circuitry which generated a nearly pure 0.07-cps sine wave. The noise is sufficient to mask completely the signal (a), and even with the 10 cps cut-off filter (b), the nature of the intended signal is not clear. However, proper filtration does finally produce the desired pure tone. Proper filtering should be a key element in any wave and seakeeping instrumentation. In addition to eliminating unwanted elements in the signal, it supplies the cut-off frequency and thereby automatically defines Δt .

8.44 Sampling variability. The subject of sampling variability is relegated to a subsection not because of its

order of importance but because of the particular sequence of discussion in Section 8. The importance of sampling variability was hinted at earlier by the oft-repeated qualifying statement, "... if the record is long enough." Simply, this means that no matter how carefully the spectrum analysis is carried out and how sincerely it is qualified, the results will only define the state of the sea within the limitations of the length of the sample. The length of sample has been discussed in Section 8.43 but it will be worth while to illustrate the effects of variable sampling length and at the same time to review the consequences of choice of computational parameters.

Consider the record of a particular seakeeping event (not waves) which was originally 30 min. long. Ten different spectrum analyses of this basic record were made (Marks, 1959). The results appear in Fig. 79. Five analyses were made with $m = 30$ lags and five with $m = 60$ lags. The five analyses in each set were made for different lengths of the record as specified in the figure. Since the problem of aliasing is straightforward, a good choice of Δt eliminates consideration of this aspect here. It is obvious that the more detailed spectra, computed for 60 lags, Fig. 79(A), have much more variability than the spectra computed for 30 lags, Fig. 79(B). This is to be expected, in view of equation (126) and Table 12 which predict wider confidence bands (less degrees of freedom) for the spectra associated with 60 lags.

It should be pointed out here that the entire discussion relating to degrees of freedom and confidence bands always refers to sampling from a population in which the sample is one member of an ensemble that might have been taken from the population. Description of the population is derived through estimation, from the sample, of the average statistical properties of the ensemble. Care must be exercised in analyzing such a sample to be certain that the results reflect the characteristics of the ensemble rather than those of the particular sample. Longer records and less resolution tend to give results in this direction. However, if the wave record is the *population*, such as is often the case when generating the same irregular waves in the towing tank for successive tests, then the foregoing discussion does not apply. The "sample" record completely represents the waves and its energy spectrum will describe the process; the greater the resolution (more lags), the better the description.

Examination of the individual sets of spectra reveals that the shortest records result in highly variable estimates of spectral density while the longer ones tend toward stability, with the spectrum of the 30-min record appearing to give a good approximation of the averages of the other records, and which may now be considered to be a good estimate of the population from which the sample record was taken. Comparison of the spectra of the longest records for 30 and 60 lags, Fig. 75, shows little variability as a function of the number of lags, a further indication of stability, in this case.

Blackman and Tukey point up the difficulties of using

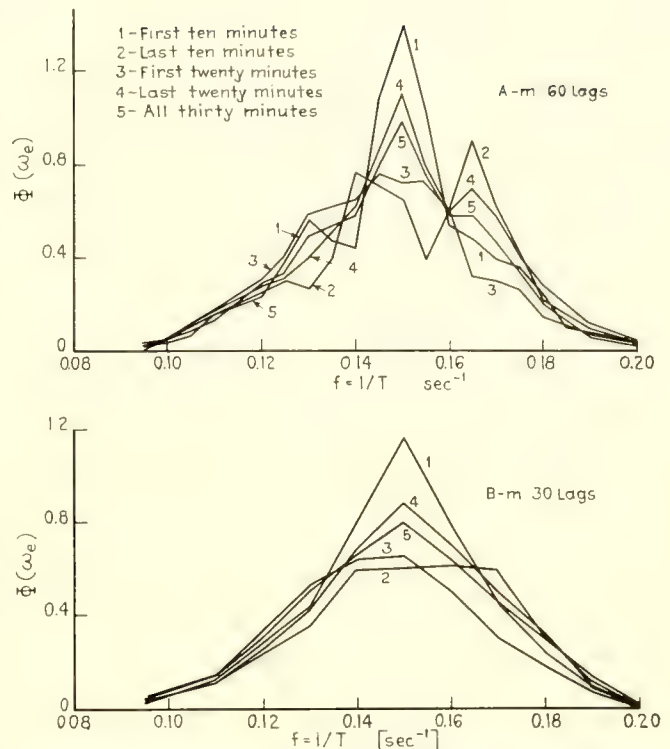


Fig. 79 Spectra computed from various lengths of same seakeeping record (courtesy David Taylor Model Basin)

theoretical methods to estimate variability and end in saying: "... The precision of final over-all values is ordinarily far more wisely judged from the observed consistency of repeated measurements than from any theoretical variability based on a Gaussian assumption."

This is sound advice, for towing tank work, where short samples demonstrate high variability and where repeated tests in different waves reveal a safe number of such tests to provide a stable ensemble average. For full-scale trials (wave measurements at sea), it is hardly possible to repeat tests and so experience gained by the type of experiment discussed in this section is used in the design of criteria for sampling at sea.

The work of Tucker (1957) lends itself well as an example in the study of sampling variability. Two continuous records, each of about 10 hr duration were made and the mean-squared wave height for consecutive 10-min intervals was computed and the results plotted in Fig. 80. The means for groups of 20 of these estimates are drawn in as reference levels. The upper figure demonstrates nearly stationary conditions over the 10-hr recording period yet the mean-squared values for 10-min samples, which may be taken as a measure of the total energy in the related spectra, demonstrate large variability. Averages of pairs of 10-min squared means of the wave height are seen to reduce the variability bounds. Averages of trios decrease the variability even more. It is clear then that in conditions which are known to be stationary for 10 hr, a 10-min sample will

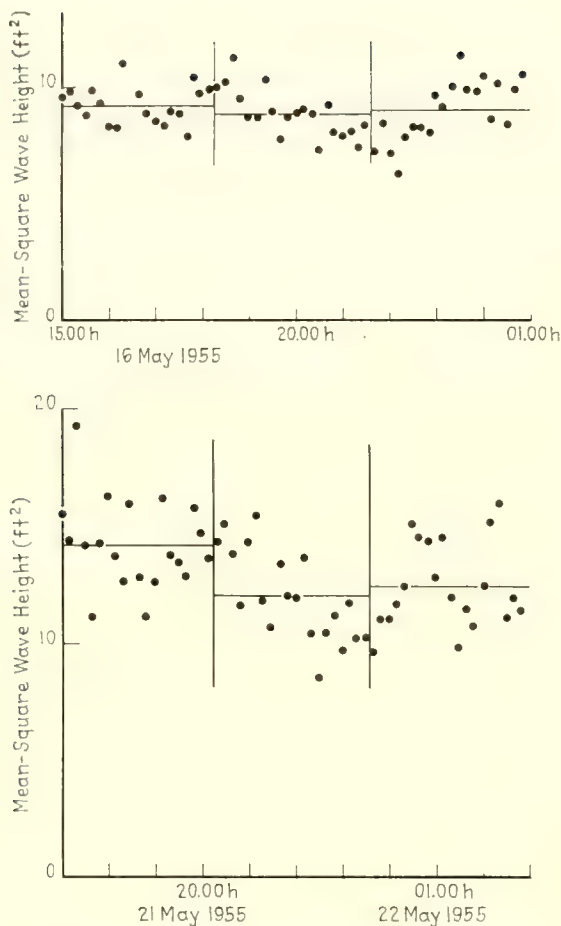


Fig. 80 Values of mean square wave-height averaged over 10-min intervals. Mean value of each group of twenty is shown (from Tucker, 1957)

have a low probability of describing the seaway, a 20-min sample will be a better estimate, and a 30-min sample is bound to be very good.

The lower graph in Fig. 80 is essentially the same, except that the waves are somewhat higher and the state of the sea not nearly so stationary over the 10-hr recording period. The 10-min samples are even more inadequate in this case. An extension of the experiment of Tucker is suggested to provide more information on sampling variability versus sea state. This will lend itself to intelligent planning of any seakeeping measurement.

8.5 Scalar Sea Spectrum—Calculation by Electronic Analog Filter Method. Section 8.4 discussed a computational procedure for obtaining the energy spectrum by numerical methods. The particular method, that of calculating the autocovariance function and Fourier cosine transform, is not exclusive to digital computation but may be done electronically as well (Brooks and Smith, 1956). A more direct electronic method for obtaining the energy spectrum is the use of filter techniques. Of course, the filter method may be adapted for numerical use as well.

This introductory comment is meant to point up the possibility of confusion among naval architects (and others) on the notion of which kinds of equipment and which methods may most efficiently be used to compute spectra. In principle, whatever can be done electronically (analog) can be done numerically (digital). Further, there are general and special purpose digital computers as well as general and special purpose analog computers so that no distinction can be made on that account. Still further, there is little to choose in speed and accuracy in so far as the requirements of the spectrum are concerned. It is possible to produce a spectrum almost as soon as the data are collected by either analog or digital means. If accuracy implies high resolution, then there is still little difference considering that “ultra-high” resolution of a finite sample defeats the purpose of describing the ensemble average or population. Certainly, large masses of data can be handled both ways, and again there is a practical upper bound beyond which additional data add little to the description of a stationary process.

The fact is that choice of analog or digital methods depends largely on what is available to the investigator. The choice of either autocovariance-transform or filter-squaring is dependent on what is convenient for the machine. Most investigators involved with relatively few spectra are content to use available general purpose digital computers, because it only involves supplying a list of numbers and a decision on the number of lags. The computational equations are available and programming is done by the custodians of the equipment. Also, there is no capital investment. General purpose analog computers are not as readily available. When the number of spectra is large so that computing time is involved and the program desires frequent changes for special treatment of each record to be analyzed, a special purpose computer may be a worth-while investment. The analog computer lends itself well to specialization because the components necessary for amplification, filtering, and squaring are commercially available. A special purpose digital computer would have to be built from the ground up.

For the reasons given here, there have developed lately in seakeeping analysis, two primary techniques for spectrum analysis: *a*) Autocovariance-Fourier transform-digital, and *b*) filter-square-analog. The first of these has been treated thoroughly in Section 8.4; the second will be discussed here.

Unlike the digital input of equally spaced ordinate values, the input to the analog computer is a continuously varying signal either recorded on film (or paper) and scanned by a photocell or recorded on magnetic tape and fed into the analyzer as a variable voltage.

The theory and practical operation of photocell-type analyzers have been described by the following authors: Barber, Ursell, Darbyshire, and Tucker (1946); Tucker and Collins (1947) (see ref. p. 105); Barber and Ursell (1948b); Barber (1949); Tucker, Pierce, and Smith (1950); Tucker (1955); Klebba (1949); Rudnick (1949).

The principles of the analog-filter type analyzer have

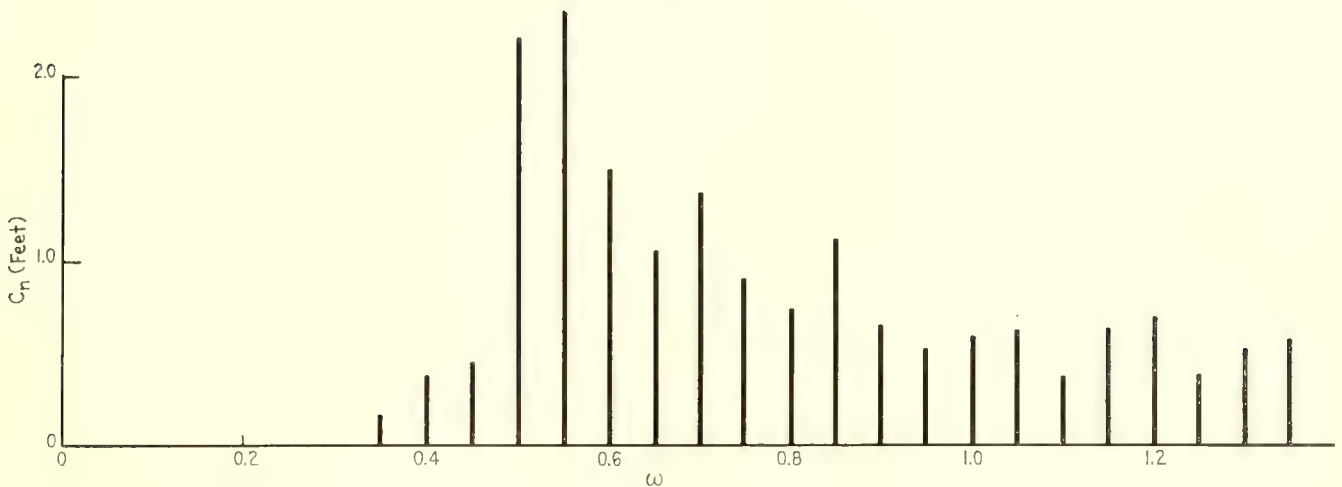


Fig. 81 Wave-amplitude spectrum measured by Cartwright and Rydill (1957) on a ship at sea

been reported variously by: Chang (1954, 1955a, b); Pierson and Chang (1954); Smith (1955); Marks and Strausser (1959); Chadwick and Chang (1957).

The first successful analog ocean-wave analyzer was developed at the National Institute of Oceanography (Barber, 1949). Narrow filters were used to approximate a Fourier-series analysis of the record (rather than an energy-spectrum analysis). The amplitudes of several adjacent harmonics in a band are averaged in the analog computer and the display, Fig. 81, is one of average amplitude (for 0.05 rad/sec band) against circular frequency. Strictly speaking, this type of analysis will obtain spectral estimates, with two degrees of freedom, which are additive with the number of such estimates. The same process obtains from modern wave analyzers only the weighting and averaging is done continuously over the frequency band and is consequently not so obvious.

There are numerous ways of performing the random-signal filtering process. The most efficient method is a bank of filters, each filter centered at a different frequency and with a prescribed width (not necessarily the same for all filters). The bank covers the entire frequency range of the signal being analyzed and the analysis is completed with one passage of the record. Single-filter techniques require many passages of the input signal (spliced into a loop) as the filter continuously looks at different bands in the frequency content of the signal. Such an analysis may be performed by holding the center frequency of the filter constant and then varying the frequencies it looks at by varying the speed of passage of the input signal. In this way, the filter always sees a different range of apparent frequencies. Still another method, the one employed in the SEADAC (Marks and Strausser, 1959) involves the modulation (mixing) of a local oscillator wave with the random signal being analyzed. The oscillator emits a continually varying signal which is the carrier frequency (say 97,000 cps), plus the frequency that is being examined (say 100 cps). The

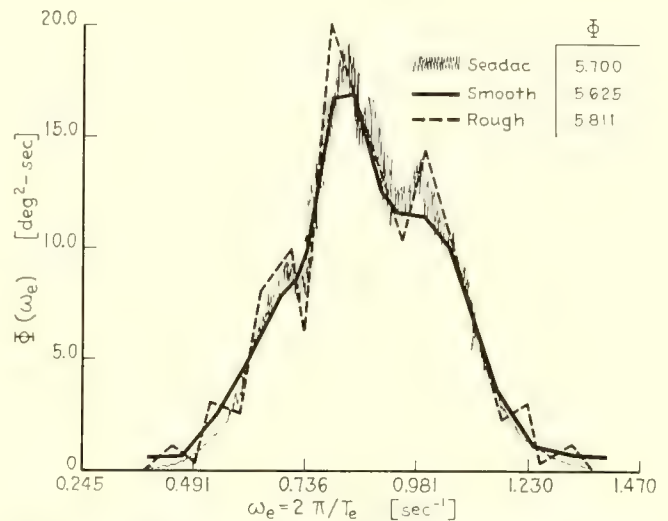


Fig. 82 Energy spectrum of a seakeeping record calculated by digital computer and SEADAC (from Marks and Strausser, 1959)

modulation process results in a new signal which has as its components the two signals being mixed (97,100 cps and 100 cps from the random signal), as well as their sum (97,200 cps) and difference (97,000 cps). One of these frequencies must be the standard carrier (97,000 cps) and this is the only component the narrow-band filter permits to pass. This standard carrier has an amplitude proportional to the amplitude of the frequency that is being examined and it is this quantity which is squared and reproduced in graphical form.

Since digital procedures are well established, the burden of proof of equality or superiority is on the analog computer. An early experiment in the SEADAC involved the comparison of spectra made by analog and digital computations. One result is shown in Fig. 82 where the analog spectrum (oscillatory curve) is superim-

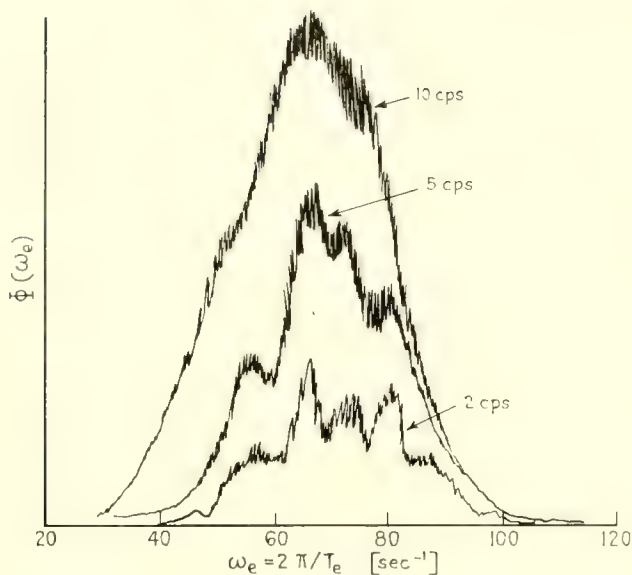


Fig. 83 Spectra of same seakeeping event resulting from analysis by different filters (from Marks and Strausser, 1959)

posed on the smooth and rough digital spectra. The fit is excellent, as the graph shows. The SEADAC analysis took 5 min. The record was 20 min long and the spectrum was first computed for 60 lags numerically and then analyzed by a 5-cps filter in the SEADAC. The areas, as tabulated, are in good agreement.

Subsequent tests, for the purpose of studying resolution, resulted in Fig. 83 where the same record was subjected to analysis by filters of different width (corresponding to different lags numerically). The 5-cps analysis corresponds, of course, to the 60-lag analysis shown in Fig. 82. The 2-cps analysis shows more resolution than is probably required to define the ensemble properties, since the 60-lag analysis has about 50 degrees of freedom. The 10-cps analysis has lost most of the detail in the spectrum and the use of still broader filters will result in complete loss of detail as the filter's characteristics become apparent in the spectrum.

Choice of filter bandwidth is subjective in the same way as is choice of lags in the numerical process. By and large, the problems of experimental design are the same as for numerical computation. Pierson (1954a) discusses at length the mathematical theory of analog-spectrum computers and Spetner (1954) treats the errors resulting from analysis of a finite length of data spliced in a loop.

8.6 Information Derivable From the Wave Spectrum.

The wave spectrum, once it is computed, may be an end in itself or a means to an end, depending on the problem associated with it. Visual examination reveals certain useful information, such as:

- 1 Significant range of frequencies outside of which there is no appreciable energy contribution to the seaway.
- 2 Frequency of maximum energy.
- 3 Existence of low-frequency swell.

4 Energy content in different frequency bands.

Beyond these directly observable "statistics," information obtained from the wave spectrum requires additional calculations, some simple, some lengthy. Chief among the easily obtainable and most desired information is the wave-height distribution. Since energy and wave height (squared) are related, it is natural that averages of wave height should relate to the total energy in the spectrum. The first computational step then is an area measurement to determine the total energy; that is,

$$E = m_0 = \frac{1}{n} \sum y_n^2 = \int_0^\infty E(\omega) d\omega \quad (127)$$

Jasper (1956) found that wave and seakeeping data (in the form of peak-to-peak measurements) behaved according to the Rayleigh distribution, equation (112). Longuet-Higgins (1952) assumed a Rayleigh distribution and a narrow frequency spectrum to derive the following statistical relationships:

$$\begin{aligned} \bar{H} &= 1.772\sqrt{E} \\ \bar{H}(1/3) &= 2.832\sqrt{E} \\ \bar{H}(1/10) &= 3.600\sqrt{E} \end{aligned} \quad (128)$$

already given in Table 7. The work of Longuet-Higgins lends itself to extraction of a number of presentations of wave height such as the percentage of waves between height strata and the average height of the highest wave out of a total of N -waves. This information is conveniently tabulated in Pierson, Neumann and James (H).

The symbol E was introduced by Pierson (1952) to designate the double mean square of the water-surface elevations in waves. The integral in equation (127) represents the total area under the curve of spectral densities. The use of the integral expression is permissible because the smoothed spectrum is assumed to represent the true continuous spectrum of waves (or of the ship's motion) under consideration. A summation is used when the true spectrum is approximated by the average energies in a series of frequency bands. The "root-mean-square" (abbreviated rms) wave amplitude is now defined simply as \sqrt{E} . It is the amplitude of an imaginary simple harmonic wave which has the same mean energy per unit of sea surface as the complex wave system represented by the spectrum.

Cartwright and Longuet-Higgins investigated a correction factor to take into account the broadness of the spectrum. This factor

$$(1 - \epsilon^2)^{1/2}$$

multiplies the quantities in (128). The broadness parameter, ϵ , depends on the even moments of the spectrum and is given by

$$\epsilon^2 = \frac{m_0 m_4 - m_2^2}{m_0 m_4} \quad (128a)$$

where

$$m_n = \int_0^\infty \omega^n E(\omega) d\omega$$

The constant by which \sqrt{E} is to be multiplied in order to obtain the average of $1/n$ highest waves (or amplitudes) is shown in Fig. 84 as a function of ϵ . Cartwright and Longuet-Higgins applied their calculations to five sample records; a pressure record of swell, two records of waves measured by a ship-borne wave recorder, a record of a ship's pitching, and a record of a ship's rolling. The values of ϵ in these five examples were found to be:

Swell pressure.....	0.41
Wave-height records.....	0.57 and 0.67
Ship's pitching record.....	0.48
Ship's rolling record.....	0.20

It is seen that in all of these cases the narrow-spectrum calculations are severely reduced by the broad-spectrum corrections. This is strange, in view of the fact that Jasper (1956) found excellent agreement between height distributions of spectra of all shapes (and different parameters) and the Rayleigh distribution, without any corrective measures. This may be explained by considering that, in the case of swell, where high frequencies are not prominent, the height distribution approaches the Rayleigh law. While in wind-generated sea (wide band spectra), high frequencies are very prominent and the distribution approaches the Gaussian law. If the high frequencies are ignored, the Rayleigh law will apply.

Since m_4 grows faster than m_0 and m_2 , as higher frequencies are added, and since $\epsilon \rightarrow 0$ as $m_4 \rightarrow m_2^2/m_0$, it is seen that the correction factor for an ideally narrow spectrum will not be unity unless all the high frequencies in the spectrum are resolved. The correction term of Cartwright and Longuet-Higgins is well defined; aliasing of energy and filtering of the signal seem to defeat its purpose.

Rice (K), derived the expression for the average number N_0 of zero up-crosses per second. In terms of moments of the spectrum function

$$N_0 = \frac{1}{2\pi} (m_2/m_0)^{1/2} \quad (129)$$

and by definition the average apparent period

$$\bar{T} = \frac{1}{N_0} = 2\pi(m_0/m_2)^{1/2} \quad (130)$$

The total number of maxima N_{\max} is expected to be

$$N_{\max} = \frac{1}{2\pi} (m_4/m_2)^{1/2} \quad (131)$$

The proportion, r , of negative maxima to the total number of maxima (from Cartwright and Longuet-Higgins, 1956) is

$$r = \frac{1}{2} \left[1 - \frac{m_2}{(m_0 m_4)^{1/2}} \right] = \frac{1}{2} [1 - (1 - \epsilon^2)^{1/2}] \quad (132)$$

For a narrow spectrum, $r \rightarrow 0$ as $\epsilon \rightarrow 0$; i.e., all the maxima occur above the mean level. The sea surface is represented in this case by a uniform, regular swell with gradually varying amplitude and length. For a broad

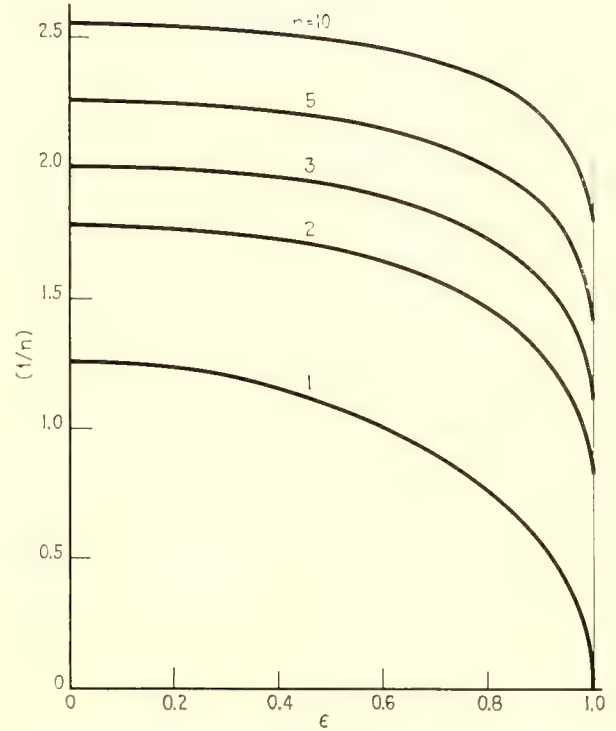


Fig. 84 Graphs of $\eta(1/n)$, the mean height of the $1/n$ th highest maxima as a function of ϵ , and $n = 1, 2, 3, 5$ and 10

spectrum $r \rightarrow 0.5$ as $\epsilon \rightarrow 1.0$. In this case, ripples ride crests and troughs of larger waves so that there are equal numbers of maxima above and below the mean level.

Pierson (1954b) applied Equation (130) to Neumann's spectrum and expressed \bar{T} as a function of wind speed, U , appropriate to the fully developed sea state

$$\bar{T} = 0.866(2\pi U/g) \quad (\text{Neumann}) \quad (133)$$

and

$$T_{\max} = \sqrt{2} \bar{T} \quad (\text{Neumann}) \quad (134)$$

where T_{\max} denotes the period at which the spectral density is a maximum.

Pierson (1954b) also considered the case of a sea defined by a Neumann scalar spectrum modified by inclusion of the factor $\cos^2\Theta$, an estimate of angular dispersion of wave energy which is assumed to be bounded by $(-\pi/2 < \Theta < \pi/2)$. For this case, Pierson derived the expression

$$\bar{L}_0 = \frac{2}{3} \frac{g}{2\pi} \bar{T}^2 \quad (135)$$

which states that the average distance between the successive dominant crests of the sea surface, when observed at a fixed time as a function of space, is equal to two thirds of the value computed from the classical formula for the harmonic wave period $T = \bar{T}$.

Some of the statistics just given are only of general interest and others may be computed directly from the

wave record. The strongest justification for computing wave spectra however lies in the description of the ship motions induced by the waves. This subject will be discussed in detail in Chapter 3, Section 3.

Before the subject of wave (and seakeeping) statistics is closed, some qualifying remarks on their validity, in view of the confidence bounds discussed in Section 8.4.2 should be made. The height statistics given in (128) depend only on E as given in (127). The record from which E is computed may be regarded as a sum of sinusoids whose amplitudes and phases are unrelated. Each sinusoid may be in turn considered as a chance sample from a population whose variance depends on the true energy density at that particular frequency. The total energy, E , is the sum of the energies in all of the component sinusoids in the spectrum. The deviation of the measured spectral densities from the true spectral densities is given by the degrees of freedom (126) and by Table 12. Since the spectral densities are independent estimates, their deviations from the true estimates are random and their addition is a much better estimate of the total energy than the error associated with the individual spectral densities. Consequently, confidence bands resulting from say 50 degrees of freedom which may appear to exhibit wide variation in the estimate of the true spectrum, Fig. 71, actually give a very good estimate of the total energy in the population from which the sample was drawn. The remaining statistics, (129) to (132), depend on moments associated with E and accuracies should be of the same order of magnitude as for E .

The proportional standard error (see the example, Tucker, 1957) of the variance (total energy) of a record in terms of the squares of the amplitudes in the harmonic components (S_n^2) is given by

$$\frac{1}{\sqrt{2}} \frac{(\sum S_n^2)^{1/2}}{\sum S_n}$$

From this, it is seen that the proportional standard error is inversely proportional to the square root of the length of the record.

8.7 The Directional Sea Spectrum. In the words of Marks (1954): "The sea surface is believed to be made up of a sum of an infinite number of infinitesimally high sine waves traveling in different directions with different frequencies added together in random phase (Pierson 1952). This surface, at any point, has a certain amount of energy associated with it, and this energy is distributed among the component waves according to frequency and direction. It is this distribution of energy and the particular direction of travel of each of the component waves which defines the two-dimensional energy spectrum $E(\omega, \theta)$ where $\omega = 2\pi/T$ is the spectral frequency and θ is the direction of propagation. This is the property of the ocean surface which must be measured."

As in the case of the one-dimensional or scalar spectrum, it is desired to find an analytical representation that will best define the directional qualities of waves. The most profitable way, at this time, appears to be the

brute-force method; that is, the directional spectrum is measured at sea enough times to provide an empirical evaluation of the general angular spreading characteristics of ocean waves and this information is used to determine an empirical operator which may then be applied to one of the spectrum formulations in Section 6. A substantial amount of research has been done on the method of collecting and reducing wave data to directional spectra and some of this work will be reported here.

Pierson (1952) was the first to try to show theoretically how wave records could be used to obtain information on the directional properties of waves. As a simple case, he suggested that a plane carrying a radar altimeter could make a variety of passes over the sea surface such that the record showing the shortest waves would be indicative of the mean wave travel. Flying very slowly and observing the waves visually would resolve the problem, through the Doppler effect, of estimating wrong by 180 deg. Cartwright (1956) used this principle on a ship to determine mean wave direction; more will be said on this later. Pierson extended his idea of an airplane collecting wave data, by flying rapidly over the surface ($t \sim 0$), to give a spatial definition of the sea surface. He derived an elaborate transformation and inverse transformation between the frequency and wave-number domains to show that it was possible to compute the directional spectrum if one could evaluate 77 linear inhomogeneous simultaneous equations with 77 unknowns. It is believed that no attempt has been made to apply any data to the matrix of this set of equations.

Pierson (1952) further suggested (after a conference with Tukey) that aerial-stereophotography would yield a grid of points that might lend itself to determination of the directional spectrum. The possible methodology was not discussed, but it did materialize a few years later in the work of Longuet-Higgins (1957) and in Project SWOP (1957). Both of these treatments will be discussed.

One final method suggested by Pierson involved the recording of waves from distant storms at shore stations spaced several hundred miles apart. With knowledge of the dispersive properties of waves and the geometry of the originating storm, one conceivably could extrapolate back from the measured swell spectra to the two-dimensional spectrum at the edge of the wave-generating area. An investigation not unlike the one suggested by Pierson here is being undertaken at the Scripps Institution of Oceanography where attention is being paid to the frictional dissipative processes in wave dispersion with distance, which was considered by Pierson to be negligible. This work will also be discussed later in this section.

The embryo ideas set forth by Pierson have since been prosecuted by other investigators (independently), but aside from these, no novel contributions have been made in measuring the directional spectrum except by N. F. Barber (1957) who proposed no less than eleven methods for such measurement. These methods, through the mechanics of measurement, lend themselves best to ir-

regular waves in model tanks and indeed are so specified. Barber's work also will be reviewed here.

Once there developed an appreciation for the energy-spectrum description of waves, theoreticians found it difficult to wait for a definite analytical formulation of the directional parameter. St. Denis and Pierson (1953) concluded from the work of Arthur (1949), on variability of direction of wave travel, that the energy in a wave spectrum travels directionwise according to a $\cos^2\theta$ law where θ is the deviation from the dominant direction of travel of wave energy. This angular-dispersion factor coupled with Neumann's version of the wave spectrum resulted in an expression for the directional-energy spectrum of the form

$$E(\omega, \theta) = \frac{c}{\omega^6} e^{-(2g^2/U) \omega \cos^2\theta}, \quad (-\pi/2 < \theta < \pi/2) \quad (136)$$

This expression was used as the excitation in a statistical approach to solution of the equations of ship motions (see Chapter 3, Section 3). Subsequently, there was some belief that the angular spreading of wave energy was not so great and the factor $\cos^2\theta$ in (136) was replaced by $\cos^4\theta$. This was accepted tentatively until actual measurements suggested directional measurements which are not so universal but which depend on the wave frequency (ω). The matter is by no means settled.

8.71 Extension of the covariance transform method. Section 8.3 discussed a formulation of the wave spectrum through the autocovariance and Fourier transform concept. Longuet-Higgins (1957) treated the extension of this method for the case of a short-crested Gaussian sea surface as a function of time and space. In particular, he showed that the Wiener-Khinchine relations, established for a time base, could be transformed to a spatial co-ordinate system without loss of generality. This is to be expected, in view of the ergodic theorem.

Consider the Fourier-series representation of the sea surface as a function of time, given in (133). If this is written in the general form

$$\eta_n = c_n \cos(u_n x + v_n y + \omega_n t + \epsilon_n) \quad (137)$$

where $c_n^2 = a_n^2 + b_n^2$ and $\epsilon = \tan^{-1}b_n/a_n$, and then extended to a spatial cartesian co-ordinate system, the resulting expression given by Longuet-Higgins for a continuous two-dimensional spectrum is

$$\eta(x, y, t) = \sum_n c_n \cos(u_n x + v_n y + \omega_n t + \epsilon_n) \quad (138)^{39}$$

where

$$u = \omega^2 \cos \theta / g \quad \text{and} \quad v = \omega^2 \sin \theta / g$$

are the projections of the wave number ($k = 2\pi/\lambda$) on the x and y -axes, respectively, and ω_n is a function of both u_n and v_n . The amplitudes c_n are random variables such that, in any element $du dv$ it is assumed that

³⁹ This is an abbreviated notation for the equivalent expression

$$\eta(x, y, t) = \sum_m \sum_n c_{mn} \cos(u_m x + v_n y + \omega_{mn} t + \epsilon_{mn}) \quad (138a)$$

used by some investigators. The formulation in (138) is that of Longuet-Higgins and is retained in this discussion of his work.

$$\sum_n \frac{1}{2} c_n^2 = E(u, v) du dv \quad (139)$$

The quantity $E(u, v)$ is the two-dimensional or directional spectrum of the waves.

According to Longuet-Higgins: "the mean square value of η per unit area of the sea surface per unit time is given by

$$\begin{aligned} \lim_{X, Y, T \rightarrow \infty} \frac{1}{8XYT} \int_{-X}^X \int_{-Y}^Y \int_{-T}^T \eta^2 dx dy dt \\ = \sum_n \frac{1}{2} c_n^2 = \iint_{-\infty}^{\infty} E(u, v) du dv \end{aligned} \quad (140)$$

Thus the contribution to the mean energy from an element $du dv$ is proportional to $E du dv$.

We shall write

$$\iint_{-\infty}^{\infty} E(u, v) du dv = m_{00} \quad (141)$$

and in general for the (p, q) th moment of $E(u, v)$ about the origin we write

$$\iint_{-\infty}^{\infty} u^p v^q E(u, v) du dv = m_{pq} \quad (142)$$

These quantities will occur repeatedly throughout the following analysis. It is assumed that they exist up to all orders required.

"The function $E(u, v)$ is closely related to the correlation function $\psi(x, y, t)$ defined by

$$\begin{aligned} \psi(x, y, t) = \lim_{X, Y, T \rightarrow \infty} \frac{1}{8XYT} \int_{-X}^X \int_{-Y}^Y \int_{-T}^T \eta(x', y', t') \\ \times \eta(x' + x, y' + y, t' + t) dx' dy' dt' \end{aligned} \quad (143)$$

On substituting from (138) in the foregoing we find

$$\psi(x, y, t) = \sum_n \frac{1}{2} c_n^2 \cos(u_n x + v_n y + \omega_n t)$$

which can be written

$$\psi(x, y, t) = \iint_{-\infty}^{\infty} E(u, v) \cos(ux + vy + \omega t) du dv \quad (144)$$

so that ψ is the cosine transform of E .⁷⁷

If $y = t = 0$, the result is the spectrum of a record obtained as a line drawn on the stereophotograph of the sea surface. When the wave record is obtained from a pair of stereophotographs, the elevations $\eta = \eta(x, y)$ are for a fixed instant of time ($t = 0$) and the spectrum can be expressed as

$$E(u, v) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \psi(x, y, 0) \cos(ux + vy) dx dy \quad (145)$$

In the foregoing exposition taken from Longuet-Higgins (1957), the directional spectrum was represented in terms of projections of wave numbers on the x and y -axes. The presentation was oriented to show that the measurement of the directional spectrum $E(u, v)$ lends itself to determination from a rectangular grid consisting

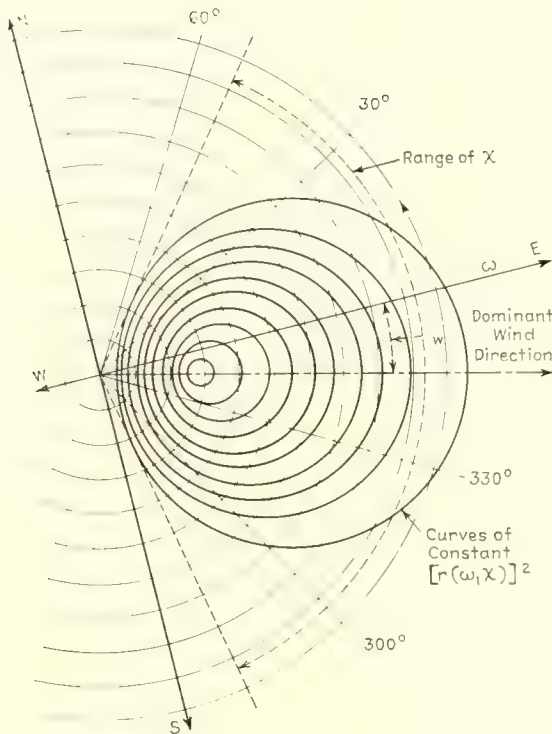


Fig. 85. Energy spectrum for a sea (idealized) (from St. Denis and Pierson, 1953)

of lines parallel to the x and y -axes of a spatial picture of the ocean. This method is essentially that derived earlier and used in the observed sea spectrum determination by Chase, et al (1957) which will now be discussed.

The first attempt to measure directional spectra at sea is known as project SWOP, the complete designation of which is Stereo Wave Observation Project (Chase, et al, 1957).

Several pairs of aerial stereophotographs of an open-sea surface, were made from two airplanes flying upwind in tandem 2000 ft apart at an altitude of 3000 ft. The cameras in the two airplanes were simultaneously triggered by a radio link. In all, each camera had taken 100 pictures. The scale of distances was provided by the RV *Atlantis*, which towed a raft at a distance of 500 ft from it, approximately in the middle of the photographed area. RV *Atlantis* also took wave records by means of a wave pole.

After examining all photographs, two overlapping pairs were chosen for analysis with rectangular usable areas of 1800×2700 ft. This area was subdivided into intervals of 30 ft, giving a 60×90 grid. The grid intersection points were used both for stereophotogrammetric measurements and for subsequent spectral analyses. The use of the same points for both purposes was important since the point height accuracy was estimated at ± 0.5 ft while the contours drawn at the 3-ft intervals were accurate to ± 2.0 ft.

The spectral analyses were made in terms of wave-number projections u, v , on the x and y -axes, as shown by

equations (144) and (145). These equations represent, of course, only the mathematical foundation and were converted to summations of finite intervals for actual use. Twenty lags were used in the analysis for each of two stereo-pairs.

A good deal of difficulty was experienced, because the first analysis indicated a tilted sea surface. A large amount of corrective calculations was thereby made necessary. The completed spectrum, $E(u, v)$, was converted to an $E(\omega, \theta)$ -spectrum and was integrated with respect to θ in order to obtain a scalar spectrum for comparison with the wave-pole data and with the corresponding Neumann spectrum. The final expression for the directional operator is given by

$$0.25(1 - \gamma) + (0.50 - 0.46 \gamma) \cos^2 \theta + 1.288 \cos^4 \theta \quad (146)$$

where

$$\gamma = \exp[-(\omega U/g)^4/2]$$

Quoting from Pierson (Chase, et al, 1957, p. 240): "If ω is small, the angular term in (146) becomes

$$0.04 \cos^2 \theta + 1.28 \cos^4 \theta \quad (147)$$

which shows that the spectrum is more peaked at low frequencies than has been assumed in (136). Conversely if ω is large, the angular term in (146) becomes

$$0.25 + 0.50 \cos^2 \theta \quad (148)$$

which shows that the spectrum is more evenly spread out at high frequencies than has been assumed previously." Fig. 85 shows an idealized directional spectrum. Fig. 86 shows the actual directional spectrum obtained by the SWOP measurements.

The SWOP report gives valuable data on one particular set of sea measurements. It is the only instance to date in which the directional spectrum was measured in an open sea. The organization of the measuring expedition involved a ship and two airplanes; joint efforts of several organizations and of many persons were involved. Up to the time of writing the final report, 33 months' time was spent in planning, data collecting and data reduction, and presumably, the enterprise was very expensive. There is little question that the methods of SWOP leave much to be desired from the economics (time, effort, money) point of view. It is hoped that the methods of Cartwright and of Barber, which are far simpler in nature, can be made to yield more information on the directional parameter of a wind-generated sea.

8.72 Miscellaneous investigations of the directional spectrum. Marks (1954) outlined a mathematical method for evaluating the directional spectrum from stereo-movies of the sea surface. A fast, low-flying aircraft, photographs the sea surface with a stereophotographic movie camera which effectively records the three-dimensional wave form of a narrow path through the

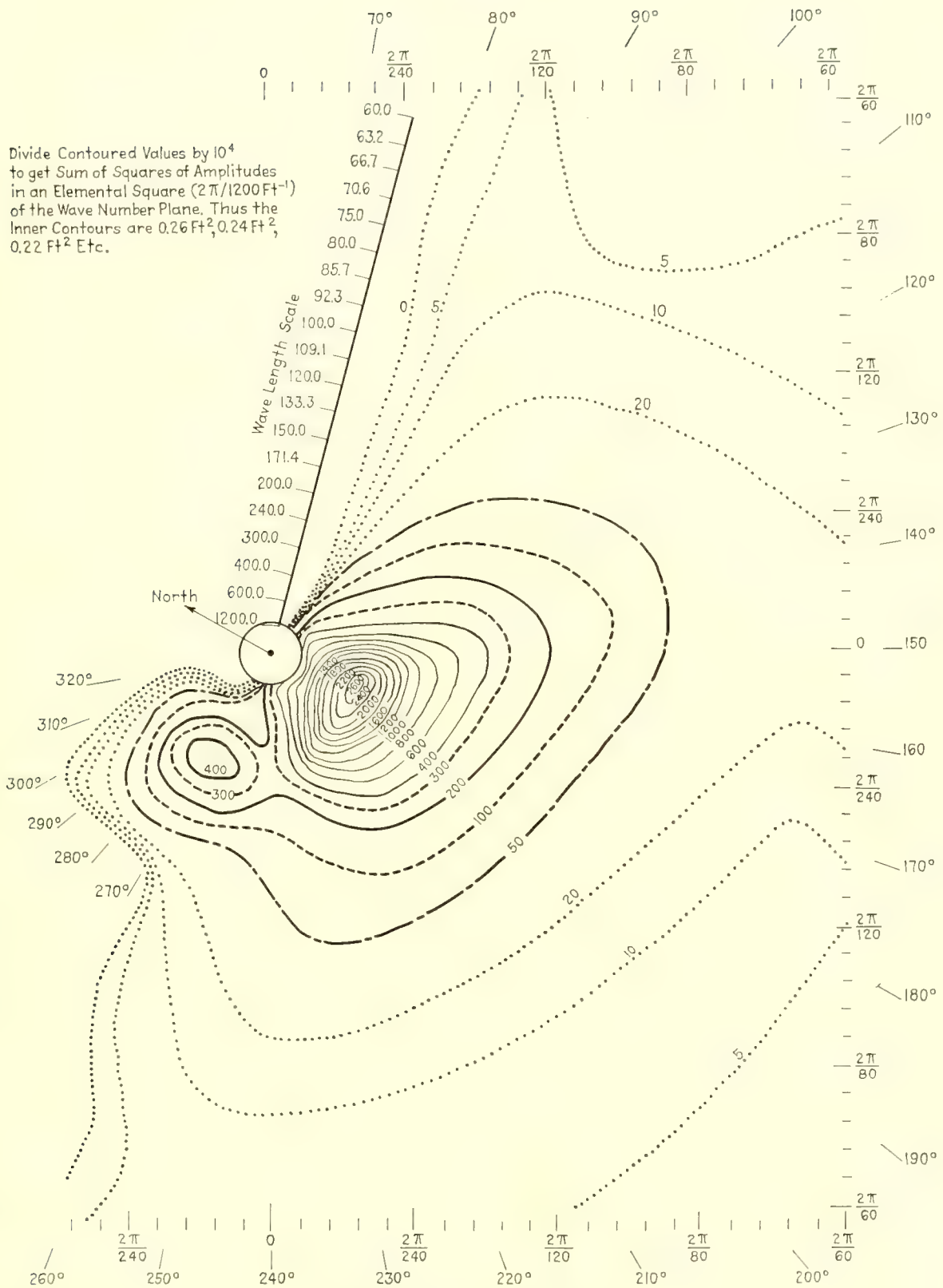


Fig. 86 (From Chase, et al, 1957)

waves. Equispaced lines are drawn parallel to the direction of travel and points are read on these lines as if they were wave records made as a function of time at a fixed point. These records are averaged and a mathematical filter is constructed which depends on the number of original space records, the distance between them, and the point reading interval. The filter is then tuned angularly through the averaged space record represented by the original component space records. This method has never been put into practice.

Cartwright (1956) attempted to measure the directional distribution of waves in the open sea using a shipborne wave recorder on the RRS *Discovery II*. The instrument gave a wave record which could be analyzed to obtain a scalar spectrum. Directional sensitivity was obtained by operating the ship so that each wave component (ω, θ) was subjected to a "Doppler shift." The ship was steered round a regular dodecagonal circuit at a constant speed of 7 knots and waves were recorded for about 12 min on each of the 12 courses. Cartwright presented his analytical theory and samples of two sets of observations. Theoretically, a complete evaluation of the directional spectrum $E(\omega, \theta)$ is possible, but actually the statistical variability in the spectrum estimates is too great (12-min records are too short) to make such an evaluation practical. The situation is simplified if the spectrum contains some isolated peaks at well-separated values of ω and θ which are not lost in the short samples. Then the mean directions of the dominant wave systems can be determined.

St. Denis (1957) proposed another method, based on records to be obtained from several probes arranged along a straight line. The row of probes is to be oriented alternately in several directions. St. Denis' analysis leads to the suggested solution of a 48th-order matrix which can be solved on a large electronic computing machine. The proposed method has not yet been tried experimentally.

N. F. Barber (1954) measured the directional wave spectrum by means of a row of detectors, in connection with a mechanical frequency filter (a resonant pendulum). Quoting Barber: "In the experiment, each detector was a pair of parallel vertical copper strips, partly immersed in the sea and fed with 2 volts (mains frequency) from a transformer. The current that passed was proportional to the depth of immersion, varying with the waves, and a corresponding voltage signal was conveyed to the analyzer. Two such instruments were hung from a pile wharf at separation of 2, 9, 18 and 27 feet in succession."

The theory of the analysis was based on the correlation coefficient

$$R_{xy} = \iint_{-\infty}^{\infty} E(u, v) \exp [i(ux + vy)] du dv \quad (149)$$

where x, y are the components of the distance between two points at which water-surface elevations are measured and u, v are the projections of the wave number k on the x and y -axes. The average of expression (149)

was to be taken over a long period of time. In the experiment a 15-min record was taken for each pair of detectors in succession. Since a very narrow frequency filter was used, equation (149) applies to a single wave number, $k = k_0$.

Quoting further from Barber (1954): "It was desired to restrict the wave measurements to discrete points upon a line taken as the x -axis. Such results can give E so long as attention is restricted to waves very near some frequency ω_0 with corresponding wave length $2\pi/k_0$. It then appears, to a sufficient approximation, that equation (149) inverts to a Fourier series:

$$E(k_0, \theta) = E_0 \sin \theta \left[\frac{1}{2} + \sum_{n=1}^{\infty} c_n \cos (unD - \epsilon_n) \right] \quad (150)$$

where $\theta = \cos^{-1}u/k_0$, the direction of wave progress relative to the line of recorders, $E(k_0, \theta)\delta\theta$ is the energy of waves with number k_0 and direction between θ and $\theta + \delta\theta$, and c_n, ϵ_n are the modulus and phase of the complex correlation coefficient between wave elevations at points separated by distance nD . The distance D should not exceed π/k_0 or half a wave-length. The expression is valid so long as no waves have directions outside the range 0° – 180° . The factor $\sin \theta$ is an approximation the error of which causes an underestimate of the wave energy in directions near 0° and 180° ."

"A pendulum of period near 2 sec was used to select a narrow frequency-band and to produce the pictures in Fig. 87. Its free motions in all azimuths had equal period and damping, and it was electrically driven through amplifiers fed with the wave signals. The motion of the pendulum was recorded optically as a spot of light moving over a photographic plate, an exposure of 15 min being required for each of the pictures in Fig. 87."

The electrical connections were such that combined signals from two wave detectors caused a spot of light to move on a straight line. Variations of the amplitude and direction of irregular waves of the filtered frequency ω_0 caused the exposure of an approximately elliptical area. The major axis of this area was inclined at an angle $\epsilon/2$, and the modulus C was given in terms of the principal diameters A and B ,

$$C = (A^2 - B^2)/(A^2 + B^2) \quad (151)$$

The theory leading to these results, the design of the analyzer-pendulum, and the electrical network were described more completely in later papers by Barber and Doyle (1956) and Barber (1957).

The foregoing outline can be completed by two additional quotations from Barber (1954): "Some preliminary experiments have been made in the Waitemata Harbor, Auckland, measuring wave direction by the correlogram. The photographs in Fig. 87 show statistical relations between wind waves of period near 2 sec at points separated by distances of 2, 9, 18 and 27 ft along a fixed line. From them may be obtained the curve in Fig. 88 showing how the energy is distributed among the various directions of travel. The wind was about 15 knots and 2-sec waves were dominant; but because the fetch in the

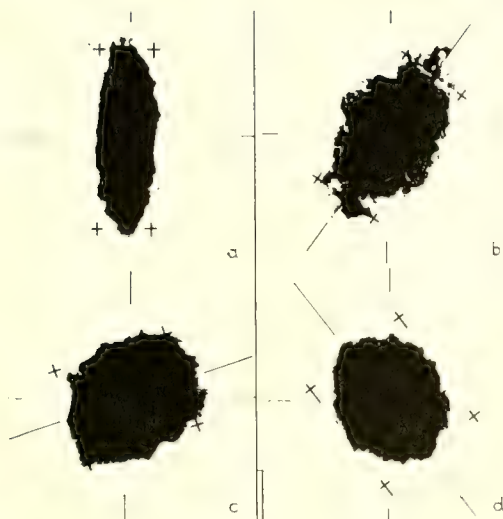


Fig. 87 Photographs showing statistical phase relations between waves at points separated by 2, 9, 18 and 27 ft along a fixed line. Attention is confined to waves of period near 2 sec (from Barber, 1954)

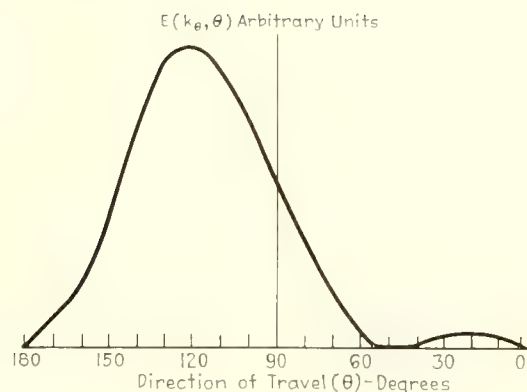


Fig. 88 Distribution of wave energy with respect to direction of travel relative to line of instruments. Curve is deduced from records in Fig. 87 (from Barber, 1954)

wind direction was much greater than elsewhere, it is not expected that the curve in Fig. 88 will apply to open water." "The $E(k_0, \theta)$ curve in Fig. 88 was computed from the last three photographs in Fig. 87. It shows that the highest waves are traveling in the direction 120° to the positive sense of the line joining the instruments. This was, in fact, the direction of the wind at the time of the measurements, so the method has given reasonable results."

It appears that Barber developed the first simple and practical method for measuring the directional wave spectrum. By the nature of a mechanical pendulum, he was limited to measuring the directional characteristics of waves near a single frequency ω_0 . Simultaneous measurements at several frequencies would be possible, however, by using several electronic filters.

Barber is also the originator of eleven methods for measuring the directional spectrum in the model tank. Since these methods speak of specifically spaced detectors making time histories, certain complications in application to open-sea work arise. Certain of the methods may conveniently be transformed to spatial systems which suggests the use of aerial stereophotography at sea. It is also possible that a fixed network of probes at sea can be arranged.

The first four methods deal with line arrays of detectors. They are, in the different methods: a) Rotated; b) remain fixed but treated with time delays; c) remain fixed but modulated by cosine operators; d) remain fixed and modulated, but measure horizontal water motions as well.

Each method results in a set of records which is then averaged and filtered. The first method is in fact the temporal analogy of the method of Marks (1954). Only

the fourth method distinguishes the proper directions of the waves; the others cannot judge whether the waves are coming or going, so to speak. The fourth method uses as few as six double detectors placed in a straight line.

The remaining seven methods are correlation techniques which depend on the geometrical orientation of the probes. They require fewer detectors and considerably less computational effort; the results suffer, as a consequence. In the words of Barber: "In my opinion, correlation methods are likely to be most useful where they aim to use comparatively few measurements to find the outline or general character of the spectrum. Here, they are more powerful than in the case of more orthodox "array" methods.

"For routine analysis in the Model Basin, an 'array' method would be best. It could be almost automatic. If the wave directions lie within an arc of 150° , then Methods 2 or 3 would serve to give an analysis in about 40 minutes. Method 1 would serve if the waves were in an arc of 170° and if a long sample were not necessary. If random waves may come from any direction, Method 4 is necessary, but it calls for a detector both for the vertical and the horizontal water motion.

"It may be worth using a correlation method while the apparatus for a more routine method is being built."

Gelei, Cazalé, and Vassal (1957) evaluated an energy spectrum by an analysis of ocean swells. These arrived at Dakar and Casablanca from many generating areas with wind velocities at various angles to the direction of the swell propagation. The spectrum, $E(T, \theta)$, in terms of wave period, T , and direction θ , was presented in the form

$$E(T, \theta) = f(T) \varphi(\theta) \quad (149)$$

where

$$\begin{aligned} \varphi(\theta) &= 9/4\pi = 0.72 & \text{if } 0 < \theta < 20^\circ \\ &= 9/8\pi = 0.36 & \text{if } 20^\circ < \theta < 60^\circ \\ &= 0 & \text{if } \theta > 60^\circ \end{aligned}$$

For a fully arisen sea

$$\begin{aligned} f(T) &= 209 T^4 && \text{if } T < 0.315 U \\ f(T) &= 209 T^4 \exp [-329(T/U - 0.315)^2] && \text{if } T > 0.315 U \end{aligned} \quad (150)$$

where the component wave period, T , is in seconds, and the ground wind velocity U is in knots. The spectral density $E(T, \theta)$ is in ergs per cm^2 per sec.

8.73 Present work. There are, at present, two known attempts to provide further information on the directional-wave spectrum. The Woods Hole Oceanographic Institution is using Barber's method 1, that of a rotating-line array, affixed to a tower in Buzzards Bay. The object is to learn more of the characteristics of the directional operator. In addition similar measurements are planned for a fixed tower offshore in relatively deep water. Here, open-sea waves are available, and in addition to measuring the direction of wave propagation, it is hoped to learn something of the scaling properties between small and large waves in order to determine whether sheltered waterways, generating relatively small seas, are natural test facilities for ship models.

At the Scripps Institution of Oceanography, swell waves are measured at two points near the California coast. The swell spectra result from waves traveling several thousand miles from the generating areas in the South Pacific. Additional detectors are planned for installation along the route of wave propagation to study further the angular decomposition of waves and their dispersive characteristics as well. Trade-wind effects on wave attenuation are of special interest in this work.

8.74 Summary of directional spectrum. Only three estimates of the directional distribution of open sea waves are available to date:

1 Arthur's (1949) derivation and measurements, interpreted by St. Denis and Pierson as the $\cos^2\theta$ law.

2 Gelci, Cazalé, and Vassal's (1957) evaluation from swell observations. The rough estimate of the directional energy distribution is given by equation (149).

3 SWOP project's (Chase, et al, 1957); a directional function is given by equation (146).

Gelci, Cazalé, and Vassal's evaluation appears to be the only one based on statistics of many observations over a period of several years. Since it is based on swells arriving from a distance, it can be expected to be deficient in high-frequency components.

There is no reason to expect a uniform directional spread of wave energies in different conditions at sea. Eckart (see Section 4.1) indicates that directional spread depends on the observer's position in the generating area. Project SWOP indicated a broader spread for high frequencies and a narrower one for low frequencies. On this basis a plausible hypothesis can be made that a "young sea" with large ratio of wind speed to wave celerity, U/c , will have a broad directional spread and that as U/c ratio decreases, the directional spread will become narrow.

9 Research Suggestions

It is hoped that ideas for further research will stem primarily from exposition of the shortcomings of the existing knowledge which were emphasized throughout the previous text. Generally speaking, two parallel directions of research must be pursued:

(a) Development of semi-empirical methods.

(b) A long-range development of rational methods.

A large amount of observations of natural phenomena and simulative experimentation is needed to form the basis for the first, and to give orientation to the second, as well as to provide the data for verification of theoretical methods.

The description of a sea surface by a spectrum has been the most valuable practical achievement of the last decade. Through its use great steps have been made in understanding a ship's behavior in the natural (always irregular) seaway. The discussion of energy balance in Section 3, and the work of Munk, Section 4.4, and Phillips, Section 4.2, have also indicated that a spectral concept of the seaway is necessary in developing a rational theory of wave growth.

In discussing spectra a distinction should be made between

A Spectra defining waves as a function of meteorological conditions.

B Spectra defining the sea state at a given time and place, regardless of its causes.

Descriptive spectra B are needed because an observer on board a ship rarely has the information to determine spectra A. Usually, rather tedious work is required of a meteorologist before the waves can be forecast and there is considerable uncertainty in this process. Although spectra A are needed for forecasting waves, they are not suitable for describing an observed seaway because a sea state is seldom defined by the simple wind conditions assumed in deriving these spectra.

9.1 Development of Semi-Empirical Methods. At the time of this writing there are four spectra formulations of Type A based on independent observations. They differ widely both in the indicated rate of wave growth and in the ultimate spectrum of fully arisen sea. Each spectrum is an empirical evaluation of the observed data available to the particular researchers. Often extrapolation is resorted to, to eke out insufficient data. The demonstrated discrepancies between the different spectra are evidence that a large amount of additional observations is needed.

The magnitude of the discrepancies, however, make it improbable that a universally valid spectrum formulation can be arrived at by the purely data-fitting technique which has been used heretofore. The various conditions under which the wave data were measured must be given more consideration, and, inasmuch as possible, quantitatively appraised. The three separate parameters used now, wind velocity, fetch, and duration, do not appear to be adequate for unequivocal spectrum definition. A search must be made for other possible

factors. The reasonably indicated effect of air-water temperature difference should be included in the basic data, rather than left to be averaged out. The effect of limited water depth on long wave components may have to be considered. Instrumental corrections may have to be given more attention.

The author doubts, however, whether a simple empirical data-fitting technique can be successful even with the increased care suggested in the foregoing paragraph. A semi-empirical approach based on the rational concept of energy transfer from wind and dissipation of it in waves may well give more nearly universally valid relationships. The prespectrum work of Sverdrup and Munk, Section 5.1, was of this type and was outstandingly successful. So also was the prespectrum work of Neumann, Section 5.2. Reasonable energy-balance equations can be set up, leaving the values of certain coefficients and parameters to be derived from the observed sea data. Often the expedient of grouping certain parameters permits a better defined arrangement of observed material.

While spectral definitions are a great advance in seaway description, they tend to concentrate attention on a static concept of energy per unit area. The dynamic (energy-flow) concept of the prespectrum work of Sverdrup and Munk, and of Neumann should be blended with the contemporary spectral-seaway concept.

The semi-empirical approach suggested in the foregoing requires data gathering in a suitable form. Wave spectra determined simultaneously for several positions along the fetch will be needed. It will be difficult to arrange such observations in the open ocean, and the development of suitable spectrum formulations may depend on small-scale experiments in limited water areas. A method of scaling to larger ocean waves may be formulated on a semi-rational, semi-empirical basis. The necessarily more limited observations in the open sea may then be used either for verification or for evaluation of isolated crucial parameters.

9.2 Rational Approach. The word "rational" is relative. Essentially all research has to start from a certain empirical basis. There is, however, a rather definite distinction between semi-empirical and rational approaches in their objectives. The semi-empirical approach can be thought of as a stop-gap; the objective is to arrive at a workable solution as quickly as possible, aiming at the best available approximation to the natural events but not necessarily seeking a profound understanding of these events.⁴⁰

In the rational methods, on the other hand, a substantial understanding of wave generation and wave behavior is sought. There is no desire to arrive at a solution quickly at any cost, rather, emphasis is put on a thorough investigation of the phenomena involved and on steadfast adherence to the principles of hydromechanics. The results of such investigations are freer of uncertainties of empirical observations.

The phenomenon of wave growth under wind action is a complicated one and research must be directed to uncovering its many facets before the whole solution can be formulated. Bowden's work cited in Section 3.1 and the work outlined in several subsections of Section 4 are on isolated facets. The evident weakness of the current activity is the lack of a comprehensive guide which would indicate the proper place for all facets. While the validity of the results would still depend on improved knowledge of individual facets, such a guide would be of great value in organizing and channeling miscellaneous activities. Apparently the only attempt in this direction was made by Neumann in his prespectrum work, Section 5.2 and Appendix B. Despite the fact that his study has for its prime objective arrival at a directly usable semi-empirical solution, it could serve well as a guide for the new rational exposition. Its drawbacks are that it has not been translated into English and that it is overburdened with detail considerations.

One difficulty in developing a rational wave-growth theory is described by the following free translation from Neumann (1954) attributed to A. Defant: "the unsatisfactory condition appears here in that theoreticians apply themselves to the advancement of theories, and with completion of these their interest fades; investigators working on the observational material, on the other hand, have mostly too little theoretical training to apply achievements of theory to particular cases. Close co-operative developments of observations and theory would here, as everywhere, be very beneficial." Since it is extremely difficult to find an individual possessing all required qualities, it appears that best progress can be achieved by well-matched pairs or groups of investigators.

It should not be assumed that "rational" research is wholly theoretical, nor that it is limited to the use of high-level mathematics. The multiplicity of the facets of the problem permits utilizing theoretical work of all degrees of complexity, and calls for a wide variety of experiments and observations.

Only one thing is universally required; namely, to be at all times conscious of the ultimate use of the results of a particular research, and to visualize how a particular facet fits into the whole. This appears to have been disregarded often in wind-flume experiments. The primary objective there has usually been to evaluate the tangential drag force of a wavy water surface, but this surface has been only crudely described by an average wave height, ignoring other wave-surface characteristics of the wide variety which exists. In particular, in these experiments the wind speed-to-wave celerity ratio, U/c , is usually too high and the experimental wave steepness H/λ is also too high in comparison with the natural large-scale phenomena.

The empirically obtained spectra formulations, the theoretical work on spectrum properties by Pierson, Longuet-Higgins and Cartwright, and the theoretical work on wave generation by Munk, Miles, and Phillips indicate that the results of past work in wind flumes and in limited water areas are of little usefulness. Essentially

⁴⁰ The very useful pragmatic activity in spectral representation of the sea surface has been a regression in this respect from the prespectrum work of Sverdrup and Munk and of Neumann.

all of this work should now be repeated with proper regard for the new theoretical requirements. As a minimum requirement, the observed waves must be presented in spectral form. Consideration must be given to short-crestedness; i.e., to the directional spectrum of waves. There are indications that this may strongly differ in a wind flume from that of the larger natural waves; a too high U/c ratio and wall reflections appear to exaggerate the short-crestedness of the flume waves. The surface-drag characteristics may be strongly affected by the degree of short-crestedness.

Velocity-gradient measurements in the air flow should be made simultaneously with drag measurements by means of water-surface inclination. Although the use of such measurements as an alternative method of wave-drag determination has been known for long, the recent work of Miles and Phillips has intensified interest in them for a more complete investigation of air-flow properties, including the evaluation of air turbulence.

Corresponding measurements of water turbulence also may be useful in evaluating the transfer of energy from wind and its dissipation in waves. Presumably, Miles' theory pertaining to the air flow can be applied also to the conditions existing in the upper layers of water. A distinction must be made between the potential air-flow energy transmitted to the potential wave energy and the turbulent energies of air and water which are caused by viscosities.

Comprehensive measurements in large sea waves are difficult to make and are expensive. Wind-flume experiments have the wall effects on water waves and on air flow which are difficult to evaluate with certainty. While both of these activities should be continued, it appears to the author that the most rapid progress can be made by observations in moderately large but limited water areas and in predominating waves of, say, 10 to 30 ft long. The geography of the United States offers many opportunities of locating and utilizing such areas. The water must be sufficiently deep to avoid wave distortion, yet shallow enough to permit installations of poles or towers carrying measuring instruments. Measurements should be made simultaneously at several stations along the wind direction, and the simultaneous recording technique must permit correlation analyses. This is particularly important for the eventual separation of the energy-transfer problems from those of energy dissipation. Care should be taken in choosing a site so as to assure freedom from swells arriving from elsewhere and from wave reflections. The results of observations should be used in development of semi-empirical and rational relationships, but *it should not be assumed that small-scale seas are directly equivalent to ocean waves*. The U/c ratio appears here to be the prime controlling factor, together with the fact that surface tension and viscosity makes it impractical to conduct model observations at sufficiently low U/c ratios.

Condensed List of Suggested Research Topics⁴¹

1 Wind-Flume Experiments, as described in Section 2.3 should be repeated and more comprehensive measurements taken as outlined in Section 9.2. As a minimum, measurements at three stations along the flume are needed to provide a possible means of separating analytically the simultaneous transmission of energy from the wind and dissipation of energy in waves. Particular care is needed to prevent wave contamination by waves reflected from the leeward beach. If an artificial wave generator is used to increase the effective fetch (as by Francis, 1951), the generator should be adjusted to provide initial waves identical with the wind waves previously found at the leeward beach without generator action. Sufficient flume height is needed to assure a normal wind-velocity gradient near the water surface.

2 Wind-Tunnel Experiments should be made on wave surface drag, similar to Motzfeld's, Section 2.3, but with short-crested waves and oblique long-crested waves.

3 Co-ordinated Wind-Tunnel and Wind-Flume Experiments. It would be desirable to have correlated wind-tunnel and wind-flume experiments, where the wind-tunnel model corresponds to the wave structure instantaneously recorded in the flume (refer to Sections 2.3 and 2.4). Particular care must be taken to repro-

duce sharp-crested wavelets. The measurements made for model reproduction will also serve to estimate the percentage of the water surface covered by such wavelets. Proper attention to short-crestedness is important because the drag of a sharp-crested wavelet can be expected to diminish rapidly with obliqueness of the crest to the normal to wind direction.

4 Observations on Limited Natural Bodies of Water (such as Van Dorn's, Section 2.5), but with a complete record of wave and air conditions, Section 9.2, are recommended. These can also, with advantage, be made on larger water areas than were used by Van Dorn and in larger waves. The limit here is set only by the feasibility and convenience of instrument installation. Larger waves are suggested in the hope of obtaining c/U values more nearly corresponding to ocean-wave conditions than is practicable with small waves. It is suggested that two intermediate observation stations be used in addition to the two at the beginning and the end of a fetch. The end stations must be located at reasonable distances from the shore in order to record sufficiently developed waves at the beginning,⁴² and to avoid

⁴¹ Approximately in the order of presentation in the body of the monograph.

⁴² The author can see little practical usefulness in the study of wave initiation. In his opinion, Roll's (1951) observations of waves, which were irregular from the beginning and traveled in oblique directions to the wind, exhaust the usefulness of initial wave study. The author realizes, however, that many oceanog-

contamination by reflections and by water shallowness at the end of the fetch. The need for a complete air-flow description (velocity gradient and turbulence) at each station must again be emphasized. Measurements of water turbulence and mean drift velocities are also desirable.

5 Manual of Applied Boundary Layer and Turbulence Theories. It is unlikely that either practical oceanographers or naval architects will become fully conversant with all aspects of turbulence or boundary-layer theory. It is desirable that a brief and simple exposition of these theories be prepared. The primary objectives of such exposition are to further intelligent planning of wave-propagation experiments and to guide the analysis of observed results (not the development of advanced theories). Only relevant sections of the basic theories needed in handling practical problems are to be discussed.

6 Boundary Layer Over Waves. Effort should be applied to establish air boundary-layer characteristics over sea waves of different types. The sea types can probably be roughly characterized by H/λ and c/U ratios, more completely by Voznessensky and Firsoff's spectrum and most completely by the actual spectrum of the measured waves. The project may consist of a survey of published data (further expanding the work of Ellison, 1956) as well as analyses of the data of projects suggested under 3) and 4) (refer to sections 2.7, 4.2 and 4.5).

7 Energy Dissipation in Waves. A poor understanding of the mechanism of energy dissipation in waves is an outstanding obstacle to development of the theory of wave growth under action of the wind. Two basic parts of this mechanism appear to be: (a) Energy dissipation by internal friction, and (b) energy dissipation in wave-breaking. The first was quantitatively, although quite inadequately, investigated in the classical theory (Appendix A, Section 4.4 and as described in Sections 3 and 3.1). The second appears to exist now merely as an idea. The whole subject is complicated and several separate facets probably could be investigated in different projects. Some of these will be listed in the following. The author believes that an evaluation of the energy transmitted by wind and the energy dissipated in water may be obtained by analysis of the data obtained from projects 3) and 4) for several sections of the fetch. This belief is based on the apparently different rates of growth of the two energies with wave steepness and with absolute wave size. The effects of the two different functions in the algebraic summation may be estimated if their sum is evaluated at several fetch values. This empirical separation of the two functions will be greatly facilitated and made more certain by prior development of theoretically established forms of these functions.

8 Energy Dissipation Based on von Karman. It is advisable to investigate use of von Karman's (1930)

relationships in a turbulent fluid flow in evaluating the energy dissipation in waves by internal (turbulent) friction, verifying and extending the work of Bowden, Section 3.1. The objective is to derive functional relationships between energy dissipated by waves and the size and proportions of waves and then to generalize the relationships found for harmonic waves to a wave spectrum.

9 Effect of Water Viscosity in Waves. It is generally known that the importance of the fluid viscosity increases with increase of the velocity gradient in a fluid flow. The viscosity is significant only when the velocity gradient is large, usually in limited regions. A project can be formulated in which the velocity gradient, known for the orbital water-particle velocity in harmonic waves, is generalized by statistical methods to apply to wave spectra. Both long-crested and short-crested waves must be considered. The results of this evaluation can be applied to the problem of energy dissipation in two ways:

(a) Expression for the probability of exceeding arbitrary levels of water-velocity gradient can be established. The energy dissipation by viscous forces can be expressed in terms of velocity gradient by an assumed (sufficiently broad) functional relationship with undetermined parameters. The parameters are to be determined later by analysis of the empirical data obtained in project 7).

(b) In the narrow regions of high-velocity gradient the Navier-Stokes' equation can be applied neglecting inertial forces and solving for the pure viscous flow. This may permit quantitative evaluation of the relationship between velocity gradient and the work of viscous forces. The project is to be completed as under a) by estimating the probability of various levels of velocity gradient, and by applying the resultant functional relationships to the empirical data of 7).

10 Viscous Conditions at Wave Crests. The projects outlined under 9 can be carried out with particular emphasis on the conditions at the crests of waves (including wavelets) of maximum steepness.

11 Energy Dissipation by Breaking Wave Crests. A rough approximation of energy losses by the breaking of wave crests may be possible by the following method: First compute the wave energy in a wave spectrum by a conventional linear superposition theory. By the use of spectral relationships for the root-mean-square of this energy, determine the height of waves exceeding the theoretical maximum steepness corresponding to an included angle of 120 deg at the crest (this is characterized by a water acceleration of $-g$). Evaluate the excess of energy resulting from linear superposition over the energy corresponding to limiting wave steepness and assume this to represent the energy lost in breaking waves. Cox and Munk's observations, Section 4.3, may be used as empirical information on the increased steepness of the leeward slopes of the wind-driven waves.

12 Towing-Tank Measurements of Wave-Energy Dissipation as a function of wave steepness are recom-

ographers hold different views on the subject. At any rate, the problems of initiation and of development of waves must be separated.

mended. The tank should be of rectangular cross section with a good wave generator, so that regular waves of low steepness progress without appreciable change of form and amplitude. A good absorbing beach must be provided so that progressive waves are not contaminated by beach reflection. The wave generator should be programmed to make irregular waves with a spectrum not too different from a typical sea spectrum. The wave-energy spectrum is to be measured at two or more stations in the part of the tank where the good conditions called for above obtain. It can be expected that the difference in the spectral energy at the different stations will grow as the steepness of the generated waves is increased and as an increasing number of breaking crests appears. It will be remembered that the rate of energy dissipation in waves is low, and therefore high accuracy is necessary in the experiments. These towing-tank experiments, in which no transfer of energy from wind is present, will provide valuable data for separation of the wind-transferred and wave-dissipated energies in the master project 7).

13 Energy Dissipation in Irregular Waves. The results of project 12) obtained in long-crested waves should be generalized to short-crested waves by means of the data of project 9).

14 Energy Transmission from Wind to Waves. None of the heretofore published work on the transmission of energy from wind to water has presented a complete solution but each has been an advance in a certain direction. All of these can be listed, therefore, as good directions of further research: The work of

(a) Eckart, Section 4.1, and Phillips, Section 4.2, based on the random fluctuations of air pressures in a gusty wind. More observations on air turbulence are needed and can be obtained from project 4).

(b) Munk, Section 4.4, based on the air pressure as a function of wave form and the wind velocity at anemometer height. Improvement of the pressure-distribution function and introduction of wave-facet velocity are suggested. Longuet-Higgins' (1956, 1957) work will be valuable in defining the facet velocity.

(c) Miles, Section 4.5, based on the air pressure as a function of wave form, but evaluated by the potential solution for air flow possessing a velocity gradient. Consideration should be given to variation of the velocity-gradient profile with location along the wave contour in order to make the solution applicable to waves of significant steepness. The shape of small, usually sharp-crested, wavelets carried by larger waves is governed by the algebraic difference between the gravity acceleration and the acceleration of the large-wave motions. This leads to expectation of steeper wavelets at the crests and less steep ones in the hollows of the larger carrier waves. This difference can be expected to be aggravated further by the higher ambient air velocity over wave crests and the reduced one over troughs. These changes of local water-surface characteristics can be expected to affect the roughness parameter and the velocity-gradient profile.

These should be measured on rigid models in wind tunnels and also in observations of large natural waves.

15 Air-Velocity Profile Measurements. More measurements of the air-velocity profiles over waves are needed for practical implementation of Miles-based projects.

General Remarks on the Problem of Wave-Energy Transmission. It should not be assumed that only mathematically complicated work such as Phillips' and Miles' is valuable. Useful results can be expected at all levels of simplicity or sophistication. Generally speaking, complex solutions have to be limited to a narrow facet of the broad problem. Simpler and cruder solutions, on the other hand, can usually be expected to cover wider ground and to lead sooner to usable (if less refined) results because they are not overburdened by details regarding any one aspect of the problem. Although work on refined and sophisticated solutions must be pursued in order to gain a more complete understanding of the phenomena, a researcher (and his sponsor) must realize that refinement in an isolated facet of a problem does not necessarily lead to improvement in the over-all result. Quite the contrary, it often upsets the balance obtained in a cruder approach. Refined solutions of individual aspects of a problem are expected to produce improved results only after several aspects are cleared and the broader problem is covered. Meantime, sufficient research effort should also be applied to simpler and broader approaches.

16 Combined Theory of Wave Generation by Random and Wave-Correlated Pressures. The three research projects covered under 14) apply to several individual aspects of the problem. Effort should be made to formulate a theory of the energy transfer from wind to water in which both the random atmospheric turbulence and the effect of the wave shape on the air-pressure distribution are considered.

17 Simple Evaluation of the Wind-Energy Transfer. A relatively simple evaluation of the energy transfer from wind to waves can probably be obtained by neglecting the drag of waves of low steepness and considering only the high drag coefficient of sharp-crested waves (Motzfeld, Section 2.3; Johnson and Rice, Section 2.4) together with the distribution probability of these waves and a suitable statistical definition of the mean facet velocity. It should not be forgotten that Motzfeld's data apply to long-crested waves, (project 2). Calculations can be made for typical wave spectra.

18 Directional Spectrum of Wind-Generated Waves. The work of Phillips, Section 4.2, should be extended to provide a clear statement of the directional spectrum of generated waves. Certain information on the directional distribution of waves was given in the work of Eckart.

19 A Combined Hydromechanic and Statistical Investigation is needed to clarify the generalization to short-crested waves of the energy transfer from wind initially formulated for long-crested waves. Mathematically, short-crested waves are described by super-

position of long crested ones. In making this superposition, the air-pressure coefficient cannot be assumed to remain constant. A short-crested wave form is expected to have a lower pressure coefficient than the long-crested one. Jeffreys indicated a ratio of 1:2. This feature appears to have been neglected by Miles, Section 4.5. The usual spectral definition of the energy per unit sea area does not appear to be sufficient in questions of energy transfer from wind. It appears to be necessary to consider the flow of energy, i.e., energy transport, which occurs not only in down-wind but in oblique directions as well. The mean transverse-energy flow can be neglected in an infinitely wide fetch and uniform wind, but becomes important in narrow fetches and in fetches with wind-velocity gradient in the lateral direction.

20 Wave Growth in Cyclonic Wind. In the projects on energy transfer suggested in the foregoing, rectilinear wind and predominating waves traveling in wind direction were considered. The action of a cyclonic wind should also be investigated. In this case, at each increment of the fetch, new wind-generated waves appear with dominating direction at an angle to the previously generated waves. Also there is a lateral wind-velocity gradient. The problem of the total wave-energy growth and the wave-directional spectrum must be established for this condition. The problem presumably can be approached by a suitable extension of any of the methods listed under project 14).

Wind-tunnel tests on pressure distributions on wave models with oblique crest directions (both long and short), needed to implement this project, were listed under project 2).

21 Experiments on Wave Growth in Cyclonic Wind. Experiments with model-tank wave generation by cyclonic wind are desirable in connection with project 20). The scale of experiments should be sufficiently large so that gravity waves can be generated and capillary waves could be neglected. Cyclonic wind can be provided by an upward suction through a bell-mouthed duct placed at a suitable distance above water. This is to be supplemented by a few guide vanes or jets at the outer periphery in order to initiate the spiral air flow. This project can be considered as an extension of Wiegel, Snyder and Williams' (1958) work.

22 Additional Wave-Spectra Measurements. There is a wide disparity among the various spectra discussed in Section 6, both in regard to the indicated rates of wave growth and in the ultimate wave characteristics. Additional ocean-wave measurements and analyses are recommended, under ideal conditions corresponding closely to the conditions of spectrum formulation. The visual observation-data analysis of Walden, Section 6.52 should be repeated using new wave data, instrumentally measured and spectrally analyzed. This large project requires, of course, prior completion of several lesser projects to be outlined.

23 Theory of Shipborne Wave Gage. At the time of writing, Tucker's (1952*b*, 1956) shipborne wave gage appears to be the most suitable for ocean-wave recording

on weather-observation ships. A theoretical analysis of its performance was given by Pierson (3-1957), and it appears to the author that such errors as are unavoidable are not important in the case of a small ship on large waves. On the other hand, a theory of the relationship between the water pressure at a submerged gage and the wave height has heretofore been used only in a crude form with a rule-of-thumb correction for hull interference. Development of this theory to include a more accurate evaluation of the hull-wave interference is recommended. It is particularly important for the measurement of the high-frequency wave components.

At the same time, the choice of the most favorable position for the gage installation may be investigated. In particular, proximity of the gage to bilge keels should be guarded against.

24 Calibration of Tucker's Gage installation on each ship against an independent standard appears to be an evident requirement. It has not heretofore been practical because of the lack of such a standard. The problem may be solved by installing wave gages on suitable off-shore structures (Texas Towers) against which a ship's gage can be calibrated.

25 Installation and Routine Use of Tucker's Gages on all weather-observation ships is recommended.

26 Shipborne Analyzing Equipment. In observations made heretofore with Tucker's gage, analysis of the records was subsequently made on shore. The collection of a large amount of wave data (and ultimately synoptic observations) requires that analyses be made aboard ships and the spectral wave description be radioed in a compact form. The development of suitable, simple, compact and rugged analyzing equipment for routine shipborne use is therefore needed. Application of the rapidly developing transistor techniques appears to be indicated.

27 Use of Wave-Measuring Buoys (Dorrestein, 1957; Voznessensky and Firsoff, 1957) appears to be practical on a sampling basis in a not-too-rough sea. With additional development, such buoys may be adapted to measuring the directional-wave spectrum. Developments in this direction are recommended. They can take the form of simple and cheap disposable devices, or the more elaborate ones which will have to be set out and subsequently retrieved.

28 Installation and Use of Wave-Measuring Devices on Off-Shore Structures (Texas Towers) is recommended. By extending the techniques developed by Barber (Section 8.72), the determination of a large number of directional wave spectra is feasible and should be made. Again, it is recommended that the analyzing equipment outlined in project 26) be used on the same structure and that accumulation of a great mass of data for later analysis be avoided. However, the practice of accumulating measurements for data analysis on shore may be resorted to temporarily pending the availability of the analyzing equipment. The nature of the recording must be matched, however, to the analyzing equipment to be used. Data are needed on a statistical basis

and must be of the type permitting mass analysis with the minimum of labor. The variability of sea conditions indicates that simplicity of data collecting and analyzing far outweighs the desire for extreme accuracy. In particular, the wave-energy distribution can be measured in only a few discrete directions. Gelci, Casalé, and Vassal expressed the mean directional spectrum by only two bands, Section 8.72.

29 Pending the collection of a sufficient amount of new wave measurements, the Material Used for Spectrum Formulation in the Past Should be Re-examined. In particular the following two projects can be suggested:

(a) The shape of Neumann's, Section 6.2, spectrum was obtained by intuitive analysis of a few wave records (see legend in Fig. 36). A formal spectral analysis of these records is recommended. In particular, this would help to clarify the evaluation of the constant C in Neumann's spectrum formulation, Section 6.23.

(b) Darbyshire's (1955), Section 6.11, wave data represent the only large collection of open-ocean instrumental wave measurements available to date. Re-examination of this material is recommended with particular regard to (i) verifying wind velocity over relevant fetches by re-examining meteorological conditions; (ii) including the necessary (theoretical) corrections of Tucker's gage indications, particularly for high-frequency waves; (iii) making independent spectral analyses, and (iv) experimenting with replacement of Darbyshire's empirical formula by alternate, possibly more sophisticated, formulations. In particular it is desired to bring out more clearly the effects of fetch length and of wind duration. Also, it is desirable to establish the spectrum form for a sea in the development stage, following the example of Gelci, Casalé, and Vassal (Sections 6.3 and 6.4; Fig. 45).

30 The collection and analysis of open-ocean data suggested in project 22) probably will take considerable time. Meantime appreciable progress can be made by Spectral Analysis of Observations in Restricted Water Areas of various sizes and at various wind velocities, project 14). A particular objective of this analysis would be to find out if the constant defining the spectrum area is truly a constant or whether it depends on other factors, particularly on "wave age" (i.e., the ratio of predominating wave celerity to wind velocity), verifying the conjecture made by the author in the fourth paragraph of Section 6.23.

31 Development of Descriptive Wave Spectra, Section 6.6 The development of a compact description of a sea surface is recommended. This should be suitable for ship-motion analysis or prediction. It appears that a three-parameter definition indicative of the wave height, wave period, and sea irregularity can be useful. Such a definition was given by Voznessensky and Firsoff's spectrum, Section 6.6. Additional work connected with it may consist of

(a) Evaluation of the three parameters for certain spectra of measured waves at sea, such as, for instance, Walden and Farmer's, Section 6.53(b). Attention

should be called to the fact that the spectrum in this case describes an observed sea which is usually generated by many separate causes. This spectrum therefore is basically different from the spectra based on wind velocity and a specified simple fetch and duration. In particular, the descriptive spectrum must have flexibility in specifying the dominant wave period.

(b) Preparation of a photographic album of various spectrally analyzed sea conditions for guidance in visual sea observations.⁴³ The photographs can be labeled and classified by the three Voznessensky and Firsoff parameters.

(c) Establishing relationships among the actual spectrum of waves, the Voznessensky and Firsoff three-parameter spectrum, simple measurements of wave heights and periods on a wave record, and the visually observed significant wave height and period. This can be accomplished by theoretical considerations based on mathematical statistics, in conjunction with the spectral evaluation of wave records (in subprojects *a* and *b*). The primary objective of this project is to enlarge the collection of wave data reducible to spectral presentation, which is needed in the prediction of ship motions.

(d) A special study to bring out the physical significance of the sea irregularity parameter α in Voznessensky and Firsoff's formulation. There is some evidence that in a "young sea" (large U/c) both the scalar spectrum and the directional energy distribution are broader than in the case of a fully developed sea. This observation apparently permits the grouping of period irregularity and of short-crestedness under one parameter. The foregoing is a conjecture, however, and the subject must be investigated.

32 Synoptic Wave Data. Collection of ocean-wave data on a synoptic basis is needed and is sure to come with time. A compact (and at the same time significant) method of reporting the wave data must be developed. To describe a complete spectrum a large number of ordinates must be reported. Any number of discrete ordinates will create uncertainty because the spectral form resulting from a computational procedure is usually irregular. Furthermore classifying a large number of reported spectra presents a problem. It does not appear to the author that it will be practical to report the complete spectra. The spectrum forms most often observed may be reported with sufficient accuracy by the three Voznessensky and Firsoff parameters which also provide a simple means of wave classification. The less frequent two-maxima spectra can be approximated by superposition of two simple spectra; i.e., by a total of six numbers. This demonstrates a possible approach. A broad study of the problem is recommended.

33 Spectra of Very High-Frequency Wave Components. The spectra discussed in Section 6 are macroscopic descriptions of sea waves. They give information on waves sufficiently long to be significant for ship

⁴³ This idea of a photographic album was advanced at the August 6 and 7, 1958 meeting of the Seakeeping Characteristics Panel of SNAME.

motions and for swell forecasting. The resolution power of the instrumentation used in their measurement and of their spectral analysis is not sufficient to describe small wavelets by which the surface of the larger waves is covered. These spectra, therefore, appear to be of doubtful value in problems of energy transfer from wind to waves. This transfer appears to be primarily dependent on the small sharp-crested waves. Projects on the measurement and analysis of these small waves are therefore recommended as a prerequisite to understanding the wave growth under wind action.

How small is "small" in this connection is not known now, and it is probable that it is defined not by absolute value but in relation to the total spectrum energy. It appears to the author, however, that waves in the gravity range are involved here, and one should not arbitrarily identify "small" waves with capillary waves.

A study of properties of the high-frequency end of the wave spectrum by Phillips (1958) can be mentioned as an example. The prospective investigator should be warned that the facet velocity of small waves depends not on their properties alone, but on the whole wave spectrum. Longuet-Higgins' (1956, 1957) papers can be used for defining the facet velocity in any spectrum. It also should be remembered that the very high-frequency ends of all of the spectra listed in Section 6 represent an extrapolation of the empirical data, and are therefore not reliable. Pending actual measurement of very high-frequency components of moderate and high seas, it is suggested that hypothetical spectra be used in theoretical studies. These would be composed of the spectra listed in Section 6 with the high-frequency ends replaced by Phillips' (1958) formulation. As sub-projects under the foregoing, the following can be listed:

(a) Measurements of the very high-frequency end of the spectra by auxiliary instrumentation at the time the usual wave measurements are made. In particular, the small wavelets can be measured with respect to a fairly large buoy riding on larger sea waves which in turn are recorded by accelerometers.

(b) Verification of Phillips' (1958) formulation by application to Cox and Munk's, Section 4.3, and Schooley's (1958) wave slope spectra.

(c) Carrying out preliminary work on the wind-energy transfer to waves using spectra as indicated in project (14-b) but with a hypothetical spectrum possessing Phillips' high-frequency extension.

34 Transformation of Small Waves into Large Ones.

In the development of waves under the action of the wind there appears to exist a perpetual change from small waves to large ones. Sverdrup and Munk and Neumann (prespectrum) have shown that waves must grow in length in absorbing the wind energy since they cannot grow indefinitely in height. The detail mechanism by which the small waves are transformed into large ones is, however, not known. Efforts to formulate and to demonstrate a suitable theory are recommended.

The energy transfer from wind appears to depend on

the action of small waves. In comparison with these, the long waves of moderate sharpness and approximately trochoidal form have very small ability to absorb wind energy. The theory of large-wave growth must apparently depend on understanding the processes by which the wind energy absorbed by small sharp-crested waves is transformed into the energy finally appearing in large waves.

The reverse problem also has been observed in towing tanks. Sometimes an apparently regular wave train, after running through a certain distance, is transformed into an irregular wave pattern containing components of much higher frequency than the original waves.

35 Manuals of Applied Mathematical Statistics. The introduction of spectral descriptions of waves and ship motions brings about the need for knowledge of mathematical statistics. It can hardly be expected that practical oceanographers, ships' officers, and naval architects will have the time and preparation for a profound study of this subject. It is recommended, therefore, that short and simple texts on the relevant aspects of mathematical statistics be prepared. The text should have direct practical use as an objective, and should avoid theoretical discussions beyond those immediately needed for understanding the practical procedures. Those engaged in research requiring a deeper theoretical insight would be directed to the many existing textbooks and articles on a higher mathematical level. The notation and expressions familiar to oceanographers and naval architects should be used as far as possible, and the unfamiliar terminology of the specialized statistical texts should be avoided. The text should preferably be arranged in a graded form, starting with the simplest possible use of mathematical statistics in oceanographers' and naval architects' problems, and progressing to the more elaborate ones.

36 Provision of Recording and Analyzing Facilities. The practical application of the spectral concepts of the sea surface requires widespread use of suitable recording and analyzing equipment. Heretofore, the work in this field has been carried out only on a pilot-research basis, making use of the few available computing centers and often resorting to tedious manual measurements of various records. Significant progress in practical utilization of the modern irregular-sea concepts depends to a large extent on widespread availability of suitable recording and analyzing equipment. The need for ship-borne equipment has already been mentioned under project (26). Suitable equipment also must be made available to experimenters in wind flumes, towing tanks, and oceanographic institutions. It is often impractical to develop the necessary instrumentation at each individual establishment because of the lack of specialized knowledge and because of the cost involved. There is also the danger that heterogeneity of the methods and of the forms of reported results will hinder progress. It is, therefore, recommended that steps to develop and make available suitable instrumentation be taken by the proper professional organizations singly or jointly. If

suitable equipment specifications were developed, avoiding unnecessary complications and costs, private industry may be able to produce the equipment in quantity. Simplicity, reasonable universality, and low cost are essential, since much of the valuable research is expected to come from many small laboratories. In general, two broad types of equipment are visualized:

(a) Magnetic-tape recorders and analyzers of the (moderately broad) filtering type.

(b) Digitizers which translate the continuous electric signals from the sensing elements into the punched-card or tape records suitable for direct use in universal computing machines. This equipment is particularly appropriate for small laboratories connected with universities or other institutions possessing high-speed digital computing machines.

The author believes that the most rapid progress in research in oceanography and naval architecture will be made if the analyzed test data could be available to a researcher while the physical observations are still clear in his mind. The electronic filtering technique listed under *a*) and recommended for project 26) gives promise of such a rapid analysis. By using transistor techniques it also gives promise of a compact and rugged equipment suitable for use on location in natural-wave observations.

The author realizes that recorders, digitizers, and analyzers have been developed by various laboratories and that many electronic components are available in the open market. Nevertheless, no complete, compact and workable instrument package appears to exist, and the cost and the needed specialized knowledge severely limit the activity in this field.

37 Clarification of Confidence Limits. The confidence limits of spectral analysis are defined with respect to certain, rather narrow, filters of the digital analysis or electronic devices. When these limits are given in the literature, as for instance in Fig. 71, it is often difficult to find the frequency band widths to which they apply. Furthermore, these particular frequency band widths may or may not be relevant to the problem at hand. No distinction has been made between confidence in an analysis of a particular wave record and confidence in this particular record considered as a sample of the random sea conditions. Finally, there appears to be some confusion in the literature between expressing the confidence in terms of wave-record-measurement subdivision and in terms of wave lengths available in a sample. Further research to clarify the situation is recommended.

It is emphasized that in practical use the confidence limits of the spectrum must be closely connected with the objective for which the spectrum is to be used. For instance, confidence limits may be desired in evaluating the significant wave height, i.e., the zero moment of the spectrum, or the mean wave slope (the second moment), or the mean wave period. A tabulation of such confidence limits appears to the author to be more valuable in practical problems than drawing the usual statistical confidence-limit curves. The problem of clarifying the

meaning of statistical confidence limits in application to practical problems requires the joint work of mathematical statisticians and oceanographers or naval architects.

38 Instrumentation for the Measurement of Directional Wave Spectra. The need for measurement of wave directional spectra on a quantity, i.e., statistical, basis has been indicated under project 28) and has been mentioned several times before. Development of the necessary instrumentation can be listed, however, as an independent project. To date, Barber's methods appear to be the only ones suitable for mass collection of data. Barber proposed several methods, but only one of these, the correlation one, was outlined in some detail in Section 8.72 where references also were given to all of Barber's papers. A certain rather obvious development of Barber's correlation method is needed for the collection of the data suggested in project 28):

(a) The directional spectrum should be obtained for several wave frequencies.

(b) The variability of sea conditions, demonstrated by Tucker, Section 8.44, requires that measurements for all frequencies be obtained simultaneously. Also it would be desirable to obtain simultaneously the records for several pairs of gages needed for correlation analysis, instead of the consecutive measurements used by Barber.

It appears to the author that available electronic techniques will permit the following scheme: (a) The usual wave-height recording can be made simultaneously for several pairs of gages on the same magnetic tape thus permitting a cross-spectral analysis to be made later; (b) the multiple record should be passed alternately through several frequency filters, yielding several single-frequency multiple-gage records; (c) each single-frequency multiple-gage record should be subjected to the correlation analysis described by Barber, except that electronic methods of analysis would be used instead of Barber's pendulum; (d) the randomly distributed two-dimensional correlation function, equation (149), would be projected on the oscilloscope screen and photographed; (e) the photograph for each frequency would be interpreted in the same way as Barber's photograph, Fig. 87.

It should be emphasized that sea-surface variability makes it unnecessary to describe in excessive detail a directional spectrum from a single observational run. The spectrum is a statistical concept and the typical spectrum is to be obtained as the mean of many individual records. This being the case, an excessive number of gage pairs and of calculated wave directions should be avoided. It appears to the author that four pairs of gages and four wave directions will be sufficient.

The instrumentation outlined in the foregoing may be adapted to the measurements made with conventional wave-height gages as well as to the wave-slope and wave-acceleration measurements by floating buoys. In the latter case the use of telemetering equipment will be required.

39 Directional Wave Spectrum Measurement From Ships at Sea. Apparently no method of measuring the directional wave spectrum on ships has been proposed to date. Nevertheless, effort should be applied to this problem.

40 Energy Transport in Irregular Waves (Section 8.8). The static concept of the wave-energy spectra may not be adequate in problems of the energy transfer from wind to waves. Thought must be given to the mathematical and physical consequences of the energy transport by irregular waves. Defined with respect to harmonic-wave components by the classical theory, the energy-transport concept should be generalized by statistical theory. The work suggested by this project can be considered as a further development of Longuet-Higgins' (1956, 1957) work with particular emphasis on flow (or transport) of energy in various directions.

41 Waves of Extreme Steepness and Their Properties. In the analysis of dangerous ship stresses it is important to know the maximum steepness of ocean waves of various lengths. A maximum height-to-length ratio of 0.14 and a minimum included angle of 120 deg at the crest are indicated by classical theory (Section 3.2 of Appendix A) for simple gravity waves. A minimum included angle of 90 deg is indicated (Taylor, 1953) for standing waves.

(a) Theoretical research is needed to establish the maximum steepness and the mean wave height for short-crested irregular waves. Conceivably, the interaction of various wave trains may bring about the reduction of the 120 deg angle. This angle is reduced to 90 deg for standing waves which are represented mathematically by a summation of two simple wave trains.

(b) Ship-stress analysis requires not only knowledge of the wave steepness as a function of wave length but also of the pressure distribution in waves. The methods by which the limiting crest angles were determined in simple gravity waves involved only local conditions and not the general flow description. A project in evaluation of pressure distributions in waves of limiting steepness is therefore recommended both for long-crested and for short-crested irregular waves. While the problem may prove to be prohibitively difficult for sharp-crested waves, the computations for Stokes' waves (Section 3.1 of Appendix A) are simple and will yield valuable data.

(c) The spectral sea description is based on the linear superposition of simple wave trains and, strictly speaking, is valid only for very low waves. Development of a nonlinear statistical wave description is needed to represent the waves approaching limiting steepness. This project consists of: (i) the basic development of nonlinear methods and (ii) their application to typical sea spectra. In defining the latter, Bowden's, Section 3.1 limitation of the wave steepness by the balance of the energy received from wind and dissipated in waves and Phillips' (1958) definition of sharp wave crests by the condition that $\ddot{\eta} = -g$ may be useful.

(d) Expressions for the wave slopes are available in the statistical work of Pierson and Longuet-Higgins. These,

however, are based on the linear theory. A study of storm-wave records (for instance, Darbyshire's, 1955) is recommended in order to verify empirically the degree of agreement between large wave slopes as observed and as derived from linear spectra. Wave steepness appears to be connected with wave age, c/U , so that small-scale data, as in Cox and Munk's, Section 4.3 sun-glitter measurements, are not applicable to the present project. It must be based on full-scale storm conditions.

42 Shape of Wind-Driven Waves. The shape of wind-driven storm waves may be of significance in evaluating the bending moments acting on ships in waves. The increased steepness of leeward slopes can be expected to cause appreciable increase in the bending moments. Three subprojects are indicated here:

(a) Efforts to formulate and solve the problem theoretically.

(b) Empirical evaluation of the increase of the observed leeward slopes of storm waves over the slopes predicted from linear spectra.

(c) Empirical modification of the descriptive spectrum formulation (such as Voznessensky and Firsoff's, Section 6.6) to generate an unsymmetric wave form.

43 Restricted-Water Waves. Increased steepness of storm waves progressing into restricted waters (reduced depth, channel constriction, head current) may cause increase of ship stresses and is, in fact, suspected to be the cause of certain ship failures. Theoretical and empirical studies of the properties of these waves are desired. It is necessary to know the pressure-distribution pattern in these waves as well as their forms. The increase of sea severity near steep shores (for instance in the Bay of Biscay) is well known to mariners. It can generally be attributed to the standing-wave system caused by wave reflections from shore, but a more thorough quantitative investigation of the wave properties is needed.

44 Natural and Ship-Wave Interaction. The interaction of ship-made waves with natural wind waves can often lead to weird wave forms, excessive wave steepness and dangerous surf-like breakers. Two particular cases of interference-caused waves can be cited:

a) Interference of the following sea with the transverse stern wave of a ship. This may lead to the breaking of a large wave over a ship's stern (Möckel, 3-1953).

b) Interference of a ship's bow waves with oblique or beam seas. The interference breakers can often be seen over a large distance in the directions of the oblique bow and stern ship waves. This interference often increases ship wetness and may be dangerous for a ship's superstructure. A ship in a formation may be endangered by the combined interference of its own and other ships' waves with ocean waves.

Theoretical and experimental work on the properties of interference waves is recommended. This study can be expected to lead to the development of operational rules for increase of safety of fast (naval) ships operating in formation in rough weather.

Nomenclature for Chapter 1

- a = harmonic wave amplitude, half-height of trochoidal and Stokes' waves
 a_n, b_n, c_n = coefficients of the n th term of the Fourier series expansion of a function
 A_n, B_n, C_n = coefficients of the air-pressure-function expansion
 A, B, C = coefficients
 c = wave celerity (also known as phase velocity)
 C = a coefficient
 C_d = dynamic (i.e., pressure) drag coefficient
 C_p = pressure coefficient
 C_r = frictional-drag coefficient
 $C_d^* = C_d + C_r$ = total drag coefficient
 D = gust diameter (Eckart, Section 4.1)
 E = energy
 E = wave-energy content per unit of sea surface per second. Energy indicated by the spectrum area (true energy divided by ρg)
 $E(T), E(f), E(\omega)$ = spectral energy (divided by ρg) density in terms of wave period, frequency and circular frequency, respectively
 $E(u, v)$ = directional spectrum energy density in terms of wave number projections on x and y -axes
 $E(\omega, \theta)$ = directional spectrum energy density in terms of circular frequency and wave-propagation direction
 E_p = wave energy per unit sea area per second transmitted from wind to waves by normal air pressures
 E_{dk} = wave energy per unit area per second dissipated by waves
 $E_{1,2}$ = wave energies carried through reference planes at fetches F_1 and F_2
 f = frequency $1/T = \omega/2\pi$
 f = number of degrees of freedom in spectral analysis (equation 126)
 f_c = cut-off (Nyquist) frequency
 F = fetch
 g = acceleration of gravity
 h = water depth
 h = number of integrations of an autocovariance function in computing spectral density
 H = wave height
 \hat{H} = apparent wave height
 \bar{H} = mean apparent wave height
 $H_{1/3}, H_{1/10}$, etc. = mean height of $1/3, 1/10$, etc., of highest waves
 H = equivalent wave height (Darbyshire, Section 6.1)
 $i = \sqrt{-1}$
 k = wave number $= 2\pi/\lambda = \omega^2/g$
 L_h = "raw" value of spectral density
 m = maximum number of time lags in autocovariance analysis
 m_n = moments of scalar spectrum defined by equation (128a)
 m_{pq} = moments of the directional spectrum defined by equation (142)
 n = order of harmonic components in a Fourier Series
 n = number of intervals Δt in a sample of duration T
 n = generally an index or a subscript of a meaning to be specified
 n = ratio of turbulent and molecular viscosities, μ^*/μ
 N_0 = number of zero up crosses of a wave record
 N_1 = number of maxima of a wave record
 p = pressure
 p = number of lags, $\tau/\Delta t$
 p = index of the wave number u in directional-spectrum analysis
 q = index of the wave number v in directional-spectrum analysis
 q = successive numbering of ordinates of record of a random function
 r = proportion of negative maxima in a wave record
 $R(\tau)$ = autocovariance function
 $R(\tau)/R(0)$ = autocorrelation function
 R, θ = polar co-ordinates
 rms = root-mean-square
 s = Jeffreys' sheltering coefficient
 s = Miles' (Section 4.5) mass parameter
 S = setup of water surface caused by wind drag
 S_1 = setup of water surface smoothed by a detergent
 S_2 = setup of water surface caused by wave resistance
 t = time
 T = period of harmonic waves
 T = duration of a wave record, sec
 T = duration or life of a gust (Eckart, Section 4.1)
 T = surface tension
 \bar{T} = mean apparent period of irregular waves defined by number of zero up-crosses per second
 \tilde{T}^* = mean apparent period of irregular waves defined as time intervals between dominant wave crests
 u = horizontal component of orbital velocity
 u = projection of the wave number k on x -axis
 $u(z)$ = wind velocity at the height z
 U = air velocity affected by the proximity of a body. Air velocity at anemometer height
 U = energy contained in a band of frequencies $\omega_2 - \omega_1$
 ΔU_T = spectral wave energy (in energy units) contained in the band ΔT of wave periods (Neumann, Section 6.2)
 U_c = gust travel velocity (Phillips, Section 4.2)
 v = vertical component of the orbital velocity
 v = projection of the wave number k on y axis
 V = velocity of undisturbed air. Gradient wind velocity
 V_c = formula velocity (Section 2.5)
 x, y = Cartesian co-ordinates
 $y(t)$ = an ordinate of a stochastic function record
 z = height of wind-velocity measurement
 z_r = roughness parameter (Section 2.7)
 α = coefficient of Voznessensky and Firsoff's (Section 6.6) spectrum, characteristic of sea irregularity (spectrum broadness)
 β = coefficient of Voznessensky and Firsoff's (Section 6.6) spectrum, characteristic of wave frequency

β = Jeffrey's sheltering coefficient (Section 2.1)	μ = molecular viscosity
β = wave age, i.e., the ratio c/U	μ^* = turbulent viscosity
γ = tangential-drag-force coefficient; $\gamma = C_d^*/2$	ν = kinematic viscosity μ/ρ
δ, Δ = an increment	ρ = water density
ϵ = phase angle	ρ' = air density
ϵ = parameter characteristic of a spectrum's broadness (Section 8.6)	τ = tangential drag
$\eta(t)$ = wave elevation; ordinates of wave record measured from mean level	τ = time lag in spectral analysis
θ = direction of wave-component propagation with respect to predominating direction	$\psi(x, y, t)$ = three-dimensional correlation function
θ_0 = spread of wave directions (Cox and Munk, Section 4.3)	ω = circular frequency = $2\pi/T = 2\pi f$
λ = wave length	σ = standard error
	σ = total wave slope
	σ_c = cross-wind wave slope
	σ_u = up/down wind-wave slope

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Code of Abbreviations

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 An. Met.—Annalen der Meteorologie (Hamburg).
 An. N. Y. Acad. Sc.—Annals of the New York Academy of Sciences.
 DTMB—David Taylor Model Basin.
 Deut. Hydr. Zeit.—Deutsche Hydrographische Zeitschrift.
 Hansa—Die "HANSA," Zeitschrift für Schiffahrt-Schiffbau-Hafen.
 JSTG—Jahrbuch der Schiffbautechnischen Gesellschaft.
 J. Aero/Sp. Sc.—Journal of the Aero/Space Sciences.
 J. Mar. Res.—Journal of Marine Research.
 J. Sc. Inst.—Journal of Scientific Instruments.
 J. Fluid Mech.—Journal of Fluid Mechanics.
 J. Met. Soc. Japan—Journal of the Meteorological Society of Japan.
 INA—Transactions of the Royal Institution of Naval Architects, London, England.
 NACA—National Advisory Committee for Aeronautics.
 NBS—National Bureau of Standards.
 NEC—Transactions of the North East Coast Institution of Engineers and Shipbuilders.
 NSMB—Proceedings, Symposium on the Behavior of Ships in a Seaway, 25th Anniversary of the Netherlands Model Basin, Wageningen, 1957, final two-volume edition.
 New Zeal. J. Sc. Tech.—New Zealand Journal of Science and Technology.
 Phil. Mag.—Philosophical Magazine.
 Phil. Trans. R. Soc. A—Philosophical Transactions of the Royal Society of London, Series A.
 Proc. Am. Soc. C. E.—Proceedings of the American Society of Civil Engineers.
 Proc. IRE—Proceedings of the Institute of Radio Engineers.
 Proc. R. Soc. A—Proceedings of the Royal Society, Series A, London.
 Q. J. R. Met. Soc.—Quarterly Journal of the Royal Meteorological Society.
 Symp. Autocor.—Symposium on Applications of Autocorrelation Analysis to Physical Problems, Woods

Hole, Massachusetts June 13–14, 1949, Office of Naval Research, Dept. of the Navy, Washington, D. C.
 SNAME—Transactions of the Society of Naval Architects and Marine Engineers, New York.
 Tr. Am. Geo. Un.—Transactions of the American Geophysical Union.
 WRH—Werft-Reederei-Hafen.
 ZAMM—Zeitschrift für Angewandte Mathematik und Mechanik.
 Z. Met.—Zeitschrift für Meteorologie.
 Ships and Waves—Proceedings of the First Conference on Ships and Waves at Hoboken, N. J., October 1954, SNAME, N. Y.
 ETT—Experimental Towing Tank (now Davidson Laboratory—DL), Stevens Institute of Technology, Hoboken, N. J.
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1 Introduction

Chapter 2 deals with definition and evaluation of hydrodynamic forces acting on the hull of an oscillating ship in waves. The oscillating motion of a ship will be discussed in detail in Chapter 3. However, the forces and motions are so closely interconnected that a complete separation of these two subjects is not possible, and a certain minimum information on motion has to be included in Chapter 2 as well.

The exposition given in Chapter 2, as indeed in all subsequent chapters, follows the policy outlined in the Introduction to the Monograph (pages iv and v). An attempt has been made at a critical summary of the existing state of the art. It is expected that the reader will be stimulated to further research by the realization of the scope and the shortcomings of present knowledge of the hydrodynamic forces acting on a ship oscillating in waves. A summary of suggestions for research will be provided at the end of the chapter.

Because of the close relationship between the subject matter of Chapters 2 and 3, the bibliography for both is placed at the end of Chapter 3. The reader is asked to refer to it whenever the reference gives only the year of publication, thus "Davidson and Schiff (1946)." When a reference is made to other chapters it is preceded by the chapter number; thus, "Pierson (1-1957)."

1.1 Forces Acting on a Body Oscillating in a Fluid.

A continuously changing pattern of water velocities relative to the hull is created when a ship oscillates in waves. By virtue of the Bernoulli theorem, these water velocities and their rates of change cause changes of the water pressure on the hull. These pressures, acting in various directions, always normal to elements of the hull surface, can be resolved along three axes, x , y , and z , and the components can be integrated over the entire area of the hull so as to give the total resultant force in each of these directions. The force components also can be multiplied by the distance to the center of gravity of a ship, and integrated to give the total moment about each axis. It has been found that once the detailed derivation has been carried out, the actual evaluation of the forces often can be accomplished by a much simpler procedure in terms of the body volume.

The actual mechanism of a ship oscillating along three axes—surging, side sway and heave, and rotating about three axes, rolling, pitching and yawing—can be complicated. Nevertheless, the basic concepts and terminology are defined in the same way as for a simple harmonic oscillator. A can buoy in heaving motion in

low, long waves is a good example of a simple forced oscillation. Its motion is described by a linear differential equation of the second order

$$a\ddot{z} + b\dot{z} + cz = F_0 \cos \omega t \quad (1)$$

Here the first term $a\ddot{z}$ denotes the forces connected with the acceleration d^2z/dt^2 , and the coefficient a is a mass. It is in reality the mass m of the buoy itself plus a certain imaginary water mass m_z , the acceleration of which gives a heaving force equal to the vertical resultant of all fluid pressures due to actual acceleration of water particles in many directions. This imaginary mass is known as "added mass" or "hydrodynamic mass" and the coefficient a is written as $m + m_z = m(1 + k_z)$, where k_z is the "coefficient of accession to inertia" in the vertical plane. The total mass represented by the coefficient a is known as "apparent mass" or "virtual mass."

The second term $b\dot{z}$ denotes the force proportional to the instantaneous vertical velocity dz/dt . The coefficient b is known as "damping coefficient" for a reason to be discussed shortly. In most cases it is assumed to be constant. In reality it is often not constant and in application to ship rolling, for instance, it often has been taken as depending on velocity squared \dot{z}^2 as well as on \dot{z} . However, a satisfactory description of many forms of oscillation in nature is given by the linear form of Equation (1).

The term cz is the force proportional to displacement, and is usually known as the "restoring force," while the coefficient c is often referred to as a "spring constant." This is a force exerted per unit of displacement z . In the present example of a can buoy, the constant c is the heaving force caused by a change of draft of one unit; i.e., 1 ft in the foot-pound system.

The term $F_0 \cos \omega t$ on the right-hand side of equation (1) is the "exciting force." In the present simple example, F_0 is the amplitude of the buoyant force due to wave height. In a ship's case it also will depend on water velocities.

In the forced motion with harmonic exciting force, equation (1), the motion, after sufficient time, is also a simple harmonic so that body position at any instant is

$$z = z_0 \cos (\omega t + \epsilon) \quad (2)$$

where ω is the circular frequency, and ϵ is the "phase lag angle." Term z_0 is the amplitude of motion defined in its relationship to the amplitude of exciting force F_0 by

$$z_0 = F_0 [(c - a\omega^2)^2 + b^2\omega^2]^{-1/2} \quad (3)$$

It is of interest to establish what work is done by an oscillating body on the fluid per cycle of oscillation. On the basis of equation (2):

$$\begin{aligned} z &= z_0 \cos(\omega t + \epsilon) \\ dz &= -\omega z_0 \sin(\omega t + \epsilon) dt \\ \dot{z} &= -\omega z_0 \sin(\omega t + \epsilon) \\ \ddot{z} &= -\omega^2 z_0 \cos(\omega t + \epsilon) \end{aligned} \quad (4)$$

and the work done, in the period T , by the acceleration forces

$$\begin{aligned} a \int_0^T \ddot{z} dz &= a \omega^3 z_0^2 \\ &\times \int_0^T \sin(\omega t + \epsilon) \cos(\omega t + \epsilon) dt = 0 \end{aligned} \quad (5)$$

by damping forces proportional to \dot{z}

$$\begin{aligned} b \int_0^T \dot{z} dz &= b \omega^2 z_0^2 \int_0^T \sin^2(\omega t + \epsilon) dt \\ &= b \omega^2 z_0^2 T/2 \end{aligned} \quad (6)$$

by restoring forces, proportional to z ,

$$\begin{aligned} c \int_0^T z dz &= -c \omega z_0^2 \\ &\times \int_0^T \sin(\omega t + \epsilon) \cos(\omega t + \epsilon) dt = 0 \end{aligned} \quad (7)$$

Thus, it is seen that the average amount of work done on a fluid by acceleration and by restoring forces is nil. A body does the work on the fluid during a half cycle, and the fluid does an equal amount of work on the body during another half cycle. Only damping forces do a net amount of work on the fluid, and therefore take the energy out of the body and dissipate it in the fluid. In an oscillation of a free body this causes gradual diminution of the amplitude of motion, from which the term "damping" has been derived. In a continued forced oscillation the energy necessary to maintain it is supplied by the exciting forces.

If the frequency of the oscillation is low enough, the phase lag is negligibly small and the displacement of a body z is in phase with the exciting force. Equations (4) show that the velocity \dot{z} is 90 deg out of phase and the acceleration \ddot{z} is 180 deg out of phase.

The forces caused by water pressures can be divided into two groups. The restoring force cz is caused by hydrostatic water pressures. The hydrodynamic forces $b\dot{z}$ and $m_z\ddot{z}$ result from the velocities and accelerations of water particles. These two forces are in reality two components of the resultant of all hydrodynamic (i.e., exclusive of hydrostatic) water pressures.

Confusion has occasionally resulted from the descriptive definitions of the damping force and the inertial (acceleration of the hydrodynamic mass) force given earlier. In the recent literature there has been, therefore, a tendency to define these forces merely as the out-of-phase (by 90 deg) and in phase (in reality 180 deg out-of-phase) components of the hydrodynamic force.

1.2 Order of Exposition. Equation (1) was introduced in order to define four categories of forces acting on an oscillating body, namely, inertial, damping, restor-

ing, and exciting. The following sections of Chapter 2 will be devoted to the evaluation of these forces by theoretical and experimental means. Theoretical evaluation of hydrodynamic forces in harmonic oscillations has been approached in three ways:

- Comparison with ellipsoids (in Section 2)
- Strip theory (in Sections 3, 4 and 5)
- Direct three-dimensional solution for mathematically defined ship forms (Section 6).

In Sections 7 and 8 the forces in transient (slamming) conditions will be discussed.

2 Estimates of Hydrodynamic Forces and Moments by Comparison With Ellipsoids

The problem of forces and moments exerted by a fluid on a body moving in it received the attention of hydrodynamicists at a very early date, and chapters on this subject are found in all major books on hydrodynamics (see Chapter 1: References C, pp. 353–393; D, pp. 160–201; F, pp. 464–485). The problem is usually formulated for a body moving within an infinite expanse of a fluid initially at rest and assumed to be nonviscous. Only the forces due to the fluid inertia can therefore be present.

The forces and moments acting on a body can be evaluated by two methods. In the first method the pressure p acting on each element of a body surface is computed by Bernoulli's theorem

$$p = p_0 + \rho \frac{\partial \phi}{\partial t} - \rho \frac{q^2}{2} \quad (8)$$

where ϕ is the velocity potential, and q is the local fluid velocity at the surface of the body, induced by its motion. By taking components of pressures p in the desired direction and integrating over the surface of the body, the total force is obtained.

The second method consists of expressing the rate of change of the kinetic energy contained in a volume of fluid between the body surface and an imaginary control surface taken at a sufficiently large distance from the body. The kinetic energy T is given by the expression (Lamb, 1-D, p. 46)

$$T = \frac{1}{2} \rho \int_S \phi \frac{\partial \phi}{\partial n} dS, \quad (9)$$

where n denotes the outward normal and S the surface of a body over which the integral is taken. The force is then found by differentiation of the energy with respect to the body displacement; for instance, the force X in the direction of the x -axis is

$$X = \partial T / \partial x \quad (10)$$

In the application of either of the foregoing methods it is necessary to obtain the velocity potential ϕ . Also it is necessary to have the mathematical description of a body in order to formulate expressions for the directions of the normals, and to permit the integration over a sur-

face. The needed mathematical expressions reduce to tractable forms only for deeply submerged ellipsoids. Since the forces in this case are inertial, they can be expressed in terms of the "coefficients of accession to inertia" k , defined as

$$1 + k = \frac{\text{total inertia of a body floating in a fluid}}{\text{inertia of fluid displaced by body}} \quad (11)$$

In connection with the objectives of the present monograph, interest is concentrated on prolate ellipsoids in which the major semi-axis a is taken to coincide with the x -axis in which also the mean body-velocity vector V lies. The minor semi-axes b and c (not necessarily equal) are then taken to coincide with the y and z -axes. The oscillatory motion of the body may include translations along any of the three axes and rotations about any of these axes. The coefficients of accession to inertia have different values for any of those motions, and the symbol k is supplemented by a suitable subscript. Treating the motions of an ellipsoid of revolution (a spheroid), Lamb (Chapter 1-D) designated by k_1 the coefficient of accession to inertia for accelerations along the major semi-axis a (i.e., x -direction), by k_2 that for accelerations along a minor axis, and by k' that for rotation about a minor axis. These designations were used (among others) by Davidson and Schiff (1946), Korvin-Kroukovsky and Jacobs (1957, also Appendix C to this monograph) and Macagno and Landweber (1958). It has been recommended¹ that symbols k_x , k_y , and k_z be used for translations along axes and k_{xx} , k_{yy} , and k_{zz} for rotation about axes indicated by subscripts. This notation was used by Weinblum and St. Denis (1950). While convenient for treatment of multicomponent motion of three-dimensional bodies this notation may be confusing in discussing two-dimensional flows in the strip theory of slender bodies. In this case it is customary to take the x -axis laterally in the plane of water and the y -axis vertically. In order to avoid confusion with the three-dimensional analysis the notation k_v and k_h can be used² for the coefficients of accession to inertia in the vertical and horizontal (lateral) directions. Here k_v is identical with k_2 of Appendix C.

A brief table of coefficients for spheroids will be found in Lamb (Chapter 1, D, p. 155). Curves of the coefficients of accession to inertia or to moment of inertia for various proportions of the ellipsoid axes can be found in Zahm (1929), Kochin, Kibel and Rose (Chapter 1, C, pp. 385-389), and Weinblum and St. Denis (1950).

Since the exact evaluation of the coefficients of accession to inertia of ship forms is practically impossible, it has been customary to estimate them by comparison with ellipsoids of similar length, beam and draft. A typical application of this method is found in the work of Weinblum and St. Denis.

¹ Minutes of the first meeting of the Nomenclature Task Group of the Seakeeping Panel, SNAME.

² Subscripts v and h were used by Landweber and de Macagno (1957).

In making these estimates for surface ships an assumption is introduced that the coefficients of accession to inertia, initially derived for a deeply submerged body, are still valid for a body floating on the surface. In other words, the effects of wavemaking on the free water surface are neglected. These effects have been investigated in the simpler "strip theory" to be discussed in the next section. It appears that, within the practical frequency range, the coefficient k_z for heaving oscillations of a floating body may be, on the average, 80 per cent of that computed by comparison with a deeply submerged ellipsoid.

It is clear that comparisons with ellipsoids are limited to investigations of ship motions of a general nature, in which only the over-all proportions are involved and the details of the hull form are not considered. In addition, the results are evidently applicable to investigation of the motion of a ship, but provide no information on distribution of forces along the length of a ship. Knowledge of this distribution is necessary in calculating the bending moments acting on a ship in waves.

Theories and computations made for ellipsoids have been important in bringing out certain trends or laws of action of hydrodynamic forces which are indicative of what can be expected in ships and submarines. As typical examples of this theoretical activity, the work of Havelock (1954, 1955, 1956) and Wigley (1953) can be cited.

3 Evaluation of Forces in Heaving and Pitching by Strip Theory

As has been mentioned earlier, solutions of three-dimensional hydrodynamic problems have been limited to ellipsoids, and are practically impossible when dealing with ships.³ The strip theory has been introduced in order to replace a three-dimensional hydrodynamic problem by a summation of two-dimensional ones. Using this method, solutions are possible for a much wider range of problems and actual hydrodynamic conditions connected with ship motions can be represented more completely. F. M. Lewis (1929) appears to be the first to apply this theory in connection with evaluation of hydrodynamic forces acting on a vibrating ship. Hazen and Nims (1940), St. Denis (1951), and St. Denis and Pierson (1953) used the strip theory in connection with the analysis of ship motions. This theory was described more explicitly later by Korvin-Kroukovsky (1955c) and Korvin-Kroukovsky and Jacobs (1957). Quoting from the latter work:

"Consider a ship moving with a constant forward velocity (V) (i.e., neglecting surging motion) with a train of regular waves of celerity (c). Assume the set of co-ordinate axes fixed in the undisturbed water surface, with the origin instantaneously located at the wave nodal point preceding the wave rise, as shown in Fig. 1 [here-with]. With increase in time t the axes remain fixed in space, so that the water surface rises and falls in relation

³ Solutions of hydrodynamic problems related to special mathematically defined ship forms will be discussed in Section 6.

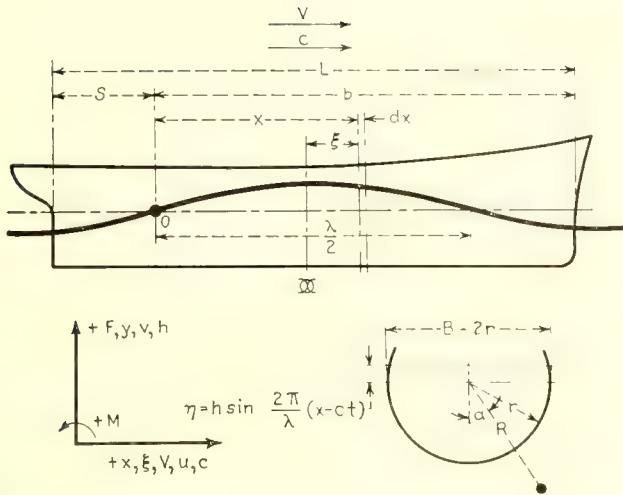


Fig. 1 Sketch illustrating notation used in connection with strip theory (from Korvin-Kroukovsky, 1955c)

to them. This vertical displacement at any instant and at any distance x is designated η . Imagine two control planes spaced dx apart at a distance x from the origin, and assume that the ship and water with orbital velocities of wave motion penetrate these control surfaces. Assume that the perturbation velocities due to the presence of the body are confined to the two-dimensional flow between control planes; i.e., neglect the fore-and-aft components of the perturbation velocities due to the body, as in the 'slender body theory' of aerodynamics. This form of analysis, also known as the 'strip method' or 'cross flow hypothesis' is thus an approximate one in the sense that a certain degree of interaction between adjacent sections is neglected."

In analyzing ship motions it is generally necessary to stipulate two systems of axes, one fixed in space and one fixed in the body. Thus, considering heaving and pitching of a ship, Korvin-Kroukovsky and Jacobs (1957)⁴ stipulated an x, y, z -system fixed in space (with the x - y -plane in the undisturbed water surface) and a ξ, η, ζ -system fixed in the ship. The location in the ship of the origin of the latter system is arbitrary, but the mathematical work is simplified considerably if the origin is placed at the center of gravity. A primary step in the strip method of analysis is to evaluate the hydrodynamic forces caused by the relative ship-wave vertical motion at a ship section located at a distance ξ from the origin. The vertical velocity of this section is the summation of the velocity components in heaving \dot{z} and in pitching $\xi\dot{\theta}$. When a ship is at a small angle of trim θ , the draft of ship sections, passing through the water slice dx , increases with time. This gives an added vertical velocity component θV .

After the forces acting on individual ship sections are evaluated, they are integrated over the ship length.

The integral forms used to obtain various coefficients for the equations of motion are given in Appendix C and are discussed in Chapter 3. Use of the sectional forces in computations of the hull bending moments are discussed in Chapter 5, by Jacobs (5-1958) and by Dalzell (5-1959).

The forces produced by water pressures on ship sections can be classified by their nature as inertial, damping, and displacement. They also can be classified by their cause as resulting from a ship's oscillation in smooth water or from wave action on a restrained ship. Kriloff (1896, 1898), considering only displacement forces, demonstrated that the total force acting on a ship in waves can be considered as the sum of these two components. Korvin-Kroukovsky and Jacobs (1957) demonstrated that this subdivision of forces also holds (within linear theory) when the pressures are generated by water acceleration. This is the direct consequence of the linear superposition of velocity potentials defining various water flows. It can be added here that the forces involved in pitching and heaving are caused primarily by potential flows, and water viscosity does not appear to be important in this connection.

The sectional inertial forces will be discussed in the following Section 3.1 and the damping forces⁵ in Section 3.2. The action of displacement forces in a ship oscillating in smooth water is obvious and needs no discussion. The displacement effect caused by waves will be brought out in the consideration of inertial forces, since the wave elevations are inseparably connected with water accelerations.

3.1 Inertial Forces Acting on a Body Oscillating in Smooth Water: 3.11 Conformal transformations.

The work on added mass in vertical oscillations most often referred to is that of F. M. Lewis (1929). Lewis assumed that water flow around a circular cylinder floating half immersed on the water surface is identical with that around a deeply immersed cylinder. Simple expressions for the latter are available in standard textbooks (Chapter 1, References C, D, and F). The added mass of a cylinder is found from these expressions to be equal to the mass of water displaced by it. The coefficient of accession to inertia is, therefore, unity. Lewis devised a conformal transformation by means of which a circle is transformed into ship-like sections of various beam/draft ratios and sectional coefficients. Water flows corresponding to these sections were derived and coefficients of accession to inertia were determined. In addition to Lewis' (1929) original work, the procedure was described (with various extensions) by Prohaska (1947), Wendel (1950), and Landweber and de Macagno (1957). The resultant relationships were also given by Grim (1956).⁶

The original half-immersed circle of radius r is defined in complex form

⁴ The mathematical part of this reference is included in this monograph as Appendix C.

⁵ It will be shown in Section 3.2 that damping forces are also of inertial origin.

⁶ An independent evaluation of added masses was also made by J. Lockwood Taylor (1930b).

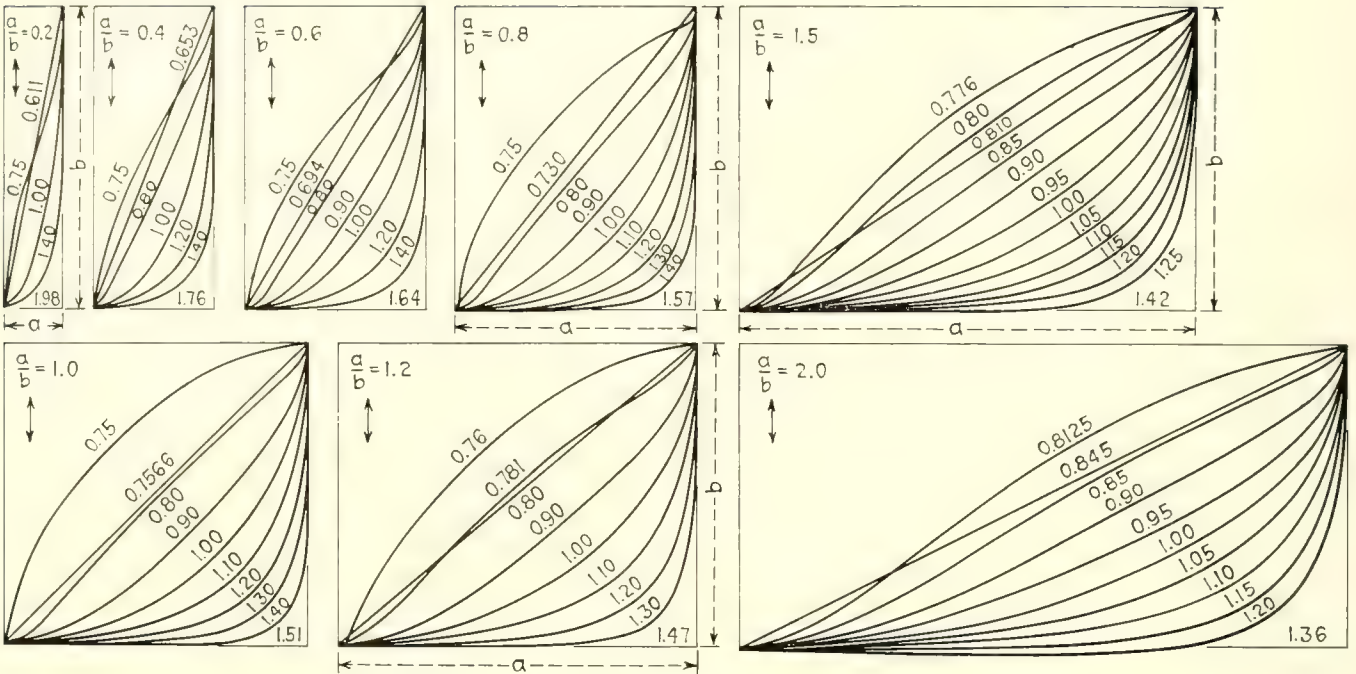


Fig. 2 Transverse sections and inertia coefficients obtained by Lewis (from Wendel, 1950)

$$\zeta = \xi + i\eta = re^{i\theta}$$

where θ is the angle which the radius vector makes with the water level. The transformed figure is described by F. M. Lewis as

$$z = x + iy = \zeta + \frac{a_1}{\zeta} + \frac{a_3}{\zeta^3} \quad (12)$$

$$= re^{i\theta} + \frac{a_1}{r} e^{-i\theta} + \frac{a_3}{r^3} e^{-3i\theta}$$

where a_1 and a_3 are real coefficients. After separation of real and imaginary parts, the co-ordinates x and y are given by parametric equations

$$\begin{aligned} x/r &= (1 + a_1) \cos \theta + a_3 \cos 3\theta \\ y/r &= (1 - a_1) \sin \theta - a_3 \sin 3\theta \end{aligned} \quad (13)$$

The half-beam b and the draft d of the resultant figure are related to the coefficients a_1 and a_3 by

$$\begin{aligned} b/r &= 1 + \frac{a_1}{r^2} + \frac{a_3}{r^4} \\ d/r &= 1 - \frac{a_1}{r^2} + \frac{a_3}{r^4} \end{aligned} \quad (14)$$

The section shapes resulting from this transformation are shown on Fig. 2. Each separate set of curves corresponds to a different b/d ratio (labelled on the figure as a/b). The curves are labelled by the values of the added mass coefficient C (defined later). The validity of this transformation is limited to the range of parameter values shown in Table 1. Fig. 3 shows the values of the

Table 1

(From Landweber and de Macagno, 1957)

d/b	$\beta = A/2bd^*$
0.6	0.412-0.93
0.8	0.353-0.942
1.0	0.294-0.957
1.4	0.379-0.937
1.8	0.425-0.925
2.5	0.471-0.914
5.0	0.530-0.898

* A is sectional area.

added mass coefficient C of Lewis forms as a function of the section coefficient β and the draft/beam ratio.

F. M. Lewis expressed the added mass in terms of the water mass enclosed within a semi-circular contour of radius b as the coefficient

$$C_v = \frac{\text{added mass}}{\pi \rho b^2 / 2} \quad (15)$$

For the ship sections obtained from the foregoing transformation the coefficient is evaluated as

$$C_v = \frac{(1 + a_1)^2 + 3a_3^2}{(1 + a_1 + a_3)^2} \quad (16)$$

The computed values of the coefficient C_v are shown in Fig. 3. The coefficient C_v is related to k_v the coefficient of accession to inertia by

$$k_v = C_v \pi B^3 / 8A \quad (17)$$

where B is the sectional beam, and A is the sectional area $k_v = C_v$ for a semi-circular profile.

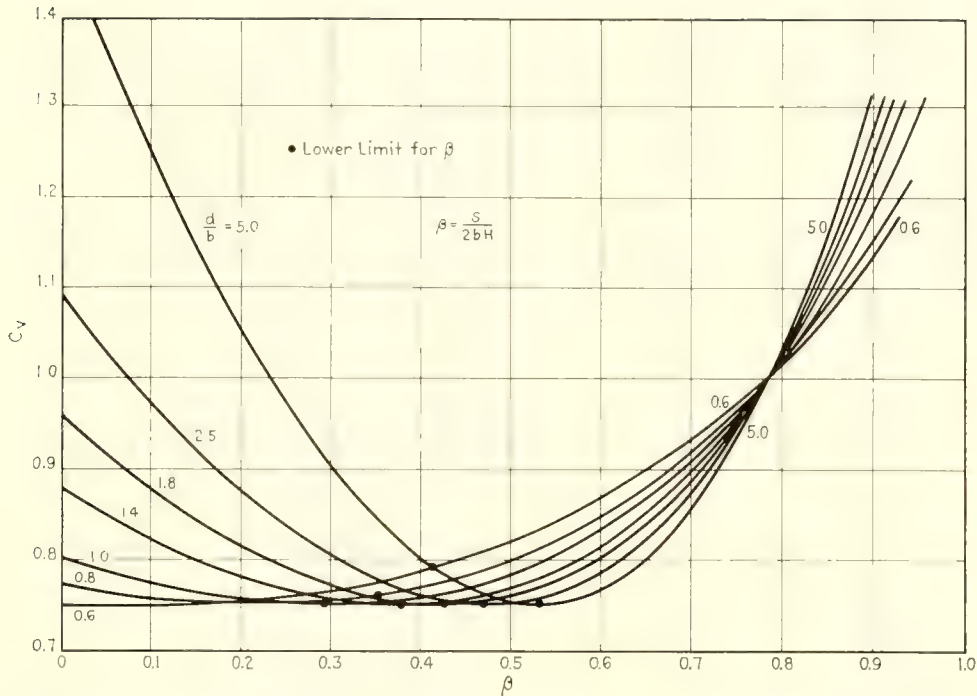


Fig. 3 Added mass coefficients for vertical motion (from Landweber and de Macagno, 1957)

Landweber and de Macagno (1957) have shown that F. M. Lewis' transformation is a particular case of a more general transformation form

$$z = \zeta + a_1/\zeta^m + a_2/\zeta^n \dots \quad (18)$$

in which the indices m , n , etc., are odd numbers in the case of symmetrical sections. Prohaska (1947) computed the properties of sections with indices (m, n) of (1,5), (1,7), and (3,7).

The transformation from a circle, described in the foregoing, is not suitable for section forms with sharp edges. The water-flow pattern around polygonal forms and the resulting added masses can be obtained by means of the Schwartz-Christoffel transformation. A good description of this with examples of application will be found in Wendel (1950). Rectangles, rectangles with bilge keels, and rhombuses were analyzed by this method. The data on these will be found in Figs. 4 and 5.

Experiments, based on the analogy between the electric potential and the velocity potential of a fluid flow (Koch, 1933), give results identical in principle to those obtained from conformal transformations. They can be useful where conformal transformation becomes laborious, as for instance in the case of the added mass of a floating body in shallow water. They could also be useful in cases to which the available computational methods do not apply. A ship section composed of curves and equipped with bilge keels can be cited as an example, as can also typical sections in the stern portions of commercial ships. These sections are usually bounded by curved lines and possess either a large vertical

deadwood, or propeller-shaft bossings, or both. The hydrodynamic properties of such sections are not known, and their evaluation by means of electrical analogy can be recommended.

An extended discussion of added mass (neglecting free-surface effects) will be found on pages 417-441 of Volume II of "Hydrodynamics of Ship Design" by Harold E. Saunders published by The Society of Naval Architects and Marine Engineers, 1957. A listing of 54 references is included in this work.

3.12 Effect of the free water surface. The evaluation of the added masses in the foregoing was based on the assumption that the water flow about the submerged part of a floating profile is identical with the flow about a submerged profile consisting of the original one and its reflection in the water surface. This means that the wavemaking by an oscillating floating body was neglected. Subsequent mathematical work (Havelock, Ursell, Haskind) has shown that the foregoing assumption is valid for a high oscillation frequency. The values obtained on this basis are therefore directly applicable to ship vibrations, which was indeed the application intended by F. M. Lewis and Prohaska.

A different situation is found at the low frequency of a ship's pitching and heaving in waves. Two wave systems are generated by these motions: The standing-wave system in proximity to the ship, and the progressive waves running away from the ship. The mean energy content of the first of these systems remains constant and defines the added mass. The progressive waves carry the energy away from the ship and represent the


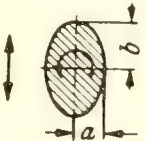

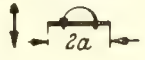
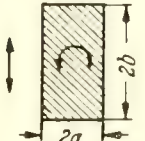


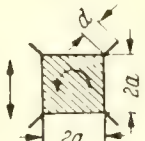
Cross-Sectional Forms		Hydrodynamic Mass per Unit Length	Inertia Coefficient $C = \frac{m''}{m''_{\text{circle}}} =$	Hydrodynamic Moment of Inertia per Unit Length	Inertia Coefficient for Rotation $D'' = \frac{J''}{J''_{\text{plate}}} =$
Direction of Motion	Axis of Rotation				
	$\frac{a}{b} = 1$ (circle)	$m'' = \pi \rho a^2$ [1]	1	$J'' = 0$	0
		$m'' = \pi \rho a^2$ [1]	1	$J'' = 0.125 \pi \rho (a^2 - b^2)^2$ [1]	$\left[\left(\frac{b}{a} \right)^2 - 1 \right]^2$
					
	Plate	$m'' = \pi \rho a^2$ [1]	1	$J'' = 0.125 \pi \rho a^4$ [1]	1
	$\frac{a}{b} = \frac{1}{2}$	$m'' = 1.7 \pi \rho a^2$ [2]	1.7	$J'' = 0.15 \pi \rho b^4$ [5]	$\left. \begin{array}{l} 1.2 \\ 1.18 \end{array} \right\} \text{Referred to Plate of Width } 2b$
	$\frac{a}{b} = \frac{1}{5}$	$m'' = 1.98 \pi \rho a^2$ [2]	1.98	$J'' = 0.15 \pi \rho b^4$ [5]	
	$\frac{a}{b} = \frac{1}{10}$	$m'' = 2.23 \pi \rho a^2$ [2]	2.23	$J'' = 0.147 \pi \rho b^4$ [5]	
	Square	$m'' = 1.51 \pi \rho a^2$ [2]	1.51	$J'' = 0.234 \pi \rho a^4$ [3, 5]	1.872
	$\frac{a}{b} = 2$	$m'' = 1.36 \pi \rho a^2$ [2]	1.36	$J'' = 0.15 \pi \rho a^4$ [5]	$\left. \begin{array}{l} 1.2 \\ 1.18 \end{array} \right\} \text{Referred to Plate of Width } 2a$
	$\frac{a}{b} = 5$	$m'' = 1.21 \pi \rho a^2$ [2]	1.21		
	$\frac{a}{b} = 10$	$m'' = 1.14 \pi \rho a^2$ [2]	1.14		
	$\frac{d}{a} = 0.05$	$m'' = 1.61 \pi \rho a^2$ [5]	1.61	$J'' = 0.31 \pi \rho a^4$ [5]	$\left. \begin{array}{l} 2.4 \\ 3.2 \\ 5.5 \end{array} \right\} \text{Referred to Plate of Width } 2a$
	$\frac{d}{a} = 0.1$	$m'' = 1.72 \pi \rho a^2$ [5]	1.72	$J'' = 0.4 \pi \rho a^4$ [5]	
	$\frac{d}{a} = 0.25$	$m'' = 2.19 \pi \rho a^2$ [5]	2.19	$J'' = 0.69 \pi \rho a^4$ [5]	

Fig. 4 Tabulation of hydrodynamic masses, hydrodynamic moments of inertia, and inertia coefficients as calculated by [1] Lamb, [2] Lewis, [3] Proudman, [4] Weinblum, [5] Wendel, [6] determined experimentally (electrical analog) by Koch (from Wendel, 1950)

primary cause of damping. The standing-wave system is very complex and the mathematical solution for the added mass has so far been obtained only for a floating

circular cylinder (Ursell, 1949b, 1953, 1955).⁷ The

⁷ Three-dimensional solutions for special mathematical ship forms by Haskind and Hanaoka will be discussed in Section 6.


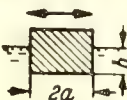
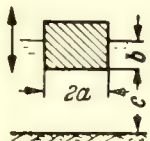
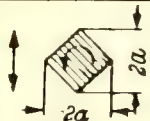
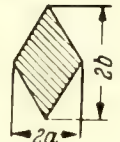


Cross-Sectional Forms		Hydrodynamic Mass per Unit Length	Inertia Coefficient $C = \frac{m''}{m''_{\text{circle}}} =$	Hydrodynamic Moment of Inertia per Unit Length	Inertia Coefficient for Rotation $D'' = \frac{J''}{J''_{\text{plate}}} =$
Direction of Motion	Axis of Rotation				
	$\frac{a}{b} = 1$	^[2] $= 0,75 \pi \rho a^2$ $= \frac{1}{2} \text{ square}$	0.75	^[5] $= 0.117 \pi \rho a^4$	0.936
	$\frac{a}{b} = 1$	^[5] $= 0,25 \pi \rho a^2$	0.25		
	$\frac{e}{b} = \infty$	^[5] $= 0.755 \pi \rho a^2$	0.75		
	$\frac{e}{b} = 2.6$	^[5] $= 0.83 \pi \rho a^2$	0.83		
	$\frac{e}{b} = 1.8$	^[5] $= 0.89 \pi \rho a^2$	0.89		
	$\frac{e}{b} = 1.5$	^[6] $\approx 1 \pi \rho a^2$	≈ 1		
	$\frac{e}{b} = \frac{1}{2}$	^[6] $\approx 1.35 \pi \rho a^2$	≈ 1.35		
	$\frac{e}{b} = \frac{1}{4}$	^[6] $\approx 2 \pi \rho a^2$	≈ 2		
		^[2] $= 0.76 \pi \rho a^2$	0.76	^[5] $= 0.059 \pi \rho a^4$	0.47
	$\frac{a}{b} = \frac{1}{2}$	^[2] $= 0.67 \pi \rho a^2$	0.67		
	$\frac{a}{b} = \frac{1}{5}$	^[2] $= 0.61 \pi \rho a^2$	0.61		
	$\frac{a}{b} = 2$	^[2] $= 0.85 \pi \rho a^2$	0.85		
	Regular Octagon			^[4] $\approx 0.055 \pi \rho a^4$	0.44

Fig. 5 Tabulation of hydrodynamic masses, hydrodynamic moments of inertia, and inertia coefficients as calculated by [1] Lamb, [2] Lewis, [3] Proudman, [4] Weinblum, [5] Wendel, [6] determined experimentally (electrical analog) by Koch (from Wendel, 1950)

added-mass coefficient was expressed as a function of a nondimensional frequency parameter $\omega^2 r / \pi g$. The results can be conveniently interpreted as a correction coefficient⁸

$$k_4 = \frac{\text{added mass of a body floating on water surface}}{\text{added mass of a body deeply submerged}} \quad (19)$$

⁸ This correction coefficient was first used in ship-motion analysis by B. V. Korvin-Kroukovsky (1955c). Having assigned (following Lamb) the notation $k_{1,2,3}$ to the coefficient of accession

to inertia of a submerged body in three directions, he used the notation k_4 for the added mass correction of a body floating on the water surface.

The values of coefficient k_4 , computed by Ursell for a floating cylinder, are listed in Table 2.

Table 2

(From Ursell, 1955)

$\omega^2 r / \pi g$	k_4
$1/6$	0.632
$1/4$	0.592
$1/2$	0.673
$2/3$	0.738
$3/4$	0.762
1	0.818
$5/4$	0.859
$3/2$	0.883

NOTE: For very high-frequency parameters the coefficient k_4 asymptotically approaches unity.

In the absence of data for other ship sections, Korvin-Kroukovsky (1955c) and Korvin-Kroukovsky and Jacobs (1957) have assumed that the coefficient k_4 , initially computed by Ursell for a circular cylinder, will apply to other profiles. Additional theoretical research is evidently needed in order to provide values of k_4 for F. M. Lewis' and other ship sections.

Attention should be called to the particular surface effect occurring in the case of sections with inclined sides. Such inclinations exist in the bow sections of V-form ships and are particularly pronounced in stern sections of most ships. When such a section penetrates the water surface, the adjacent water level rises and the wetted beam becomes greater than the one indicated by the intersection with the undisturbed water level. This causes an increase in added mass and probably in damping forces. A solution of this problem was given by Wagner⁹ for the asymptotic case of an impact in which gravity forces are neglected; i.e., the instantaneous water rise is taken into account but not the subsequent wave-making. In Wagner's solution, the effect of a deep draft in conjunction with a V-section has not been considered. J. D. Pierson (1950, 1951) used a computational method (initially suggested by Wagner) in which the draft of a V-section is included, subject to the gravity-free assumption. No information is available on the water flow, and the added masses and damping connected with it, for the water-surface penetration by a wedge at the frequencies of a ship's heaving and pitching in waves. Also, the available impact theories treat only the hydrodynamic force acting on a body when it penetrates the water. The theory of ship motions and bending moments requires also knowledge of the forces during body emergence from the water. The added masses involved in these motions will evidently be functions of oscillation frequency. Research in this field is needed.

3.13 Three-dimensional effects. In the evaluation of hydrodynamic forces by the strip theory, the initial assumption is made that the water flow at each strip is two-dimensional and is not influenced by the adjacent strips. Subsequently, it is desired to verify this assumption and, if practical, to establish a correction fac-

tor for the deviation of the physical conditions from the initially assumed ones. This problem can be considered in connection with three applications:

a) A body vibrating with various numbers of nodal points.

b) A body of a certain L/B ratio pitching and heaving, in smooth water.

c) A body subjected to waves.

The first problem (a) was treated by F. M. Lewis (1929) and J. Lockwood Taylor (1930b) with varying results. Macagno and Landweber (1958) have investigated these solutions and demonstrated that the results are strongly affected by the completeness with which the shear and flexural deflections of a body are described.

The second problem (b) has apparently received no attention. The third problem (c) was solved for a spheroid under waves by Havelock (1954) and Cummins (1954a, b) using advanced mathematical methods, and by Korvin-Kroukovsky (1955b) using the strip theory. The agreement between calculational methods was satisfactory (as shown in Figs. 15 and 16), and apparently no correction for three-dimensional effect is needed for the length/beam ratios normally used in ships and for waves of length approximately equal to a ship's length. This conclusion applies, however, to the coefficient of accession to inertia k_z for the entire body in the analysis of body motions. Three-dimensional effects, in all probability, do affect the distribution of hydrodynamic forces along a ship; i.e., the k_x -values for individual strips. These effects are, therefore, significant in the analysis of ship bending moments and research toward their evaluation is recommended.

3.14 Inertial forces caused by waves. It appears that satisfactory estimates of forces exerted by waves on a submerged body can be made considering inertial forces alone and neglecting viscous forces. This means that hydrodynamic-force components in phase with the waves can be expressed in terms of added mass. The added mass is expressed, in turn, in terms of body displacement. In a strip theory the displacement is taken per unit length. In considering the wave action on a body it is necessary to remember that a velocity gradient and a corresponding pressure gradient exist in waves.

The force acting on a small submerged body in long waves can be calculated most conveniently on the basis of this gradient and the body's volume. The force exerted by a hydrostatic pressure gradient is equal to the product of the gradient and the body's volume. G. I. Taylor (1928) showed that this relationship is modified when the pressure gradient in a fluid results from the acceleration of fluid particles. The relationship becomes

$$\text{Force} = (1 + k) \times (\text{pressure gradient}) \times (\text{body volume}) \quad (20)$$

where k is the coefficient of accession to inertia. The factor $(1 + k)$ is the result of modification of an accelerating fluid flow by the presence of a body.¹⁰

⁹ To be discussed in greater detail in Section 7 on slamming.

¹⁰ Forces exerted on a body by fluid accelerations were also investigated by Tollmien (1938).

In estimating the force acting on a body by the strip theory, the foregoing relationship is applied to each section using the appropriate value of the coefficient k .

A detailed derivation of the wave-caused force by means of surface-pressure integration was given by Korvin-Kroukovsky and Jacobs (1957) and is reprinted in Appendix C. In the derivation carried out for a semi-circular ship section, neglecting surface effects, the product of the sectional volume and the mean pressure gradient was found to be multiplied by 2. Since $k_v = 1$ for a semi-circular section, the factor of 2 was interpreted as $1 + k_v$ on the basis of G. I. Taylor's result. Grim (1957c) has confirmed this intuitive conclusion by application of F. M. Lewis' transformation. To correct for the surface effects neglected in the formal analysis, Korvin-Kroukovsky and Jacobs (1957) interpreted k_v as $k_1 k_2$ in evaluating the force exerted by the vertical wave pressure gradient on a surface ship. The calculated wave forces on a ship's model were confirmed by a towing tank test (Appendix 2 to Korvin-Kroukovsky, 1955c).

Attention should be called to the fact that the direction of the pressure gradient is such that the vertical force is acting downward on a submerged body under the wave crest, and upward under a trough. In a body floating on the water surface these pressure gradient (or inertial) forces are subtracted from the displacement force caused by water-surface rise in waves. The net force is thereby considerably reduced.

It is often convenient to think of water acceleration in waves as algebraically added to the acceleration of gravity. The water at wave crests appears then to be lighter and at wave troughs heavier than normal.

This modification of the effective weight of water in waves is often referred to as the "Smith effect," since attention was called to it by Smith (1883) in connection with ship bending-moment evaluation. Estimation of the wave forces acting on a ship by the buoyancy forces modified by the Smith effect is referred to as the "Froude-Kriloff hypothesis." The effect of the ship in disturbing waves is neglected in this case; i.e., the added term k in equation (20) is not taken into account. Since its inclusion is a simple procedure, there is no justification for neglecting it in the future.

3.15 Experimental data on inertial forces. Very few experimental data are available on added masses, and these, while confirming the general ideas outlined in the previous paragraphs, do not provide exact information. Experiments have been made for the following cases:

- a) Deeply submerged prisms and cylinders.
- b) Prisms oscillating on the free water surface.
- c) Ship forms oscillating on the water surface.
- d) Restrained ship forms and other bodies acted upon by waves.

1 Deeply submerged prisms and cylinders. Tests in the first category (a) are of interest for confirmation of the classical theory. The reasonableness of neglecting viscosity is the particular assumption to be verified. The frequency and amplitude of oscillations would be

irrelevant if water were a truly nonviscous fluid. The existence of a small viscosity, however, may cause eddy-making in certain experimental conditions, particularly in the case of a body with sharp edges. In such a case scale relationships may become significant.

Moullin and Browne (1928) experimented with two-node vibrations of flat steel bars submerged in water. The bars were from $1/4$ to 1 in. thick, 2 in. wide, and 78 in. long, so that three-dimensional effects probably were insignificant. The vibrations were excited by an electromagnet, and the added mass was obtained by comparison of the resonant frequencies in air and in water. It was concluded that the added mass is equal to the water mass of the cylinder circumscribing the rectangular profile of the bar. This is in agreement with the theoretically indicated added mass of a thin plate considering the expected increase with thickness of the rectangular section.

2 Prisms oscillating on the water surface. Moullin and Browne (1928) also experimented with bars $1/2$ in. thick and 3 in. wide, set on edge and partially submerged. They concluded that the added mass is independent of the vibration frequency. This conclusion is in agreement with theoretical expectations for high frequencies. Browne, Moullin and Perkins (1930) tested the vertical vibrations of rectangular and triangular prisms partly immersed in water. The prisms were attached to a flat steel spring and were vibrated by an electromagnet. A $6 \times 6 \times 54$ -in. prism vibrated at a frequency of about 15 cps. This frequency corresponds approximately (to scale) to the usual two-node frequency of ship vibrations. The theoretical added mass (for a submerged double profile) was computed by a Schwartz-Christoffel transformation. The experimentally determined added mass was found to be about 90 per cent of the theoretical one. Experiments were made with various lengths of prisms and the authors stated that, above a length/beam ratio of 4, the added mass was independent of the length. Todd (1933), in applying these and Lewis' (1929) results to ship-vibration analysis, attributed the reduction in added mass to the effect of the length/beam ratio.

It should be emphasized that the frequencies in the experiments just outlined correspond to ship-vibration frequencies. These are about 10 times as much as the usual frequency of a ship's pitching in head seas. Theory (Section 3.12) indicates that at lower frequencies a pronounced dependence of added mass on frequency can be expected.

Prohaska (1947) reported on oscillation tests of prisms of several profiles partly submerged in water. The experimental data indicated added-mass values about 90 per cent of the theoretical ones computed for the submerged double profiles. Unfortunately no information was furnished as to the frequency of oscillations. The description indicates that the test apparatus was of the same type used by Dimpker (1934) and Holstein (1936). With such an apparatus, the high frequency of Moullin's experiments can hardly be expected. Without information on the frequency, Prohaska's tests can be ac-

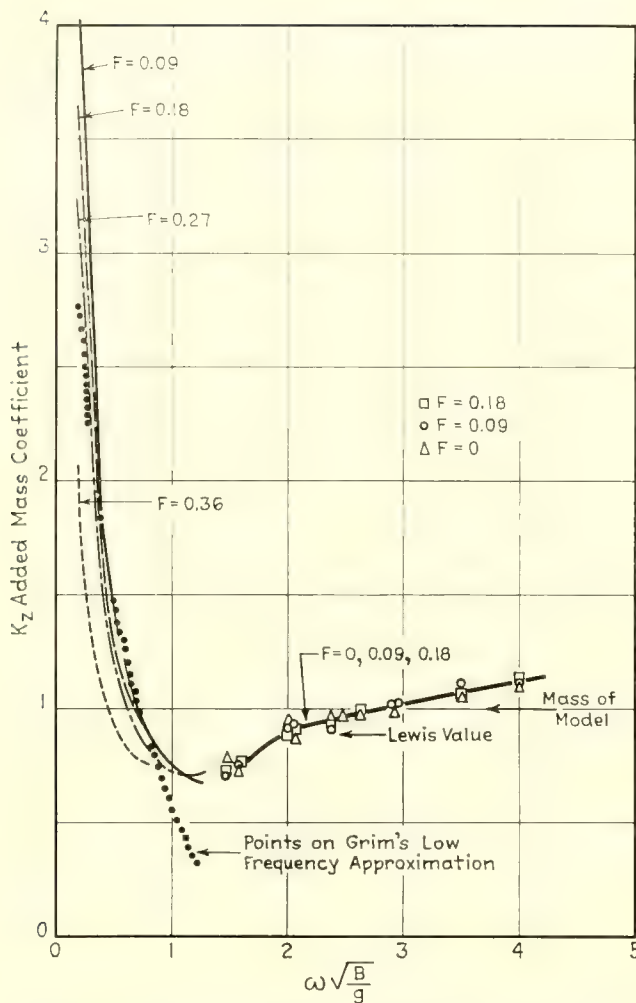


Fig. 6 Variation of added mass coefficient with frequency (from Golovato, 1957)

cepted only as a rough confirmation of the order of magnitude of the added masses. The roughly indicated 10 per cent reduction in added masses is clearly caused by the surface effects, since Prohaska's models spanned the width of the test tank and three-dimensional effects were absent.

Dimpker's (1934) and Holstein's (1936) tests were made at frequencies in the range estimated for ship motions in waves. These tests will be discussed in greater detail later in connection with damping (in Section 3.2) since damping forces as well as added masses were determined. Holstein's measurements of added masses appear to be too erratic to be useful. Wendel (1950), estimated the experimental errors in these measurements and demonstrated that added masses are considerably smaller than indicated by the submerged-prism theory. Dimpker, like Holstein, made tests at a series of frequencies governed by the stiffness of the retaining springs. He did not publish information on frequencies directly, but on spring stiffnesses. The data given in

the published paper do not appear to be sufficient to calculate the frequencies.

To summarize the results on the oscillation of partially submerged prisms: None of the tests made so far gives sufficient information on the frequency of oscillations to permit evaluation of the added mass versus frequency relationship. The tests have generally indicated that experimentally measured added-mass coefficients of bodies on the water surface are smaller than those for deeply submerged prisms. Evidently, additional experimental research is needed. All of the tests described in the foregoing were made in small tanks and it can be questioned whether the test data were not affected by wave reflections from tank ends. While wave-absorbing beaches have been used in towing tanks for many years, it was not realized until recently how difficult it is to prevent wave reflections.

In the work just described a strictly pragmatical approach was taken. Reference should be made to Weinblum (1952) and Keulegan and Carpenter (1956) for the less evident aspects of the inertial force and added-mass concepts. In particular, for bodies at the water surface the hydrodynamic force is connected with wave formation. It depends therefore not only on instantaneous conditions but on the past history of motions as well. Added mass becomes a definite concept only when correlated with a definite type of motion. The added masses in harmonic oscillation are not necessarily identical with the added masses in, for instance, uniform acceleration of a body. In the tests of Dimpker, Holstein, and Prohaska, the added masses were derived from the natural period of decaying oscillations. It can be questioned whether added masses so obtained are identical with those occurring in sustained harmonic oscillations.

3 Ship forms oscillating on the water surface. Golovato (1957a,b) reported on experiments with a harmonically heaving ship model restrained from pitching.¹¹ The model had lines composed of parabolic arcs, following Weinblum (1953), and had a prismatic coefficient of 0.655. The inertial and damping forces in heaving were calculated from the amplitude and phase lag of the motion records as compared with the records of the harmonic exciting force. Fig. 6 shows the coefficient of accession to inertia k_z plotted versus nondimensional frequency $\omega(B/g)^{1/2}$. A horizontal arrow at about $k_z = 0.93$ shows the value calculated by using F. M. Lewis' (1929) data; i.e., neglecting surface wave effects. The curve shown by heavy dots is Grim's (1953 a,b) asymptotic evaluation of the added mass for low frequencies. The experimental data at low frequencies are somewhat uncertain because of model interference with waves reflected from the sides of the towing tank. At higher frequencies the coefficient k_z is shown to be independent of the Froude number.

Fig. 6 covers a wide range of frequencies and the picture may be misleading unless the range important in

¹¹ The author understands that similar experiments also were made with pitching oscillations, but the results have not yet been published.

ship operations is kept in mind. Data on the pitching-oscillation frequencies of several ships will be found in Korvin-Kroukovsky and Jacobs (1957). For the Series 60 model of 0.60 block coefficient in head waves of $\lambda/L = 1$, the parameter $\omega(B/g)^{1/2}$ varies from 0.9 to 1.5 at ship speeds from zero to the maximum expected in smooth water. At synchronism in pitching the parameter is equal to 1.25. This narrow range of frequency parameters straddles the minimum of the curve of added mass coefficients in Fig. 6.

The reader will find it instructive to plot the Ursell data for the circular cylinder from Table 2 on Fig. 6, remembering that $r = B/2$. Both k_2 -curves plotted as a function of $\omega(B/g)^{1/2}$ are similar in form but the curve shown for Golovato's model is seen to be considerably above Ursell's curve. It should be noted that Golovato's curve does not asymptotically approach the F. M. Lewis value but is directed much higher. This raises the question of experimental reliability or perhaps the presence of physical features not accounted for by the theory. Reference to Keulegan and Carpenter (1956) indicates that the added mass may have been increased on the up-stroke of the oscillator by the separation of the water flow. If so, the data would be subject to the degree of model roughness and to a scale effect.

Gerritsma (1957*c, d*) tested a Series 60, 0.60 block coefficient model, 8 ft long, by subjecting it to forced oscillations in heaving and pitching alternately. Fig. 7 shows a comparison of measured and calculated virtual masses and virtual moments of inertia; i.e., ship masses plus added water masses. The calculated masses were taken from Korvin-Kroukovsky and Jacobs (1957), and are based on the strip integration of the product of F. M. Lewis' k_2 coefficients and the surface-effect correction coefficient k_4 based on Ursell's data. The discrepancy between measured and calculated data is small, and is in the right direction. In the Series 60 model the after-body ship sections have large inclinations to the water surface, and the surface effect caused by these inclinations was not taken into account. A correction for this effect (not known at present, see Section 3.12) can be expected to increase the calculated added masses.

It is gratifying to see that the discrepancies in virtual masses in heaving and in virtual moments of inertia in pitching are similar. This indicates that the three-dimensional effect is not important in added-mass evaluation.

To summarize the present section: Gerritsma's tests show a satisfactory agreement between added masses as measured and as calculated by the strip theory using the product of Lewis' and Ursell's coefficients, $k_2 k_4$. The need for an additional correction for the effect of inclined sides is indicated, and a correction for three-dimensional effect may be included in the future. However, the results of the motion analysis of usual ship forms would not be significantly affected by it. Golovato's tests on the idealized ship model show a greater value of the added mass than would be indicated by the method of calculation just described. The failure of the data to approach

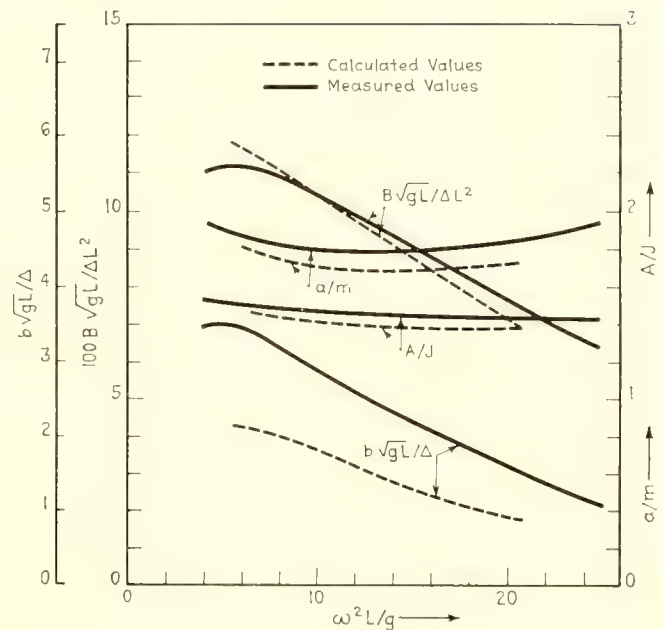


Fig. 7 Comparison between calculated and measured values of A , B , a , b , for series 60, $C_B = 0.60$ hull form (from Gerritsma, 1957*d*)

Lewis' value asymptotically at high frequencies raises suspicion and calls for added investigation. Since only two investigations have been reported, further research is evidently needed.¹²

4 Restained ship forms and other bodies subjected to wave action. Three references can be cited in connection with this subject: Keulegan and Carpenter (1956), Rechlin, Steele and Scales (1957), and Korvin-Kroukovsky (1955*c*, Appendix 2). A study of the first two, although they are primarily concerned with the forces acting on offshore structures in shallow water, should be fruitful to a researcher in naval architectural problems. The third reference appears to be the only work concerned directly with the forces acting on ships.¹³

The heaving force, pitching moment, and drag force exerted by waves on a ship model were measured. A Series 60, 0.60 block coefficient model, 5 ft long, was restrained from heaving, pitching and surging by dynamometers attached at 0.25 and 0.75 of the model's length. Tests were made in regular waves 60 in. long (i.e. $\lambda/L = 1$) and 1.5 in. high at six speeds of advance, starting with zero. Fig. 8 is a comparison of test data with calculations made by Korvin-Kroukovsky and Jacobs (1957) using strip theory and added-mass coefficient

¹² Minutes of the S-3 Panel of the SNAME indicate that such research is in progress at the Colorado State University under the guidance of Prof. E. F. Schulz. In this program added masses and damping forces are measured on individual sections of a segmented ship model so that the distribution of forces along the body length will be obtained.

¹³ Additional measurements of the wave-caused forces recently were published by Gerritsma (1958, 1960).

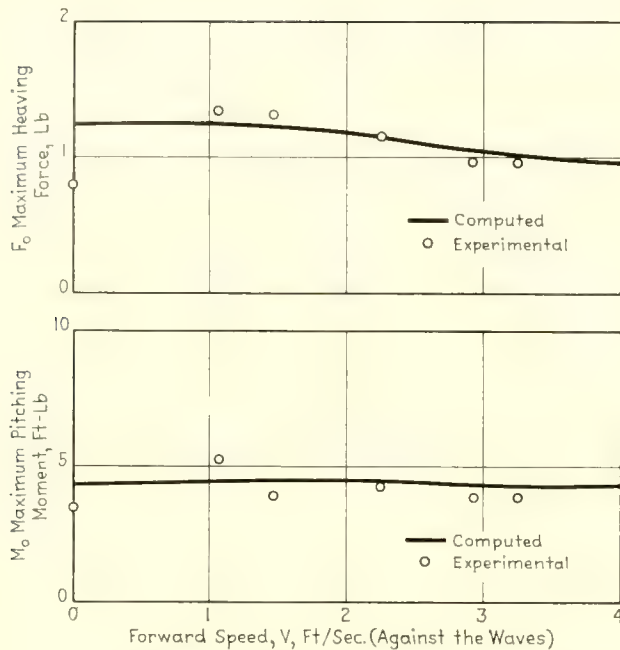


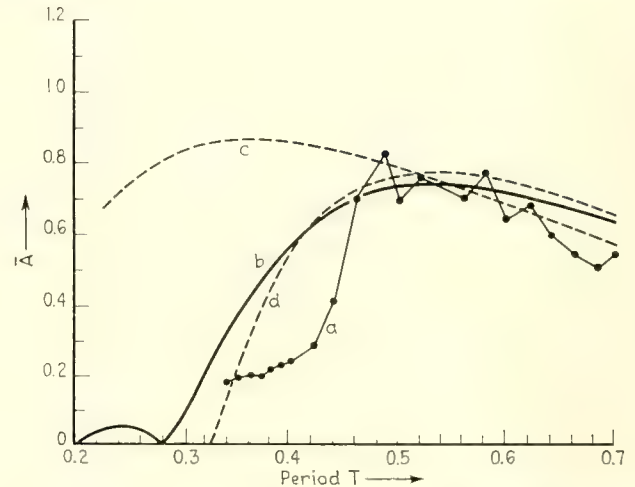
Fig. 8 Comparison of computed and experimentally measured exciting force and moment amplitudes for ETT Model 1445 in waves 5 ft by 1.5 in. (from Korvin-Kroukovsky, 1957)

k_2k_4 .¹⁴ A good agreement is demonstrated except at zero model speed. It is possible that at zero speed the reflections of model-radiated waves from towing-tank walls was affecting the test data. The influence of these reflections was not recognized at the time of the tests, but has since been found important by Golovato and Gerritsma.

While Fig. 8 appears to provide a satisfactory confirmation of the calculational procedure, it should be remembered that it is only the result of a single project of small scope. Additional tests by other investigators with several model forms and different wave lengths and heights are desirable.

The tests of Korvin-Kroukovsky just described, as well as those of Golovato and Gerritsma point to difficulties caused by wave reflections at low model speeds. Tests in wide (maneuvering) tanks are therefore recommended.

Attention should be called to the fact that the hydrodynamic force caused by waves is in composition like the force caused by ship oscillations; i.e. it has force components proportional to acceleration (inertial), to velocity and to displacement. In the analysis of ship motions, Korvin-Kroukovsky and Jacobs (1957) neglected the wave-force component proportional to velocity, since this component became very small after integration of sectional forces over a ship's length. This component cannot be neglected, however, for individual sections when the force distribution along the length is



(a)—Prism 12 cm wide, 5 cm draft—Holstein's experimental data
(b)—Same computed by source distribution along the bottom
(c)—Semi-cylinder of 6 cm radius—on basis of Ursell's (1949) computations
(d)—Semi-cylinder computed by source distribution over the surface

Fig. 9 Ratio, \bar{A} , of wave amplitude to amplitude of heaving motion of cylinder and of rectangular prism of same width and sectional area (from Korvin-Kroukovsky, 1955a)

important, as in the calculation of hull bending moments (Jacobs, 5-1958). Therefore, research planning must provide for the measurement of the distribution of velocity-proportional forces as well as the inertial ones.

3.2 Damping Forces. The work of Grim (1953) is particularly valuable in that it contains a comprehensive set of curves for estimation of damping. These curves are of \bar{A} , the ratio of the progressive waves' amplitude to the amplitude of heaving oscillation, as a function of nondimensional frequency $\omega^2 B/2g$ for Lewis-type sections. From the values of \bar{A} , the damping force coefficients $N(\xi)$ per unit length can be computed readily (Havelock, 1942b; St. Denis, 1951) by

$$N(\xi) = \rho g^2 \bar{A}^2 / \omega_e^3 \quad (21)$$

One of the drawbacks to following Grim's study is the extensive interpolation needed to obtain the values of \bar{A} and the damping-force coefficients for various ship sections. A more convenient approximate evaluation of damping had been developed earlier by Holstein (1936, 1937a, b) and Havelock (1942). Their method is based on the fact that formation of surface waves by a single pulsating source at a depth f can be evaluated as a preliminary step. The solution gives two types of wave systems, the standing waves and the progressive waves. The expression representing a progressive-wave system at a great distance from a body takes on a simple form. The action of a prismatic body in heaving oscillation is replaced by a series of pulsating sources on the body surface, and the total wave formation is obtained by integration of the effects of separate sources over the

¹⁴ The method of calculation is described in Appendix C.

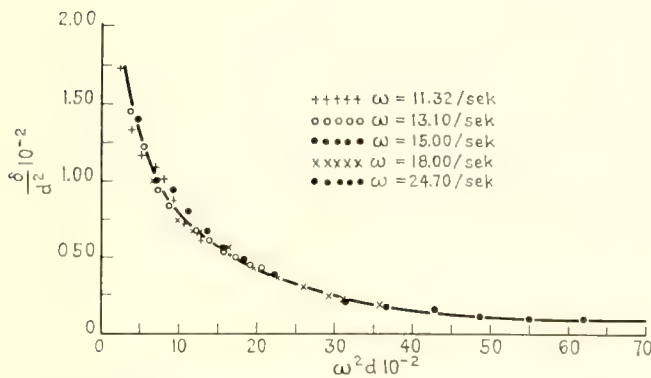


Fig. 10 Variation of logarithmic decrement, δ , with frequency, ω , in damping of heaving oscillations of 60-deg wedge at submergence d (from Dimpker, 1934)

contour. Havelock suggested that a ship section can be replaced by a rectangular one of draft f corresponding to the mean draft of the section; i.e., $f = A/B$, with sources distributed along the bottom. The resultant expression for the ratio \bar{A} is

$$\bar{A} = 2e^{-k_0 f} \sin(k_0 y) \quad (22)$$

where¹⁵ $k_0 = \omega_e^2/g$ and y is the half beam of a ship section under consideration. Fig. 9 taken from Korvin-Kroukovsky (1955a) gives a comparison of the ratio \bar{A} computed by three methods: As given for a semi-cylinder by Ursell (1949) (this agrees with Grim's values for a semi-cylinder); as computed by a source distribution over the contour; and as computed by a source distribution over the bottom of a rectangle of the same sectional area. The results of all methods are in reasonably good agreement at low frequencies, but differ considerably at high frequencies. Fortunately, the frequency of oscillation at synchronism of normal ships is generally in the region in which the disparity is not excessive, and both Havelock's and Grim's damping coefficients have been used with reasonable success. It should be remembered that damping is most important in the evaluation of motion amplitudes near synchronism, and that, at frequencies widely different from synchronism, large errors in estimated damping have relatively little effect on the amplitude. The effect of the damping on the phase lag of motions is, however, most pronounced at frequencies different from the synchronous one.

3.21 Experimental verification of the sectional damping coefficients. The complexity of advanced forms of the solution of the foregoing problem, such as Ursell's and Grim's, necessitates adopting various approximations, the effect of which is difficult to appraise. Therefore, experimental verification is desirable. In Holstein's and Havelock's method of calculation, no investigation is made of boundary conditions at a body, particularly as

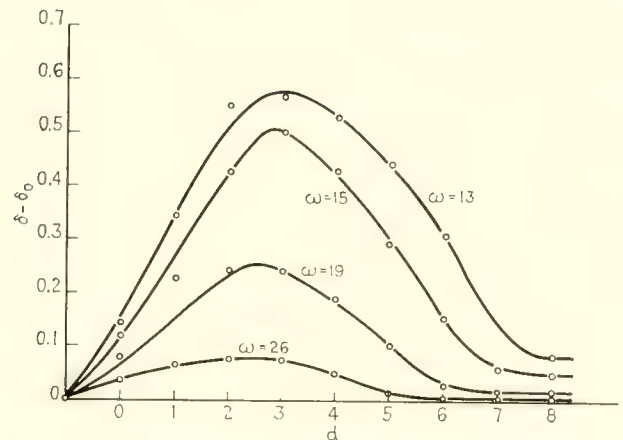


Fig. 11 Dependence of net logarithmic decrement, $\delta - \delta_0$, on submergence, d , with frequency, ω , as parameter for a cylinder 10 cm diam (from Dimpker, 1934). δ is the decrement measured in water, δ_0 is decrement measured in air in preliminary calibration

these are modified by the wave formation. Acceptance of this method depends entirely on successful experimental verification. Holstein (1936) made experiments to verify his theory. These were limited, however, to a rectangular prism varying in degrees of initial immersion, and were made in a small test tank 0.70 m (2.3 ft) wide by 3 m (10 ft) long.

The values of \bar{A} were established by comparison of directly observed wave amplitudes with the amplitudes (half strokes) of the heaving prisms. The results of a large number of experiments appear to be consistent, thus inspiring confidence. The small size of the tank, however, makes the data questionable. It should be remembered that to evaluate \bar{A} the wave amplitudes should be measured far enough from a body for the progressive wave system to be completely free from the standing waves. Furthermore, one must be certain that the progressive waves are not contaminated by reflection from the test-tank ends. These aspects of the test are not discussed sufficiently by Holstein, and, in view of the shortness of the test tank, they may be suspected as having affected the results. The use of a rectangular prism is also questionable, since a certain disturbance can emanate from its sharp edges. This effect is not provided for in the theory.

In his experiments Holstein also attempted to determine virtual masses, but the resultant data were too erratic to be useful.

Dimpker (1934) published data on experiments with a 60-deg wedge and a cylinder with various degrees of immersion. The wedge was tested with an initial immersion from 0 to 12 cm, and the 10-cm-diam cylinder with an initial immersion varying from 0 to 8 cm. The floating body was connected by springs to a motor-driven eccentric, so that either free or forced oscillations could be investigated. Only the free oscillations were dis-

¹⁵ It is necessary to distinguish between the frequency ω of the oncoming waves and the frequency ω_e of the wave encounter which is also the frequency of the ship-radiated waves.

cussed in the published paper.¹⁶ The oscillatory motions of the model were recorded on a rotating drum. The damping was defined by the logarithmic decrement δ as a function of the frequency ω of the oscillation and the mean submergence d .

For a 60-deg prism the mean submergence (over the oscillating cycle) is equal to the beam at mean waterline. Dimpker showed that data for tests at varying frequencies and submersions collapsed into a single curve when plotted as δ/d^2 versus $\omega^2 d$. These amplitude and frequency parameters were initially defined in a non-dimensional form. After the constant quantities, such as the mass involved, the acceleration of gravity g and the water density ρ were omitted, the parameters took the form indicated in the foregoing. The resultant curve is reproduced on Fig. 10. Data for the cylinder are given in Fig. 11. In this case the immersed shape varies with the draft d and it is not possible to make a generalized plot. It is interesting to note that the maximum damping occurs at an immersion of about 2.5 cm; i.e., half-radius. The decrease of damping with further immersion is in agreement with the general tendency shown by the theories of Holstein and Havelock (increase of f in equation 22).

Dimpker (1934) evaluated the virtual masses for a wedge and a cylinder on the basis of changes in the natural period resulting from changes of immersion and frequency. The frequencies were given in terms of spring constants, and the significance of results cannot be seen readily since insufficient data were given for recalculation.

Unfortunately, the experiments of Holstein and Dimpker appear to be the only published data in direct verification of the theory in regard to sectional damping coefficients. Verification of other theoretical methods is indirect. This consists of calculating ship motions using the coefficients evaluated on the basis of the previously mentioned methods and of accepting the successful motion prediction as justification for the method of calculation. However, it is far from being a reliable verification in view of the number of steps involved and the complexity of the calculations. Grim (1953) justified his theoretical damping curves by an analysis of coupled pitching and heaving oscillations of several ship models. The oscillations were induced by means of rotating unbalanced masses; i.e., with a known value of the exciting function. Korvin-Kroukovsky and Jacobs (1957) successfully used Grim's damping coefficients in the analysis of several ship models which had been tested in towing tanks. Korvin-Kroukovsky and Lewis (1955) and Korvin-Kroukovsky (1955c) previously made similar use of Havelock's coefficients.

Direct measurements of damping on a ship model were made by Golovato (1957a and b) and Gerritsma (1957c and d, 1958, 1960).¹⁷ The ship model used by Golovato was a

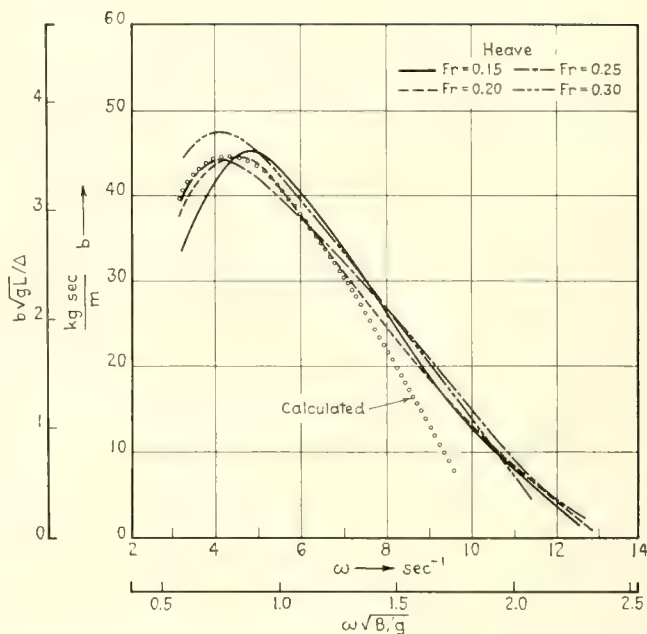


Fig. 12 Comparison of measured damping in heave with one calculated by strip theory using Holstein-Havelock method (from Gerritsma, 1957c)

mathematically defined form, symmetrical fore and aft. Various hydrodynamic forces were measured experimentally in a simple heaving motion (pitching restrained) induced by a mechanical oscillator. The results for damping are shown in Fig. 14 which also includes damping as computed by Havelock's and Grim's methods. The general trend of the damping-force variations as given by all three methods is identical, but both calculated methods give higher damping than the experimental values. (In connection with ship motion analysis, it is suggested that the reader concentrate his attention on the abscissa range 0.9 to 1.5.)

Fig. 7, taken from Gerritsma (1957d), shows a comparison of damping-force coefficients measured on a Series 60, 0.60 block coefficient model with those computed by Korvin-Kroukovsky and Jacobs (1957) by means of the strip theory, using Grim's (1953) material. Figs. 12 and 13 show a similar comparison with the damping computed by Korvin-Kroukovsky (1955c) using the Holstein-Havelock method.

In comparing the heave damping data of Golovato and Gerritsma, it is observed that the measured damping of the Series 60 model is much higher than that of the idealized model: the maximum nondimensional value in Fig. 12 is about 3.7 as against 2 in Fig. 14. A part of this drastic increase can be explained by the presence of inclined ship sides in the afterbody of the Series 60 model, while Golovato's idealized model was wall-sided.

The theoretically evaluated damping is also higher for the Series 60 model, but the increase is not as drastic as for the measured damping. This difference may result from the fact that the inclination of ship sides at the

¹⁶ Dimpker's (1934) paper is a part of a Göttingen dissertation prepared under the guidance of M. Schuler and L. Prandtl.

¹⁷ Added mass data from these tests were discussed in Section 3.15-3.

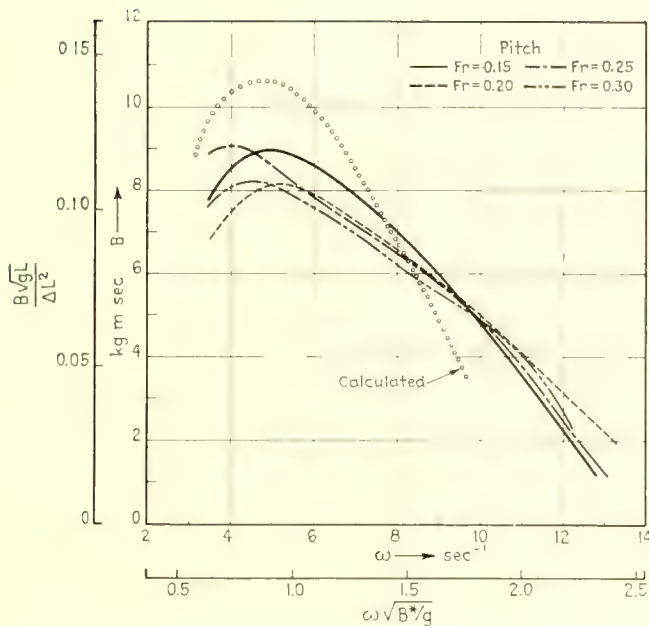


Fig. 13 Comparison of measured damping in pitch with one calculated by strip theory using Holstein-Havelock method (from Gerritsma, 1957c)

waterline is not taken into consideration in the available methods of calculation.

In Golovato's tests the calculated damping greatly exceeded the measured damping, and Grim's theoretical method gave the better approximation. With the different rates of increase of measured and calculated damping for a practical ship form, Gerritsma's heaving test indicates an apparently excellent agreement with the Holstein-Havelock method. Fig. 7 shows that Grim's method underestimated the damping in heave. The word "apparently" was used advisedly. Were the increase of damping due to inclined ship sides taken into account in calculations, both curves of theoretical damping in Figs. 7 and 12 would be displaced upwards.

Examination of Figs. 7 and 13 indicates that the relationship between calculated and measured damping in pitching is drastically different from that in heaving. In the case of pitching, Grim's method is found to agree with the measured data, while the Holstein-Havelock method exaggerates the damping. The calculated damping would be further increased if ship side inclinations were taken into account.

The shift from agreement to disagreement of the calculated and measured damping in the cases of heaving and pitching oscillations indicates a strong three-dimensional effect. However, application of the three-dimensional corrections developed by Havelock and Vossers (to be discussed in Section 3.23) would make the situation still worse. The pitch-damping curves (at synchronous frequency) would be displaced upwards, while the heave-damping curves would not be affected.

To summarize: Prediction of the damping of a ship's

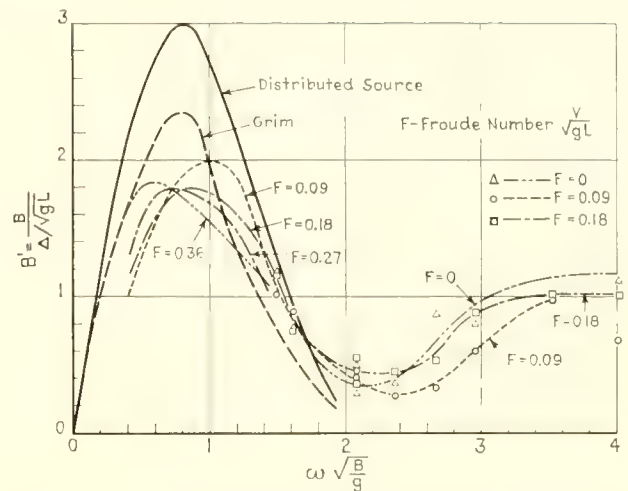


Fig. 14 Variation of damping coefficient with frequency (from Golovato, 1957)

heaving and pitching motions is rather uncertain. The order of magnitude and the functional dependence of the damping on oscillation frequency can be estimated roughly. Neither of the two available methods of calculating, Grim's or Holstein-Havelock's, gives uniformly satisfactory results. Fortunately, one or the other will be preferable in a particular case. Currently available calculations of three-dimensional effects (Section 3.23) do not correct discrepancies but apparently make the situation worse. The most pressing need in the theory of ship motions and ship bending stresses is to develop a reliable method of evaluating the damping characteristics.

Experimental measurements on idealized ship forms may be misleading if used directly as an indication of normal ship behavior. Tests on such models are, however, recommended, but only for comparison with calculated values since more advanced methods of calculation can be used for such mathematically defined ship forms than is possible for normal ship forms (Section 6). It would be desirable to develop mathematical ship lines which would be more like the normal ship form and yet permit application of the advanced calculation methods.

The erratic correlation between computed and measured damping when comparison is made of mathematical and normal ship forms, and of pitching and heaving motions, indicates that the phenomenon is caused by a complexity of conditions. A research program should be directed therefore towards resolving this complex phenomenon into its (not now known) parts and towards subsequent analysis of these component parts. Two general directions of approach can be visualized. In one, tests similar to Golovato's and Gerritsma's would be conducted on a variety of ship forms. Various factors can then be isolated intuitively after inspection of the test data, and the conclusions can be verified subsequently by synthesis of the elemental findings. The other

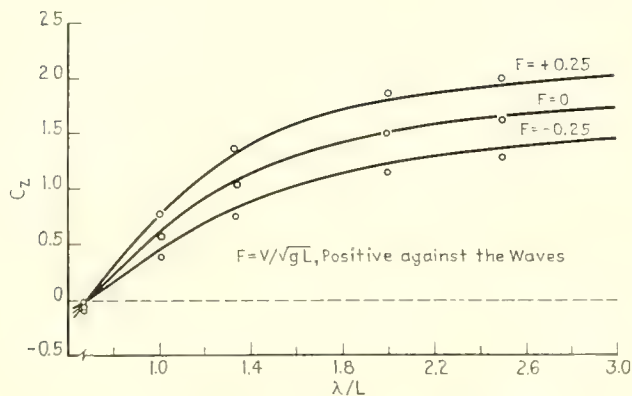


Fig. 15 Maximum heaving-force coefficient for submerged spheroid of $L/D = 5$ in varying wave lengths. Solid curves indicate Havelock's (1954) solution, circles Korvin-Kroukovsky's strip-method solution. $C_z = (\text{heaving force})/\nabla\bar{\theta}$ (from Korvin-Kroukovsky, 1955b)

method calls for tests and analyses of special ship forms designed with the specific purpose of isolating various theoretical features. Under this program only a few tests on normal ship forms are needed. These will indicate the target for the investigation and will provide the final check on the ultimately synthesized information. Under this analytical and experimental program, tests of mathematically defined ship forms will play an important part. The research program should include investigations of prismatic bodies (two-dimensional flow) as well as three-dimensional forms with various types of sections and different distributions of sectional types and areas along the ship length. This subject will be discussed further in Section 6.

It appears that inclination of ship sides at the waterline is one qualitatively evident factor which affects added masses and affects damping characteristics to a greater degree. The physical conditions of this problem were described in Section 3.12. Theoretical and experimental investigations are recommended. These can start with an analysis of the damping of the 60-deg wedge of Dimpker's (1934) experiments.

3.22 Nonlinear effects in evaluating mean damping.

Attention should be called to the fact that definitions of the added masses and damping forces in the foregoing exposition have been based on the assumption of linear differential equations of motion with constant coefficients. In reality, the coefficients are not constant. Even in the case of a wall-sided ship, such as the model used by Golovato, the damping will vary with the instantaneous mean draft of the ship sections; i.e., the factor f in the exponent of equation (22). The analyses of Golovato and Gerritsma give a certain mean value of the damping to be used in a linear theory. This mean value, however, may be affected by the nonlinearity in sectional properties.

The ultimate amplitude ratio \bar{A} is the result of interference of many wave systems which have their origin at different elements of the body area. Such an interfer-

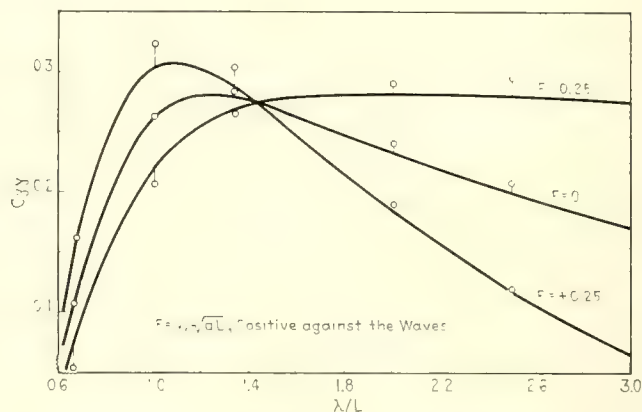


Fig. 16 Maximum pitching-moment coefficient for submerged spheroid (see caption Fig. 15). $C_{yy} = (\text{pitching moment})/L\nabla\bar{\theta}$ (from Korvin-Kroukovsky, 1955b)

ence can be very sensitive to the varying conditions at the generating elements. Not only the ratio \bar{A} for two-dimensional sections but three-dimensional corrections as well can be affected. Havelock's and Vossler's three-dimensional corrections, discussed in Section 3.23, are based on infinitely small displacements and cannot bring out the effects discussed here.

The author believes that it is important to inaugurate research on evaluating the mean damping characteristics of ship sections (two-dimensional flow) taking into account the draft and beam variations during an oscillation cycle. The oscillations themselves can still be assumed to be harmonic.¹⁸

A similar research is suggested for three-dimensional corrections, taking into account the cycle variation of conditions at the ends of a pitching ship.

3.23 Three-dimensional effects. It has been mentioned that the strip theory is approximate in that certain interaction between adjacent sections is neglected. This was realized by F. M. Lewis (1929) and J. Lockwood Taylor (1930) who developed corrective procedures for their field of interest, namely, ship vibrations with a certain defined number of nodal points. These procedures do not appear to be usable for a ship in waves.

In order to verify the accuracy of the strip method of calculations this method has been applied to a submerged spheroid moving under waves (Korvin-Kroukovsky, 1955b), and compared with results obtained by more precise methods used by Havelock (1954) and Cummins (1954). These more precise methods cannot be applied to a normal ship form in waves but can be used for ellipsoids. The results are shown in Figs. 15 and 16. The agreement appears to be satisfactory for all practical purposes. Attention should be called to the fact that in the strip method as used by Korvin-Kroukovsky (1955b, c)

¹⁸ This assumption is justified by the fact that ship motions are represented mathematically by the double integral of forces with respect to time. The motions, therefore, reflect primarily the mean conditions and are not very sensitive to instantaneous force changes.

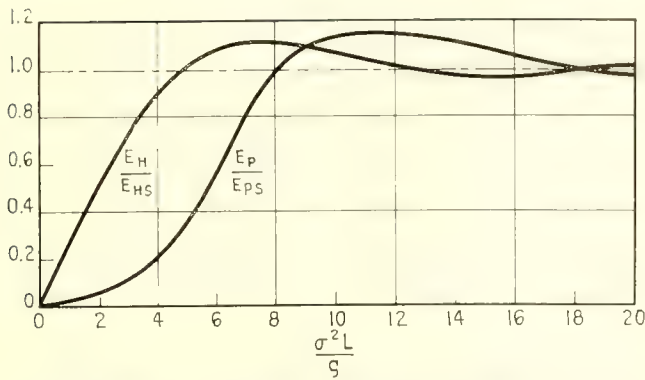


Fig. 17 Ratios of damping coefficients for heaving and for pitching (from Havelock, 1956). Damping coefficients are denoted by E with subscripts H and P for heaving and pitching by three-dimensional theory and HS and PS , respectively, by strip theory

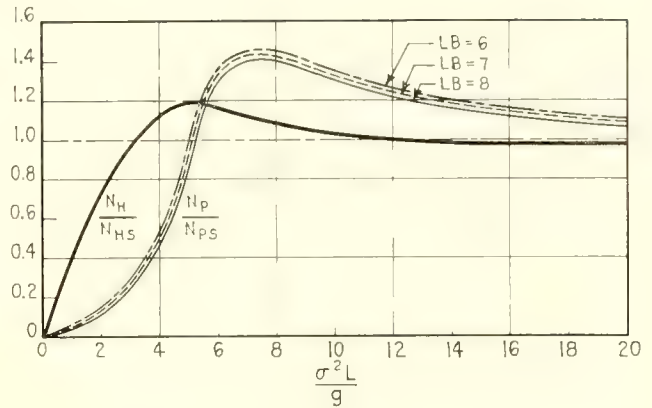


Fig. 18 Ratios of damping coefficients for heaving and for pitching (from Vossers, 1956). Damping coefficients are denoted by N with subscripts H and P for heaving and pitching by three-dimensional theory and HS and PS , respectively, by strip theory

the elements of ship length were taken to be conical, so that in passage of a body through control planes the sectional area was taken as variable and as a function of time, $A = A(t)$. This feature permitted the forward speed of a body to be taken correctly into account, and also made the results applicable to bodies of relatively small length-to-diameter ratio (the length-diameter ratio of 5 was used in the foregoing verification). The calculations confirmed the applicability of the strip theory as far as the inertial body characteristics and the exciting forces caused by waves are concerned. The damping forces were not considered.

Havelock (1956) calculated damping forces by three-dimensional theory for a spheroid of length-diameter ratio of 8 (representative of usual ship proportions) and compared the results with the strip theory. Fig. 17 shows the ratio of three-dimensional to two-dimensional damping (added subscript s) versus the frequency parameter $(\omega^2 L/g)$. Vossers (1956) made similar calculations, generally following Haskind's (1946) methods for a surface ship which was assumed to be "thin" in the sense of using the assumptions formulated by Michell (1898) in the theory of ship wave resistance. The results are shown in Fig. 18.

It has been shown by Korvin-Kroukovsky and Jacobs (1957) that in commercial ships synchronism of pitching or heaving oscillations occurs at a frequency parameter over 10, and in slender fast ships, such as destroyers, at about 18. At these frequency values the correction coefficient for heaving motion is unity, while for pitching it is between 1.1 and 1.2. It has been shown in the preceding section that application of these corrections would increase the discrepancy between calculated and measured damping on the Series 60 model. Further investigation of this question is therefore needed before these corrections are put into practical use.

3.24 Speed effect on damping. Figs. 12 and 13 contain curves of experimental damping coefficients in heaving and pitching for several values of the Froude

number. The small differences between these curves are probably within the experimental accuracy. In Fig. 7 these differences due to speed were neglected and the heave or pitch damping was represented by a single curve for all model speeds. From a practical point of view, the speed effect can be neglected. A more careful examination of the figures reveals a rather weak trend; i.e., at any given frequency the damping decreases slowly with increase of speed. A similar but more pronounced trend can be observed in Fig. 14 for the parabolic ship model within the practical range of abscissae from 0.9 to 1.5. The word "practical" refers to a commercial ship operating in head seas, in which sea condition the heaving and pitching behavior is most critical.

Most of the information found in the literature on the effect of speed on damping may be misleading¹⁹ because of emphasis on frequency ranges outside the practical interest, or because of incompleteness of the conditions yielding the theoretical or experimental conclusions. In a scientific examination of any physical phenomenon it is, of course, often advisable to go beyond the practical limits but in presenting the results of such broad investigations one should indicate the region of practical applicability. Failure to separate the speed effect in damping from the frequency effect, and failure to consider the coupling between heave and pitch in a free-model experiment are the shortcomings most frequently encountered. A free-floating model may be excited in pitching oscillation only. Because of the cross couplings, however, heaving oscillations also develop. The transfer of a part of the exciting energy to heaving motion simulates an exaggerated damping in pitch.

A pronounced dependence of the damping upon speed apparently occurs at a low frequency of oscillation in smooth water. In wave-excited oscillations such a low

¹⁹ The Gerritsma data in Figs. 7, 12, and 13 are excepted.

frequency may occur in a following sea. This condition is not critical for heaving and pitching motions, but may become of interest in an analysis of six-component motion in quartering seas. The interest in this case is indirect and is connected with the effect of bow submersion on yawing and yaw-induced rolling. However, the frequencies of encounter in quartering seas of significant wave lengths are usually a fraction of the resonant frequency of heaving and pitching. Large errors in damping estimates, therefore, have little influence on the motion estimates.

The peculiar behavior of the damping in pitch at low frequencies was determined analytically by Havelock (1958) and was recorded by Golovato (1958) in tests on a parabolic model. Golovato's results are shown on Fig. 19.

3.3 Integration With Respect to Length. An individual ship section of length $d\xi$, at a distance ξ from the origin, has a simple vertical motion both in space and in relation to the rising and falling sea surface. It is subjected to the four kinds of forces generated by water pressures, three of which are caused by ship oscillation (as in smooth water) and the fourth by wave action. The first three are:

Inertial (proportional to acceleration).

Damping (proportional to vertical velocity).

Displacement (proportional to changes of displaced water volume).

The wave-caused forces also can be considered as composed of inertial, damping and displacement components. In the classical wave theory, however, the inertial and displacement forces are interrelated and are more conveniently combined in a single force.

In order to analyze a ship's motion (considered as that of a rigid body) in waves, the sectional forces must be integrated over the ship's length. If the sum of the sectional forces (exclusive of wave forces) is designated as $dF/d\xi$, the integrations have the general form (with reference to Fig. 1):

$$\text{Heaving force} = \int \frac{dF}{d\xi} d\xi \quad (23)$$

$$\text{Pitching moment} = \int \frac{dF}{d\xi} \xi^2 d\xi$$

All integrations are carried out over the ship's length. Since $dF/d\xi$ consists of three components, the integration yields the three coefficients a , b , and c of equation (1). Likewise integration of the wave force yields the coefficient F_0 of equation (1). Two separate equations for heaving and pitching of the form of equation (1), each containing four parameters, are referred to as "uncoupled equations." In reality heaving motion also causes certain pitching moments and pitching motion causes certain heaving forces. These cross-coupling effects are represented by coefficients resulting from an integration of the form:

$$\text{Cross coupling force or moment} = \int \frac{dF}{d\xi} \xi d\xi \quad (24)$$

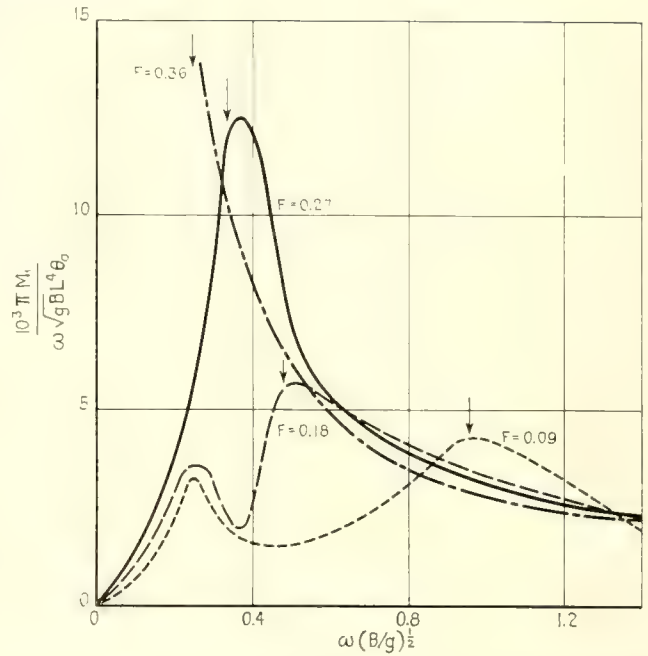


Fig. 19 Damping in pitch versus frequency in region of low frequencies (from Golovato, 1958)

A more detailed exposition of these coefficients and of their use in ship-motion analysis will be found in Korvin-Kroukovsky and Jacobs (1957). It will also be reviewed briefly in Chapter 3. The derivation of sectional forces and their integration with respect to ship's length will be found in Appendix C.

Evaluation in a closed form of the integrals just shown is possible only in special cases of mathematically expressed ship lines. An example of such a procedure can be found in Weinblum and St. Denis (1950). For normal ship forms, these integrations can be carried out either graphically or numerically by methods familiar to naval architects.

4 Forces in Lateral Motion

In the present discussion the term "lateral motion" refers to yawing and side sway. (The rolling motion is to be discussed separately.) As in heaving and pitching, the forces involved in these motions can either be estimated for a body as a whole by comparison with ellipsoids, obtained experimentally for the body as a whole, or computed theoretically by means of the strip theory. In view of the extreme scarcity of the available data, all of these will be treated together in this section.

Data on the force coefficients for the entire ship were given for six models in the work by Davidson and Schiff (1946) as part of a theoretical and experimental investigation of ship stability on course in smooth water. These data have been deduced partly from measurements made on the rotating arm and partly from observation of the turning radii of self-powered, free-running

models in conjunction with measurements of the rudder forces. Similar data were reported more recently by Lewis and Eskigian (1954/55) on additional models in which the skeg areas were varied. The foundations of the test procedure also are more completely covered in this latter work. The difficulty in using such data for practical design purposes lies in the fact that the results cannot be generalized readily, while the number and types of models tested are not sufficiently large to cover all practical requirements. Such empirical data, obtained without an underlying hydrodynamic theory, are particularly difficult to interpret because of the complicated relationship between the hull's contributions and that of the appendages, such as skegs, rudders, and propellers.

In the case of lateral motion the principle of comparison with ellipsoids is identical with that described earlier in connection with motions in the plane of symmetry; i.e., surging, heaving, and pitching. Davidson and Schiff (1946) used the coefficients of accession to inertia as given by Lamb for ellipsoids, but the effects of appendages appear to have been neglected.

An attempt to evaluate the yawing moment caused by oblique waves was made by Davidson (1948) as a part of an investigation of ship broaching in a following sea. Davidson used an intermediate step between study of the body as a whole and the strip theory. Lateral forces resulting from changes in displacement and distribution of the lateral (with respect to ship) components of wave orbital velocities were obtained on a strip basis. The final effect of these on a ship was evaluated by making use of the coefficients obtained on the rotating arm as a part of the previous work of Davidson and Schiff.

An important contribution to the study of lateral forces by means of the strip theory was made recently by Grim (1953a, 1956). Using mathematical definitions of ship sections previously developed by F. M. Lewis (1929), Grim found the solution for inertial and damping forces in motions in lateral directions and in rolling. Landweber (1957) and Landweber and de Macagno (1957) also presented derivations of added masses and added moments of inertia in lateral motions. The results were given in a simple form, suitable for calculation of forces and moments. These appear to be the only sources of information to date on the basis of which the strip theory can be applied to lateral motion. As yet no experimental verification has been made.

In the work just cited the effects of the free surface were taken into account for two asymptotic cases of very low and very high frequencies. It has not yet been possible to evaluate the hydrodynamic forces in lateral motion and in rolling for the intermediate range of frequencies. Grim (1956) considered that the derivation valid for the low frequency is directly applicable to ship motions among waves. Wendel (1950) and Landweber, on the other hand, gave added-mass data for high frequencies, and their data may be considered as directly applicable to the study of ship vibrations.

Further research, directed to evaluation of added

masses and damping in lateral and rolling motions at all frequencies, is needed.

Although Grim's work goes a long way toward supplying the desired information for normal sections, it still is necessary to obtain information on the effect of appendages. It should be noted that hydrodynamic solutions have been obtained successfully in two extreme cases: (a) A ship in heaving and pitching, where the hull can be represented by sources and sinks or doublets; and (b) an airplane wing described by a distribution of vortices. No satisfactory approach has been developed for a mixed condition, such as a ship with appendages in lateral motion. Fedyaevsky and Sobolev (1957) treated a ship in lateral motion by means of a low aspect-ratio wing theory.

The added masses of typical afterbody ship sections with large deadwood or skegs can be estimated by replacing a true section with a polygonal one and applying a Schwartz-Christoffel transformation. Following Grim (1956), the results of such an analysis may be assumed to be valid for low frequencies. Alternately, the added masses of ship sections formed of curved lines in conjunction with deadwood, skegs, or propeller bossings can be obtained by electrical analogy (Section 3.11). Unfortunately, no method of evaluating the damping properties of such sections appears to be available. Theoretical and experimental research in this field is needed.

Well-separated appendages, such as spade-type rudders, or short wing-like skegs (with leading edges) often used to support rudders on multiscrew naval ships, can be treated as separate hydrofoils, with an empirical factor for increase of effective span due to abutment against the hull. Also, an assessment must be made of local water velocities. A certain amount of information on such appendages will be found in Mandel (1953) and Becker and Brock (1958).

5 Forces and Moments in Rolling

The general introduction given in Section 1.1 with regard to pitching and heaving applies to rolling as well. In the simple equation of motion (1), the heaving displacement z is merely replaced by the angle of roll, designated by ϕ , so that it becomes

$$A\ddot{\phi} + B\dot{\phi} + C\phi = L \cos \omega t \quad (25)$$

The forces and moments are caused by changes in water pressure, in turn caused by the angle of roll ϕ , velocity of roll $\dot{\phi}$, and acceleration in roll $\ddot{\phi}$. The resultant moments are correspondingly classified as restoring, damping, and inertial ones, the latter usually expressed in terms of the added or hydrodynamic moment of inertia coefficient k_{rr} . The amplitude of the rolling moment is here designated by L . As in the case of pitching, all forces and moments involved in rolling also can be considered as the sum of forces and moments caused by ship rolling in smooth water and those exerted by waves on a restrained ship. Direct data on the forces and moments in rolling are extremely scarce. Most of the practical

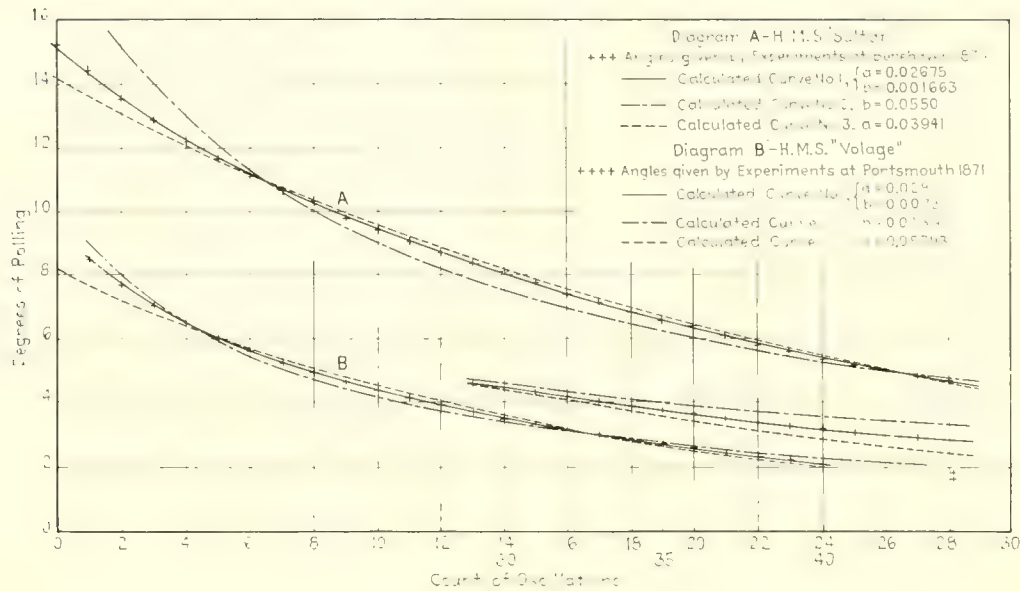


Fig. 20 Curves of declining angles in rolling oscillations of two ships in still water (from W. Froude, 1874)

data is derived from analysis of ship motion which is assumed to be generally expressed by such differential equations as (25). The literature on the subject is divided into: (a) Rolling in still water (i.e., RH term of equation 25 equal to zero), and (b) rolling in waves.

When a ship rolls freely in smooth water, a small heaving motion and side swaying occur. It has been shown, however, (Ueno, 1942; Grim, 1956) that heaving in this case is small, and side sway, although more pronounced, is also small. It is believed permissible, therefore, to treat the motion as a pure rotation about the fore-and-aft axis passing through the center of gravity. This is believed true providing the centers of gravity of all ship sections lie on this axis. If the principal axis of inertia is inclined appreciably to the fore-and-aft axis of a ship, a significant yawing component of motion also will develop.

5.1 Restoring Moment. The restoring moment is defined by the concepts of "metacentric height" \overline{GM} and of "righting arm" \overline{GZ} . These are familiar to naval architects and will not be elaborated on here. The righting moment at small roll angles is

$$L = \Delta \overline{GZ} = \Delta \overline{GM} \sin \phi \quad (26)$$

where the right-hand part of the equality is valid only for constant \overline{GM} . Here Δ is the weight of a ship.

For small angles ϕ , $\sin \phi = \phi$. Substitution of equation (26) with $\sin \phi = \phi$, for $C\phi$ in equation (25) yields a linear differential equation that is simple to solve. The initial height of the metacenter over the center of buoyancy of a ship \overline{BM} can be computed by the expression

$$\overline{BM} = I_w / \nabla \quad (27)$$

where I_w is the second moment of the waterplane about the axis of symmetry, and ∇ is the displaced volume. If the height of the center of gravity over the center of buoyancy \overline{GB} is known, the \overline{GM} is determined. Equation (26) is the basis for experimental verification of \overline{GM} by inclining a ship through small angles. If a known heeling moment L is applied and the resulting heel angle ϕ is measured, \overline{GM} is determined.

For larger heel angles, \overline{GM} does not remain constant and the righting arm \overline{GZ} follows some other law than $\sin \phi$. In order to use the simple form of equation (25), Froude (1861) assumed $\overline{GZ} = \overline{GM} \phi$, and showed that this corresponds with reality for a ship with slight tumblehome sides. This was the usual hull form of the battleship of his day. For a wall-sided ship

$$\overline{GZ} = \sin \phi \left(\overline{GM} + \frac{1}{2} \overline{BM} \tan^2 \phi \right) \quad (28)$$

and in general

$$\overline{GZ} = \overline{GM}_0 \phi + F(\phi) \quad (29)$$

where \overline{GM}_0 is the metacentric height for very small angles ϕ .

5.2 Inertial Moments. Once a metacentric height is known, the effective moment of inertia of a ship in rolling is defined by

$$T = \frac{2\pi k}{(g\overline{GM})^{1/2}} \quad (30)$$

where T is the natural rolling period and k the radius of gyration. Once \overline{GM} is known from static inclining experiments and the natural period of rolling T is observed, k and the virtual moment of inertia ($I = k^2 \Delta / g$) can be

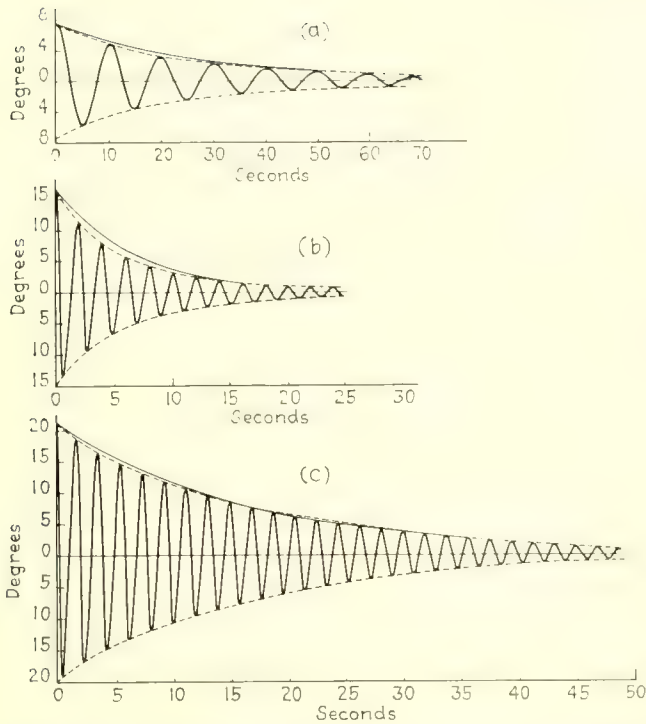


Fig. 21 Curves of declining rolling oscillations in still water for a destroyer at zero speed (from A. M. Robb, "Theory of Naval Architecture," 1952, p. 259). (a) Full-size ship, (b) complete model, (c) naked hull. Linearized envelope curves are shown by solid lines

computed. In general, $I = I_0 + I''$, where I_0 is the moment of inertia of the ship itself and I'' the added or hydrodynamic moment of inertia. Or, if I' is the moment of inertia of the water displaced by a ship, $I'' = k_{xx}I'$. The coefficient of accession to inertia in roll k_{xx} can be estimated roughly by comparison with ellipsoids, or can be computed by the strip theory. Generally, it is small, and Manning (1942, pp. 21, 27) indicated that direct use of equation (30) with ship's radius of gyration k was satisfactory. Grim (1956), considering low oscillation frequency, listed the expressions for computation of the accession to inertia k_{xx} for ship sections defined by F. M. Lewis' (1929) formulas. As an example, for a ship section with a draft 0.4 of the beam and a section coefficient of 0.93, k_{xx} is found to be 0.07. The coefficient of accession to inertia k_{xx} is increased, however, by the addition of appendages, particularly bilge keels.

In the asymptotic case of a high frequency, the rolling of a rectangular ship section of draft/beam ratio 1/2 is equivalent to rotation of a submerged square prism. For this case Wendel (see Figs. 4 and 5) computed $k_{xx} = 0.276$. The addition of bilge keels, 0.083 of the beam in width, gave a computed $k_{xx} = 0.63$ (by interpolation of the data in Fig. 4). T. B. Abell (1916) found experimentally the corresponding values of 0.285 and 0.72 for k_{xx} . The water viscosity effects did not, therefore, change significantly the values of the added masses.

The importance of taking the effect of bilge keels into account has been vividly demonstrated by these studies.

The foregoing data refer to water of infinite depth. The added mass in rolling, as well as in heaving and pitching, is strongly affected by the shallowness of water. Data on the effect of shallowness on rectangular ship sections will be found in Koch's (1933) paper. The depth of water must, therefore, be taken into account in all vibration and rolling experiments.

5.3 Damping Moments. It has been established by W. Froude (1861, 1872, 1874), and confirmed by others, that the value of damping coefficient B for a bare hull depends mostly on dissipation of energy in waves and only to a small extent on viscous forces. In Baumann's (1937) experiments with rolling circular cylinders, it was observed that, once excited, oscillations damped out very slowly. Bilge keels can cause a large dissipation of energy by vorticity, as will be shown in Section 5.33.

The primary source of information on damping to date has been the observations made on the rate of amplitude decay in rolling in smooth-water oscillation. Such experiments were first conducted by W. Froude (1874) and thereafter by many others (for instance, Gawn, 1940; Williams, 1952). Fig. 20 shows the plot of roll amplitude ϕ versus the number n of oscillations as obtained by Froude (1874) for two ships. Froude found that the experimental data were closely fitted by a curve of the type

$$-d\phi/dn = \bar{a}\phi + \bar{b}\phi^2 \quad (31)$$

where the coefficients \bar{a} and \bar{b} were different for various ships and are shown by the legend on the figure.²¹ Also shown are the best fits using only the first (linear) or the second (quadratic) terms *with a suitable adjustment of coefficients*.²² The adjusted coefficient will be designated by \bar{a} . The reasonably good fit by a single linear term should be noted. Froude and many subsequent writers emphasized the nonlinearity of the damping coefficient as shown in equation (31).

5.31 Linear approximation. The linear approximation to the extinction curve ($d\phi/dn = \bar{a}\phi$) is particularly valuable because of its direct connection with the damping coefficient B in the linear differential equation (25). This equation can be rewritten for free oscillations in calm water as

$$\ddot{\phi} + 2\kappa\omega_0\dot{\phi} + \omega_0^2\phi = 0 \quad (32)$$

where

²¹ A bar has been placed over the letters as a reminder that they should not be confused with the coefficients a, A and b, B in the differential equations of motion (1 and 25). The coefficient B in the latter case is defined as a function of equation (31); i.e., $B = B(\phi) = f(d\phi/dn)$. The symbols a and b used by Froude in Fig. 20 correspond to \bar{a} and \bar{b} used in this monograph.

²² This procedure is to be contrasted with the one used by Golovato (1957a,b) who linearized the similarly expressed damping in heaving merely by letting $b = 0$, and not changing the value of \bar{a} . It will be understood that the word "linearized" as used in this monograph implies adjustments of both coefficients so as to give the best approximation to a function in the range of the independent variable of greatest interest in practical problems.

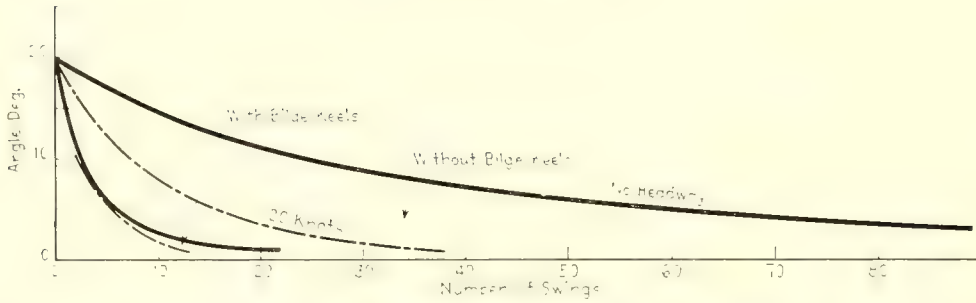


Fig. 22 Effect of forward speed on rolling of model of "Royal Sovereign" class of battleship (from A. M. Robb, "Theory of Naval Architecture," 1952, p. 278)

$$\begin{aligned} B/A &= 2\kappa\omega_0 \\ C/A &= \omega_0^2 = (2\pi/T)^2 \end{aligned}$$

ω_0 = circular frequency of free undamped oscillations

T = period of undamped oscillation.

Damping increases the period of oscillation so that it becomes

$$T_1 = T/(1 - \kappa^2)^{1/2} \quad (33)$$

The well-known solution of equation (32) for a particular case, in which $t = 0$ is taken at the maximum initial inclination ϕ_0 ; i.e., when $d\phi/dt = 0$, is

$$\phi = \phi_0 e^{-\kappa\omega_0 t} \cos \omega_0 t \quad (34)$$

The plot of roll angle ϕ versus time is shown in Fig. 21, taken from Robb (1952, p. 259). It appears to consist of a series of nearly sinusoidal half-cycles, the amplitudes of which diminish as shown by the exponential factor in equation (34). If the rate of amplitude decrease is not too rapid, the tangency points of the envelope curve to the oscillatory curve can be taken approximately at $\cos \omega_0 t = 1$. Substituting the value of ω_0 in terms of the period T and expressing the time in terms of T , $n = t/T$, the equation of the envelope is obtained as

$$\phi = \phi_0 e^{-2\pi\kappa n} \quad (35)$$

On the other hand, W. Froude (1874) shows that decremental equation (31) when linearized, $\bar{a} = \bar{a}$ and $\bar{b} = 0$, corresponds to the equation of the envelope curve

$$n = \frac{1}{2\bar{a}} \log \frac{\phi_0}{\phi} \quad (36)$$

$2\bar{a}$ has been written above instead of Froude's \bar{a} , as Froude recorded n in half-cycles while counting in whole cycles is now customary. Equation (36) also can be expressed as

$$\phi = \phi_0 e^{-2\bar{a}n} \quad (37)$$

From comparison of equations (35) and (37) it follows that

$$\bar{a} = \pi\kappa \quad (38)$$

and the coefficient of damping B in equation (25) is

$$B = A.2\kappa\omega_0 = 4A\bar{a}/T \quad (39)$$

The foregoing has been presented in detail since the decremental equation (31) is widely used in British literature, and most of the practical data on damping of ships in rolling is available as coefficients \bar{a} and \bar{b} . However, descriptions of the relationship of these coefficients to the equations of motion are few and not clear. A good but very brief one was given by Williams (1952). A much more widely used method in general vibratory problems is the "logarithmic decrement" defined as

$$\delta = \log (\phi_1/\phi_2) \quad (40)$$

where ϕ_1 and ϕ_2 are amplitudes of any two succeeding oscillations. The logarithmic decrement is measured by the slope of $\log \phi$ versus the number of cycles of oscillation n . It is related to the nondimensional damping coefficient κ , and to the coefficient \bar{a} of the linearized decremental equation by

$$\delta = 2\bar{a} = 2\pi\kappa \quad (41)$$

5.32 Empirical data on damping in roll. In the current literature on ship motions, strong emphasis is put on nonlinearity of damping, and reported values of the coefficients \bar{a} and \bar{b} are shown to vary widely and irregularly. Their relationship to a ship form is hardly ever discussed. However, if the coefficient \bar{a} of the linearized equation is considered, an idea of the order of magnitude can be established. Assuming, for instance, that the linearized equation must match equation (31) at $\phi = 7.5$ deg,²³ Table 3 can be compiled as an example from readily available data.

The range of fluctuation for the values of \bar{a} in each group shown in Table 3 is approximately 50 per cent from the mean for ships without bilge keels, and 40 per cent for those with bilge keels. This fluctuation is surprisingly small, considering the fact that neither the details of ship forms nor the details of keel and bilge keel constructions have been considered. The powerful effect of bilge keels on rolling is brought out vividly by the table and also by Figs. 21 and 22.

The effect of the forward speed on damping in roll

²³ Corresponding to the mean value for a fairly heavy rolling.

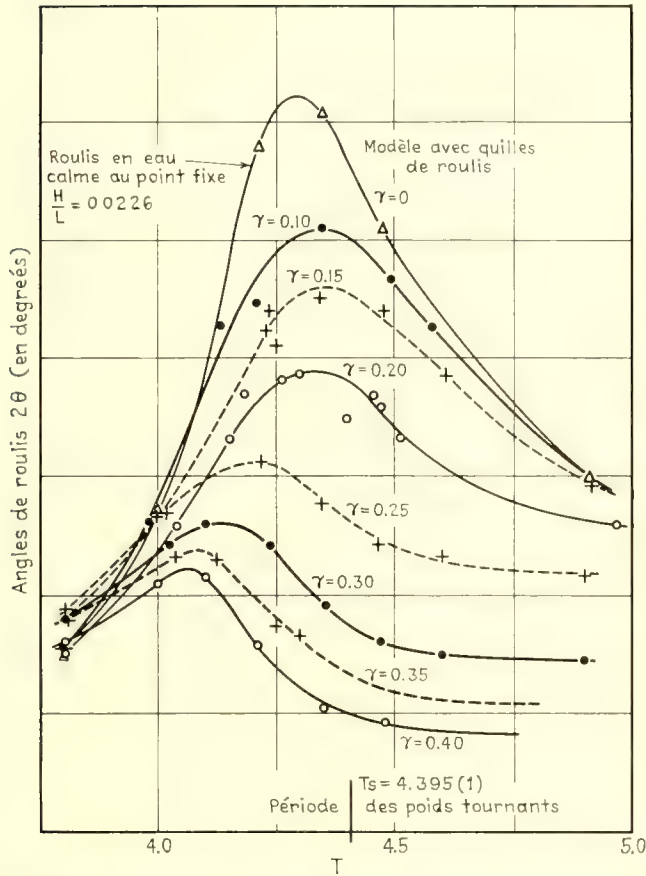


Fig. 23 Effect of forward speed on steady rolling of a ship model (from Brard, 1949). Rolling in smooth water was induced by harmonically oscillating moment approximately equal to moment produced by waves of height/length ratio of 0.0226 at zero speed. Speed parameter $\gamma = 2\pi V/gT$ where V is speed, T period of rolling. $T_s = 4.395$ is natural period for small angles of roll

Table 3 Coefficient \bar{a} of the Linearized Decremental Equation for Several Ships at Zero Forward Speed

Source of information	Ship	\bar{a}	κ
Without bilge keels			
1	<i>Inconstant</i>	0.073	
1	<i>Sultan</i>	0.050	
1	<i>Volage</i>	0.083	
2	<i>Greyhound</i>	0.068	
2	<i>Revenge</i>	0.031	
	Mean	0.061	0.020
With bilge keels			
2	<i>Greyhound</i>	0.367	
2	<i>Revenge</i>	0.242	
2	<i>King George V</i>	0.186	
2	<i>Nubian</i>	0.235	
	Mean	0.258	0.082

1 W. Froude (1874).

2 Robb (1952, p. 271). The data from W. Froude and Gawn (1940).

appears to be considerable. This is discussed briefly by Robb (1952, pp. 275 and 278). Fig. 22, taken from

Robb, shows a curve of declining angles for a model of a *Royal Sovereign* class battleship tested in a towing tank at different speeds. Fig. 23 shows the effect of the forward speed according to Brard (1949).

5.33 Theoretical knowledge of damping in roll.

Theoretical studies to date are very few and their applicability uncertain. The work of Ursell (1948a, b) can be mentioned. Cartwright and Rydill (1957) quoted an expression for damping from an unpublished work of Ursell (1946). By use of the strip theory they calculated the coefficient $\kappa = 0.088$ for the *Discovery II* as compared with the observed $\kappa = 0.0675$. The discrepancy is in fact much larger since the theoretical work of Ursell was based on smooth elliptical sections, while Table 3 shows that the experimental value corresponds (although on the low side) to ships with usual appendages. The value based on Ursell in this case is over four times the value obtained from rolling of ships without bilge keels.

From observations at low speeds at sea, Williams (1952) obtained the corresponding average value of $\bar{a} = 0.275$, i.e., $\kappa = 0.088$ for HMS *Cumberland*. Cartwright and Rydill (1957) have found $\kappa = 0.0675$ for the research ship *Discovery II*.

Ursell (1949a) presented an analytical derivation of the wave amplitude caused by rolling of noncircular cylinders on the water surface. He applied this derivation to cylinders described by a certain transformation which gave an approximately rectangular section with large rounding of corners. The wave amplitude at infinity was found to be, to the first order,

$$0.63k^2\phi(b+d)(b+1.05d)|b-1.26d| \quad (42)$$

where

$$k = \omega^2/g$$

ϕ = amplitude of rolling

b = half-beam

d = draft

$||$ denotes absolute value

An interesting feature of this expression is that wave amplitude (and therefore damping) vanishes when $b = 1.26d$ or roughly $d = 0.40B$, where B is the beam. At this d/B ratio the damping can be obtained only from bilge keels.

When $d = 0.5B$, the amplitude is

$$\frac{27}{40} k^2\phi b^3 \quad (43)$$

In a previous work, Ursell (1948) showed that for a thin vertical plate rolling about the line of intersection with the water surface the wave amplitude is

$$\frac{2}{3} k^2\phi d^3 \quad (44)$$

It was mentioned that the foregoing derivations are valid when the product kb is small. An exact definition of smallness was not given, but generally the equations

can be considered as applicable to rolling but not to vibration of ships.

The brief summary in the foregoing paragraphs is entirely inadequate to illustrate the importance of Ursell's (1949a) paper. The mathematical formulation of the water flow presented in this paper is the fundamental information on the basis of which added mass and damping forces and moments can be derived for many non-circular cylinders.

5.34 Effects of viscosity and frequency. T. B. Abell (1916) made experiments on attenuation of rotary oscillations of a square prism. A 6 x 6-in. prism was suspended vertically in a water tank with ends fitting closely to the tank bottom and to the boards covering the water surface. These boards prevented surface-wave formation, so that the water flow corresponded to that at an infinitely long prism in an infinite fluid. The prism was suspended on three wires, forming a rotary pendulum, and the oscillation frequency was governed by the prism's weight and moment of inertia and also by additional radially disposed masses. Various initial deflections and various frequencies were tried. The rate of attenuation of oscillation was expressed by equation (31), and the resulting coefficients \bar{a} and \bar{b} are shown in Table 4.

Table 4

(From T. B. Abell, 1916)

Period T'	Small amplitudes		Large amplitudes		
	\bar{a}	\bar{b}	\bar{a}	\bar{b}	$\bar{b}T^2$
No keels					
3.35	0.031	0.007	0.0007	0.01664	0.2098
2.94	0.04	0.0091	0.0007	0.02386	0.2062
2.58	0.042	0.0157	0.0007	0.0305	0.2066
2.14	0.054	0.0122	0.0007	0.0436	0.2000
With keels					
3.54	0.017	0.0954	1.196
3.15	0.026	0.1278	1.266
2.82	0.053	0.1584	1.260
2.41	0.083	0.1996	1.106

The attenuation is expressed in degrees per full cycle. The upper half of the table refers to the bare prism, and the lower one to the prism with $1/2$ -in-wide bilge keels at the four corners.

Since the time of Froude (1861, 1872, 1874) it has been understood that the linear term of equation (31) is caused by the energy dissipation in waves and the quadratic term by the effect of viscosity.²⁴ Under the conditions of Abell's experiments, therefore, a zero value for \bar{a} would be expected. No explanation has been found for the existence of the small values of \bar{a} shown in Table 4.

Ideal fluid conditions are approached in the case of the bare prism at small amplitudes and low period, in which case both \bar{a} and \bar{b} are small. The coefficient \bar{b} becomes appreciable at large amplitudes, and is greatly increased by bilge keels.

²⁴ This opinion appears to be contradicted by Watanabe and Inoue (1958).

T. B. Abell called attention to the fact that, with viscous resistance proportional to the square of the velocity, the product $\bar{b}\phi^2T^2$ is constant. For a series of experiments made with constant initial amplitude ϕ_0 , the damping coefficient \bar{b} will vary with frequency, but the product $\bar{b}T^2$ is expected to be constant. This expectation is confirmed by the experimental values listed in the last column of Table 4. This observation of T. B. Abell appears to have escaped the attention of later experimenters, and the coefficients \bar{a} and \bar{b} have usually been reported without regard to the oscillation frequency. For the sake of consistency the values $\bar{b}T^2$ should be reported instead of \bar{b} .

6 Direct Three-Dimensional Solution Including Wavemaking

Apparently the most complete analyses of the hydrodynamic forces and moments acting on a surface ship oscillating in waves have been made by Haskind and Hanaoka. Haskind's (1945a, b; 1946, 1954) work is based on a method initially developed by Katchin (1937, 1940). Attention was concentrated on forces due to body motions, while exciting forces due to waves were not analyzed. Ship-wave formation, resistance, damping and motions in waves were treated by Hanaoka (1951, 1952, 1953), but most of this work has not yet been translated from the Japanese, and therefore is largely inaccessible. At the NSMB Symposium (1957) Hanaoka presented a broad outline of his work in English, but the exposition is too sketchy for complete understanding without reference to his previous, more detailed work. This work includes also calculation of the bending moments acting on a ship's hull.

A valuable exposition of the basic principles of ship-motion analysis was given by Fritz John (1949). Other investigators have not treated the broad problem of ship motions, but have concentrated instead on elucidation of certain aspects of the broad problem. Wigley (1953) and Havelock (1954) determined the forces and moments acting on a submerged ellipsoid moving under waves without oscillations. Havelock (1955) also discussed the coupling of heave and pitch due to speed of advance, in support of the findings of Haskind (1946). Havelock (1956) and Vossers (1956) compared damping computed by three-dimensional theory with that obtained by the strip theory. Havelock (1958) discussed the effect of speed on damping.

6.1 Statement of the Problem. In the approaches taken by the foregoing writers, a frictionless fluid is assumed, so that the velocity potential ϕ exists and satisfies Laplace's equation. This velocity potential is to be evaluated subject to the "boundary conditions." The boundaries are formed by (a) the moving surface of the ship, (b) the wave-covered free surface of water, (c) the ocean bottom, and (d) the infinitely distant parts of the ocean in fore, aft, and lateral directions. The last two are simply expressed. No vertical velocity of water ($\partial\phi/\partial z = 0$) exists at the ocean bottom. At infinite distance all space derivatives (i.e., fluid perturbation

velocities) vanish, i.e. $\partial\phi/\partial x = \partial\phi/\partial y = \partial\phi/\partial z = 0$. The term "perturbation velocities" is used here to designate fluid velocities caused by the presence of a ship.

The boundary condition at the free water surface consists of statements that the air pressure is uniform and that surface particles of water remain on the surface. This is expressed in suitable mathematical form in terms of the velocity potential ϕ .

The boundary condition at the ship's surface consists of the stipulation that water flow is tangential to the surface. Mathematically, this is expressed by the statement that the fluid-velocity component normal to an element of ship surface is equal to the velocity of the surface itself in the direction of its normal. To give a simple illustration, consider a wedge moving with velocity V through initially calm water. Let α be the half-angle of the wedge. The component of the motion of the wedge surface normal to itself is then expressed on the basis of body geometry as $V \sin \alpha$. The fluid velocity induced by the foregoing motion is expressed in terms of the velocity potential by the vectorial sum of the fore-and-aft and lateral components as $(\partial\phi/\partial x) \sin \alpha + (\partial\phi/\partial y) \cos \alpha$. The boundary condition is then

$$\frac{\partial\phi}{\partial x} \sin \alpha + \frac{\partial\phi}{\partial y} \cos \alpha - V \sin \alpha = 0 \quad (44)$$

As a somewhat more complicated example, consider a surface element of a normal ship moving forward without oscillations. This can be described by the angle α of this element to the fore-and-aft x -axis, and by the angle β of the rotation of this, initially imagined as vertical, element about the x -axis. The boundary condition then becomes

$$\begin{aligned} \frac{\partial\phi}{\partial x} \sin \alpha + \frac{\partial\phi}{\partial y} \cos \alpha \cos \beta \\ + \frac{\partial\phi}{\partial z} \cos \alpha \sin \beta - V \sin \alpha = 0 \end{aligned} \quad (45)$$

The next step is to introduce oscillations of a ship usually described by six components—three translational ones along the x , y , and z -axes, known as surging, side swaying, and heaving, and three rotational ones, rolling, pitching, and yawing. Haskind (1946) wrote, in vector notation,

$$U_n(M, t) = U \cdot n + \Omega \times r \cdot n \quad (46)$$

where U_n is the normal velocity of the surface element as a function of the position of the element M and time t . U is the resultant velocity vector of all translational body motion, i.e.,

$$U = U_1 i + U_2 j + U_3 k$$

and Ω is the resultant angular velocity vector of the body rotations about three axes, i.e.,

$$\Omega = U_4 i + U_5 j + U_6 k$$

i , j , k , and n are the unit vectors along the co-ordinate axes and normal to the ship surface S , respectively. $r = OM$ is the radius vector from the center of gravity of

the ship to the surface element M on the body surface S .

Expression (46) thus is a geometrical description of the normal velocity of an element of body surface just as $V \sin \alpha$ was in the first example of a simple wedge. In the expression for the normal fluid velocity in terms of the velocity potential, it is convenient to consider the total potential Φ_0 as composed of two parts:

$$\Phi_0(x, y, z, t) = \Phi(x, y, z, t) + \Phi^*(x, y, z, t), \quad (47)$$

where Φ^* is the known potential of the simple harmonic wave train, and Φ is an as yet unknown potential due to the presence of the body and its motions. The boundary condition is then expressed as

$$\partial\Phi/\partial n = U_n(M, t) - \partial\Phi^*/\partial n \quad (48)$$

The forced oscillations of a floating body which are established after damping the free oscillations can be taken as

$$U = U^* e^{i\omega t}; \quad \Omega = \Omega^* e^{i\omega t}; \quad U_n = u_n e^{i\omega t} \quad (n = 1, 2 \dots 6)$$

and the time-dependent potential Φ can correspondingly be taken as

$$\Phi(x, y, z, t) = \phi(x, y, z, t) e^{i\omega t}, \quad (49)$$

where ω is the circular frequency.

6.2 Evaluation of the Velocity Potential ϕ . Evaluation of the function ϕ is a difficult mathematical problem for the solution of which the reader is referred to the works listed under the authors mentioned previously. Evaluation of the function ϕ apparently has been possible in only two applications; i.e., to submerged ellipsoids and to "thin" or "Michell" surface ships.

Wigley (1953) and Havelock (1954, 1956) have concentrated on analysis of forces acting on submerged ellipsoids moving under waves. The results of their work can be applied directly to submarines and torpedoes, and are qualitatively indicative of what can be expected in the case of surface ships.

Haskind (1946) first developed various basic expressions for wave formations and for hydrodynamic forces and moments acting on an oscillating surface ship in general form. These are applicable to any body form and to any mode of motion. The expressions take simple form in terms of a certain function H originally defined by Kotchin (1940). The H -function for a given frequency depends on a body's form and is given as a double integral taken over the surface of a ship. This function generally is prohibitively complicated, but is simplified and becomes tractable for either ellipsoids or Michell ships.

The term "Michell ship" designates a mathematical model of a ship for which theoretical computations become tractable after making simplifying assumptions formulated by Michell (1898) in his theory of ship-wave resistance. In the first of these assumptions a ship is considered as having small beam/length and beam/draft ratios so that it is possible to assume that angles α and β in equation (45) are small. Thus, $\sin \alpha = \tan \alpha = dy/dx$, where y is the half-breadth at the water sur-

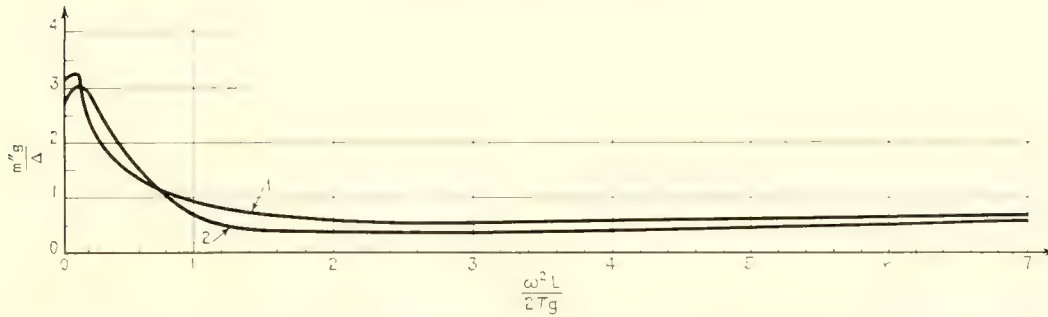


Fig. 26 Ratio of added mass to mass of ship model: 1—experimental curve, 2—theoretical curve (from Haskind and Riman, 1946)

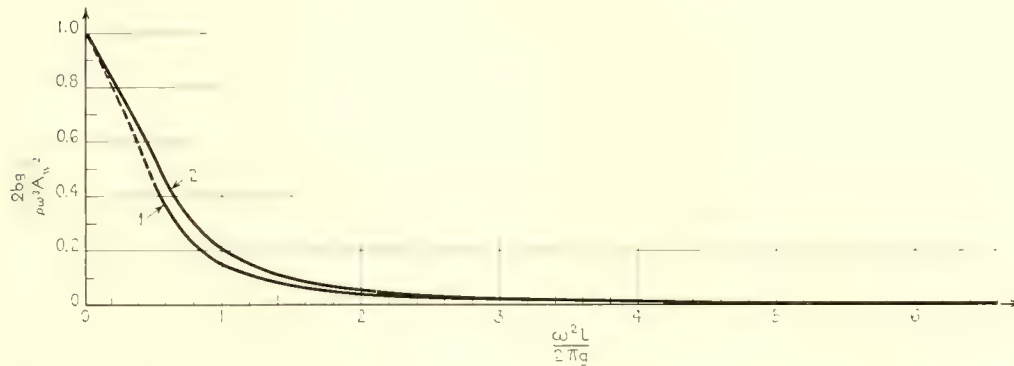


Fig. 27 Variation of damping coefficient with frequency: 1—experimental curve, 2—theoretical curve (from Haskind and Riman, 1946)

computed functional relationship between damping coefficient and ship speed was similar to the measured one. The computed values were, however, much larger than the experimental. Newmann's (1958) report is, however, a preliminary one and the subject is being investigated further.

The assumption that angle α is small corresponds closely to reality in ships of low prismatic coefficient. However, the assumption that angle β is small implies a small beam/draft ratio and a large angle of deadrise which are usually not found in ships. On the other hand, the theory of wave resistance developed in numerous publications of Havelock, Guilloton, Weinblum and Wigley, and the ship-motion theories of Haskind and Hanaka give reasonably good results despite this discrepancy. This is explained by the fact that the parts of a ship's surface nearest the free water surface are most significant in causing waves, while deeply submerged parts have less effect. In all cases where comparisons of theoretical and experimentally measured wave resistance were made, the idealized ship lines were such that a ship's sides were tangent to the vertical at LWL; i.e., the angle β approached zero. A very large angle β , which is not compatible with Michell's assumption, occurred only at the bottom of a ship's surface where elements have relatively small effect.

6.3 Experimental Verification. An idealized ship model was also used in the experimental verification of Haskind's theory by Haskind and Riman (1946). The verification was limited to heaving motion. The ship lines, shown in Fig. 24, are defined by the equation

$$y = \pm \frac{B}{2} X(x)Z(z) \quad (50)$$

where $X(x)$ is a function giving the shape at the waterline, and is taken as a fourth-degree parabola

$$X(x) = 1 - \left(\frac{2x}{L}\right)^4 \quad (51)$$

x and y are distances from the midship section and plane of symmetry, respectively. L is the length of the ship.

The function $Z(z)$ gives the shape of cross sections and is defined by equation

$$Z(z) = 1 - \left(\frac{-z}{d - \delta d}\right)^3 \quad (52)$$

where d is the draft and δd designates the extent of the vertical ship side below LWL as shown in Fig. 24, where all dimensions are in millimeters. The model used was 2000 mm (6.56 ft) long, 250 mm (0.82 ft) wide and had 135 mm (0.44 ft) draft. The model was restrained from pitching and was oscillated in heaving by means of a

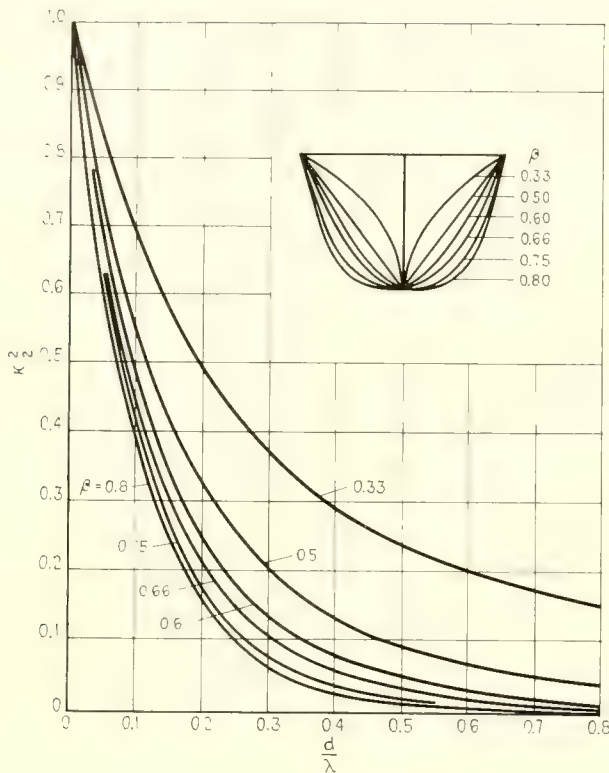


Fig. 28 Variation of correction coefficient for damping in heave in short waves, $k_z^2(\lambda)$ with ratio of draft/wave length with section coefficient, β , as parameter (from Haskind, 1946)

motor-driven scotch yoke and a spring. The theory of the test and the method of computing added masses and damping forces were described clearly by Haskind and Rimani (1946). The measured dependence of these on the circular frequency ω are shown in Fig. 25. Here m'' denotes the added mass and b the coefficient of damping force, corresponding to b in equation (1). Comparison of the calculated and experimental values is given in Figs. 26 and 27. Here the ordinate $m''g/\Delta$ is the coefficient of accession to inertia, where Δ is the displacement. The ordinate in Fig. 27 is the ratio of the damping b at a given frequency ω to the asymptotic value \tilde{b} for very long waves; i.e., small values of ω , which Haskind defined theoretically by the equation

$$\tilde{b} = \frac{\rho}{2} \frac{\omega^3}{g} A_w^2 \quad (53)$$

where A_w is the area of the waterplane. The experiments were made at zero forward velocity.

Superficially, the curves in Figs. 26 and 27 show agreement. The general trend of variation over a wide range of frequencies ω is well represented. This wide range, however, is of little practical interest. For a ship of the *Mariner* type in head seas the parameter $\omega^2 L/2\pi g$ will range from 0.6 to 2.5, with the value of 1.8 approximately corresponding to the synchronous condition. At this value of abscissa the calculated damping coefficient ap-

pears to be some 40 per cent above the experimentally measured one. The error in the added mass appears to be approximately 80 per cent.

6.4 Computations of Haskind. While the formulations of Haskind's (1946) theory are applicable to any form of oscillatory ship motion, the actual application was limited to coupled pitching and heaving. Haskind applied his method to a ship with lines defined by equation (50). The $Z(z)$ -function is defined in this case as

$$Z(z) = 1 - \left(\frac{-z}{d}\right)^n \quad (54)$$

where z is the depth below LWL and d is the draft. The index n is related to the section coefficient β by

$$\beta = \frac{n}{n+1} \quad (55)$$

The forms of ship sections corresponding to various values of β are shown in the insert in Fig. 28. (In particular, $n = 1$, $\beta = 0.5$ represents a straight-sided V-section; $n = 0.5$, $\beta = 0.33$ represents a concave-sided V.)

Attention should be called to the fact that use of equation (50) gives ship lines with the same value of the index n , or section coefficient β , throughout the ship's length. The insert in Fig. 28 should not be confused, therefore, with a conventional body plan.

Haskind (1946) expressed damping in a pure heaving motion at zero speed in the form

$$b = \tilde{b} \cdot \kappa_z^2 \left(\frac{d}{\lambda}\right) \cdot \kappa_z \left(\frac{L}{\lambda}\right), \quad (56)$$

where \tilde{b} is the damping due to very long waves given by equation (53). $\kappa_z^2(d/\lambda)$ is the correction coefficient for the wave length as compared with the draft d of a ship; values of it are given in Fig. 28. The L/λ ratios most important in ship operation in head seas are from 0.75 to 1.5. It is suggested that the reader concentrate his attention on the corresponding range of d/λ of 0.05 to 0.08. The rapid increase of damping with decrease of section coefficient shown in Fig. 28 is noteworthy.

The correction coefficient $\kappa_z(L/\lambda)$ for wave length as compared with ship length depends on the form of the waterlines. Haskind made computations for water lines defined by the equations

$$X(x) = 1 \text{ for } 0 < x < \gamma l \\ X(x) = \frac{1}{1 - \gamma^m} \left[1 - \left(\frac{x}{l}\right)^m \right] \text{ for } \gamma l \leq x \leq l, \quad (57)$$

where l is the half length $L/2$, and γ is the proportion of parallel section. The coefficients γ and m define the fullness coefficient α of the waterline

$$\alpha = \frac{m}{1+m} \frac{1 - \gamma^{1+m}}{1 + \gamma^m} \quad (58)$$

The values of the coefficients $\kappa_z(L/\lambda)$ for the case $\gamma = 0$ are reproduced in Fig. 29. Again it should be emphasized that the range of L/λ values is much wider than is needed, since the practical range of L/λ is within the limits 0.5 to 2.0.

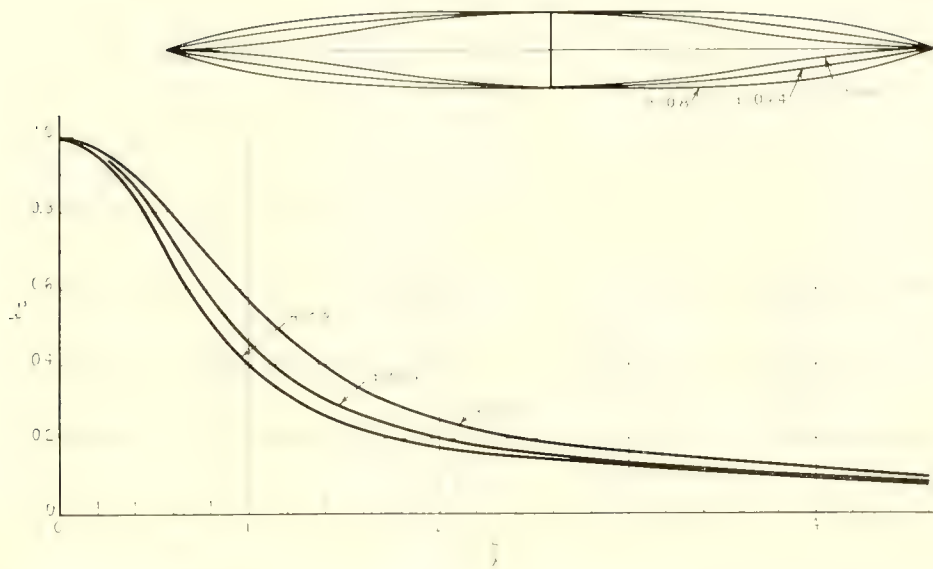


Fig. 29 Variation of correction coefficient for damping in heave in short waves, $k_z(L/\lambda)$, with ratio ship length/wave length, with waterplane fullness coefficient, α , as parameter (from Haskind, 1946)

Damping in pitching was given by Haskind as

$$B = \bar{B} \cdot \kappa_z^2 \cdot \kappa_\theta \quad (59)$$

Damping in long waves, \bar{B} , is given by him as

$$\bar{B} = \frac{1}{4} \rho \frac{\omega^7}{g^3} J_y^2, \quad (60)$$

where J_y is the moment of inertia of the water plane about midship (i.e., about the y -axis). Values of the coefficient κ_θ are given in Fig. 30. Haskind also gave the curves of variation of damping with the relative length of the parallel section γ .

These Haskind computations are valid for pure heaving and pure pitching at zero speed. It appears that within the frequency range of practical interest, damping depends but little on the speed of advance *per se*, but primarily on the frequency of wave encounter.²⁶ In a coupled pitching and heaving motion, as distinct from pure heaving and pitching, the apparent damping coefficient includes additional terms dependent on both speed of advance and added masses. These terms occurred in Haskind's equations of coupled motion, and were discussed in Havelock's 1955 work where he refers to them as "dynamic" coupling terms. These terms also were discussed by Korvin-Kroukovsky and Jacobs (1957) and in Appendix C.

6.5 Concluding Remarks. It appears that the work of Haskind is the only comprehensive one covering the subject of hydrodynamic forces resulting from ship oscillations in waves, although the wave-exciting forces are not evaluated. The only comparable work (and apparently further extended) is that of Hanaoka, but this is not yet

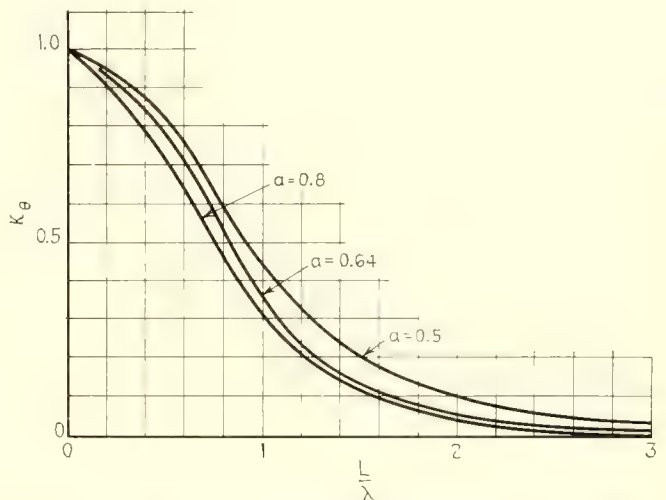


Fig. 30 Variation of correction factor, k_θ , for damping in pitch in short waves (from Haskind, 1946)

adequately available in English. In actual application Hanaoka, as well as Haskind, used the idealized parabolic type of ship to which Michell's assumptions reasonably apply. Understanding the basic derivations of Haskind requires a thorough knowledge of mathematical physics, but his final results are expressed in relatively simple form.

Agreement of calculated and experimental values of added masses and damping forces in the region of synchronism is much poorer in this advanced treatment²⁷

²⁶ See Section 3.24.

²⁷ Subject to Michell's assumptions.

than in the case of the simpler strip method (see concluding paragraph of Section 6.3). The primary value of the advanced method lies in indicating general trends which are not seen as easily in the strip method.

One of the most important needs when using the strip method (two-dimensional flow) is establishment of corrections to account for the actual three-dimensional flow. A beginning in this direction was made by Havelock (1956) and Vossers (1956) as shown by Figs. 17 and 18. Havelock treated a submerged spheroid. Vossers' treatment applied to a Michell ship and was made on the basis of Haskind's methods, but no details of the process or the exact ship model used have been published. Correction coefficients were given for the entire ship. It is also necessary to evaluate a correction for each strip for calculation of the load distribution. Presumably this evaluation can be obtained by differentiating Haskind's expressions for forces and moments with respect to ship length.

Haskind treated ships with affine sections. It would be desirable to develop a similar treatment for ships with full convex form amidships changing gradually to wedge and hollow sections at the ends. This would correspond to normal practical ships. Conceivably correction factors for conversion to three-dimensional flow would be different for such forms from those obtained by Havelock and Vossers for bodies with affine sections.

Haskind and Riman experimented with a model, Fig. 24, wall-sided at the LWL, for which Michell's assumptions appear to be admissible. It would be desirable to repeat the experiments for a model with V and concave sections, thus distinctly violating Michell's assumptions at the load waterline. Such sections are common at the stern of actual ships and appear at the bow of Maier-form ships.

The projects outlined would make direct use of Haskind's material and do not call for advanced mathematical knowledge. A project on a higher level would be to find a mathematical ship form for which the Ketchin-Haskind H -function can be evaluated without resorting to Michell's assumptions. This apparently is a straightforward process for a submerged spheroid, and conceivably a suitable mathematical definition could be formulated for a surface ship which would bear a reasonable resemblance to a normal practical form.

The general derivations of Haskind have been applied only to "longitudinal" or "symmetrical" motion of surging, heaving and pitching. The material also can be applied to combined rolling, yawing and side swaying.

7 Forces Caused by Slamming

As a definition of slamming the following quotations from J. L. Kent (1949) can be used:

"It is not an uncommon experience for ships to 'slam' when labouring in a seaway. By slamming is meant the series of blows delivered by the sea to the ship's structure at irregular intervals and generally at the forward end of the vessel. Each 'slam' causes a shudder to run

through the ship, followed by a rapid vibration of the hull structure. If the magnitudes of these blows are large, serious structural damage may occur and even if the blows are small, the hull will be weakened by fatigue, if slamming is frequent.

"Damage to the hull structure by slamming has occurred at the forward end of the ship only, in all cases within the knowledge of the author. This damage was situated between 1/9th and 1/10th of the ship's length from the forward perpendicular and occurred to the bottom plating and floors a little to one side or the other of the vertical keel. Damage to the vertical keel was not shown, probably because of the great strength of this ship's girder.

"When slamming becomes severe, the experienced shipmaster invariably reduces speed, which immediately eases or stops slamming. The author has never heard of a ship slamming when drifting unpropelled in a seaway. It would, therefore, appear that the hydrodynamic pressures on the hull, brought into existence by the ship's forward motion through the water, play an important part in creating slamming forces.

"It was the author's experience that loaded vessels did not slam with the same persistence or force as when the same ships were in the light condition; and it is believed that this is the general experience of shipmasters.

"It would appear that the ship and sea conditions conducive to heavy slamming are:

- (1) Maximum heaving into and out of the water
- (2) Considerable height of ocean waves
- (3) Light draught
- (4) High ship speed
- (5) Shape of ship form forward
- (6) Irregular sea; i. e., two or more swells occurring at the same time.

(7) The blows delivered to the ship's hull will probably occur between 1/8th and 1/10th of the ship's length aft of the stem."

In this section attention will be concentrated on the generation of hydrodynamic pressures and forces in the process of slamming. Ship motions leading to slamming will be discussed in Chapter 3. Towing-tank experiments as well as observations on ships at sea have shown that the force in slamming is of the nature of an impulse manifested by a high value of acceleration with a very short duration. Only a small amount of the ship's kinetic energy is absorbed, and recorded traces of heaving and pitching motions continue without perceptible change. Oscillations initially excited by a slam are superposed over the strain records in ship bending, which are continuous in the mean. These oscillations appear to have the natural two-mode vibration frequency of a ship. On a typical cargo ship at light draft the vibrations persist through 30 to 60 cycles, or between 15 and 30 sec. They are thus superposed over several cycles of the primary bending stress, which have periods of the order of 6 sec.

7.1 Expanding Plate Theory in Landing Impact of Seaplanes. While hydrodynamic shocks can occur at the

bow of a fast ship without forefoot emergence, severe shocks (slams) to commercial vessels, and heavy pressures, occur as the bottom strikes water on its downward motion after emergence. This is similar to impacts sustained when seaplanes land. As a consequence the principles developed in the seaplane field can be used with only a few modifications.

Seaplanes generally have V-shaped bottoms in order to alleviate landing shocks. The sections are often modified by giving them curvature. Quantitative evaluation of the impact forces is based on two theories which, for brevity, will be referred to as the "expanding-plate" and the "spray-root" theories.

The first originated with von Kármán (1929) and was given its full development by Wagner (1931). In it the water flow pattern at the wedge penetrating the water surface is taken to be comparable at each instant with the flow about a flat plate of the same width as the wetted width of the wedge. This is shown in Fig. 31. On the basis of this analogy, the velocities and accelerations, the resultant pressures, and the total water reaction are obtained. This theory has been used by Mayo (1945), Benscotter (1947), and Milwitzky (1948) for the analysis of the entire sequence of seaplane impact; i.e., in defining the wedge penetration, velocity and acceleration during impact as functions of time.

Were the water surface not disturbed, the wetted semi-breadth²⁸ C_0 would be connected with the instantaneous draft z by

$$C_0 = z/\tan \beta, \quad (61)$$

where β is the angle of deadrise. However, the water on the sides of a wedge rises as the wedge displacement increases. Wagner (1931) found, on the basis of the expanding-plate analogy, that the actual wetted semi-breadth is

$$C = \pi C_0/2 \quad (62)$$

Likewise, if the vertical velocity of the falling wedge $\dot{z} = v_0$, the rate of propagation \dot{C} at the edge of a wetted area in the horizontal direction is

$$\dot{C} = \frac{\pi}{2} v_0/\tan \beta \quad (63)$$

Thus, at small angles of deadrise β , the horizontal velocity of the point S at which the water meets a body surface is very large, Fig. 31. The local pressure at the point is approximately

$$p_s = \rho \dot{C}^2/2 = \frac{1}{2} \rho \left(\frac{\pi}{2} v_0/\tan \beta \right)^2 \quad (64)$$

This simple expression shows that the peak of the impact pressure increases rapidly in magnitude with decrease of the angle of deadrise β .

The foregoing relationships have been given for a simple V-wedge with straight sides. Wagner (1931) has shown that, if another body form can be represented

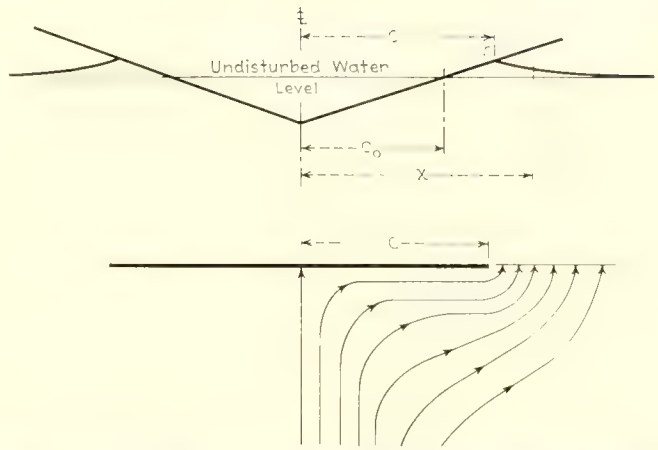


Fig. 31 Representation of water flow pattern induced by falling wedge by flow about lower half of a flat plate of half-breadth C equal to instantaneous wetted half-breadth C of wedge (following Wagner, 1931)

by the following series

$$y = B_0x + B_1x^2 + B_2x^3 + \dots + B_nx^{n+1} \quad (65)$$

the corresponding ratio v_0/\dot{C} becomes

$$v_0/\dot{C} = \frac{2}{\pi} B_0 + B_1C + \frac{1}{\pi} B_2C^2 + \frac{3}{2} B_3C^3 + \dots \quad (66)$$

M. A. Todd (1954) and Bledsoe (1956) used this principle in evaluating the impact pressures on ship bottoms. Small initial deadrise and a sharp turn of the bilge necessitated use of a few terms with high powers.

7.2 Spray Root Theory. The expanding-plate theory correctly indicates the rise of the water surface on the sides of a wedge due to pressures generated in water. It also shows that maximum pressure occurs near the edge of the wetted area and indicates with good approximation the total force exerted by water on a plate. It does not, however, describe in detail the local water flow phenomena occurring at the edge of the wetted area. These phenomena have been described by the "spray root" theory of Wagner (1932). Existence of high pressures near the edge of a wetted area is connected with formation of the spray jet in which water is quickly accelerated to a high velocity.

The complete mathematical solution of the time-dependent flow about two sides of a penetrating wedge with two spray roots, i.e., two regions of spray generation, has not been possible. Wagner (1932) treated three substitute steady-state flows about flat plates. These are valuable in describing local details of the water flow in the spray root and in giving the resultant pressure distribution over a flat planing surface. Furthermore, a numerical solution has been given in the case of a penetrating wedge with two spray roots (J. D. Pierson, 1950, 1951).

The work of Wagner was written in German and is difficult to understand because of extreme condensation

²⁸ The notation often used in aeronautical literature is retained in this section.

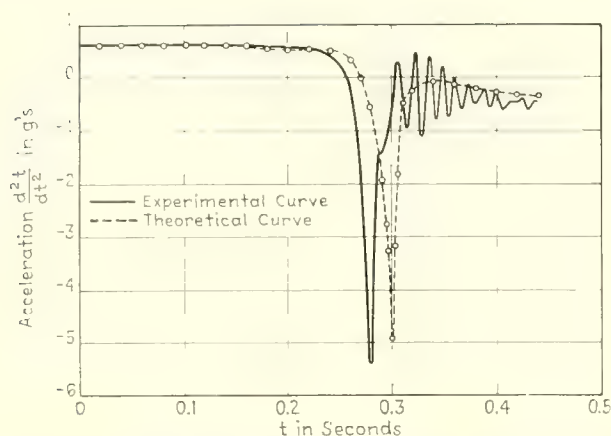


Fig. 32 Theoretical and experimental linear acceleration curves for 7.8-ft model of M/S San Francisco when rotating about a fixed point (from M. A. Todd, 1954)

of the mathematical analysis and omission of intermediate steps in the development. It has been presented in English and expanded by Pierson and Leshnover (1948) and J. D. Pierson (1950, 1951).

7.3 Adaptation of Seaplane Impact Theories to Ship Slamming. Seaplane impact theories have been adapted to the problems of ship slamming by Szebehely (1952, 1954), Szebehely, Todd and Lum (1954) and Szebehely and Todd (1955). An extensive bibliography on slamming was prepared by Szebehely (1954).

The problem of slamming can be divided into two distinct parts; i.e., the study of ship motions leading to slamming, and the generation of pressures and forces in the process of slamming. Both are treated by Szebehely and Todd (1955), but only pressures and forces will be treated in this chapter, while motions will be dealt with in the next one.

Neither the mathematical theories nor the experimental data were adequate at the time to permit Szebehely to correlate actual ship slamming in waves with theory. Therefore, it was necessary to devise artificial experiments which would be amenable to mathematical analysis. The first of these by Szebehely and Brooks (1952) consisted of dropping a ship model onto water, while maintaining it in a horizontal position. The second, a more realistic one, was devised and analyzed by M. A. Todd (1954). The experiments were performed on a model of the MS *San Francisco* (7.8 ft long BP, 12.9 in. beam.) Tests were made at several drafts and were correlated with calculations at a very shallow draft of 1.09 in. The model was pivoted at a point 2.3 ft aft of amidships. The experiment was performed in smooth water by lifting the bow until a 12.2-deg angle of trim was obtained and then letting it drop freely onto the water. Accelerations were read from an accelerometer placed 10.5 in. aft of the forward perpendicular at the model bottom. Equations of model motion were developed using Wagner's principle of series expansion to define the hull cross sections and the resultant pressures

and forces. Excellent agreement between experiment and calculations was secured as shown in Fig. 32.

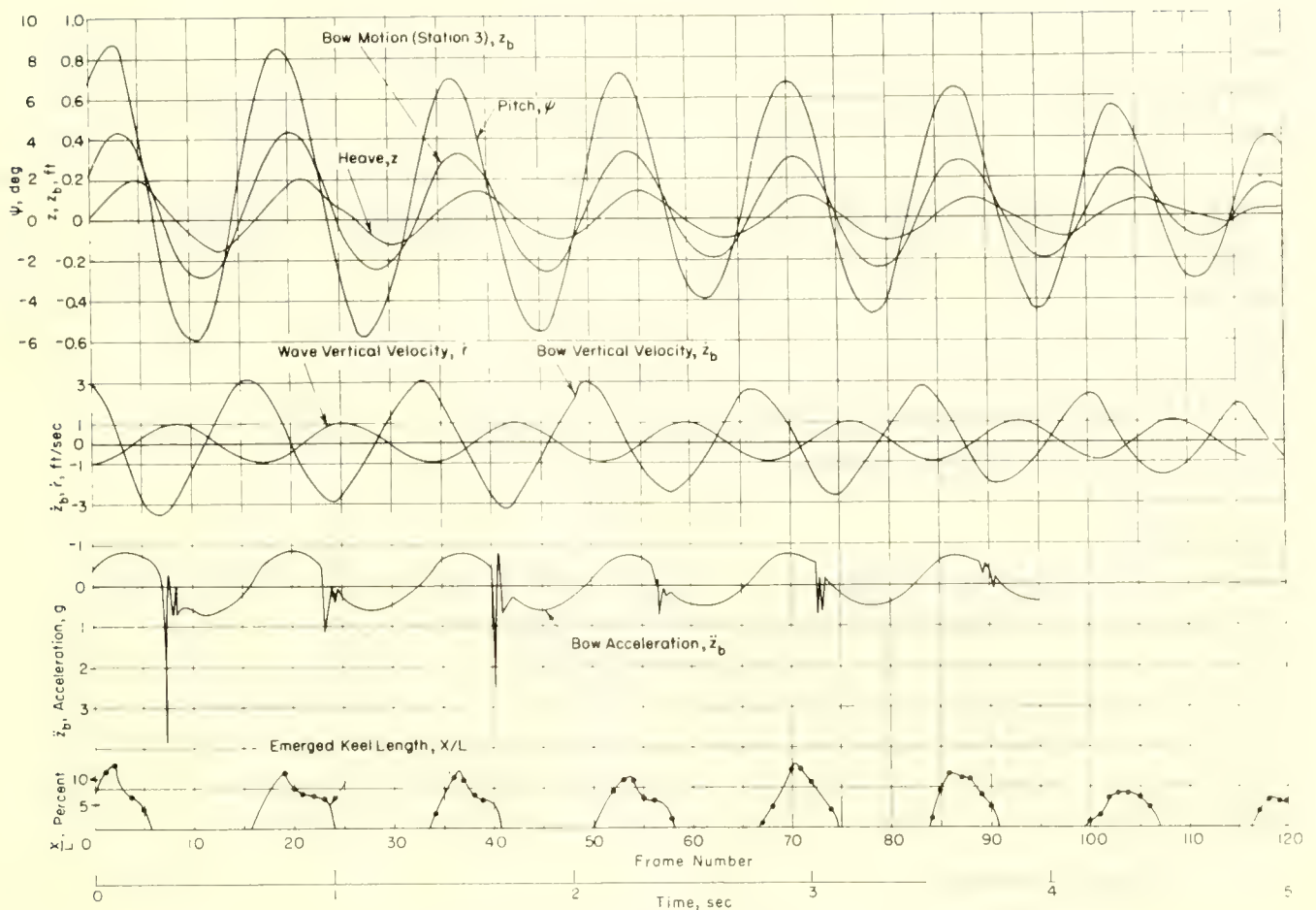
In view of the success of this correlation, a series of theoretical computations of slamming pressures was made by Bledsoe (1956) for the David Taylor Model Basin Series 60 hulls in order to establish the effect of ship fullness, or block coefficient, on slamming pressures. The results of the calculations agree with practical ship observations in that, other conditions being equal, slamming pressures increase rapidly with ship fullness.

While these artificial experiments were necessary for verification of the ship-slamming theory, other experiments were necessary in order to investigate the natural conditions under which slamming occurs. Szebehely and Lum (1955) attacked this problem by testing a model of the *Liberty Ship* in head seas. Kent (1949) previously had concluded that slamming does not occur in regular seas and that it was necessary to create a complex sea containing several wave lengths in order to make the model slam. Since facilities were not yet available for making reproducible irregular seas, Szebehely and Lum obtained slamming in regular waves by using steep regular waves with a length-to-height ratio of 16.7.

Results of two tests with towing forces of 1.2 and 0.8 lb are given in Figs. 33 and 34. The most important observation is that slamming, indicated by the disturbance of the acceleration curve, occurs at the instant when the descending bow position is nearly level. Also, the curves of displacements or velocities are not sensibly affected by the occurrence of slamming. This is important since it permits definition of the conditions leading to the slam by means of ship-motion calculations made without regard to the slam itself. The severity of slamming depends on the relative vertical velocity of a ship's bow with respect to the wave surface, which, in turn, depends on the phase relationship between the wave and ship-bow motions. At the lower speed resulting from the 0.8-lb towing weight in Fig. 34, the wave vertical velocity at the instant of slamming is either nil or slightly downward; i.e., deductive from the bow velocity. The resultant slams are very light. When speed is increased by using a 1.2-lb towing weight as shown in Fig. 33, the phase relationships change so that at the instant of slam an appreciable upward velocity of the wave surface is added to the high downward velocity of the ship bow.

These two tests bring out clearly the fact, well known in practice, that slamming can generally be eased by a reduction of speed. While the tests were made in abnormally high regular waves, the results appear to agree qualitatively with actual observations in a complex sea, as well as with E. V. Lewis' (1954) towing-tank tests in irregular waves shown in Fig. 35.

The results of Szebehely and Lum (1955) have been confirmed by tests made by Akita and Ochi (1955). In the latter case, sustained and severe slamming was obtained in regular waves in a towing tank by using a flat-bottomed ship model with full ends in plan view at an extremely shallow draft. An example of the test results is given in Fig. 36 where the sequence of events is



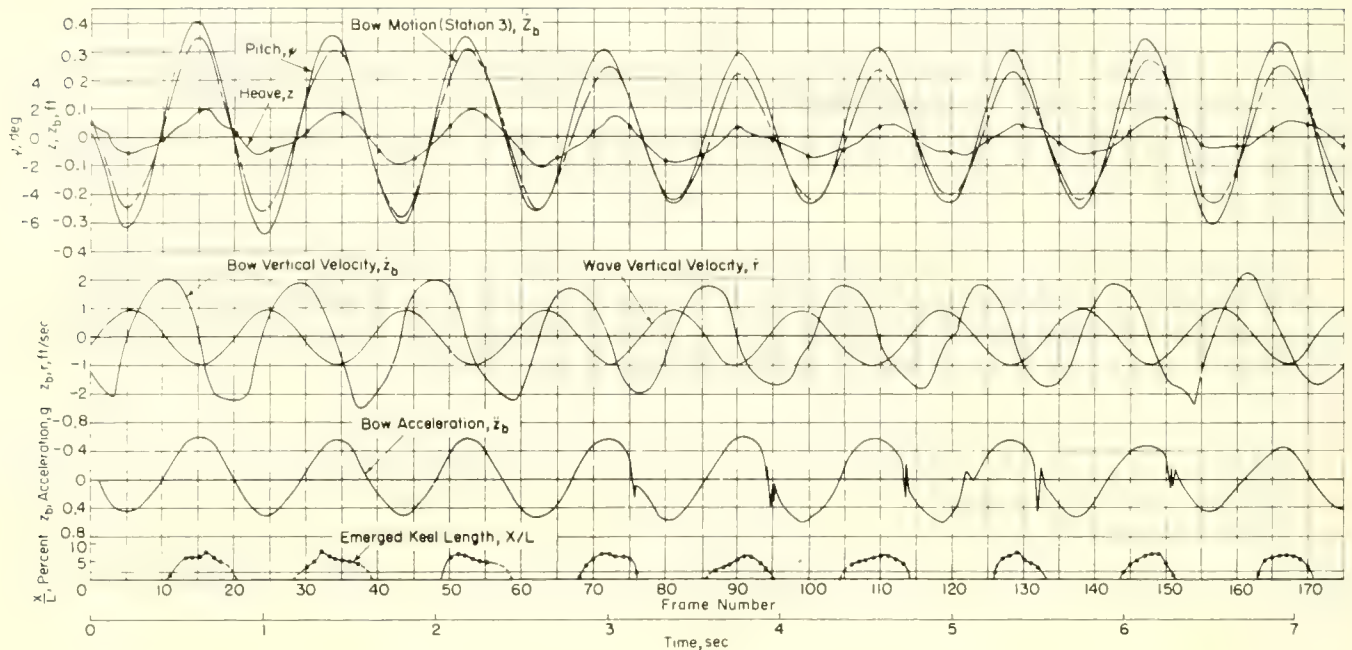
Experimental motion study, 1.2 lb towing force
 Regular seas: $\lambda = 5$ ft, $\lambda/b = 16.7$ Natural periods: $(T_n)_\theta = 0.65$ sec; $(T_n)_z = 0.70$ sec
 Ballast condition: $\Delta = 38.19$ lb Gyradius: 1.394 ft

Fig. 33 Towing tank record of slamming of a model of liberty ship in steep regular waves (from Szebehely and Lum, 1955)

to be read in the order of numbers just above pitching trace. The slam occurs at the instant 4 at which the pitch angle is barely below level; i.e., the downward bow velocity is nearly at its highest value. Heaving is on the decrease, but is still high. Maximum pitching velocity is combined with the downward heaving velocity to give a high downward velocity at the bow. In this case a large metal model was used, nevertheless the vibrations after slamming decayed rapidly. On a full-size ship they last longer, so that nearly the full value of stress due to slamming is superposed on the primary bending stress. On a small wooden model used by Lewis (1954), on the other hand, the vibrations were quickly extinguished.

The work of Szebehely and Lum (1955) and Akita and Ochi (1955) points up the intentional artificiality of the test conditions and calculations used by M. A. Todd (1954). One can consider Todd's work as composed of two parts: (a) Use of Wagner's (1931) method in evaluating hydrodynamic forces for a given vertical velocity

of a body; and (b) use of the methods employed by Mayo (1945), Bencotter (1947) and Milwitzki (1948) in formulating the equation of motion for a falling body decelerated by water forces. In the latter sense the experimental conditions were analogous to a seaplane landing in that the entire kinetic energy of the falling body was absorbed by the hydrodynamic forces generated in the process of the impact. However, although the slamming force in waves is large, its duration is so short that a negligible amount of kinetic energy is absorbed, and the pitching and heaving motions continue essentially as if the slam did not occur. Only part (a) of M. A. Todd's work should be used in an analysis of slamming in waves. Since the bottom area affected by slamming pressures will be more limited than in still water because of the curvature of the wave profile, fewer hull stations will have to be analyzed. Thus, the problem of expressing the forces occurring in slamming is basically simpler than in the artificial case of slamming due to pure pitching in



Experimental motion study, 0.8 lb towing force
 Regular seas: $\lambda = 5$ ft, $\lambda/b = 16.7$ Natural periods: $(T_n)_\theta = 0.65$ sec; $(T_n)_z = 0.70$ sec
 Ballast condition: $\Delta = 38.19$ lb Gyradius: 1.394 ft

Fig. 34 Towing tank record of slamming of a model of Liberty ship in steep regular waves (from Szebehely and Lum, 1955)

smooth water. The difficulty shifts to the evaluation of ship motions and the definition of the ship-water surface relationship at the instant of slam.

Szebehely and Todd (1955) pointed out that occurrences of high local pressures may not coincide with the total force felt at a slam, but they did not pursue this important topic further. High local pressures are generated, in accordance with Wagner's theory, whenever a ship section of small deadrise penetrates the water surface with sufficient vertical velocity. If the draft of a ship and the heaving and pitching motions are such that high pressures are generated successively at one section of a ship after another in a sufficiently slow succession, the total force will remain small and a slam will not be felt. Such conditions are found in most cases of bow emergence, since the calculations of M. A. Todd (1954) and of Bledsoe (1956) show that a high sectional force occurs at a very small instantaneous draft. This force can occur simultaneously over a large area only when a ship's bottom is almost parallel to the water surface. The high pressures then occurring simultaneously over a number of ship sections add up to a large force which is felt as a slam. In the case of a ship in waves this postulates a certain amount of up-heave as is evident in Figs. 33–36. This condition of parallelism also is more readily fulfilled at shallow draft than at deep draft.

Slamming appears to be one of the main reasons for reducing a ship's speed in bow seas. The work of Szebe-

hely and his associates has thrown a considerable amount of light on the nature of this phenomenon. The value of their work lies not so much in the successful performance of a difficult analysis as in the vivid demonstration of the harmful effects of nearly flat bottoms in bow areas. Others also have commented on the importance of ship form in slamming; e.g., King (1934–35) and Kent (1949). Ochi's (1956) experiments in a towing tank on U and V-section form ships, Fig. 37, demonstrate that slamming acceleration is much lower for the V-forms than for the U-forms. Practicing naval architects appear to be slow in adapting themselves to this effect of ship form.

7.4 Calculated Slamming Pressures. Bledsoe (1956) calculated slamming pressures for five models of the Series 60, ranging in block coefficient from 0.60 to 0.80. These calculations were made for slamming-approach conditions similar to the ones assumed in the experiments of M. A. Todd (1955). A characteristic result of these calculations was the extremely high value of the peak pressure.

Quoting Bledsoe: "The absolute magnitude of the maximum pressure (for example, 1450 psi for Station 3, of . . . ²⁹ is not in itself too significant. The susceptibility of the ship to damage depends not only on the magnitude of the slamming pressures but on their extent and

²⁹ Figure referred to is not reproduced here. It gives the pressure distribution on the 0.75 block coefficient hull.

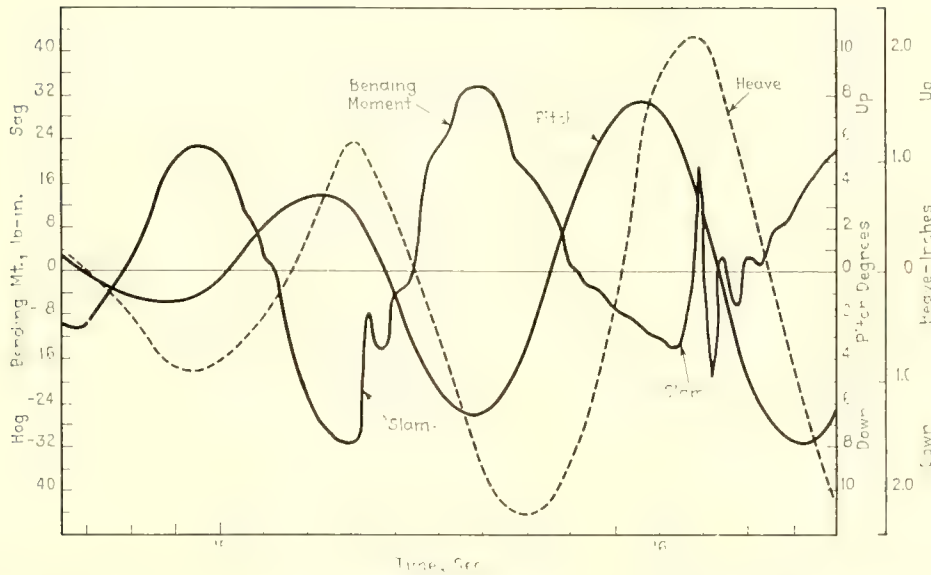


Fig. 35 Towing-tank record of slamming of a model of T2-SE-A1 tanker in irregular head waves (from E. V. Lewis, 1954)

duration as well. A study of the data shows that reduction of these extremely high pressures occurs very rapidly. For example, at a time 0.0001 sec later than that which produced a maximum pressure of 1450 psi for Station 3, the pressure has dropped to approximately half of its original value, and at a time lapse of 0.0004 sec, it had experienced an 84 percent decrease. Figure²⁹ shows further that this high pressure is restricted to an extremely small area of the ship's bottom. Therefore, due to the extreme localization and rapid reduction of these large pressures, they contribute but little to the total impact load which the ship receives."

The work of Bledsoe is valuable in demonstrating quantitatively the significance of ship fullness in the slamming process. The calculations were based, however, on the assumption of artificial approach conditions, and the absolute values of pressure do not correspond to the operating conditions of a ship. Among other things, the assumed ship draft was less than 20 per cent of the normal ship draft.

The high pressures computed theoretically probably would not occur in a physical experiment. In the first place they result from the expanding-plate theory, without consideration of the spray-root properties. Consideration of these would result in a somewhat lower and more diffused, although still narrow, peak. Furthermore, the theory is based on a quasi-static consideration; i.e., the water flow at each instant is taken to be identical with the steady state flow. It was explained in Section 3.12 that the added mass derived from the water flow at the free surface is not a fixed property of the body form but depends on the past history of the fluid motion.

The impact theory resulting from this assumption has been shown by experiments with seaplane bottoms to give

good results for deadrisers of the order of 20 deg. There is some suspicion that the theory partially breaks down at deadrisers below 8 deg. Exact experimental verification is extremely difficult because of the narrowness of the pressure peak and the rapidity with which it moves past a pressure gage. Greenspon (1956) gave the results of pressure measurements on a Coast Guard Cutter operating in a rough sea with the specific intention of developing strong slamming. Peak pressures up to 295 psi were recorded, while the mean pressure over the area of a bottom plate between structural supports was of the order of 100 psi.

It should be mentioned that, in addition to strong distinct slams, a series of almost continuous impacts of lesser magnitude also can be felt. Like slams these occur frequently on typical cargo ships in shallow-draft condition, and may maintain a ship in an almost continuous state of vibration. For fast ships, such as destroyers, they may occur at normal draft.

7.5 Impact on Wavy Water Surface. Available theoretical methods of evaluating slamming pressures fail in the case of impact of a flat plate on smooth water. In such case they yield infinite pressures. Such an impact, however, would be rare at sea. Usually the sea consists of many different waves, ranging from the smallest (ripples) to the largest, corresponding to the wind and swell conditions. While the conditions conducive to slamming are determined by the larger wave components, the actual impact takes place on a surface covered with ripples and small, sharp-crested waves. These waves have a large cushioning effect, reducing both the local pressures and the total slamming load. Some idea of the latter effect may be obtained by recollecting that in M. A. Todd's (1955) experiments the penetration of the water surface

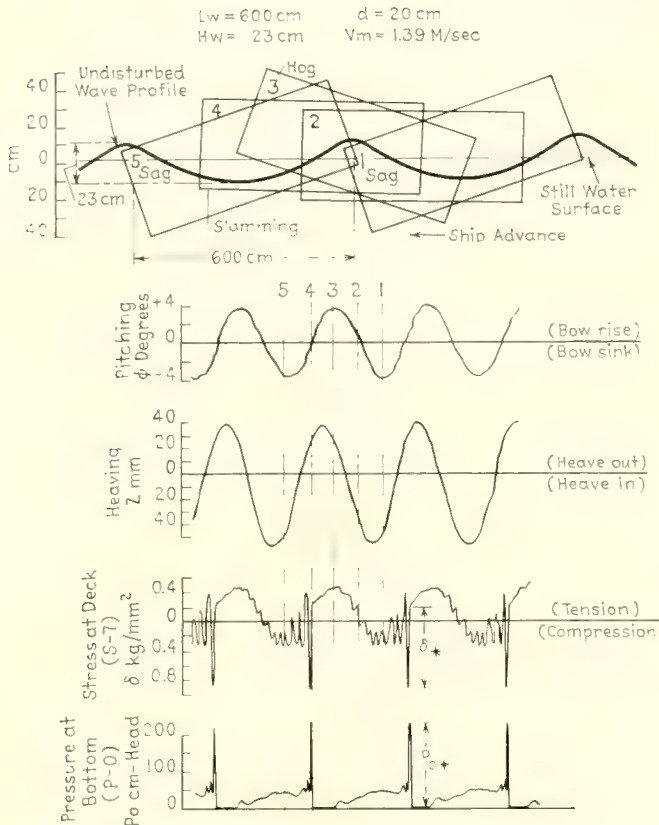


Fig. 36 Towing-tank record of slamming of a flat-bottom shallow-draft ship model in regular waves (from Akita and Ochi, 1955)

up to the time of maximum force was of the order of $2/3$ ft full scale. The presence of sharp-crested wavelets 2 ft high would have increased the length of the shock-absorbing travel threefold, and would considerably reduce the impact force. Theoretical and experimental research on the impact of flat and small-deadrise plates on a rippled water surface is suggested. The impact of a flat surface on a single wave crest of a specified included angle can be theoretically investigated. This can next be generalized to apply to a complex sea surface by means of the statistical methods discussed in Chapters 1 and 3.

7.6 Forces Generated by a Bow Flare. As has been stated earlier, strong slams usually occur when the descending velocity of the bow is high and the ship-wave phase relationship is such that the water surface is rising to meet the bow. In natural irregular seas this usually happens at the recurring periods of unusually high and steep waves. Since slamming absorbs but little energy, the downward pitching motion continues and often water is shipped over the bow. Walls of green water deflected by the flared sides of a ship near the bow are familiar to mariners. Large water pressures are produced as a reaction to this deflection. In accordance with Wagner's (1931) theory, equations (62), (63) and (64), water

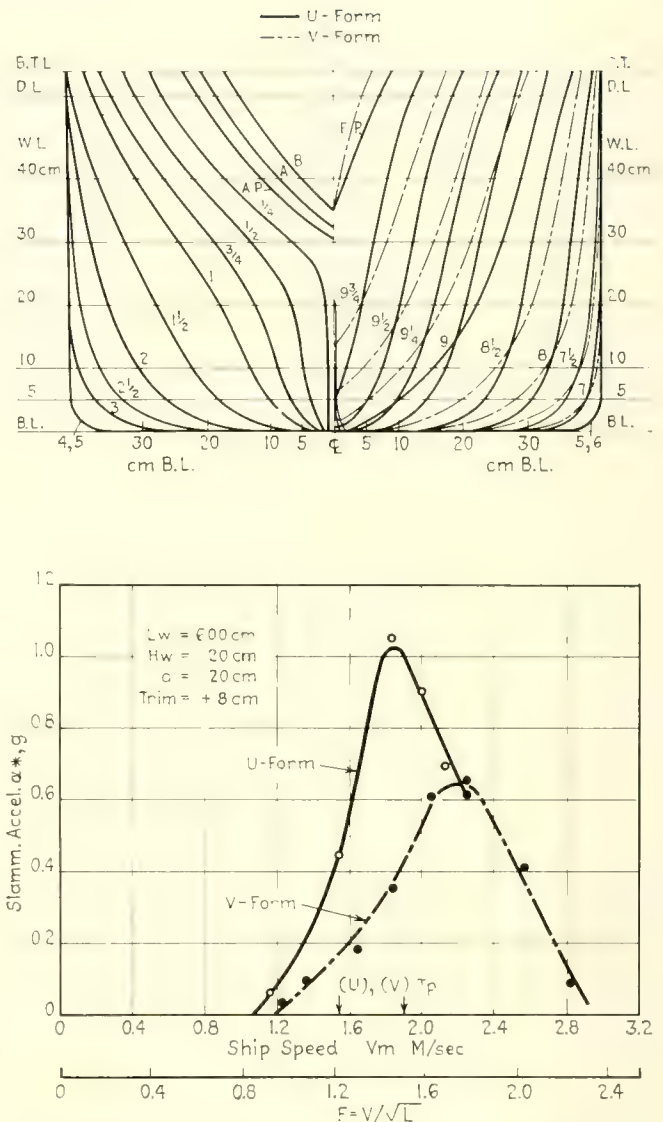


Fig. 37 Effect of bow form on slamming acceleration of a ship model in regular waves (from Ochi, 1956)

pressure increases with decrease of the angle between the tangent to the body surface and the horizontal. This angle, even in the largest flare, is never small, usually over 45 deg. Unit pressures are, therefore, not high, and the rate of force development is relatively slow. Flare pressures, however, can cover a large area and generate a large force which seriously increases the bending stress in a ship. The "flare impact" will be discussed in greater detail in Chapter 5 in connection with observations made on destroyers at sea.

The forces caused by slamming are usually applied at such a rapid rate that their effect on a ship cannot be expressed by the laws of statics. It becomes necessary to consider the elastic response of the ship's structure. Further discussion of the slamming process will be deferred therefore to Chapter 5.

8 Concluding Remarks and General Research Suggestions

It appears that analysis of ship motions³⁰ has reached its most complete development, from a practical point of view, in using hydrodynamic forces obtained by the strip theory. Strip theory, at the same time, provides the distribution of the hydrodynamic forces along the length of a ship. This distribution is needed for analysis of shear forces and bending moments acting on a ship. Use of the strip theory, likewise, appears to be necessary in the evaluation of rolling and yawing moments and of the corresponding translational forces acting on a ship in waves coming from an oblique direction. An example of a partial evaluation of rolling moments by this method can be found in the work of Cartwright and Rydill (1957). Available material on sectional hydrodynamic forces suitable for use with the strip theory, however, are meager and mostly theoretical. Broadly speaking, there is an acute need for (a) experimental data on prismatic and cylindrical bodies, (b) experimental data on complete ship models, (c) additional theoretical calculations for prismatic bodies and complete ships, relating to added masses and damping-force coefficients in oscillatory motions of all modes. The force connected with the acceleration of the added mass and the damping force are here understood to be two components of a hydrodynamic force, acting respectively in phase and 90 deg out of phase with the oscillation of the body. In theoretical work and in the analysis of experimental data these appear as real and imaginary parts of the total force, which is considered to be a complex quantity.

8.1 Experimental Research: 8.11 Methods of testing. Various types of tests have been described by Dimpker (1934), Holstein (1936), Haskind and Riman (1946), Grim (1953), Golovato (1956-57), and Gerritsma (1957c). The papers of Haskind and Riman and of Golovato give clear descriptions of the analysis of test data in connection with two different basic test methods. While they are actually applied to complete ship models, the methods are equally applicable to prismatic bodies. A third method used by Holstein (1936) consists of measuring wave profiles and is applicable only to damping forces.

In the tests of Haskind and Riman and of Gerritsma, model motion was excited harmonically through a spring actuated by an eccentric, while forces were evaluated from the recorded amplitude and phase of the model motion. A method used by Grim (1952, 1953) is similar, except that unbalanced rotating weights were used instead of springs and eccentric. In either case a known harmonically varying exciting force or moment was applied, and the analysis was based on the assumption of harmonic model motion. In practically all body forms the actual instantaneous force is not a simple harmonic but is nonlinear. The tests gave the forces and moments of an assumed linear or harmonic system of the same amplitude and phase relationship as the true physical

system at hand. It will be shown later in Chapter 3, in the section on irregular seas, that this equivalent linearization is what is needed for the present statistical theory of irregular model motions. However, since the forces and moments are in all likelihood not linear, the experiment must be repeated for several amplitudes of motion. These amplitudes, and the frequencies of oscillation as well, must cover the practical range met by a ship in waves.

In this method of testing the model is flexibly connected with the apparatus and, therefore, records are largely free from noise. The resultant forces are inferred from the model's motions, while the model itself acts as an integrator of all instantaneous forces. The records, therefore, are relatively smooth and easy to interpret. For these reasons great precision is not required in the apparatus, which thus can be simple and inexpensive.

In Golovato's tests at the David Taylor Model Basin, the model was driven positively by an apparatus in a simple harmonic motion. Dynamometers with high spring constants were placed between the model and the driving part of the apparatus. The deflection was so small that the model motion was assumed to follow directly the motion of the apparatus. The dynamometers measured the sum of hydrodynamic and model inertia forces. These were very sensitive to any irregularity in the acceleration of the driving system. The apparatus at the David Taylor Model Basin is believed to be the only one of this type in existence and has taken a number of years to develop.

While the dynamometers were rigid enough to transmit the apparatus motion to the model, their own frequency response could not be neglected, since it was of the same order of magnitude as that of the record taken. Because of the large mass of the model and of the added water mass it is usually not possible to secure a sufficiently high frequency response while retaining sensitivity. Correction for the frequency response of dynamometers, therefore, is required and was discussed by Golovato.

The advantage of this method of testing lies in that it is possible to get a record of nonlinear variations of the added masses and damping forces. In the case of Golovato's model the damping force was nonlinear.

It appears that at the present stage of ship-motion theory, and in particular the theory of ship motions in irregular seas, it is not yet possible to make use of the detailed nonlinear information provided by the DTMB apparatus as used by Golovato. It is suggested that the simpler methods described by Haskind-Riman (1946) and Gerritsma (1957c) be used in order to provide, as quickly as possible, a large amount of experimental information on hydrodynamic forces. Certain other dynamometers may be added at the model in order to measure directly the cross-coupling forces and moments. Knowledge of the nonlinear behavior of forces, however, may be needed in the near future for analysis of structural loads acting on ships. In view of this, it is recommended that the Golovato-David Taylor Model Basin method

³⁰ To be discussed in Chapter 3.

be reserved for investigation of nonlinear behavior of a few models. It is hoped that a much larger number of models will be tested by the simpler "flexible" methods.

In any case it is strongly recommended that familiarity with the Haskind-Riman, Golovato, and Gerritsma papers be required of all prospective investigators.

The prismatic models and ship models should be tested in:

- (a) Heaving oscillations.
- (b) Side-sway oscillations.
- (c) Rolling oscillations.

The complete ship models should be tested in addition in:

- (d) Pitching oscillations.
- (e) Yawing oscillations.

The series of tests intended for analysis of symmetric motion, i.e., heave-pitch, and unsymmetric motion, i.e., side sway, roll and yaw, need not necessarily be conducted by the same investigators, since as a rule different types of apparatus will be needed.

8.12 Prismatic models. Those investigators of ship motions using strip theory have usually estimated the added masses in heaving motion on the basis of the theoretical studies of F. M. Lewis (1929). Lewis devised a conformal transformation by means of which he developed potential flow patterns for a number of ship-like sections starting from the known velocity potential of the flow about a circle. Lewis assumed the water flow around a surface ship to be identical with that around a fully submerged double body, and on this basis computed added masses. Grim (1953) used Lewis' sections in computing damping forces in heaving motion and also made a certain (not yet adequate) amount of computations of added masses in the presence of a free water surface. Grim (1956) also has calculated added masses and damping forces for side-swaying and rolling motions of floating bodies of Lewis' sections.

It is believed important to obtain experimental data on added masses and damping forces in heaving, side-swaying and rolling for prismatic bodies of the type used by Lewis. This will provide experimental verification of Grim's work, and at the same time furnish data on sections of practical type with large section coefficients. Lewis' results are not realistic for small section coefficients since free-surface effects are expected to be particularly strong in sections having sloping sides at the LWL. Experiments with prismatic bodies of cross sections used by Haskind (1946) are therefore recommended. These are defined by equations (50) and (54). At small section coefficients, their sides at the LWL are tangent to an inclined line; they are pointed at the keel and represent reasonably well the V and concave V-sections at the bow and stern of many ships.

The foregoing program is particularly recommended since there is strong evidence³¹ that the slope of a ship's sides at the LWL has a large effect on hydrodynamic forces.

Use of these two families of mathematically defined lines is recommended not only because they provide an opportunity to compare theoretical and experimental data, but also because hydrodynamic forces can be expressed systematically by a plot against a pair of parameters defining the section. Were tests made on unrelated profiles, subsequent use of the data for a new ship would involve rather uncertain comparisons.

8.13 Complete ship models. Tests of complete ship models are recommended in order to provide data on the relationship between a true three-dimensional flow and the two-dimensional one assumed in strip theory. They also are valuable in cases when only ship motions are of interest and the question of force distribution is not involved. Three types of tests are recommended here:

(a) Tests of mathematically defined lines symmetrical about the midship plane, with sections of one family and the same parameter (affine sections), such as Haskind's (1946) for which theoretical computations are available.

(b) Tests of mathematically defined lines in which sections vary from full ones amidship to V or concave V at the ends; also possibly unsymmetrical fore and aft. The ship-surface equation and theoretical forces for these forms are not yet available and must be developed.

(c) Tests of selected ships of normal practical type. Comparison of test data with theory should be based on matching true ship sections to those closest to them in F. M. Lewis' or Haskind's families of sections.

8.2 Theoretical Research. It has been pointed out in the text that information on added masses of ship forms in two-dimensional flow is only available for two asymptotic cases of very high or very low frequency. Added-mass information at all frequencies is only available in the work of Ursell for a semi-cylinder. Additional theoretical work is needed to obtain the added masses for other sections at all frequencies. Ursell's (1949a) paper can serve as a foundation for this work.

It is necessary to evaluate the distribution of added masses along the length of a ship and this can possibly be done by extension of Haskind's (1946) and Hanaoka's (1957) work. Early translation of the complete set of Hanaoka's papers is desirable.

Theoretical work on hydrodynamic forces acting on three-dimensional bodies (Haskind, Hanaoka, Vossers) has been invariably based on parabolic forms with affine sections, symmetric fore and aft. It is desirable to extend it to hull forms with nonaffine sections, more nearly resembling actual ships. The availability of high-speed computing machines will permit a wider scope of activity in the future than was feasible for the original investigators.

Grim's (1953) theoretical computations of damping forces in heaving appear to be the most complete at present. However, their application to the damping on a complete ship model did not lead to good agreement with Gerritsma's test data. The discrepancy may lie in the assumptions made in the original development of the theory for cylindrical bodies, or it may have been caused by the three-dimensional effect. The corrections

³¹ Korvin-Kroukovsky and Jacobs, 1954, 1957.

for the latter indicated by the analyses of Havelock and Vossers have not diminished the discrepancy. Further work is needed in four directions: Recheck of the original mathematical work (particularly in regard to physical assumptions); experimental verification by oscillation of cylindrical bodies; investigation of the effect of nonlinear damping on the mean value, Section 3.22; and further investigation of the three-dimensional effect.

In view of damping nonlinearity, the three-dimensional investigation must also be made for a finite amplitude of ship oscillations. The mathematically simple Holstein-Havelock method of damping evaluation may possibly be extended to yield the damping of three-dimensional bodies oscillating with finite amplitudes.

In the foregoing projects, heaving and pitching were kept directly in mind. Similar projects are recommended for side-swaying and for rolling motions. Although the basic technique was indicated by Haskind (1946) and Ursell (1949a), derivations for the nonsymmetric motions were carried out to a lesser degree of completeness.

Practical evaluation of Haskind's (1946) work has been made in applying it to a Michell-type ship. An advanced physio-mathematical project is recommended in order to develop a mathematically defined family of lines for which the Ketchin-Haskind H function can be evaluated without resorting to Michell's assumptions.

8.3 Measurements on Full-Size Ships. Measurements made on full-size ships at sea involve responses of a ship to the complex (irregular) sea which occurs in nature. These will be discussed later in Chapters 3 and 5 in connection with ship motions and ship elastic responses.

8.4 Slamming Pressures and Forces. The main shortcomings of pressure measurements in slamming, both on models and on ships at sea, have been as follows:

(a) Failure to distinguish between the maximum pressure causing local plate failure and the total force felt as a slam and recorded in the bending stress amidship.

(b) Failure to consider the extremely sharp peaks of the pressure-distribution diagram, given theoretically by Wagner's theory and observed experimentally.

As an illustration of the former, in the work of M. A. Todd (1954) and of Bledsoe (1956) effort was directed only at determining the high pressures which were indicated to be at the edge of the wetted area located at the beginning of the bilge turn. These pressures were associated with the maximum value of the total load recorded as slamming by an accelerometer. On the other hand, maximum pressures causing bottom-plate damage apparently occur in actual ships at an earlier instant of impact, when the wetted width is very small and when high pressures, resulting from too small a deadrise, are present near the keel. While these pressures are high, they cover a small area and the total force does not become significant. In each research project on slamming the two separate objectives, local pressure and total force, must be clearly kept in mind.

In connection with the second shortcoming (b), it should be remembered that with a small angle of dead-

rise, common in ship forms, the pressure peak is extremely narrow and moves from keel to bilge extremely rapidly. (The distance from keel to bilge is traversed in about two thirds of the time it takes for this part of the bottom to penetrate into water.) Therefore, reliable measurements of these peak pressures on models are next to impossible, both because of the physical diameter of a pressure pick-up and of its frequency response. The expanding-plate theory also is not reliable in the matter of peak pressure for a small deadrise, although it is apparently reliable in respect to the total force.

In experiments on ships at sea it has usually been possible to provide only a very limited number of pressure gages, so that the probability of recording the peak pressures is low. Gage readings depend to a large extent on an accidental proximity to the pressure peak, so that the real meaning of the record taken is uncertain.

It is suggested, on the basis of the foregoing discussion, that a peak pressure is an academic concept to which little attention need be paid. Instead it is recommended that a convention be established for an area over which a mean pressure is of interest. In practice this corresponds to an area of a bottom plate between structural supports. Instead of using point-pressure gages in models and ships, a movable plate or a large-diameter gage can be provided, the total force on which can be measured by suitable small-deflection dynamometers.³² Records of the total force exerted by water on such a plate should be more systematic and more applicable to design problems than measurements by a few point-pressure gages. In addition, it will be possible to obtain comparable measurements by the same method on a ship at sea and on a model in a towing tank. It should be emphasized, however, that statement of the maximum of the mean pressure over an area is not sufficient. A complete time history of the pressure is needed in order to evaluate the elastic response of a structure.

Theoretical research on ship slamming in waves is recommended. The procedure of M. A. Todd (1954) for computing the slamming force will apply once evaluation is made of the relative position of ship and wave and of relative ship-water velocity. This subject will be dealt with in Chapter 3. The project should be carried out first for regular waves and then for irregular ones. Study should be made of a ship model for which towing tank data on motions and instances of slamming are available.

Condensed List of Suggested Research Topics

The following are given approximately in the order of exposition in the monograph text.

1 Evaluation of Added Masses should be undertaken for sections characterized by:

(a) The use of curved lines in conjunction with bilge keels. Such sections are common in the middle part of a ship's length.

³² In this connection attention is called to Ochi's (1958) work. A gage 1 in. diam was used in the models, corresponding to 22 in. diam on a full-size ship.

(b) The use of curved lines in conjunction with deadwood (or skegs) which are common in the stern portions of ships.

(c) The use of curved lines in conjunction with propeller-shaft bossings which are common in multi-screw ships.

It is believed that these recommended evaluations can be accomplished by electric analogy methods (Section 3.11; Koch, 1933).

2 Analytical Research for Evaluation of Added Masses and Damping in heaving oscillations of cylindrical bodies (F. M. Lewis' Sections), floating on the free water surface, is recommended for the full range of frequencies. This would be an extension of Ursell's (1949b) work on heaving oscillations of a circular cylinder (Section 3.12).

3 Analytical Investigation of Added Masses for ship sections the sides of which are inclined is also required. These inclinations are particularly prominent in the stern section of most ships, and are also present to a smaller degree in the bow sections of V-form ships. They are present throughout the length of sailing yachts. This project involves (see Section 3.12 for details) water-surface penetration at moderate vertical velocities (not permitting neglect of the gravity forces), the conjunction of a deep draft and inclined sides, and movements both into and out of water.

4 Analytical Investigation of a Three-Dimensional Correction to the two-dimensionally evaluated added masses in heaving and pitching oscillation of ships is advisable, Section 3.13. The correction is required at individual strips (sections of a ship's length) for use in the computation of bending moments. For the companion project on damping forces see item 17.

5 Experimental Measurements of Added Masses (Section 3.15) for prisms and cylinders of various cross sections are needed. The objective of the tests is experimental verification of the theoretical research listed under items 1(a), (b), and (c), 2 and 3. The test bodies must span the experimental tanks. Tests preferably should be conducted in long towing tanks and particular care must be taken to eliminate reflected waves from the tank ends. Proper control and reporting of oscillation frequencies should be provided. Inertial forces and damping forces should be recorded as 180 and 90 deg out-of-phase components of the total hydrodynamic force.

6 Experimental Measurements of the Forces and Moments Exerted by Waves on ship models are needed, Section 3.14-4. Tests at low model speeds should be conducted in wide tanks in order to avoid contamination by reflected waves. Different model forms should be used and waves of different lengths, from 0.75 to 2 times the model lengths, are suggested. Amplitudes of forces and moments and phase lags should be measured. Wave profiles at the model (but not distorted by it) should be recorded simultaneously. Inertial (acceleration) and velocity-proportional forces should be computed as in-

phase (0 or 180 deg) and 90 deg out-of-phase components of the total force or moment.

7 The Distribution of Hydrodynamic Wave-Caused Forces along the ship length should be determined. This can be done either by using a segmented ship model or by measuring the pressure distribution over the hull. The last method calls for a very large number of pressure probes if the pressure integration is to be reliable. The method can be made practical, however, if properly supplemented by theoretical considerations. Theory can be used to indicate all details of the pressure distribution. A relatively small number of pressure probes can then show the difference between theoretical and measured (or measurable) values.

8 Theoretical Evaluation of Sectional Damping Forces Appears to be the Most Pressing Need in the theory of heaving and pitching motions and in the rational evaluation of ship bending moments, Section 3.2. A critical review of Grim's (1953) work is suggested, since this study is the most advanced in this field yet appears to fail in indirect experimental verification, Section 3.21. In particular, it is desirable to assess the deviations from true physical conditions in setting up a mathematical model and if possible to reduce these deviations. Attention should be concentrated on the range of oscillation frequencies of practical interest in ship problems.

9 Alternate Evaluation of Sectional Damping Forces should be undertaken making use of the flow evaluation method given by Ursell (1949a).

10 Alternate Evaluation of Sectional Damping Forces and Their Distribution Along the Ship Length on the basis of the work of Haskind (1946) and Hanaoka (1957) is also advisable.

11 Theoretical Evaluation of Mean Values of Non-linear Sectional Damping (with respect to draft changes, Section 3.22) can probably be obtained relatively easily by a suitable extension of the Holstein-Havelock source method.

12 Three-Dimensional Damping Considering Non-linear Sectional Contributions (Section 3.22, last paragraph) can probably be attacked successfully by extending the Holstein-Havelock source method. The evaluation of three-dimensional effects by Havelock and Vossers, based on the theory of infinitesimal displacements, failed to improve the agreement between computed and experimental damping forces, Section 3.21. Since the damping intensity is indicated by the energy carried by waves at infinity, and since the amplitudes of these waves are governed by wave interference patterns, it is suspected that periodically changing drafts at the ship ends may be the most important aspect of three-dimensional effect.

13 Theoretical Investigation of Damping in Heaving of Sections With Inclined Sides is a companion project to No. 3 on added masses. The problem is outlined in Section 3.12. In regard to damping the problem is further characterized by the fact that water particles at

the LWL are given a lateral as well as vertical impulse. This is expected to favor the development of progressive waves carrying the energy away from a ship and thereby to increase damping. The problem can be attacked in two ways:

(a) By suitable formulation of a basic theory.

(b) As a correction to the water flow induced by a wall-sided section. Guilloton's "second-order" corrections in the theory of ship wavemaking resistance (Korvin-Kroukovsky and Jacobs, 1954) may prove to be applicable in this case.

This project affects project 11 inasmuch as inclined sides can be expected to intensify the non-linear draft effect.

14 Experimental Measurement of Damping Forces in heaving of prismatic and cylindrical bodies, Section 3.21, is a companion project to No. 5. It is expected that inertial and damping forces will be obtained from analysis of the measured data as real and imaginary parts of the total hydrodynamic force. It is suggested that measurements be made on Lewis' (1929) sections for which Grim's (1953) theoretical calculations are available. For section fullness greater than that of the semi-circle, Lewis' sections closely resemble those used in practical ships. Tests on these sections have, therefore, direct practical interest.

15 Experiments on Damping Forces at Low Section Coefficients are also advisable. For section types finer than semi-circular, Haskind's (1946) sections can be suggested. Theoretical information on these can be obtained by reworking Haskind's three-dimensional analysis. These sections have shapes similar to those in the bow and stern portions of actual ships, which are characterized by sloping sides at the LWL. Since the hydrodynamic characteristics of such sections are nonlinear, tests must be conducted at a series of amplitudes.

The measured result at a given amplitude will represent the damping coefficient of an equivalent linear system. In the present state of development of ship-motion analysis, and in view of the ultimate application to ship motions in irregular waves, the author sees no need for a detailed description of the nonlinear damping.

16 Evaluation of Sectional Damping Forces by Means of Wave-Amplitude Ratio (Section 3.21; Holstein, 1936) is suggested as alternate or supplemental to projects 14 and 15. Such measurements can be accomplished with the minimum of special apparatus, and therefore, it is hoped, can be collected for a large number of sections in the shortest possible time. A towing tank of rectangular cross section (for a minimum of wave distortion), a plunger-type wavemaker hugging the end wall of the tank and a good wave-absorbing beach are the required equipment, to be found in many laboratories. By measuring wave amplitudes at several frequencies and amplitudes of the existing plungers a good deal of useful information can be obtained. Much information can be obtained by mere re-examination of wavemaker cali-

bration data. Further data on various sections can be obtained by building plungers of the proper semi-section.

17 Damping-Force Measurements on Complete Ship Models should be made preferably in wide (maneuvering) tanks in order to avoid contamination of results by the wave reflection from towing-tank walls. This project can be subdivided and classified by the method of testing and by the type of model.

By the method:

(a) Tests similar to those of Haskind and Riman (1946) and Gerritsma (1957c), in which the model is restricted alternately in pitching and heaving, and is harmonically driven by a spring link. This type of test apparatus appears to the author to be the best combination of simplicity and reliability.

(b) Tests similar to Golovato's (1957a, b) in which the model is driven alternately in heaving and pitching by positive (rigid) mechanism and forces are measured directly by small-deflection dynamometers. Good quality apparatus and extreme care are needed for proper interpretation of results. Corrections for dynamometer response must be included.

(c) Analysis of the coupled pitching and heaving motion of a free model (Grim, 1953) excited in one of the modes by rotating weights. While only one mode is excited, oscillations in the other will develop under action of cross-coupling forces. In particular, the heaving induced by directly excited pitching will increase with model speed. Unless complete analysis of the coupled motion is made, the results will simulate an exaggerated effect of ship's speed on the damping in pitch.

By the type of model:

(a) A theoretically defined (parabolic) model with affine sections. The model should be chosen preferably from the Haskind (1946) or the Hanaoka (1957) series for which complete theoretical calculations are available. The object of the test is to compare the measured and calculated results on damping and added masses at various speeds. The data obtained on these idealized ship forms should not be assumed directly applicable to actual ships. This project is an extension and verification of the work of Haskind and Riman and of Hanaoka. The author believes that it deserves only low priority.

(b) A theoretically defined model with sections varying from full amidships to V or hollow V at the ends, and unsymmetrical fore and aft. The object of these tests is dual; i.e., to provide experimental comparison with theoretical calculations, and to provide data on a (mathematically tractable) model reasonably resembling actual ships. The author is inclined to assign the highest priority to this project, but only after theoretical work on the formulation and analysis of these model types, Section 6.1.

(c) Tests on models of conventional ships. The only tests available to date are those of Gerritsma (1957c, 1958, and 1960). Additional tests on other ship models are needed. These tests will provide the data on actual ship forms, and also will provide the information needed to

judge the practical value of the models under the preceding subproject (b). They will also serve as the final check on the strip method of analysis using sectional results in conjunction with three-dimensional corrections. Comparative evaluation of damping in pitch of various combinations of bow and stern sections is particularly important in view of the suspected nonlinear effects of sections typical for ship ends.

18 Theoretical Evaluation of Distribution of Sectional Damping Forces along the length of a ship is needed in view of its importance in rational calculations of bending moments. Under Michell's thin-ship assumptions, this can probably be obtained by recasting Haskind's (1946) and Hanaoka's (1957) ship-motion theories.³³

19 Distribution of Nonlinear Damping Forces (in assumed harmonic oscillation) may be of considerable importance in rational bending moment evaluation. This project implies a finite beam and finite amplitude of motion. It may well be intractable by advanced mathematical methods of the type used by Haskind and Hanaoka. The author believes, however, that a cruder intuitive approach based on Holstein and Havelock's pulsating-source distribution may produce valuable results.

20 Experimental Evaluation of Sectional Damping Distribution is necessary to provide direct data for ship bending-moment analysis as well as verification of the theoretical results of projects 17 and 18. Conventional ship models and idealized parabolic models should be used.³⁴

21 Experimental Evaluation of Sectional Distribution of Wave Forces on restrained ship models of various forms is needed. This project, listed here under damping, is a companion to project 7 on distribution of added masses. Preliminary calculations (Korvin-Kroukovsky, 1955; Korvin-Kroukovsky and Jacobs, 1957) have indicated that damping (i.e., velocity-dependent) forces caused by waves may be negligible in defining ship motions. They are important, however, in the distribution of sectional forces since they appear to have a strong influence on bending moments (Jacobs, 5-1958).

22 Theoretical Evaluation of Hydrodynamic Forces and Moments in Side Sway and Rolling is needed for the ultimate analysis of ship motions and stresses in irregular oblique seas. This broad problem evidently must be subdivided into a series of lesser projects. Essentially all of the projects listed in the foregoing under heaving and pitching can be rewritten to apply to side sway and rolling. The subject of side sway and roll has been much less developed than that of heaving and pitching. The studies of Grim (1956), Landweber (1957) and Land-

weber and de Macagno (1957) apply only to asymptotic cases of very high or very low frequencies. Evaluation of added masses and damping forces is needed for the complete frequency range of a ship moving in waves. Evaluation is needed of sectional forces, of three-dimensional corrections of these, and of forces for complete ships. While it is difficult to list all possible projects falling under this broad description, a few suggestions will be listed.

23 Research Based on Haskind (1946) is suggested. Haskind formulated the solution for a ship oscillating in all six degrees of freedom under action of harmonic (but not otherwise defined) wave forces. He has completed the solution, however, only for heaving and pitching. This project would extend Haskind's work to a complete evaluation of hydrodynamic forces in six-component ship motions.³⁵

24 Research Based on Ursell (1949a) is also advised. Ursell presented in his 1949a paper a formulation for the velocity potential and the stream function about a noncircular cylindrical body floating on the water surface. He completed the solution for the rolling of a certain family of ship sections. Research projects are suggested for:

(a) Extension of Ursell's calculations on rolling to other forms of ship sections.

(b) Extension of Ursell's basic flow expressions to lateral (side-swaying) oscillations.

(c) Extension of Ursell's basic flow expressions for pitching oscillations.

25 Experimental Evaluation of Sectional Damping of Bilge Keels on systematic series of models is needed. While it is evident that bilge keels greatly increase the damping in roll of a ship, there exists but few data to permit either rational or empirical evaluation of this damping. Noncircular cylindrical models spanning the width of a towing tank are suggested. The results of experimental work seldom cover sufficiently the variety of forms and proportions found in actual ships and such test results can be better generalized by close correlation with a theory. It is suggested therefore that ship sections analyzed by Ursell (1949a) be chosen as the first subproject. Tests should be made alternately on the bare sections and sections equipped with bilge keels of various widths. Various draft/beam ratios, should be tried, including the one for zero theoretical damping (Section 5.33; Ursell, 1949a).

(a) Tests with a fixed axis of rotation are suggested for the best correlation with theory and so the possibility of generalization. However, these tests alone may be misleading.

(b) In the rolling of a free body, the bilge keels will change the instantaneous axis of rotation and will modify the coupling between rolling, heaving and side sway. The added energy dissipation of additional motions will modify the apparent damping in roll. Alternate experi-

³³ Independent theoretical research by Dr. Paul Kaplan of S.I.T. is in progress under sponsorship of the Analytical Ship-Wave Relations Panel of the SNAME.

³⁴ A current project under the sponsorship of the S-3 panel of the SNAME was mentioned in Section 3.14-4. In this project an attempt is made to measure the distribution of sectional hydrodynamic forces.

³⁵ The subject will be further discussed under the heading of ship motions in Chapter 3.

ments on a free floating body with a suitable analysis of the coupled motion are therefore suggested.

In both types of tests just suggested the added masses and damping moments are to be obtained as real and imaginary parts of the total hydrodynamic moment.

26 Scale Effect on Damping in Roll is very uncertain and conflicting data have been given by various experimenters. Presence of the squared term in equation (31) as well as T. B. Abell's (1916; Section 5.34) measurements indicate a large viscous effect. A scale influence is therefore to be expected. On the other hand, Watanabe and Inoue (1958) demonstrated identical damping values for a ship and its model. Specific research to evaluate the scale effect is therefore needed. This can best be done by sectional damping measurements indicated under project 25; larger model sections can be used in these tests than would be practical in the same tank with a complete ship model.

27 Speed Effect on Damping in Roll is large as is indicated by Figs. 22 and 23 and as is well known from experience with ships at sea. It is known, however, only qualitatively and theoretical and experimental research for its quantitative evaluation are needed. Figs. 22 and 23 give, of course, quantitative data for particular models. The word "quantitative" was used in the previous sentence in the sense of generalized quantitative data suitable for use in ship design.

(a) Theoretical approaches to the evaluation of sectional damping do not indicate the probability of a large speed effect. It appears to the author, therefore, that these effects stem from (i) the forward (leading) edge of bilge keels, (ii) the bow sections of the hull adjacent to the stem. Theoretical analyses (at least crude ones) of the action of these ship parts can be made by analogy with the evaluation of damping of airplane surfaces. This can be found in many books on aeronautical engineering.

The local water flow at the stem of a ship probably can be estimated by assuming it to be two dimensional (in horizontal planes) and applying the Schwartz-Christoffel transformation. The instantaneous obliquity of the flow is given by the vectorial addition of the forward ship velocity and the lateral velocity of an element of the stem caused by ship rolling and a given depth of the element below the axis of rotation.

(b) The following experimental projects on the speed effect on damping in roll are suggested: (i) The relative contribution of the bilge keel's leading edge can be estimated by a successive shortening of bilge keels, cutting away the leading edge while retaining the position of the trailing edge; (ii) in experiments on the bare ship models, the effect of the entrance angle on the damping increase with speed is to be evaluated.

It is probable that the damping can be expressed empirically by a polynomial in speed with a first term independent of speed. It may be hoped that this first term can be evaluated by the strip method on the basis of Ursell's (1949a) or possibly Grim's (1956) material.

The subsequent speed-dependent terms may be evaluated by the theoretical analysis suggested under subproject (a). Thus a complicated physical relationship may possibly be replaced by a summation of assumed simple ones, and a crude theory may provide sufficient guidance for the generalization of empirical data.

28 Measurements of Pressures in Slamming Impact are needed. Pressures should be measured in the process of normal slamming in waves rather than in artificial conditions. The author considers the measurements of peak-point pressures neither reliable nor necessary for engineering purposes. He suggests instead the measurement of a mean pressure over an area typical of the unsupported area of bottom plating in actual ships. Ochi's (1958) use of 1-in-diam gages on models corresponding to 22 in. on a ship, appears to be a good example. It is emphasized that the time history of pressure growth and decay is needed. The data are to be used in computing the elastic response of a ship structure, and this requires knowledge of the time pattern of the pressure application.

29 Impact Characteristics of a Plate on a Rippled Water Surface should be evaluated both theoretically and experimentally. Theoretically, the problem of plate impact on a single crest of a wave of a certain steepness (up to the limiting case of 120 deg included angle) may be tractable. The results can be generalized by statistical methods to apply to the impact on a sea surface of a typical spectrum. Flat-plate and V-shaped sections of small deadrises may be considered.

30 Total Impact Force in Slamming should also be measured experimentally. The measurement (or rather estimate) can be obtained from simultaneous accelerations in heaving and pitching. This will give the magnitude and the position of the impact force. This method of force evaluation is applicable to ships at sea as well as models in towing tanks. A complete time history is required. The very short duration of the impact makes it necessary to pay utmost attention to the sensing and recording equipment. Because of uncertainty in the elastic response of a model, it is suggested that accelerometers be located near the estimated force position and be well secured to a solid block supporting the bottom of the model in the impact area. Measurements made in regular waves may be used to provide the easiest correlation with a theory. Measurements in irregular waves will correspond to actual ship behavior at sea and also will be useful in connection with a statistical evaluation of slamming at sea. The results of this project are expected to be used in the evaluation of ship bending moments taking the elastic response into account. This will be discussed in Chapter 5. Tests should be conducted at several values of draft and trim, corresponding to the range of these values used at sea.

31 Slamming Impact of Ship Sections of Low Section Coefficient must be evaluated theoretically and experimentally. Such ship sections are typical for bow sections of fast ships such as destroyers, cruisers, and aircraft carriers. With a sharp deadrise at the keel, the

impact force at the initial contact with water is not large, but it increases rapidly with submersion (up to the deck level) because of the summation of displacement and dynamic forces. Theoretically the problem differs from that of seaplane impact (Wagner, 1931) by the slower rate of immersion which does not permit neglect of gravity forces. Time history of the force development and decay is needed.

32 Total Impact Force in Slamming of the fast ships mentioned in the above project should also be measured. This is a project similar to 30, but it is listed separately because the relatively slow rate of acceleration development³⁶ may require different sensing and recording equipment. Experiments in towing tanks are therefore simpler with fast ship models than with the conventional cargo ship models, and a greater number of laboratories will be capable of conducting project 32 than 30. On full-size ships, however, the elasticity of slender ships makes the evaluation of the impact force from accelerometer readings less certain.

Nomenclature

NOTE 1: NASA³⁷ nomenclature shown herewith is used for ship and fluid motions in three dimensions.

NOTE 2: Symbols defined locally in connection with a particular topic are not necessarily included in the list of symbols.

NOTE 3: The symbols listed on pages 13–15 of the Proceedings of the Sixth International Conference of Ship Tank Superintendents (Washington, September 10–15, 1951, published by SNAME) are used whenever applicable. Symbols not contained therein are chosen from the ones frequently used in recent literature on ship motions.

- a = a coefficient
- a = coefficient of inertial term in differential equation of heaving motion
- \bar{a}, \bar{b} = coefficients of equation (31) defining attenuation of ship rolling amplitudes
- \tilde{a} = modified value of \bar{a} in linearized equation
- $a_{1,2,3}$ = coefficients of series expansion, equations (12), (14) and (18)
- A = a coefficient
- A = an area; sectional area of a ship
- A = coefficient of inertial term in differential equation of pitching motion of a ship (also in rolling when considered separately)
- $b = B/2$ = half-breadth
- b = a coefficient
- b = coefficient of damping term in differential equation of heaving motion of a ship
- \bar{b} = asymptotic value of damping in heave at $\omega \rightarrow 0$ equation (53)
- B = beam
- B = coefficient of damping term in differential equation of pitching motion (also rolling when considered separately)

- \bar{B} = asymptotic value of B for $\omega \rightarrow 0$, equation (60)
- c = coefficient of restoring force in differential equation of heaving motion
- c = wave celerity
- C = wetted semi-breadth in slamming impact (time dependent, Fig. 31)
- C = coefficient of restoring force in differential equation of pitching motion (also rolling when considered separately)
- C = added-mass coefficient in two-dimensional flow based on comparison with that of a circular cylinder (Lewis, 1929, equation 15). C_v in vertical oscillation, C_h in horizontal oscillation
- d = draft
- f = depth at which a source, simulating heaving oscillation, is located
- F_0 = amplitude of exciting force in heaving oscillation caused by waves
- g = acceleration of gravity
- i, j, k = unit vectors along x, y, z -axes
- $\hat{i} = \sqrt{-1}$
- I = moment of inertia
- I_0 = moment of inertia of a body
- I' = moment of inertia of water displaced by a body
- I'' = added moment of inertia
- $k = \pi/\lambda$
- $k = \omega^2/g$ = wave number
- k = radius of gyration
- k = coefficient of accession to inertia
- $k_{1,2,3}$ or preferably $k_{x,y,z}$ } = coefficient of accession to inertia along x, y, z -axis
- k' = coefficient of accession to inertia in rotation of a spheroid about the minor axis (Lamb's notation)
- $k_{xx,yy,zz}$ = coefficient of accession to moment of inertia in rotation about x, y, z -axis
- k_4 = correction coefficient for free surface effect on added masses in heaving motion, equation (19)
- $l = L/2$ = half-length of a ship
- L = ship length
- L = rolling moment
- m = an index or subscript
- m = a mass; mass of a ship
- m' = virtual mass
- m'' = added mass
- M = a moment; pitching moment
- M_0 = amplitude of time-dependent pitching moment caused by waves
- n = an index or subscript
- n = unit vector in direction of normal to a surface element
- n = number of oscillations
- $N(\xi)$ = damping coefficient per unit of a ship's length, equation (21)
- p = pressure
- q = total fluid velocity (equivalent to U often used for total fluid velocity)
- r = radius; radius vector
- R = radius in polar co-ordinates
- S = wetted-surface area
- S = stagnation point (slamming, Fig. 31)
- T = kinetic energy of a fluid
- T = wave period; period of undamped oscillation of a body
- T_1 = period of damped oscillations

³⁶ Some data on this will be found in Chapter 5.

³⁷ National Aeronautics and Space Administration.

Axis			Moment about axis			Angle		Velocities	
Destination	Sym- bol	Force (parallel to axis) symbol	Designation	Sym- bol	Positive direction	Designa- tion	Sym- bol	Linear (compo- nent along axis)	Angular
Longitudinal....	X	X	Rolling.....	L	Y → Z	Roll.....	ϕ	u	p
Lateral.....	Y	Y	Pitching.....	M	Z → X	Pitch.....	θ	v	q
Normal.....	Z	Z	Yawing.....	N	X → Y	Yaw.....	ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{qbS}$$

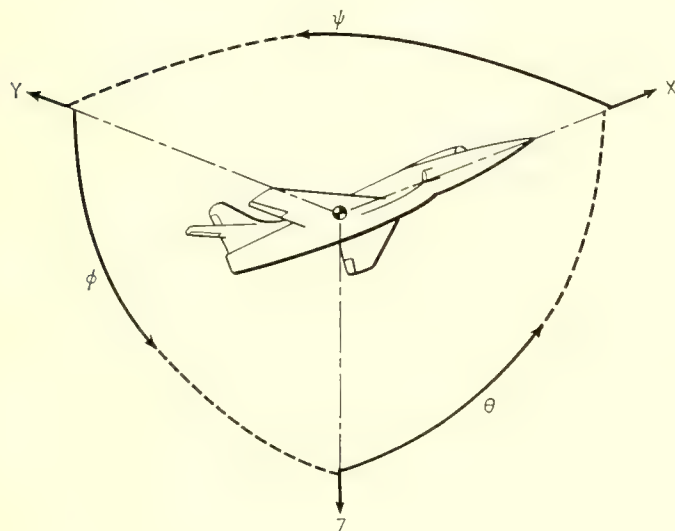
(rolling)

$$C_m = \frac{M}{qcS}$$

(pitching)

$$C_n = \frac{N}{qbS}$$

(yawing)

Angle of set of control surface (relative to neutral position), δ .
(Indicate surface by proper subscript.) U = total fluid velocity U = velocity vector in body motions $U_{1,2,3}$ = velocity components of body motions
along three axes $U_{4,5,6}$ = angular velocity components U_n = velocity of a body surface element along
normal to itself V = ship speed; velocity of undisturbed water flow x, y, z = Cartesian co-ordinate axes x, y = real and imaginary parts of a complex number z X, Y, Z = forces along x, y, z -axes z = a complex number α = an angle α = waterplane area coefficient β = an angle β = sectional-area coefficient γ = length of prismatic section in terms of ship half-length l δ = an increment δ = logarithmic decrement δ = phase angle ϵ = phase angle ζ = a complex number θ = an angle; pitching angle; polar co-ordinate κ = nondimensional damping coefficient, equation (32)
$$\left. \begin{matrix} \kappa_2(d/\lambda) \\ \kappa_2(L/\lambda) \\ \kappa_\theta \end{matrix} \right\} = \text{Haskind's coefficients defined in Section 6.4}$$
 λ = wave length ξ, η = real and imaginary parts of a complex number ξ, η, ζ = Cartesian co-ordinate system fixed in a ship ρ = density (in mass units) ϕ = angle of roll or list ϕ = velocity potential; modulus of time-dependent
velocity potential Φ, Φ_0, Φ^* = time-dependent velocity potentials defined in Sec-
tion 6.1 ψ = stream function ω = circular frequency (used for frequency in all cases
in which no ambiguity is involved) ω_e = frequency of wave encounter (used only when dis-
tinction from wave frequency is necessary) ω_0 = natural frequency of an oscillating body Ω = total angular velocity in body rotation about three
axes (Section 6.1)

1 Introduction

It was shown in Chapter 1 that observed sea waves can be classified by various statistical methods without regard to the weather conditions causing them. Likewise, ship motions, observed visually or with the help of various instruments, can be recorded without reference to sea waves which caused them. Such records can be useful, for instance, in the design of equipment for which a certain operational range must be specified. The statistical methods of handling ship-motion data of this type are identical with those already described for sea waves in Section 1-7.¹ A short reference to ship-motion records of this type will be made in Section 5.2, but the subject will not be developed further.

The plan of this monograph is to trace the quantitative effect of waves on ship motions and on ship stresses. Attention is concentrated, therefore, on formulation of the functional relationships on the basis of which ship motions and stresses can be predicted once the sea waves are defined. The definition of sea waves as a function of weather conditions was the subject of Chapter 1. The waves cause ship motions, reduce a ship's speed and cause bending stress in a ship's hull. These effects will be covered respectively in Chapters 3, 4 and 5. An outline of the hydrodynamic information needed in this connection was given in Chapter 2.

The ultimate aim of the activity to be surveyed in Chapter 3 is to predict quantitatively all motions of a ship in a natural (always irregular) sea with various directions of wave propagation. This broad and difficult problem can be solved only by dividing it into a series of sub-problems, each of which is sufficiently simple to be tractable. The major subdivisions, established only within the last few years, are (a) ship motions in regular long-crested waves, and (b) motions in an irregular sea described by its spectrum. The first of these sub-problems is attacked by means of hydrodynamics and the dynamics of rigid bodies, and the second mostly by the methods of mathematical statistics. This second sub-problem consists of mathematical operations on the results of the first one. All physical characteristics of waves and ships are considered only in the first sub-problem. The development of these two subdivisions will be outlined in Sections 2 and 3 of this chapter.

¹ Reference to sections, equations, figures and bibliography date in preceding chapters will be designated by chapter number and section, equation, reference or figure number; reference above, for example, is to chapter 1, section 7.

2 Ship Motions in Long-Crested Harmonic Waves

A ship traveling obliquely to the direction of wave crests will experience a complicated series of translational and rotational oscillations. In the analysis, these motions are considered as the summation of six components, three translational, and three rotational. The translational motions are surging along the x -axis, side-swaying along the lateral or y -axis and heaving along the vertical or z -axis. The rotations about these axes are rolling, pitching, and yawing. In the mathematical analysis of such a motion, a differential equation is written for each mode; i.e., six simultaneous equations are formed.

Generally motion in any one of these six modes brings into play forces and moments affecting all other modes, so that the analysis becomes rather complicated. The general principles have been stated and analysis has been made of the much simpler problem of the motions of airplanes and airships, but has not yet been accomplished for surface ships. Two special simplified cases, however, have been investigated:

(a) A ship traveling in a direction normal to the wave crests and experiencing only motions in the plane of symmetry; i.e., surging, heaving, and pitching.

(b) A ship at zero speed in beam seas experiencing only rolling, side-swaying, and heaving.

The two cases have been listed as physically and mathematically consistent. Additional simplifications, to be outlined later, are obtained by an arbitrary disregard of certain interactions between modes. These are not justifiable *a priori* but can be accepted either on the basis of experimental confirmation or because they permit evaluation of certain broad trends.

The two consistent simplified cases will be considered first in Sections 2.1 and 2.2. The complete six-component motion will be discussed in Section 2.3.

The motion composed of the six components just described is often referred to as one in six degrees of freedom. The motion of a rigid body is completely described by its six degrees of freedom, but in a practical application this description is adequate only when the *stability* of a body with fixed controls is to be investigated. It is not adequate for investigation of the *path* or *trajectory* of a controlled body's motion. Uncontrolled submerged bodies eventually orient themselves broadside to the direction of motion and an uncontrolled and unpropelled ship will orient itself parallel to the wave crests. Maintenance of a useful trajectory or path of a ship requires use of the rudder. A practical ship is, therefore, not a

rigid body but has an additional degree of freedom in movements of the rudder. This subject will be further discussed in Section 2.32.

2.1 Ship Motions in the Plane of Symmetry: 2.11 Linear theory of coupled pitching and heaving. Ship motions in the plane of symmetry involve surging, heaving, and pitching. The oscillatory surging motion, along the x -axis, does not appear to be of direct interest in defining the seagoing qualities of ships. Previous investigations of the stability of airplanes and seaplanes have also indicated that surging motion has no effect important in practice on the heaving and pitching motions of free ships.² Two mutually interconnected, or "coupled," differential equations of motion, expressing Newton's second law for heaving and pitching, are usually sufficient. These equations can be written (on the habitual assumption of infinitely small motions) as:

$$\begin{aligned} m\ddot{z} &= Z \\ J\ddot{\theta} &= M \end{aligned} \quad (1)$$

where m and J are the mass and mass moment of inertia of a ship, z is the displacement along the vertical or Z -axis, and θ is the angular displacement in pitching. Z is the component of the total hydrodynamic force in the vertical direction and M the total hydrodynamic pitching moment. By "hydrodynamic force" is usually meant the resultant of those water pressures acting on the hull which are connected with relative water velocities and accelerations. However the force Z and moment M in equation (1) include changes in the buoyancy force resulting from ship oscillations. Only the velocities and accelerations connected with waves and the oscillatory body motions are considered. Water pressures resulting from the steady forward speed of a ship and the hydrostatic pressures in normal flotation are not considered. These are balanced by the propeller thrust and the weight of a ship. The oscillatory fluctuation of the propeller thrust in waves and the moments connected with it are insignificant in comparison with other oscillatory forces acting on a normal surface ship. They would have to be taken into account in a planing-type craft, but this is outside the scope of this monograph.

The hydrodynamic force Z and moment M can be evaluated by several methods which will be discussed later. The results of this evaluation take the form of a polynomial with various terms proportional to displacements, z , θ , η (wave elevation), velocities, \dot{z} , $\dot{\theta}$, $\dot{\eta}$ and accelerations, \ddot{z} , $\ddot{\theta}$, $\ddot{\eta}$. The terms in z and θ and their derivatives represent the forces connected with a body oscillating in smooth water. The terms in η and its derivatives correspond to wave forces acting on a ship restrained from heaving or pitching. When this second group is left on the right-hand side and all other terms are transferred to the left-hand side of equations (1), these equations take the expanded form:

² It has been shown theoretically and experimentally (Reiss, 1956; Sibul, 1956) that surging motion occasionally becomes important in towing tanks because of synchronism with the model towing system. By the same token it may be important in towed and in moored craft. References for chapters 2 and 3 appear at the end of this chapter.

$$\begin{aligned} a\ddot{z} + b\dot{z} + cz + d\ddot{\theta} + e\dot{\theta} + g\theta &= \bar{F}e^{i\omega t} \\ A\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{z} + E\dot{z} + Gz &= \bar{M}e^{i\omega t} \end{aligned} \quad (2)$$

The first three terms on the left-hand sides of equation (2) are identical with equation (1) of Chapter 2 representing a simple oscillator.

The last three terms on the left-hand sides are known as "cross-coupling" terms and express the influence of pitching on heaving motion and the influence of heaving on pitching. The coefficients e and E contain simple (dissipative) damping terms arising from the fore-and-aft asymmetry of the hull, and also contain inertial contributions indicating the transfer of energy from one mode of motion to another. Havelock (1955) referred to these contributions as "dynamic damping." This is in agreement with the fact, well known in the study of vibrations of coupled mechanical systems, that oscillations can be controlled by a certain disposition of masses and springs, without introducing dissipative damping.

The forces caused by waves on the right-hand sides of equations (2) could have been presented in the same form as the first three terms on the left-hand sides; i.e., as forces proportional to acceleration, velocity and displacement. In a simple harmonic wave, however, the relationships among these terms are rigidly defined, and the solution of equation (2) is simplified by a compact complex notation, with the understanding that real parts are to be taken. These terms can be written as

$$\begin{aligned} Re\bar{F}e^{i\omega t} &= Re(F_0e^{i\sigma})e^{i\omega t} = F_0\cos(\omega t + \sigma) \\ Re\bar{M}e^{i\omega t} &= Re(M_0e^{i\tau})e^{i\omega t} = M_0\cos(\omega t + \tau) \end{aligned} \quad (3)$$

\bar{F} and \bar{M} are the "complex amplitudes"; i.e., the quantities defining both the amplitudes of force and moment (F_0 and M_0) and the phase lag angles (σ and τ). These latter can be expressed with reference to an arbitrary origin, but the same convention must be retained for both the force and the moment. The symbol ω is used in the foregoing equations to represent the frequency of the wave encounter, ω_e , but the value of \bar{F} and \bar{M} also will depend on the wave length; i.e., on the wave's own frequency.

The solution of the coupled differential equations of motion (2) is given for the steady-state oscillations (Korvin-Kroukovsky and Lewis, 1955; Korvin-Kroukovsky and Jacobs, 1957) as

$$\begin{aligned} \bar{Z} &= \frac{\bar{M}Q - \bar{F}S}{QR - PS} \\ \bar{\theta} &= \frac{\bar{F}R - \bar{M}P}{QR - PS} \end{aligned} \quad (4)$$

where P , Q , R , and S represent the groupings of the coefficients of equations (2) as follows:

$$\begin{aligned} P &= -a\omega^2 + ib\omega + c \\ Q &= -d\omega^2 + ie\omega + g \\ R &= -D\omega^2 + iE\omega + G \\ S &= -A\omega^2 + iB\omega + C \end{aligned} \quad (5)$$

The symbols \bar{Z} and $\bar{\theta}$ in equations (4) are the complex

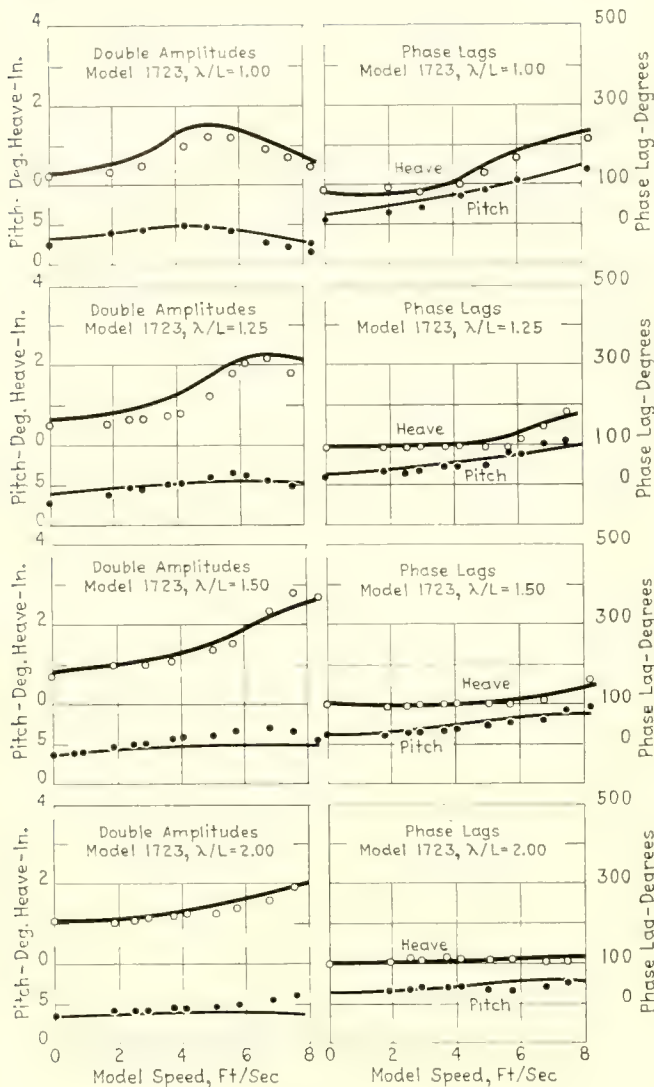


Fig. 1 Motions of a destroyer model in waves 1.0, 1.25, 1.5 and 2.0 model lengths by 1.43 in. high (from Korvin-Kroukovsky and Jacobs, 1957). Circles indicate experimental data—open for heave, closed for pitch—and curves show calculated motions

amplitudes of motion, expressing both the amplitude and the phase lag. Expressed mathematically

$$\begin{aligned}\bar{Z} &= Z_0 e^{i\delta} \\ \bar{\theta} &= \theta_0 e^{i\epsilon}\end{aligned}\quad (6)$$

where \bar{Z} and $\bar{\theta}$ are complex numbers given by the solution of equations (2) in (4). Z_0 and θ_0 are real numbers expressing the amplitudes of heaving and pitching motions, respectively. The $-\delta$ and $-\epsilon$ indicate lag of the maximum of ship motion behind the maximum of the wave.

Equations (2) are general and represent the dynamics of a two-mode coupled oscillation of a body. Hydrodynamics is involved in evaluating the coefficients of these equations. The various theories of ship motions

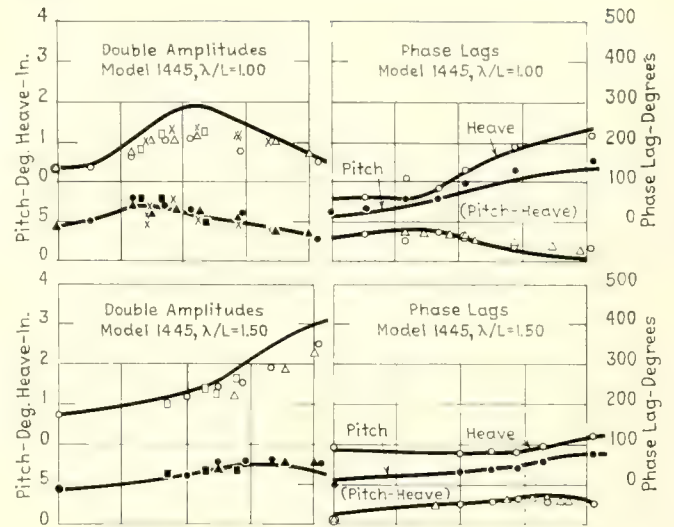


Fig. 2 Motions of a 5-ft model of Series 60-0.60 block coefficient in waves of 1.0 and 1.5 model lengths by 1.25 in. high (from Korvin-Kroukovsky and Jacobs, 1957). Circles indicate Davidson Laboratory experimental data—open for heave, solid for pitch—and curves show calculated motions. Triangles are Taylor Model Basin experimental data, and exes University of California data

differ by the methods used in evaluating the coefficients and by the approximations involved in these methods.

Kriloff (1896, 1898), Hazen and Nims (1940), Weinblum and St. Denis (1950), and St. Denis (1951) neglected the cross coupling between heaving and pitching; i.e., assumed $d = e = g = D = E = G = 0$. Kriloff assumed the coefficients a and A as equal to the ship mass m and moment of inertia I , respectively. Weinblum and St. Denis included added water masses which were estimated by comparison with ellipsoids. They estimated the damping coefficients b and B on the basis of Haskind's (1946) derivations. St. Denis evaluated the coefficients by the strip theory using F. M. Lewis' (1929) material for the added masses in coefficients a and A (neglecting surface effects), and using Havelock's (1942) method for evaluating the coefficients b and B . All of the aforementioned authors used the Froude-Kriloff hypothesis (see Chapter 2, Section 3.14) in evaluating wave forces F_0 and moments M_0 .

The coupled equations (2) were used by Haskind (1946), Hanaoka (1957) and Korvin-Kroukovsky and Jacobs (1957). A brief outline of the principles used by Haskind and by Hanaoka in evaluating the coefficients was given in Section 2-6. The mathematical complexity limits the applicability of these methods to ships of idealized form expressible by simple mathematical formulae.

Korvin-Kroukovsky and Lewis (1955), and Korvin-Kroukovsky and Jacobs (1957) used a strip method in which the coefficients of equations (2) were expressed by the integration of sectional properties. The details of this procedure will be found in Appendix C. In this method

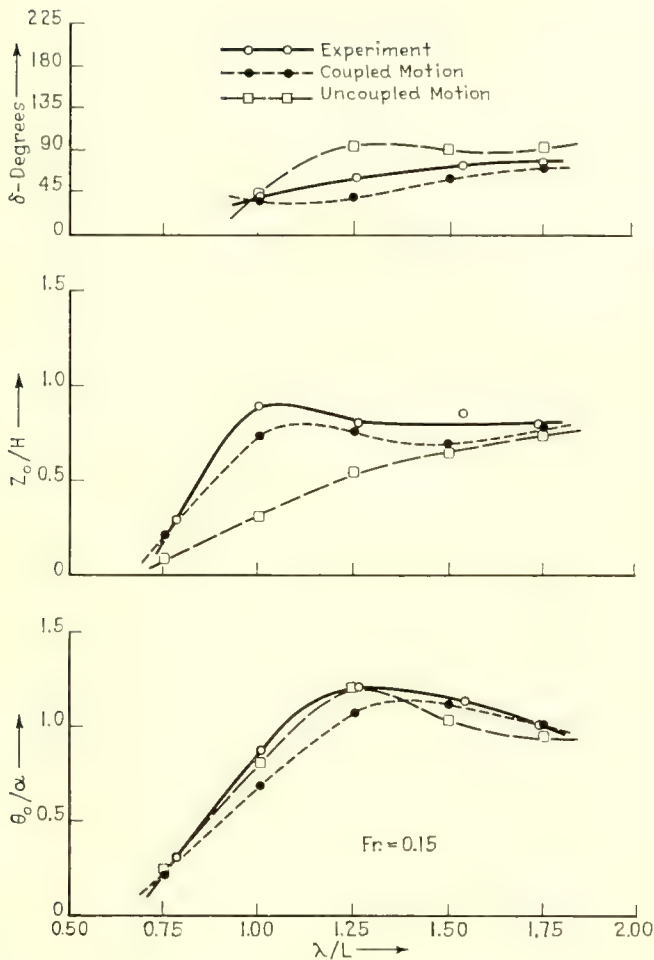


Fig. 3(a) Comparison of computed and observed motion amplitudes and phases of Series 60, 0.60-block-coefficient model: a wave amplitude, Z_0 amplitude of heaving motion, θ_0 amplitude of pitching angle, α maximum wave slope δ phase lag of heaving after pitching motions (from Gerritsma, 1958)

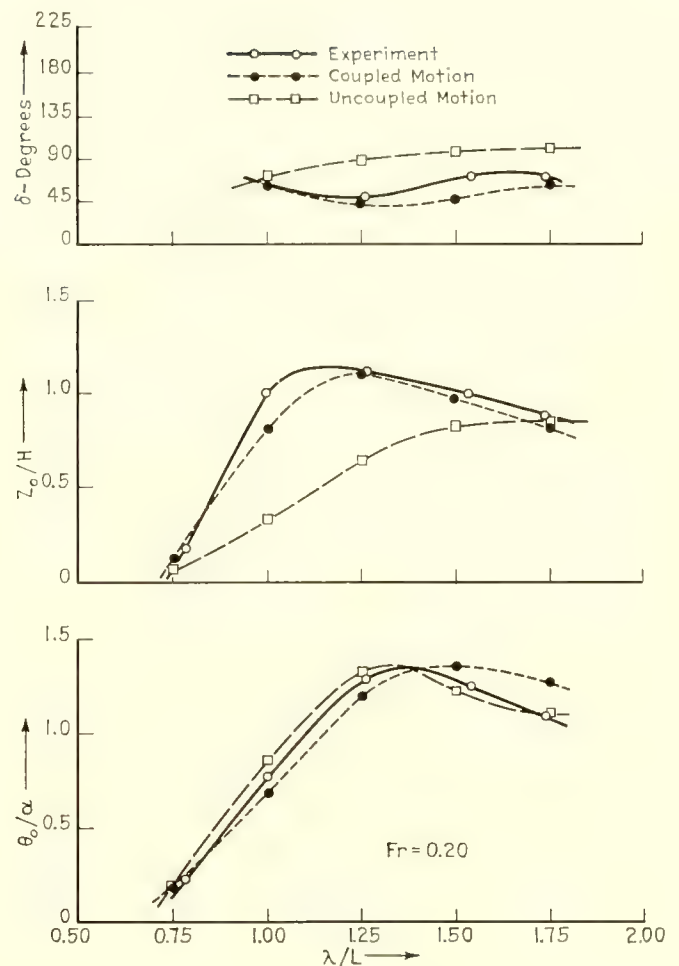


Fig. 3(b) Comparison of computed and observed motion amplitudes and phases of Series 60, 0.60-block-coefficient model: a wave amplitude, Z_0 amplitude of heaving motion, θ_0 amplitude of pitching angle, α maximum wave slope δ phase lag of heaving after pitching motions (from Gerritsma, 1958)

the sectional added masses (derived by F. M. Lewis, 1929) were corrected for the free surface effect on the basis of Ursell's data (see Section 2-3.12). The damping-force coefficients b and B were estimated by the methods of Holstein (1933) and Havelock (1942) in the 1955 paper and by the method of Grim (1953) in the 1957 paper. The Froude-Kriloff hypothesis, often used in the past for evaluation of the wave force F and moment M , was abandoned and these forces and moments were calculated instead, taking into account the interference of the ship's hull with the water flow in waves. The use of the strip theory permits calculations to be made for ships of arbitrary form and also provides the data for subsequent calculation of bending moments

2.12 Comparison of computed and observed ship model motions. The regular long-crested sea appears to be a theoretical concept. It can be well approximated in towing tanks but it is not found in nature. The records of even "regular" swells are found in reality to consist of a

spectrum of waves of many frequencies. Any comparison of the calculated and observed motions of a ship at sea must be deferred, therefore, to Section 3 on irregular ship motions. The present section must necessarily be limited to comparison of calculated and observed motions of ship models in towing tanks.

It appears that prior to 1955 no comparison of calculated and measured ship model motions had been made. The first such comparison (for two ship models) was given by Korvin-Kroukovsky and Lewis (1955); comparative data for eight ship models were given by Korvin-Kroukovsky and Jacobs (1957). Figs. 1 and 2 are taken from the latter work and refer to a destroyer model and a series 60, 0.60 block coefficient model. The experimental data and calculations³ were compared over a range of model speeds and at several wave lengths. The comparison covers the amplitudes of the heaving and pitching

³ Calculated by methods described in Appendix C. Additional test data were published by Gerritsma (1960).

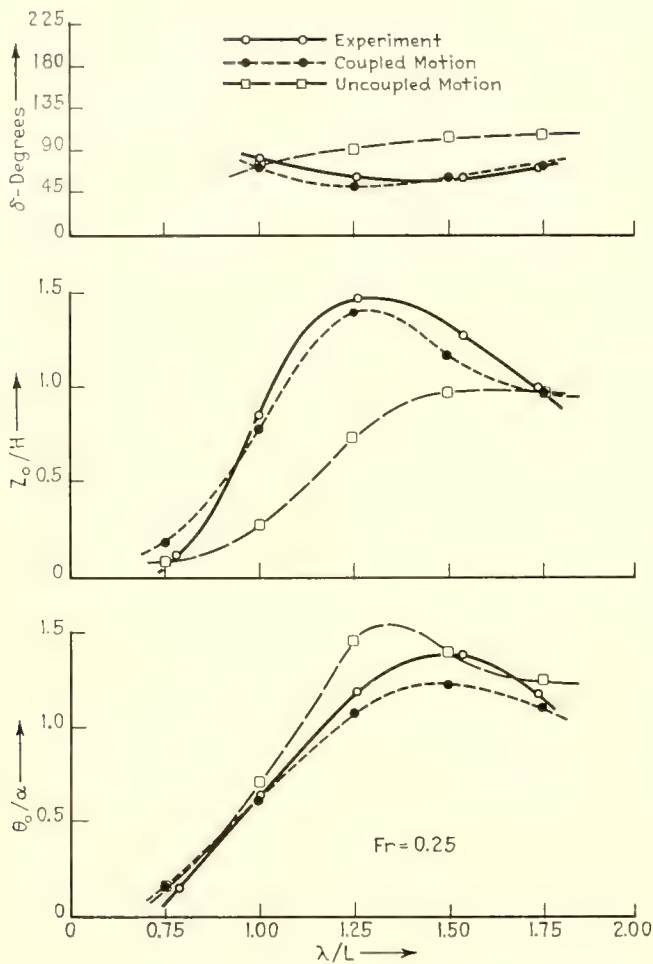


Fig. 3(c) Comparison of computed and observed motion amplitudes and phases of Series 60, 0.60-block-coefficient model: α wave amplitude, Z_0 amplitude of heaving motion, θ_0 amplitude of pitching angle, α maximum wave slope, δ phase lag of heaving after pitching motions (from Gerritsma, 1958)

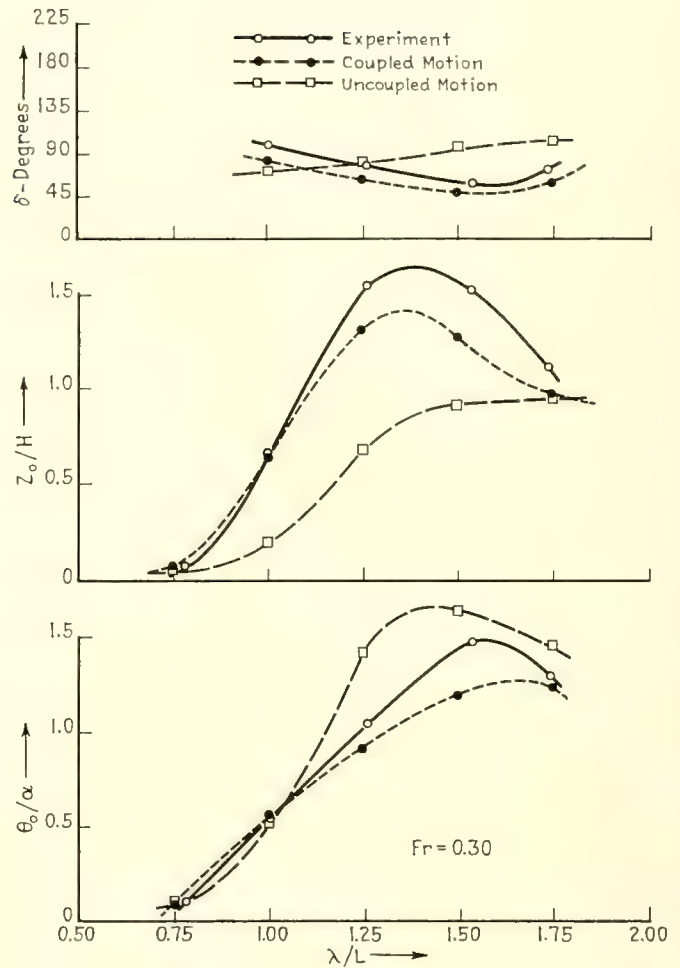


Fig. 3(d) Comparison of computed and observed motion amplitudes and phases of Series 60, 0.60-block-coefficient model: α wave amplitude, Z_0 amplitude of heaving motion, θ_0 amplitude of pitching angle, α maximum wave slope, δ phase lag of heaving after pitching motions (from Gerritsma, 1958)

motions and the phase lags of these motions. The phase lags in particular have been found sensitive to computational errors

It appears that the destroyer form most closely approximates the theoretical linearizing assumptions and there is good agreement between computed and measured data. The agreement is generally satisfactory also in the case of the series 60 model, shown on Fig. 2. In this case, however, a well-defined discrepancy is observed in the amplitude of heaving in the vicinity of synchronism. Subsequently, Gerritsma's (1957c and d, 1958; see Section 2-3.21) experiments indicated that the discrepancy was caused by underestimating the damping in the heaving motion. It was stated in Chapter 2 that the development of more reliable methods of estimating damping is the most important need in the prediction of ship motions.

Fig. 2 shows a comparison of the motions of the series 60, 0.60 block coefficient model as computed by strip

theory and as measured in a towing tank. Gerritsma (1958) repeated tests of this model and made motion calculations indicated by equations (2), (4) and (5), using his experimentally determined coefficients. He also made alternate calculations, in which the cross-coupling terms were neglected. The comparison of the measured motions, the motions calculated by equations (2), and those calculated without the cross-coupling terms is shown in Figs. 3(a), (b), (c), and (d). A good agreement is demonstrated between motions as measured and as calculated with coupling effects. Neglect of coupling results in severe discrepancies in heaving amplitudes and in phase relationships. At low Froude numbers, Gerritsma's tests confirmed Fay's (1958) theoretical conclusion that, through the coupling, pitching strongly affects heaving but heaving has a small effect on pitching. At the higher Froude numbers, however, both heaving and pitching are affected. In particular, Fig. 3(d) clearly

shows the coupling-caused transfer of oscillatory motion energy from pitching to heaving.

The most conspicuous deviation of the series 60 form from the assumptions of the linearized theory is in the large inclination of ship sides in the stern region. In this connection it is interesting to note that out of eight models used by Korvin-Kroukovsky and Jacobs (1957), the two, for which calculations were found to be completely invalid, were models of sailing yachts. The lines of these models show a large inclination of ship sides throughout the model length. The need of investigating the added masses and damping of such forms was emphasized in Chapter 2.

To summarize: Reasonably reliable calculations of the coupled heaving and pitching motions of conventional ships can be made on the basis of the linearized theory using the strip method described in Appendix C for evaluation of coefficients. Further research on the supporting material for damping is, however, needed. Various projects, directed to this objective, were listed in Section 2-9.

2.13 Variable coefficients and nonlinearities. At the end of the previous section, attention was called to deficiencies of the theoretical damping coefficients, the values of which are assumed to be constant throughout the motion cycle. Discrepancies between theoretically computed and experimental ship motions may also be a result of the latter assumption. In the linear theory, the equations are solved for very small wave heights and very small motions, in which case the coefficients are essentially constant. In the actual fairly large motions of models, however, the coefficients are not constant but are functions of instantaneous ship position and, therefore, functions of time. A simple explicit solution can be obtained only when the coefficients are assumed to be constant and independent of time. The coefficients are evaluated in this case with ship sections submerged to the normal still waterline (for instance as in Appendix C, equation 42). These coefficients can be evaluated, however, for any instantaneous position of the waterline at a ship section, and thus can be expressed as functions of displacement from a normal position. The solution of equations (2) can be obtained in this case only numerically by means of step-by-step integration.

An important deviation from the assumptions of the linear theory occurs in the restoring-force coefficients c and C which depend directly on the width of the waterline. For sloping sides, usually found in stern sections of ships, these coefficients have a higher value than mean at a deeper submergence and a lower value at a shallower submergence. There is also an important nonlinearity in the case of the damping coefficients b and B because of a triple effect: (a) Changes in waterline width with submergence, (b) changes of mean draft, (c) a certain dependence of damping on the square of the vertical velocity in addition to the first power of velocity i.e., $b=b(\dot{z})$.

The significance of item (b) in a ship's motion was demonstrated in an exaggerated form by a towing-tank

test of Akita and Ochi (1955), as shown in Fig. 4. The solid line with black dots indicates the phase angle (designated here as δ_{p-H}) between pitching and heaving motions. The model was 19.7 ft long, had a flat bottom and vertical sides, and was tested at a very shallow draft of 9.5 in. in waves 7 in. high. Because of the vertical sides and fore-and-aft symmetry, the model behavior should have been linear in all respects except item (b). It is clear from the discussion of damping forces, expressions 2-(21 and 22), that the damping coefficient is sensitive to draft and increases rapidly with decrease of draft. In the present case, relative changes of draft and damping force with model pitching are large because of a small mean draft. Furthermore, since in pitching a decrease of draft at one end is accompanied by an increase at the other, the cross-coupling coefficients e and E of equation (2) should be affected. Fig. 4 shows that at zero forward speed, the heave lags behind pitch by nearly 90 deg, as it would in a simple noncoupled system. At the speed near 1.4 m per sec, in the vicinity of synchronism, the heave-pitch lag, however, is reduced to about 20 deg. Apparently the only cause of this behavior is the nonlinearity of damping, which caused large cyclically fluctuating cross-coupling terms. It is important to realize that nonlinearity of terms of the equations of motion becomes particularly important because it brings about cyclically varying changes in cross-coupling terms. The effect of nonlinear damping is unusually strong in the Akita and Ochi model because of its shallow draft. It will occur, however, in cases of all ships but to a smaller degree.

Demonstrated in the foregoing for damping, the effects noted occur in the displacement cross-coupling terms (g and G) in ships having inclined sides. This effect was discussed by Radosavljevic (1957b).

The nonlinear behavior of the coefficients of equations (2) can be evaluated without difficulty by applying the expressions given in Appendix C to instantaneous section drafts. While formal solution of equation (2) becomes impossible, numerical solutions can be made easily with the help of electronic computing machines. A research project to carry out such computations is recommended because of the possibility of a large effect of nonlinearities on motions for many ship forms. In particular, phase relationships appear to be strongly affected.

So far computation of nonlinear motion has been carried out only by Hazen and Nims (1940) using a semi-mechanical process. The nonlinearity was limited to restoring moment coefficients, C . This attempt was essentially premature in that it was applied to an uncoupled pitching motion and based on displacement computed according to the Froude-Kriloff hypothesis. The damping coefficient was crudely estimated and was taken as constant. Hazen and Nims' work showed that even in harmonic waves the motions deviated considerably from the harmonic. However, while these deviations distorted the sinusoidal oscillatory trajectory, it has not been demonstrated to what extent the amplitudes were af-

feeted. The phases were not discussed, and, indeed, such a discussion would have been meaningless without taking heave-pitch coupling into account.

2.14 Significant characteristics of ship motions; significance of phase relationships. It appears that motions of a ship can reasonably be predicted either by model tests or by calculations. The resultant quantitative data must be interpreted, however, so as to obtain a somewhat intangible description of the "seagoing qualities" or "seakindliness" of a ship. This description must evidently be connected with the nature of a ship's service. The amplitude of motion is important in certain special cases; the minimum motion is desired for an aircraft carrier in order to facilitate airplane landing, and it may be desired for naval ships in order to provide a more stable gun and missile platform. In commercial ships the amplitudes of motion do not appear to be important by themselves, and other phenomena connected with motions may be more decisive in determining the seakindliness of a ship. Accelerations, which are proportional to the square of the frequency as well as to the amplitude, are more important for passenger and transport ships. Geller (1940) and Shaw (1954) proved that there is a direct connection between accelerations and sea-sickness. On fishing trawlers accelerations impose hardships on the crew at work (Möckel, 1953). On modern cargo ships, on the other hand, crew accommodations are not far from amidships and accelerations in pitch and heave are of minor importance in regard to the crew convenience. These accelerations are significant, however, in the design of the structures supporting cargoes in the No. 1 hold. The critical conditions limiting the sea speed of cargo ships appear to be shipping water and slamming, the latter occurring mostly in light ship conditions. These factors probably also limit the operational speed of naval ships.

Shipping of water and slamming are affected as much by phase relationships as by amplitudes of motions. As can be seen in Figs. 1, 2 and 3 the phase lag continuously increases with increase of a ship's speed, and so with increase in the frequency of wave encounter. In the case of a simple harmonic oscillator, i.e., equation 2-(1), the phase is near zero at low frequency, is 90 deg at synchronism, and asymptotically approaches 180 deg at high frequency. A similar pattern occurs in coupled motion, except that the synchronous speeds in pitching and heaving are different and phase angles are somewhat modified. The important fact brought out by this phase-angle behavior is that a ship tends to follow wave motion at a speed well below synchronism with the result that slamming and shipping of water do not occur. At a speed above the synchronous one a ship may have the same amplitude of motion, but because of a large phase angle the descending bow is likely to impinge on the flank of an oncoming wave. Thus, conditions are favorable for slamming and shipping of water. This has already been discussed in Section 2-7 and made evident in Figs. 2-33 and 34.

2.2 Rolling, Heaving, and Side Sway. Motion in the

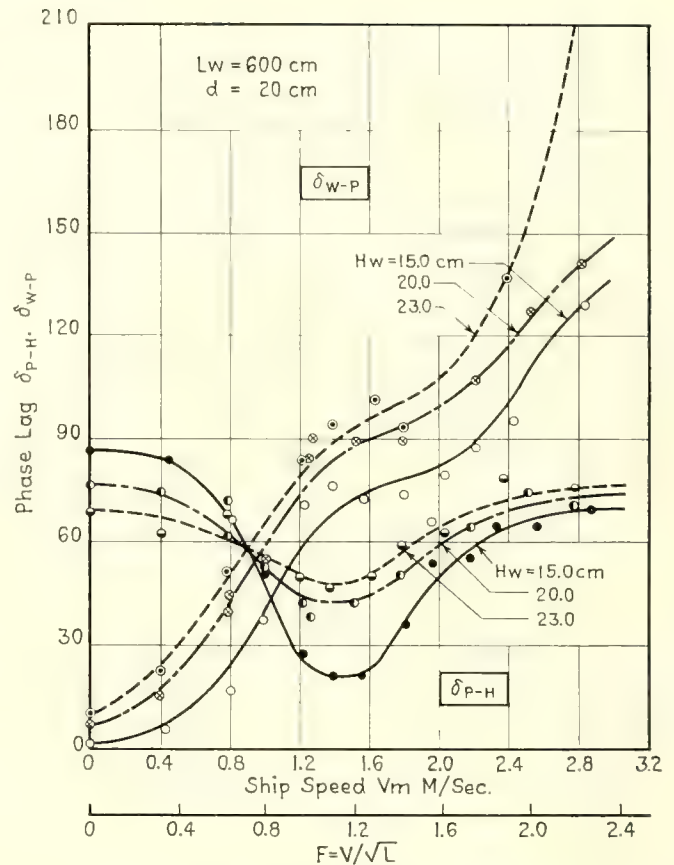


Fig. 4 Phase lags heave to pitch, δ_{P-H} , and pitch to wave, δ_{W-P} (from Akita and Ochi, 1955)

three modes of rolling, heaving and side sway, with no pitching, yawing, and surging, is approximated by a ship in side swells without forward speed. Were each mode of the motion independent, it would be represented by equation 2-(25). The motions, however, affect each other and therefore are represented by a system of three coupled equations of nine terms each on the left-hand sides, similar in structure to the pair of heave-pitch equations (2). In the present case, predominating parts of the heaving and side-sway motions are caused by the orbital motion of water in waves which is essentially of equal magnitude in the vertical and horizontal directions. It is not definitely known whether or not one of these motions can be neglected, thus simplifying this problem. The coupling of motions has usually been disregarded in the literature on ship rolling in waves except in the case of roll stabilization. In the latter case (Chadwick and Klotter, 1955a and b), heaving motion was neglected and coupled rolling and side-sway equations were treated.

W. Froude (1861), who first developed a theory of ship rolling, is largely responsible for the lack of an explicit treatment of coupling. Froude's simple and elegant solution satisfied his immediate needs, and, in fact, appears to have provided all the rolling information of prac-

tical value to date. However, theoretical research into rolling since Froude seems to have been rendered ineffective by adherence to his methods. This does not apply to the more advanced theory of roll stabilization.

By intuitive reasoning W. Froude reduced the three conventional coupled equations to the form for a simple harmonic oscillator, equation 2-(25), in which he initially neglected the damping (coefficient $B = 0$). He assumed that a ship with a small beam and draft in comparison to the wave length moves in the same manner as the water which it displaces; i.e., that its center of buoyancy has the same orbital motion as the orbital motion of water particles at its depth. He observed that the resultant of the gravity and acceleration forces in orbital motion is always normal to the water surface at any point on the wave. The exciting moment in rolling is then simply expressed as the product of this resultant force, the metacentric height and the roll angle measured between the ship mast and the local normal to the water surface. However, Froude neglected the contributions of acceleration forces to the magnitude of the resultant and used the constant gravity force in computing the righting moment. His final result is approximately equivalent to roll-side sway coupling, while partially accounting for the heaving motion.

In the form used by W. Froude (1861) the resultant equation of a simple harmonic oscillator without damping takes the form⁴

$$\frac{d^2\phi}{dt^2} = \frac{4\pi^2}{T_0^2} (\phi - \alpha) \quad (7)$$

where ϕ is the angle between the ship mast and the vertical and α is the local inclination of the water surface. Assuming that a ship is upright and at rest before waves take effect, the subsequent motions are given by the equation

$$\phi = \frac{\pi}{2} \frac{H}{\lambda} \frac{1}{1 - (T_0/T_e)^2} \left(\sin^2 \frac{\pi t}{T_e} - \frac{T_0}{T_e} \sin \frac{2\pi t}{T_0} \right) \quad (8)$$

where T_0 is the natural period of undamped ship rolling and T_e is the wave-encounter period. The motions of a ship are shown, therefore, to consist of two superposed systems of oscillations, one at a ship's natural period T_0 , the other at the wave encounter period T_e . In general, the resultant motion will show a series of beats of increasing and decreasing amplitude of individual oscillations. As the ratio T_0/T_e approaches unity, however, the oscillations rapidly grow with each consecutive cycle, reaching catastrophic proportions. Introduction of damping checks the unlimited growth of oscillations, but damping of ships in roll is generally small at zero speed and certain characteristics of an undamped motion persist to a large extent. The most important of these is the existence of a high and sharp peak on the curve of the magnification factor ($\phi_{\max}/\alpha_{\max}$) at synchronism ($T_0 = T_e$).

Most of the usable information on ship rolling comes from the foregoing simple relationships. It consists of recognition of the importance of avoiding synchronism with the waves, $T_0/T_e = 1$, and of having a ship's natural period T_0 sufficiently large, i.e., having a metacentric height sufficiently small, to avoid synchronism with waves frequently met in Nature. These basic conclusions remain valid even when physical conditions of ship operation deviate widely from the idealized conditions assumed by Froude. Ship speed and wave direction were later included as factors by others, but only in the sense of affecting the period of wave encounter T_e .⁵ Charts were constructed (Niedermair, 1936; Manning, 1942) clearly showing the favorable and unfavorable combinations of ship speed and heading. A large amount of observations at sea (for instance Hebecker, 1940, Möckel, 1941) clearly demonstrated the advantage of large natural roll periods for normal types of surface ships as recommended first by W. Froude.

Under the assumptions made by Froude, a ship moves with the surrounding water and, therefore, there is no flow of water relative to the ship. The water flow in waves is not interfered with and the pressure acting on the ship is the same as that which would exist in the water if the ship were not there. This assumption which later became known as the "Froude-Kriloff hypothesis," is the direct consequence of physical conditions for a small ship on long regular waves coming exactly from a beam direction.

In the foregoing treatment by Froude, water disturbances caused by a rolling ship and all ship side-sway motions with respect to water (i.e., apart from the horizontal component of orbital wave motion) were neglected. Rankine (1864a) showed that there is a certain effect on the water flow in the case of deep draft or short waves. Rankine (1864e) also commented on the effects of the vertical acceleration neglected by Froude. Under the physical conditions assumed by Froude, these considerations were only second-order corrections and did not modify the simple basic conclusions. Apparently the matter did not receive further attention.

2.21 Deviations from Froude's assumptions. The fact that physical conditions change drastically for a long ship in oblique waves has largely been overlooked. Under such conditions a section of a ship's length, which is in a certain relation to a wave, cannot freely participate in the wave motion, being restrained by other parts of the ship which are located differently in respect to the wave form. The flow of water around each section is unavoidable and it brings about changes in pressure distribution and in the forces acting on a ship. These forces have not yet been taken into account in the analysis of ship rolling. A certain amount of theoretical supporting material for the calculation of forces caused by lateral water flows can be found in the work of Ursell (1946, 1948a

⁴ The effects of the mass and restoring force are here represented in terms of the natural period T_0 . This corresponds to equation 2-(32) with $\kappa = 0$ and $\omega_0 = 2\pi/T_0$.

⁵ Neither changes of hydrodynamic forces nor couplings with other modes of motion were taken into account. These inevitably develop with forward speed and with obliquity of wave crests.

and *b*, 1949) and Grim (1956), and also can be derived by an extension of Haskind's (1946) work.⁶

Work oriented in this direction was presented in papers by Suyehiro (1920, 1924) and Watanabe (1938), but as yet, a straightforward formulation of the problem in the form of a set of conventional coupled differential equations of motion has not been presented.

Three-mode rolling, heaving and side-swaying motion in smooth water was described by Ueno (1942) and, in connection with evaluation of hydrodynamic pressures, by Grim (1956). It was shown that, in this case, heaving motions do not exert a significant influence on roll and side sway. This, however, has no significance as far as the influence of heaving on rolling in waves is concerned. This was discussed as an isolated feature by Rankine (1864a).⁷ Side sway is significant in defining the total amount of motion damping. In general, without a fully stated mathematical formulation for rolling in waves, similarities and differences between rolling in waves and rolling in smooth water cannot be brought to light.

Suyehiro (1920, 1924) and Watanabe (1938) presented experimental data on ship rolling in side waves with particular attention to side sway and side drift. The horizontal component of the orbital motion of a ship's center of gravity in beam waves was found to vary from 0.71 to 1.24 of the movement of water particles in waves; i.e., of the amount assumed by W. Froude (1861). As a result, the apparent center of rolling shifted over a range from below the keel to above LWL. It also shifted to the lee side of a ship, indicating a heaving component in the motion.

Under Froude's assumption, conditions are symmetrical on the lee and windward sides of a rolling ship since the passing simple gravity waves are assumed to be undisturbed by the ship. In reality, there is a large amount of ship-wave interference. The energy dissipated in damping as a result of rolling and side-swaying is taken out of the wave, reducing its amplitude. Visually, a relatively becalmed area is found on the lee side of a ship. The forces exerted by waves are, therefore, greater on the windward side, and a net force causing a ship to drift to leeward is generated in addition to that caused by the wind.

As has been mentioned earlier, the only features of the rolling problem considered in practice are the significance of synchronism and the importance of damping in controlling ship motions. Realization of the significance of synchronism leads to specification of the period of roll which is controlled by choosing a suitable metacentric height. In the past other aspects of the rolling motion had only academic interest and were not visibly connected with practical needs. They become important,

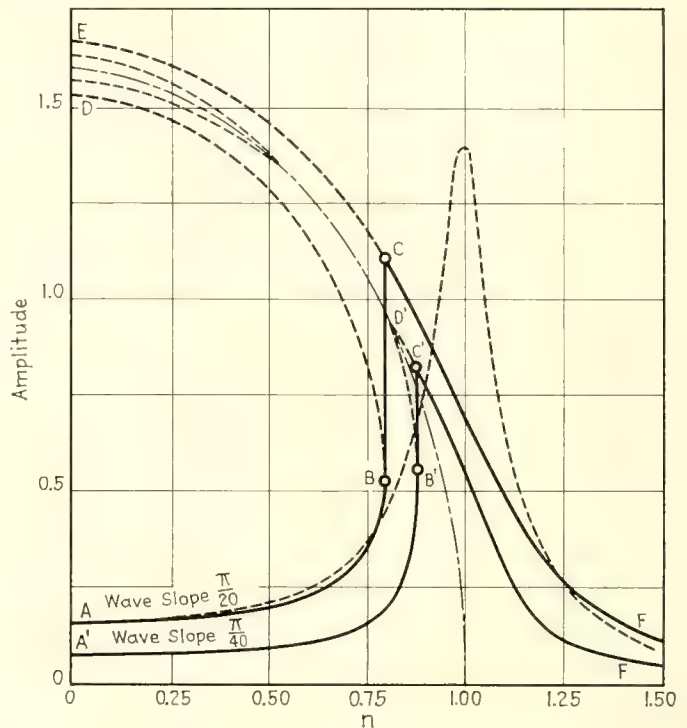


Fig. 5 Magnification factor plotted versus tuning factor in calculated steady nonlinear rolling of a ship in regular waves (from Vedeler, 1953). Here "amplitude" designates ratio of rolling amplitude to maximum wave slope, and $n = T_0/T_e$, where T_0 is natural period of rolling for very small amplitudes and T_e is wave period

however, if coupling with other modes of motion is considered. This will be discussed later.

2.22 Nonlinearities in rolling in side waves. Later studies of the rolling of ships concentrated almost entirely on the effects of nonlinearities. As will be discussed later in connection with motions in irregular seas, this does not appear to be a fruitful direction. The experimental work concentrated on nonlinearity of damping while the theoretical work emphasized the nonlinearity of the righting moment. The first has been discussed in Sections 2-5.3 and 5.32, and Fig. 2-21 shows that nonlinearity of damping does not affect significantly the nature of free rolling.

The righting arm h is connected with metacentric height \overline{GM} by the expression

$$h = \overline{GM} \sin \phi'$$

where ϕ' is the angle of inclination of a ship with respect to the water surface. For simplicity, W. Froude (1861) made the usual assumption that for small angles of roll

$$h = \overline{GM} \phi' \quad (9)$$

Froude showed that this expression (with \overline{GM} assumed as constant) was true in fact for a ship form closely resembling the battleships of his day. The rolling motion resulting from the foregoing assumption is "isochro-

⁶ Investigations of a long ship in oblique waves, neglecting the water flow effects, were made by Kriloff (1898) and Cartwright and Rydill 1957. In the last reference cross couplings were not considered.

⁷ The effect of heaving may be important for a small ship in steep beam waves. The reduction of the apparent gravity force by the water acceleration at wave crests may drastically reduce the righting moment.

nous"; i.e., the natural period T_0 is constant and independent of the angle of roll.

Subsequent theoretical studies concentrated on the dynamics of nonisochronous motion; i.e., the motion resulting from a righting arm differing from the one shown by equation (9). Two relatively recent examples of such work will be cited.⁸ Vedeler (1953) assumed the equation of motion for rolling in smooth water to be

$$\ddot{\phi} + b_1\dot{\phi} + b_2\phi^2 + \omega^2(\phi - k\phi^3) = 0 \quad (10)$$

In this case nonlinearity in damping as well as in righting arm is included. When a simple harmonic exciting function is put on the right-hand side and the equation is solved, the "magnification factor" (the ratio of the amplitude of rolling to the wave slope) is found to be (after sufficient time to damp out transients) that shown in Fig. 5. In this figure n designates the "tuning factor" $n = T_0/T_c$.

The dotted curve, peaked at $n = 1$, indicates the usual form of the magnification-factor curve for a simple harmonic oscillator. The dotted curves swinging upward and to the left result from nonlinear equation (10). To the left of $n = 0.80$ the curves indicate an unstable condition and only the part to the right, shown by a solid line, is considered to apply in reality. This part is characterized by the discontinuous jumps from one branch of theoretical curve to the other along vertical lines $B-C$ and $B'-C'$.

The other example is from Baumann (1955). The equation of rolling in regular side waves is written as

$$J\ddot{\phi} + B(T_s) + \Delta[1 + \beta(t)]h(\phi') = 0, \quad (11)$$

where

J = moment of inertia (including added water masses)

ϕ = angle of ship with respect to vertical

ϕ' = angle of ship with respect to the normal to wave surface (i.e., to "apparent vertical")

B = damping coefficient as a function of period of ship oscillation of a given amplitude

Δ = weight of a ship

β = vertical acceleration in waves in terms of gravity acceleration

$h(\phi')$ = righting arm as a function of inclination ϕ'

T_s = ship period as a function of amplitude of oscillation

Froude investigated the motion only for small amplitudes of rolling for which equation (9) can be assumed valid. Baumann investigated equation (11) for all angles up to the angle ϕ_u at which the righting arm vanishes. Baumann evaluated the equation numerically and presented the results in numerous graphs for different forms of righting-arm curves. An example is reproduced in Fig. 6. The abscissa is the ratio of the square of the period of rolling in waves T_w to that in smooth water at small angles T_0 . The ordinate is the ratio of the amplitude of the angle ϕ , designated as $\bar{\phi}$,

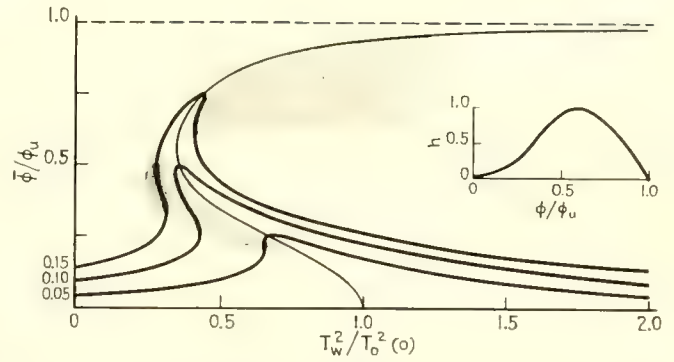


Fig. 6. Calculated steady nonlinear rolling of a ship in regular waves (from Baumann, 1955). $\bar{\phi}$ is rolling amplitude ϕ_u angle of list at which righting arm vanishes, T_w is period of rolling in waves and T_0 (O) is natural period of rolling at very small amplitudes

to the angle ϕ_u for the vanishing righting arm. The insert sketch shows the curve of righting arm for which computations were made. The curves are labelled according to the wave height-to-length ratio, and the data represent the steady ship rolling after transients are damped out.

The appearance of the curves in Figs. 5 and 6 is striking and has been used by some to explain lurching. An extremely important feature is, however, overlooked; namely, these figures describe a fully established motion, which results from a sustained action of regular waves after all transients are damped out. However, waves in nature are never regular, and the sustained excitation needed to develop the indicated behavior does not occur. Even the so-called "regular swells" show on the records a large degree of irregularity. It should be emphasized that the behavior indicated by Figs. 5 and 6 will not manifest itself after a few oscillations, but requires a continuous sustained excitation. In realistic sea conditions, high waves usually are encountered in small groups, and two or three waves in such a group may exhibit an apparent regularity. This is not sufficient to bring about the odd behavior described in the figures. It is, therefore, the author's opinion that this type of research in rolling has taken an artificial direction. It cannot be too strongly emphasized that investigations of ship motions in regular waves are meaningful only when considered as the material to be further operated upon by statistical methods in order to represent the ship behavior in the natural irregular sea.

W. Froude (1861-1875) did not discuss explicitly the behavior of ships in irregular seas but he did emphasize transient behavior in rolling. In this respect his conclusions appear to be more realistic than the conclusions of later writers based on the implied regularity. The author suspects that if transient instead of sustained behavior were considered in the foregoing examples, the action of a nonlinear system in the first few oscillations would not differ materially from a linear one, with suitably chosen constant coefficients.

⁸ An additional paper by Robb (1958) has just appeared.

2.3 Six-Component Ship Motions in Waves: 2.31 Motions with rudder fixed. In Sections 2.1 and 2.2 two simple and yet realistic cases were considered in which a ship's motions are confined either to the longitudinal plane of symmetry or to the transverse yz -plane. For both of these cases the motion was described completely by three coupled differential equations, and was well approximated by two. A ship traveling obliquely to the direction of wave crests, on the other hand, moves in all six modes. Assuming that the rudder is fixed in neutral position, such a motion can be described by six coupled differential equations. Kriloff (1898) formulated these equations and presented a thorough discussion of their solution together with numerical examples. This analysis appears to have been ahead of its time and no further use of it was made in naval architecture. Later, the subject was developed by Bairstow, et al (1913-1914, 1916-1917) in connection with aeronautical engineering and the information on it can be found conveniently in Jones (1933). These methods of analysis have subsequently been applied to the investigation of stability of submarines and torpedoes. A reference to the SNAME T. and R. Bulletin No. 1-5⁹ will be useful in this connection. It appears, however, that in all references certain features not essential to the particular problem on hand were left out and the reader is warned to be cautious in this connection.

The large number of parameters involved in the six-mode motion makes it necessary to deviate from the previously used simple notation for the coefficients and to follow a notation generally similar to the one used in the above-mentioned references. The symbols will correspond to the NACA convention included in the list of symbols in Chapter 2. The complexity of the motion and the many angles involved make it advisable to establish a co-ordinate axes system fixed in the ship with the origin at the center of gravity. The accelerations, defining the inertial forces in the following equations, are, therefore, those measured by accelerometers installed in a ship so as to read translational accelerations along the x , y , and z -axes of the ship's co-ordinate system or rotational accelerations about these axes. Equations (1) representing Newton's second law for heaving and pitching, are now extended to:

$$\left. \begin{aligned} m(\ddot{u} - vr + wq) &= X \\ m(\ddot{v} - wp + ur) &= Y \\ m(\ddot{w} - uq + rp) &= Z \\ I_x \dot{p} + (I_z - I_y)qr &= L \\ I_y \dot{q} + (I_x - I_z)rp &= M \\ I_z \dot{r} + (I_y - I_x)pq &= N \end{aligned} \right\} \quad (12)$$

The left-hand sides of the equations now include dynamic (often referred to as gyroscopic) coupling terms. In the case of a surface ship, the hydrodynamic forces X , Y , Z and moments L , M , N depend on positions, velocities and accelerations so that, for instance,

$$X = X(x, y, z, \theta, \phi, \psi, u, v, w, p, q, r, \dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}) \quad (13)^{10}$$

and similarly for Y , Z , L , M , N .

If a hydrodynamic force, for instance X , is known and designated X_0 at a certain instant t , its value at $t + dt$ is expressed by a linearized Taylor expansion as

$$X = X_0 + \frac{\partial X}{\partial x} dx + \frac{\partial X}{\partial y} dy + \frac{\partial X}{\partial z} dz \dots \text{etc.} \quad (14)$$

covering all terms in equation (13).

There are, therefore, 18 coefficients (derivatives) defining hydrodynamic forces on the right-hand side of each of equations (12) or 108 coefficients in the set of six simultaneous equations. These coefficients refer to a ship oscillating in smooth water. Additional terms must be added to represent the wave-excited forces.

Equations (12) are written for the body axes, so that, for instance, moments of inertia remain constant despite changing attitude of a ship. The quantities on the right-hand sides of these equations depend on the relative instantaneous orientation of body axes with respect to the moving axes in which the xy -plane remains parallel to the mean water surface, or, in other words, on the angle between the z -axes of these two co-ordinate systems. A suitable transformation of the hydrodynamic and hydrostatic forces into the forces with respect to body co-ordinates must, therefore, be carried out. For instance, an added vertical buoyancy force acting on a rolled and heaved ship must be resolved into its components along the ship's z and y -axes.¹¹

In application to airplanes the problem is simplified in that the forces do not depend on position, and aerodynamic forces acting on wings (with certain exceptions) do not depend on accelerations. Coefficients of the form $\partial X / \partial x$, and $\partial X / \partial \dot{u}$ vanish, therefore, and only six coefficients depending on linear and angular velocities u , v , w , p , q , r remain in each equation.

For a submarine the forces do not depend on position but do depend on accelerations. Derivatives with respect to \dot{u} , \dot{v} , \dot{w} , \dot{p} , \dot{q} , \dot{r} are therefore retained.

For a ship floating on the water surface, the forces also depend on position, and the complete set of derivatives indicated by equations (13) and (14) must be retained in principle. In all applications, however, a few derivatives can be omitted, by inspection, as having zero or near zero value.

Even in the simplest case of an airplane which involves only 37 derivatives (Jones, 1933), it is not possible to solve formally the set of six equations (12) as this has been done for the simple two-mode system shown by equation (2). Only a numerical step-by-step integration is possible, the laboriousness of which did not permit

⁹ Technical and Research Bulletin No. 1-5, "Nomenclature for Treating the Motion of a Submerged Body Through a Fluid," SNAME, April 1950.

¹⁰ The reader's attention is called to the two systems of frequently used notations: $u \equiv \dot{x}$, $\dot{u} \equiv \ddot{x}$; $q \equiv \dot{\theta}$, $\dot{q} \equiv \ddot{\theta}$; and so on.

¹¹ The data on co-ordinate transformations will be found in SNAME T. and R. Bulletin 1-5 and in St. Denis and Craven, 1958. A thorough discussion of co-ordinates will be found in Kriloff (1898).

such a solution in the past.¹² With the present availability of high speed computing machines the solution probably can be accomplished.

In application to a surface ship, it is necessary to evaluate the complete trajectory of a ship moving in waves, since the submersion of the bow or of the gunwale, for instance, must be found, as well as accelerations in various directions. In the case of airplanes the problem was further simplified by applying the equations of motion to the study of stability rather than trajectory. Furthermore, the most important practical airplane problem is stability in straight flight. For infinitesimal deviations from straight flight, equations (12) separate into two sets of three coupled equations. The first set represents the motion in the plane of symmetry and involves the first, third, and fifth equations. The second set represents asymmetric motions governed by the second, fourth, and sixth equations. Alternately, the properties of steady three-dimensional motion, such as spinning of airplanes, were investigated considering the balance of forces, but not the stability. This eliminated the dotted terms on the left-hand sides of equations (12).

Despite the complications involved in application of equations (12) to surface-ship motion in waves, the author believes that an effort in this direction should be made. The primary objective is to establish the relative significance of various derivatives and of the degree of coupling between equations. Large labor saving in future work will result from the elimination of any equation which will be demonstrated to have but a weak effect on other equations. Since only orders of magnitude are required in this connection, the work can be attempted despite the fact that many derivatives can be only crudely estimated at present. A complete step-by-step integration once accomplished for a typical ship will permit the devising of simplified calculations in the future by justifying neglect of certain cross-couplings and of many derivatives.

In using step-by-step integration, equations (12) are not limited to small disturbances, only each increment of time dt must be small. Large total motion can be represented by assigning suitable values of coefficients for each successive step. One of the most important effects in a six-component motion is the drastic cyclic variations of the coefficient $\partial N/\partial \psi$ (i.e., the yawing moment due to the angle of yaw) caused by the bow submergence and emergence. It becomes particularly important in a following sea and at $\omega \rightarrow 0$ it may possibly lead to broaching.

¹² Except in a few limited cases (Jones, 1933, p. 125). Kriloff (1898) presented an example of a complete solution for the motions of a cruiser at three forward speeds. This was based on the assumption of the fore-and-aft mass distribution symmetry and on the "Froude-Kriloff hypothesis." Under this hypothesis all external forces acting on a ship were assumed to be caused by hydrostatic water pressures corrected for the pressure gradient in waves (for the "Smith effect"). The distortion of the water flow by the presence of a ship was neglected. Kriloff's work is particularly important in that the trajectory of the motion was computed and transient responses were included.

2.32 A degree of freedom added by the use of a rudder. For investigations of the directional stability of ships, sinusoidal motions of the rudder with a predetermined amplitude were sometimes used, producing a sinusoidal ship path and a harmonic rolling. The effect of the rudder so used can be represented in the equations of motions as an added exciting side-swaying force and a yawing moment. Such a predetermined motion of the rudder would no more assure the maintenance of the specified mean heading in waves than would a stationary rudder.

To maintain a specified mean heading, the rudder must be moved in response to ship motions induced by waves and the nature of the rudder motions is not known in advance. Ordinarily the rudder is moved only in response to a yawing disturbance, but its motion generates a side-swaying force as well as the desired yawing moment.¹³ Furthermore, the rudder neither is capable of immediately checking the yawing motion, nor as a rule, produces the corrective yawing moment at the same time the exciting moment due to waves occurs. The time relationship between the wave-excited yawing moment and the rudder-produced corrective moment is a complicated function involving the phase lag in the ship response to waves as well as the properties of the rudder-controlling devices. The force and moment exerted by the rudder are generally proportional to the rudder angle of attack and therefore are not only functions of the rudder angle δ but also of the ship yawing velocity r . They are, furthermore, affected to some extent by the pitching attitude θ and velocity $\dot{\theta} = q$. The rudder angle δ is, therefore, a time-dependent variable (not known in advance) of the same significance as the ship-displacement variables, x , y , z , θ , ϕ , and ψ . Seven degrees of freedom are involved here.

The six equations (12), must be supplemented by an equation defining the rudder angle δ as a function of the ship yawing angle, $\delta = \delta(\psi)$, in which the integral and first and second derivatives of ψ may be included. Right-hand sides of the first six equations will have terms dependent on δ of greater or lesser importance. Right-hand sides of the second and sixth equations will have the important added terms $(\partial Y/\partial \delta)d\delta$ and $(\partial N/\partial \delta)d\delta$. Addition of only these two terms will suffice in linear stability analysis based on infinitesimal ship deviations from steady rectilinear motion. In a step-by-step integration for large deviation from steady conditions, and in the analysis of a ship's turning, the foregoing terms will be modified by the obliqueness of the water flow at the rudder. These derivatives become functions of the yawing velocity $(\partial Y/\partial \delta) = Y_\delta = Y_\delta(r)$ and $(\partial N/\partial \delta) = N_\delta = N_\delta(r)$. The effect of a ship's trim on the rudder-force derivative also may be taken into account; i.e., $Y_\delta = Y_\delta(\theta)$ and $N_\delta = N_\delta(\theta)$. This effect of ship's trim θ becomes particularly important during a part of the oscillating cycle

¹³ The importance of the side-swaying force in a ship's response to rudder motions is brought out by Davidson and Schiff's (1946) theory of dynamic stability of ships on course and of a ship's turning.

in which $(\partial N/\partial \psi)$ acquires a large positive (i.e., destabilizing) value because of the submergence of the bow and emergence of the stern, while the value of the $(\partial N/\partial \delta)$ is reduced.

The presence of the gyroscopic coupling terms on the left-hand sides of equations (12), in addition to the less evident couplings due to derivatives on the right-hand sides of equations, spreads the effects of the rudder motions through all six equations.

A limited problem of rudder-controlled motion will be discussed in the next section.

2.33 Approximations based on a limited number of degrees of freedom. In towing-tank tests, artificial limitation of the degrees of freedom of model motion can often be useful. This may permit a relatively simple analysis of a few free modes and evaluation of certain derivatives or wave-excited forces. However, the subject of this section will be a simplified analysis of the motion of *free models* in regular long-crested waves, based on a more or less justifiable neglect of certain cross-couplings or, in other words, an analysis based on the assumption that the motion can be described by a few selected degrees of freedom.

(a) *Two quasi-rational cases.* First, two cases will be cited which, although not mathematically rigorous, can be accepted intuitively as mathematically and physically compatible. In the first of these cases, it is assumed that surging motion has no significant effect on other motion modes. This is an extension of the practice of neglecting the effect of surging on coupled heave-pitch motion. The six equations (12) with 108 stability derivatives are thereby reduced to five with 75 derivatives. An illustration of practical application of the five-degrees-of-freedom analysis to a relatively simple airplane case will be found in the paper by Westervick (1957), in which further references are also given. This paper is an excellent example of the application of the five equations of motions (with some derivatives omitted by judgment) to the determination of the desirable automatic control function to achieve a specific objective.

The second quasi-rational multi-mode case is obtained by further assuming absence of rolling; i.e., $\phi = \dot{\phi} = \ddot{\phi} = 0$.¹⁴ The system of equations is thus reduced to four with 48 derivatives on the right-hand sides. Gyroscopic cross-coupling moments on the left-hand sides of equations (12) are also eliminated since p (i.e., $\dot{\phi}$) = 0. Moreover, a good deal of the difficulties in relationships between co-ordinate systems fixed in a ship and fixed in water surface is eliminated, because the rolling angle is the only angle which normally reaches large values. The author feels that application of this simplified analysis to model experiments in oblique waves is one of the most useful projects which can be undertaken with currently available theoretical and physical facilities.

The case just outlined corresponds closely to physical reality in the case of roll-stabilized ships. Various means of roll-stabilization reduce the roll angles to such small

values that the cross-coupling effects of rolling become quantities of second order. Stabilizing water tanks evidently have no effect on hydrodynamic derivatives. Anti-rolling fins, in a conventional arrangement near amidships, would cause such a small lateral force and yawing moment as to be negligible in comparison with other forces and moments acting on a ship.

After the solution for motions is completed with $\phi = \dot{\phi} = \ddot{\phi} = 0$, the values of the variables determined can be inserted into the fourth equation of (12). This will yield the amplitude and phase of the rolling moment which must be developed by the roll-stabilization system; i.e., will lead to a rational design of this system.

(b) *Arbitrary selection of motion modes.* Often an analysis limited to a few degrees of freedom will furnish information on a particular aspect of the motion despite the lack of physical reality in such a limitation. It is necessary, however, to remember the nature of the limitation, to be satisfied with an answer limited to certain conditions and not to expect a universally valid solution.

Rydill (1959) has undertaken such a limited approach. His objective was to find the amplitude of yawing oscillation of a ship in long-crested waves with and without rudder control, to evaluate the amount of rudder motion and to appraise the effect of alternate rudder-control functions with $\delta = \delta(\int \psi dt, \psi, \dot{\psi}, \ddot{\psi})$. Two linearized coupled equations in side sway and yaw, i.e., the second and sixth equations of (12), were considered and the possible effect of all others was neglected. Since the motions are limited in the linearized theory to small values of the variables, the rudder forces and moments were taken as proportional to the rudder angle δ , and the (in this case) second-order effect of the yawing velocity $\dot{\psi} = r$ on the rudder angle of attack was neglected. The analysis was first made for long-crested regular waves and was subsequently generalized to long-crested irregular waves. This last step was based on the spectral analysis methods which will be described in Section 3.

The action of the rudder involves a chain of successive events. First, there is a lag in a ship's response to wave excitation which implies lag in the reading of the sensing elements of a control mechanism. Next, there are various lags of other responses to the signal before the desired rudder motion is accomplished. Finally, there is a lag in the ship's response to the rudder movement. While these features can be incorporated in the differential equations of motion, Rydill demonstrated how they can be better analyzed by the servomechanism theory.

Probably the most valuable result of the paper is the demonstration that ship responses to rudder movements are large at low wave-encounter frequencies and become small at high frequencies. The frequent and rapid rudder motions induced by conventional control systems in head seas are, therefore, practically useless in reducing ship-yawing motions. The motions with and without rudder control are essentially the same in this case. The rudder control becomes indispensable in quartering seas

¹⁴ This case was suggested to the author by Prof. E. V. Lewis of the Davidson Laboratory, Stevens Institute of Technology.

at low frequencies of wave encounter. Various characteristics of the rudder-control function $\delta = \delta(\int \psi dt, \psi, \dot{\psi})$ were brought out, verifying and extending the previous work of Schiff and Gimprich (1949).

While giving a large amount of useful information, the results demonstrated at the same time the apparent inadequacy of the linear theory and of neglecting coupling with the pitching motion. The amplitude of ship yawing in waves found by the above analysis appears to be too small judging by observations at sea. The tendency of a ship to yaw in quartering and following seas appears too weak in the theoretical results and does not explain the steering difficulties encountered on ships. In particular theoretically found yawing does not lead to broaching. The conclusions therefore appear doubtful at $\omega_e \rightarrow 0$.

As stated earlier, the author believes that practical steering conditions at sea depend primarily on the cyclical variation of the derivatives $\partial N/\partial \psi$ with bow and stern immersion and emersion. From Rydill's findings, the effect of this variation becomes more important with the decrease of frequency in following seas. These effects can be brought out by a numerical step-by-step integration of four coupled equations of motions; i.e., neglecting surging and rolling. The work of Davidson (1948) on broaching can be cited as an example of the limiting case $\omega_e = 0$, and the use of step-by-step integration. The equation for surging was not formally considered, but the action of a wave in increasing the speed of a ship poised on its advancing flank was taken into account.

The aim of Rydill's project was determination of the most advantageous rudder-control method. Rydill called attention to the fact that De Santis and Russo (1936) and Allan (1945) cited sea observations indicating the influence of rolling on the directional control of ships. Nevertheless this effect was considered less important than other yawing moments, and rolling was neglected. Other limited groupings of degrees of freedom may be chosen in order to bring out the rolling characteristics of ships. Rolling is recognized as one of the most disturbing motions of ships at sea and its more pronounced manifestations were forcefully described by Captain Patterson (1955). While rolling can seldom be relieved by reduction of speed, it frequently delays ships by necessitating course changes. Even if it were assumed that most of the modern passenger liners will be equipped with antirolling devices, most of the cargo ships probably will not and improvement of rolling characteristics must remain an important item of research.

In Section 2.2 analysis was made of rolling in side waves at zero speed and in these conditions many coupling effects are inactive. They all come into play with increase of a ship's forward speed and with obliqueness of a ship's heading to wave crests. The couplings may act as added excitation forces and moments or as restraining ones. These latter may cause more rapid attenuation of rolling amplitudes than can be expected from the damping in roll alone. From observations at sea and during tests in oblique waves in a towing tank, the author has

gained the strong impression that rolling motion was *controlled* (rather than excited) by waves and differed in nature from the free rolling in smooth water. An investigation of coupling effects in a ship's rolling motion appears, therefore, necessary for a realistic presentation of rolling at sea. The coupling effects result not only from the hydrodynamic derivatives on the right-hand sides of equations (12), but also from the gyroscopic couplings indicated on the left-hand sides of these equations. These latter are suspected to be important in view of the generally small damping in pure rolling and the small moment of inertia I_x .

It is known from empirical experience in connection with ship behavior at sea that cross-coupling of rolling and yawing motions is important. The significance of strong yaw-heel (and therefore yaw-roll) coupling has been forcefully brought out by the experience of gyro-stabilizing the SS *Conte-di-Savoia* (De Santis and Russo, 1936). The stabilization system was not adequate to control rolling of the ship as designed, but apparently became much more effective when a skeg was added, increasing the directional stability of the ship.

The investigation of rolling of ships in waves can be approached by two methods. In the first method, it may be assumed that rolling is strongly influenced by other motions, but that it has little effect on these other motions. Analysis of a four-mode motion (five degrees of freedom including the rudder motion) can first be made as outlined in Section 2.33 (a); i.e., omitting surging and rolling. The resulting values of all time variables can subsequently be inserted into the fourth of equations (12), expressing the equilibrium of moments about the x -axis. The solution of the differential equation in ϕ will describe the rolling motion. The author believes this approach to be a close approximation to physical reality.

A second and reasonably realistic method of roll determination may consist of omitting the equations in surging and heaving and considering an assumed four-mode motion in side sway, rolling, pitching and yawing, only. The author considers this approach less realistic and at the same time much more difficult than the first because of the complications connected with various co-ordinate systems at large angles of rolling.

The author has already emphasized the important effect of the ship bow submersion on yawing motion; strong roll excitation by the yawing can also be expected in this case. The usefulness of the small-displacement solution of the differential equations of motions, i.e., with constant coefficients, is therefore questioned. The analyses are expected to yield realistic results if step-by-step integration is used with variable coefficients appropriate to the state of motion at each particular instant.

Among other aspects of realistic motion in waves, the described analyses will yield information for estimating the probability of a ship's capsizing in following or quartering seas. Deep bow submergence and stern emergence create strong yawing instability simultaneously with decreased rudder effectiveness. Violent yawing motion can therefore develop leading to a large yaw-induced

rolling moment. The situation is particularly dangerous for a small ship on an advancing flank near the crest of a large wave. The subtraction of the vertical acceleration of orbital water motion from the force of gravity leads to a reduction of the righting moment at the time when the above mentioned rolling moment occurs. The wind force acting on the superstructure will further aggravate the situation.

The analyses can also be expected to lead to improvement in the rolling characteristics of ships through better proportioning of appendages. Since rolling is caused by waves directly and also is induced by yawing, the resultant amplitude of roll depends on the magnitudes and relative timing of these two effects. Certain proportions of the appendages (skegs and bilge keels) may secure the optimum relationship of these effects. Thus, considering the stability of an airplane in circular flight, Korvin-Kroukovsky (1929) pointed out the fact that the ratio (yawing moment due to yawing velocity)/(yawing moment due to side slip) strongly predominates over all other factors in determining the airplane behavior. Furthermore this ratio is much more important in defining the stability of an airplane than the absolute values of the moments. Simple empirical design rules were then formulated, based on this ratio, for proper proportioning of wing dihedral and vertical tail surfaces. This simple action was possible, however, only because a complete investigation of the problem by Bairstow and his associates was available.

Information on miscellaneous coefficients of equations (13) and (14) can be obtained by:

i Experiments in towing tanks with suitable restraints on models and the use of oscillators and/or rotating arms.

ii Theoretical methods, following the procedure formulated by Haskind (1946, section 2-6) and extending it to evaluate derivatives in unsymmetrical motion. This is limited to idealized ship forms.

iii Theoretical method using strip theory, closely following the procedure used by Korvin-Kroukovsky and Jacobs (1957) for motions in the plane of symmetry and extending it to unsymmetrical motions. The supporting work of Ursell (1949), Grim (1956), and Landweber and de Macagno (1957) was described in Chapter 2. This method is applicable to normal ship forms.

3 Ship Motions in Irregular Seas

Chapter 1 of this monograph is a digest of all the information currently available on the nature of ocean waves. After the sum total of the knowledge on generation and propagation of waves is distilled, there emerge several basic tenets upon which ship motions investigations may be carried forth.

There is, first of all, no existing analytical expression that describes the behavior of the surface of the sea for any specified time and/or space. There are however several semi-empirical expressions which purport to define the energy-frequency characteristics of waves. Of

these wave-spectrum descriptions (Section 1-6) none has gained widespread acceptance, although each has been verified to some extent by measurements.

Then too there are fairly reliable methods for observation and spectrum analysis of waves at sea and these have been thoroughly discussed in Section 1-8.

Finally, there is a hypothesis which suggests that a good model of the sea surface is achieved by the linear superposition, in random phase, of an infinite number of sinusoidal wave components of all frequencies and infinitesimal amplitudes.

Perhaps it will be worth while to discuss the implications of these results as they apply to the three methods of studying ship-motions problems (theory, model tests and full-scale tests). The spectral representation of the sea surface forces statistical treatment of ship motions in all three investigative media. Wave measurement supplies a time history of the waves, made in a certain way, which can be reduced to an estimated spectrum of the seaway which in turn may define some statistical characteristics of the waves. The wave profile lends itself to a restricted deterministic treatment of ship motions both theoretically and in the model tank. The hypothesis defining the composition of surface waves complements the hypothesis that the response of a ship to a sum of sine waves equals the sum of the ship responses to the individual sine waves (if the system is linear). Thus the discussion in Section 2 of this chapter, on the solution of hydrodynamic equations of motion for sinusoidal inputs, becomes a basic building block, for ship-motions prediction based on the linear superposition principle.

The intent of this section is to review the work done in the application of present knowledge of waves to ship motions in irregular seas.

3.1 Probabilistic Versus Deterministic Methods.

These two philosophies were touched on briefly in Section 1-8 where the probabilistic approach to the description of the seaway was justified on the grounds that a general deterministic representation was unknown and that statistics of waves derived from the energy spectrum were adequate to define the state of the sea. In the case of ship behavior there are arguments in favor of a deterministic solution to the problem of ship-wave interaction. There is little that may be said against the derivation of the time history of a ship motion from the given time history of the waves at some point. This is particularly useful in predicting the occurrence of short-duration phenomena such as slamming and bending moments. There is also the notion of reproducing an actual time history of waves in a towing tank (Fuchs, 1956) so that models may be tested in the most realistic conditions appropriate to long-crested irregular waves. Advocates of the statistical approach argue that the basic pertinent information on ship behavior is available through energy spectrum considerations, for considerably less effort, and that any predicted time history of a ship motion neglects the short-crestedness of the waves and is still unwieldy and must be reduced to statistics before the essence of its potential is realized.

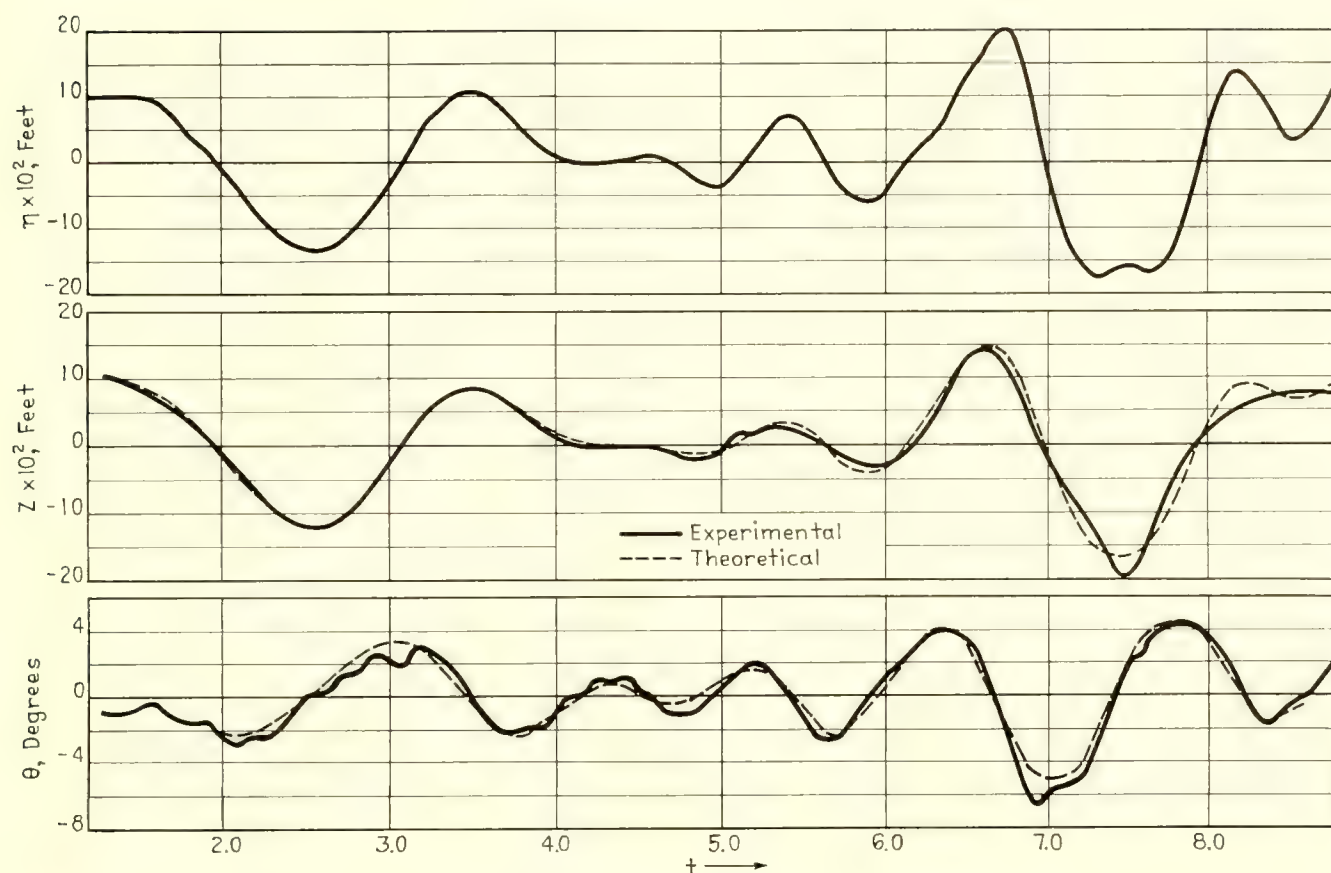


Fig. 7 Analysis of pitching and heaving of a ship model in irregular waves (from Fuchs and MacCamy, 1953)

In the statistical treatment of waves, Section 1-8, sea conditions were assumed to be stationary; i.e., the statistics of the waves remain unchanged with translations in time and/or space. In reality, wave conditions change rather slowly with respect to the observation time and under such conditions the change in the seaway may be approximated by a succession of discrete stationary events. Ship motions, since they result from such stationary sea conditions, are considered (and are observed) to be likewise stationary and are treated as such. The deterministic method does not require the stationary assumption, but like the statistical approach, interpretation of nonstationary events produced by nonstationary events is a problem of no small magnitude.

A deterministic method of calculating ship response will be discussed in the next subsection. The bulk of this section will be devoted to the review of statistical methods of treating ship motions, because almost all research effort to date has been in this direction and it appears to be the most fruitful approach at this time.

3.2 A Deterministic Method for Studying Ship Motions in Irregular Seas. The only known deterministic treatment of ship responses is due to Fuchs (1952) and Fuchs and MacCamy (1953) based on previous work by Kreisel (1945).

Fuchs' (1952) paper provides the mathematics for establishing the relationship between two simultaneously occurring phenomena. Following Kreisel, the relationship between the surface wave elevation and the pressure record measured at a certain water depth is discussed. The relationship between wave elevations at two nearby points, the forces acting on piles, and motions of a rectangular block in waves are also discussed. In Fuchs and MacCamy's (1953) and Fuchs' (1954) papers, these mathematical methods are applied to a ship's heaving and pitching motions at zero speed in irregular long-crested waves. The phase relationships, as well as the amplitudes of motions, are evaluated.

The work of Fuchs and MacCamy is a special case of the earlier (1952) work of Fuchs in linear-prediction theory. Fuchs' theory is extended to accommodate irregular waves and thereby results in the less general linear case. Their treatment is best described by the following quote from their 1953 paper: "...the oscillations of a ship in a complex system of progressive waves of small steepness can be expressed as the convolution type integral of the recorded wave motion and a kernel function which is the Fourier integral of the response of the ship to a sinusoidal forcing function. Kernels for pitching and heaving are computed explicitly for a freely

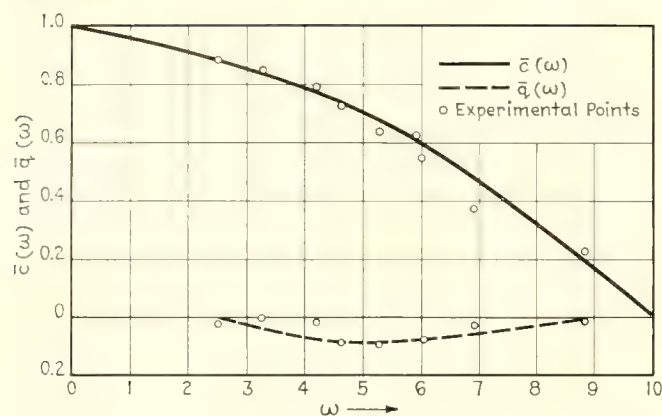


Fig. 8 Empirically determined response factors for heaving of ship model (from Fuchs and MacCamy, 1953)

floating rectangular block using experimental values of added masses and damping coefficients, as determined from oscillations in still water. These kernel functions are applied to recorded irregular wave motions and the predictions are compared with values read from 35-mm motion picture records of the motion. The experimental work has been reported separately by Sibul (1953). Similar investigations have been made with a model ship. In order to avoid the laborious numerical integrations involved in computing the response functions, these were determined experimentally."

The theory of Fuchs (1952) is applied to a ship motion in long-crested irregular waves so that the heaving and pitching motions are expressed by the aforementioned convolution integral

$$E(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \eta(t) K_E(t - \tau) d\tau \quad (15)$$

where the kernel function is given by

$$K_E(\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} T_i(\omega) e^{-i\omega\tau} d\omega \quad (16)$$

$T_i(\omega)$ is the response of the block to a unit amplitude sinusoid of frequency ω . Since the kernel function is a form of transfer function by virtue of $T_i(\omega)$, it is seen that the heave or pitch function, $E(t)$, is a deterministic response which results from linear superposition of a number of simple responses to regular wave trains.

Fig. 7 shows the results of computations made from equation (15) where $K_E(\tau)$ was determined for a towing-tank model. The upper curve is the record of the water surface elevation, the middle one shows heaving of the model, and the lower one pitching of the model. Solid lines show the measured model motions and the broken lines show the motions computed by equation (15) using the responses $T_i(\omega)$ measured in a series of sinusoidal waves. Figs. 8 and 9 show the real and imaginary parts of these responses for heaving and pitching respectively.

In equation (15), the kernel is independent of the particular wave system encountered and consequently needs

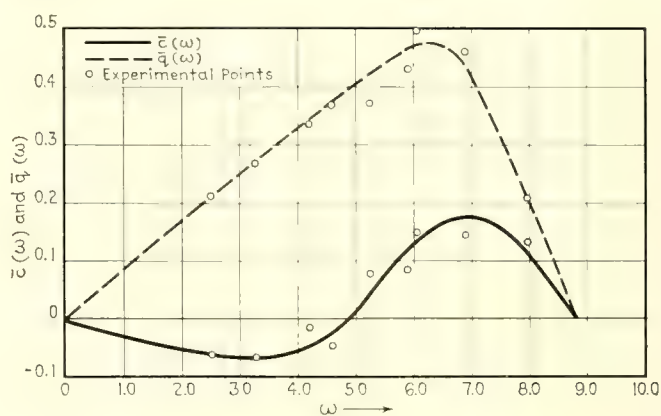


Fig. 9 Empirically determined response factors for pitching of model (from Fuchs and MacCamy, 1953)

to be computed or measured only once for each sea-keeping event of each ship. This is the case in the statistical treatment as well, as will be shown. The appeal of equation (15) lies in the fact that in one operation it separates $\eta(t)$ into its Fourier components, multiplies the Fourier components by their respective unit responses and synthesizes the results into the predicted motion $E(t)$.

The solution of the inverse problem, that of evaluating the model's frequency-response function $T_i(\omega)$ from tests in irregular long-crested waves is accomplished by noting that the recorded wave surface elevation can be represented by

$$\eta(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} a(\omega) e^{-i\omega t} d\omega \quad (17)$$

while the dependent function $E(t)$, shown by equation (15) can, in view of (16) and (17), be written as

$$E(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} T_i(\omega) a(\omega) e^{i\omega t} d\omega \quad (18)$$

Designating the Fourier transform of equation (18) by $b_E(\omega)$, the frequency-response function is obtained as

$$T_i(\omega) = \frac{b_E(\omega)}{a(\omega)} \quad (19)$$

The application of Fourier transform pairs here is reminiscent of the statistical treatment of ocean waves in Section 1-8 and indeed it will be shown that an expression similar to (19) is obtained from the wave and motion spectra, both of which may be considered as Fourier transforms of particular realizations of their respective time histories. In fact, it will be seen that this mathematical model of ship response to waves is essentially equivalent to the statistical model. It fails, however, to achieve the generality of the statistical model in that it treats only the case of long-crested irregular waves.

3.3 Ship Motion Studies Based on Linear Superposition. R. E. Froude (1905) wrote: "Irregular waves such as those commonly met when at sea...are only a compound of a number of regular systems (individually of

comparatively small magnitude) of various periods, ranging through the whole gamut (so to speak) represented by our diagrams [of behavior in regular waves] and more. And the effect of such a compound wave series on the model would be more or less a compound of the effects proper to the individual units comprising it."

This pronouncement of Froude, which antedates the conclusion of oceanographers concerning a realistic model of the sea surface, has become the basic hypothesis for ship motions studies. Fuchs and MacCamy (1953) used it in their deterministic approach, as has been discussed. These, however, were not the first applications of the idea of linear superposition.

Legendre (1933) used this principle in investigating the rolling characteristics of two cruisers by means of theory, towing-tank tests and observations on ships at sea. He summarized his findings as follows:

1 "Rolling computed on the basis of the mean observed wave does not have the character and is much smaller than the observed rolling.

2 "The summation of roll angles, computed on the basis of the observed wave, decomposed into its sinusoidal components, has for its period the natural period of the ship. It is of the same order of magnitude as the observed rolling.

3 "In order to predict the rolling of a projected ship it is necessary to investigate the general characteristics of the actual wave."

Actually the work of Legendre was ahead of his time since neither mathematical nor physical tools were then developed sufficiently to permit a thorough analysis.

Section 2.1 of this chapter treated the ship response to simple harmonic waves and such results in conjunction with a given input (either an energy spectrum or a time history) lead to an output which represents the ship motion. The transfer function, which depends on the geometry and physical characteristics of the ship as well as ship speed and relative heading, is determined either theoretically or by model tests. It will be worth while to discuss the transfer function, before the statistical method is demonstrated.

It was shown in Section 2.1 that the heaving and pitching amplitudes of a ship's motion in sinusoidal waves are expressed as complex amplitudes

$$\begin{aligned}\bar{Z} &= Z_0 e^{i\delta} = Z_0 (\cos \delta + i \sin \delta) \\ \bar{\theta} &= \theta_0 e^{i\epsilon} = \theta_0 (\cos \epsilon + i \sin \epsilon)\end{aligned}\quad (20)$$

where Z_0 and θ_0 are the real amplitudes and δ and ϵ are phase lags of a ship's motions with respect to wave form. When a ship's response to waves is obtained by model tests, the real amplitudes Z_0 and θ_0 and phase lags δ and ϵ are measured directly from the test records. When the amplitudes are computed by the methods outlined in Section 2.1 and Appendix C, the result is first obtained in the form, $C + iQ$, and is subsequently converted to the form indicated by the left-hand sides of equations (20).

For use in connection with irregular wave inputs, the foregoing ship-motion amplitudes are referred to the unit

sinusoidal wave amplitude to obtain

$$\begin{aligned}T_z(\omega) &= \frac{Z_0}{a} e^{i\delta} = \frac{Z_0}{a} \cos \delta + i \frac{Z_0}{a} \sin \delta = \bar{e}_z(\omega) + i \bar{q}_z(\omega) \\ T_\theta(\omega) &= \frac{\theta_0}{a} e^{i\epsilon} = \frac{\theta_0}{a} \cos \epsilon + i \frac{\theta_0}{a} \sin \epsilon = \bar{e}_\theta(\omega) + i \bar{q}_\theta(\omega)\end{aligned}\quad (21)$$

and in this form are known as "frequency-response functions" (Press and Tukey, 1-1956). St. Denis and Pierson (1-1953) used the term "response amplitude operator" for the square of the absolute value of the frequency-response function. In what follows, one mode of a ship's motion will be considered at a time, and the subscripts z and θ will be dropped. It should be emphasized, however, that the single mode response includes all the effects of coupled motions. The symbol $T(\omega)$ was used by Press and Tukey for the frequency-response function. In the present exposition it will be used only for the absolute value:

$$T(\omega) = Z_0/a \text{ or } \theta_0/a \quad (22)$$

The symbol $T_i(\omega)$ will be used for the complex form shown by equations (21); i.e., indicating both the amplitude and the phase lag.

3.4 Statistical Methods For Studying Motions in Irregular Waves. St. Denis and Pierson (1-1953) treated the relationship between the wave spectrum and a ship's motion spectrum as given by

$$\Phi(\omega_e) = E(\omega_e) T^2(\omega_e) \quad (23)$$

which states that the energy spectrum of a particular seakeeping variable, Φ (ship motion, strain, etc.), at a given heading and speed, equals the product of the encountered wave spectrum and the response operator (transfer function) for those conditions. This presentation introduced, for the first time, probability concepts in conjunction with the linear-superposition theory.

A practical use of this expression is illustrated by Fig. 10. The upper part (A) of this figure shows a wave spectrum, in this case a Neumann spectrum for a 40-knot wind, corrected to the ship's speed of 12 knots (this will be explained subsequently). The symbol ω_e designates the frequency of wave encounter. The middle part of the figure (B) is a plot of $T^2(\omega_e)$, i.e., the square of the absolute value of the frequency response function. The lower part of the diagram (C) is the spectrum of the ship's pitching, $\Phi(\omega_e)$. At each abscissa, corresponding to a given frequency $\omega_e = 2\pi/T_e$, the ordinate of the lower section is the product of the ordinates of the upper and middle sections. Once the spectrum $\Phi(\omega_e)$ is computed, various average characteristics of the ship's motion are obtained from it, by means of relationships given in Section 1-8.6.

Care must be taken in the interpretation of the events depicted in Fig. 10. The Neumann spectrum given there is a scalar wave spectrum; that is, it embodies the directional properties of the waves it describes but does not permit a quantitative evaluation of energy distribution

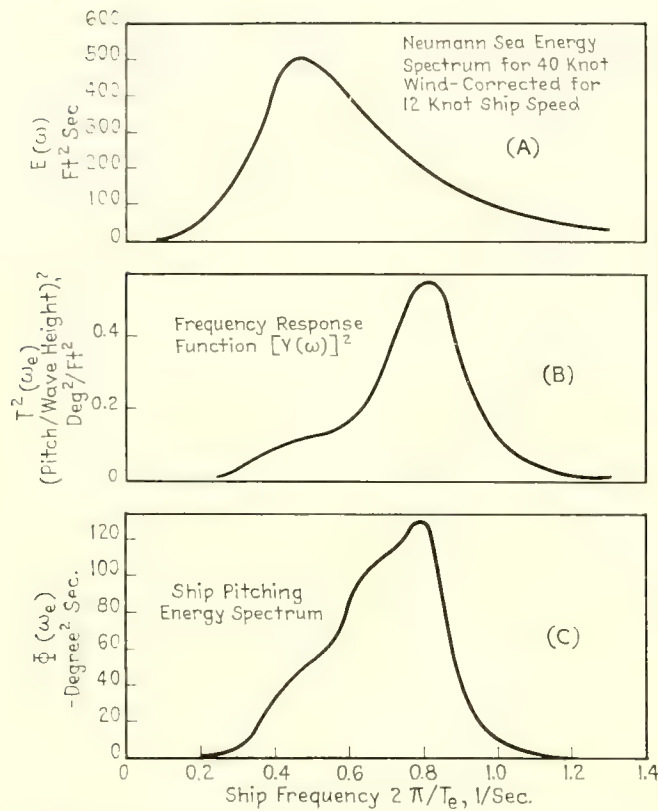


Fig. 10 Energy spectra and response of a ship model in an irregular sea. Series 60, 0.60-block-coefficient model 5 ft long, expanded to 500-ft ship (from Korvin-Kroukovsky, 1956; abstracted from Lewis and Numata, 1956)

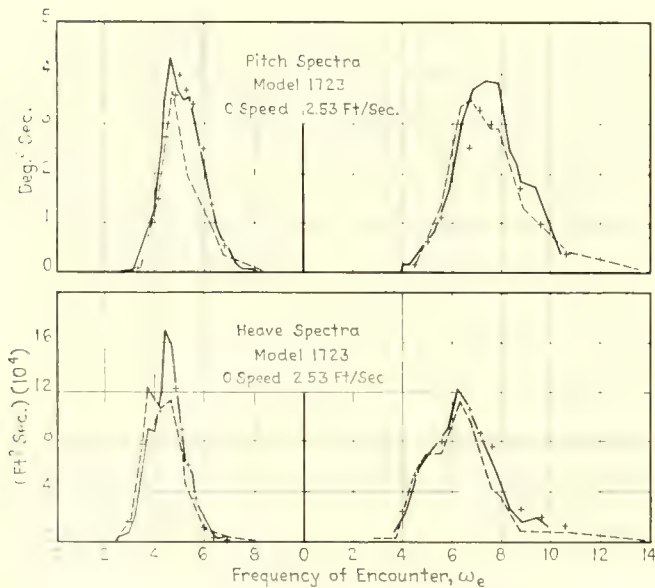


Fig. 11 Motion spectra predicted by three different methods for destroyer model 1723 in irregular long-crested sea: (a) By direct analysis of motions in irregular waves (solid lines). (b) By calculation from wave spectra and experimentally measured responses to regular waves (dash lines). (c) By calculation from wave spectra and analytically computed responses to regular waves (+) (from Korvin-Kroukovsky and Jacobs, 1957)

with direction. This is equivalent to measurement of the wave spectrum at a fixed point or to measurement of the directional spectrum and then integration with respect to direction. In either event, the directional aspects are not recoverable from the scalar spectrum. When the wave spectrum is transformed to a wave-encounter spectrum (for instance, for a ship speed of 12 knots in head seas), the directional implications of the spectrum are ignored so that the computed response presupposes that all the waves traveled in the same direction. If that is the case (a spectrum of swell, for example), the work represented in Fig. 10 is valid; if not, the wave spectrum is inadequate and the transfer function is likewise inadequate, since it was only made for the head-seas condition. Fig. 10 applies to towing-tank tests in irregular, long-crested waves so the procedure is here valid.

The most significant feature of the St. Denis-Pierson work is that it permits evaluation of ship motions in irregular short-crested seas by embodying the product of a directional spectrum with a "directional transfer function." In so doing, it was necessary to develop a complex frequency mapping based on

$$\omega_e = \omega - \omega^2 \frac{v \cos \chi}{g} \quad (24)^{15}$$

in order for all terms of (23) to be in the ω_e domain. The difficulty in such a mapping arises in the inverse transformation which is not unique but results in a quadratic expression which forces the authors to make their transformation in three separate regions to avoid confusion. A further contribution to the elegant treatment of the frequency mapping is the inclusion of the Jacobian in the transformation in order to retain the total energy in the spectrum being mapped.

The true significance of equation (23) is accomplished by a decomposition and resynthesis of its elements to arrive at the response spectrum.

The directional spectrum (either measured or assumed) is divided into three nonoverlapping parts appropriate to the regional divisions just mentioned, so that unique inverses obtain when mapping into the $\omega_e - \chi_e$ plane by equation (24)

$$E(\omega, \chi) = E_1(\omega, \chi) + E_2(\omega, \chi) + E_3(\omega, \chi) \quad (25)$$

Each term in equation (25) is multiplied by the square of the response-amplitude operator (transfer function), such that three expressions of the form

$$E_i(\omega, \chi) T_{\Phi_i}(\omega, \chi, v), \quad (i = 1, 2, 3) \quad (26)$$

result. Each of the terms in equation (26) is then mapped into the $\omega_e - \chi_e$ domain to give three terms of the form

$$E_j(\omega_e, \chi_e) T_{\Phi_j}(\omega_e, \chi_e), \quad (j = \text{I, II, III}) \quad (27)$$

The spectrum of the ship response is then obtained by integration of the terms in (27)

¹⁵ The symbol χ represents the relative heading of ship to wave as given in St. Denis and Pierson (1953). This is equivalent to θ in Chapter 1 and is retained in this account of their work.

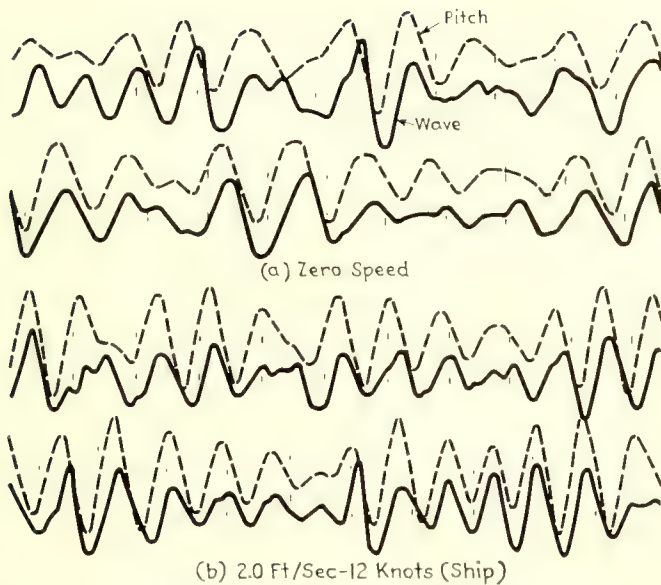


Fig. 12(a) Samples of ship pitching and wave records in irregular head seas (from Lewis and Numata, 1956)

$$\Phi(\omega_e) = \int_I E(\omega_e, \chi_e) T_{\Phi}(\omega_e, \chi_e) d\chi_e + \int_{II} E(\omega_e, \chi_e) T_{\Phi}(\omega_e, \chi_e) d\chi_e + \int_{III} E(\omega_e, \chi_e) T_{\Phi}(\omega_e, \chi_e) d\chi_e \quad (28)$$

where the subscript on the integral sign indicates the transformed region where the operation is performed. Equation (28) is the realization of the hypothesis given in equation (23) for short-crested irregular waves.

The transfer function for certain ship motions may be estimated analytically or measured in a towing tank which permits oblique seas testing. A formal description of the foregoing procedure directly applicable to towing tanks was given by Marks (1957). In this case, $T_{\Phi}(\omega_e, \chi_e)$ is obtained directly from model tests and the frequency mapping is performed on $E(\omega, \chi)$ alone, before the product in equation (27) is taken.

Equation (23) assumes essentially that an irregular sea is a stationary random process which can be represented as a summation of a very large number (or as an integral of an infinite number) of sinusoidal waves of different periods and directions superimposed in random phase. A ship's response is likewise represented as a steady-state response to these regular waves; i.e., it is assumed that no transient responses exist. Finally, the relationships are assumed to be linear so that the end result can be obtained by superposition.

The practical applicability of these assumptions, for irregular long-crested waves only, was verified by Lewis (1954a and 1954b, 1955), Lewis and Numata (1956) and Lewis and Dalzell (1957) by towing tank tests. Typical results are shown in Fig. 11 and in Table 1. It will be observed that good agreement was secured between ship

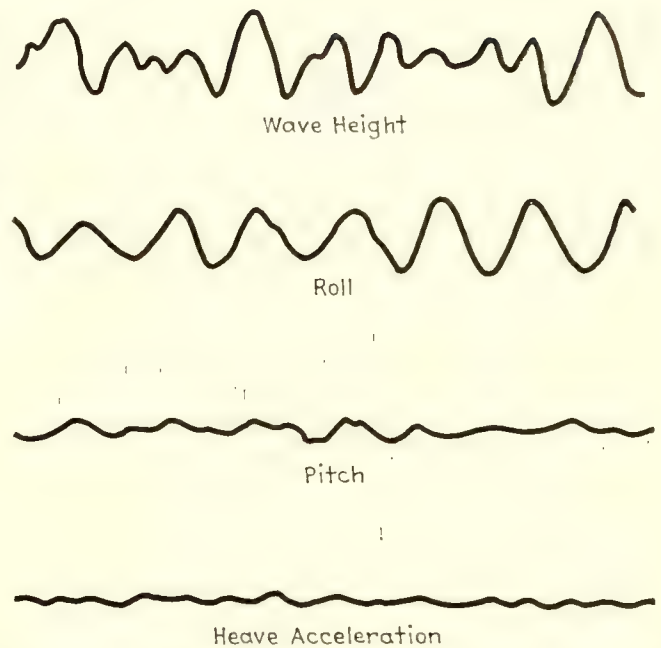


Fig. 12(b) Samples of ship motions recorded aboard an aircraft carrier in a state 5 quartering sea at a ship speed of 16 knots (courtesy Taylor Model Basin)

Table 1 Comparison of Near-Maximum^a Destroyer Model Motions in Towing Tank Irregular Head Seas, as Obtained by Three Different Methods

	Heave, in.		Pitch, deg	
	0	2.53	0	2.53
	speed	fps	speed	fps
By direct analysis of model motions in irregular waves.....	1.1	1.3	5.2	6.7
By calculation from wave spectra and experimentally measured responses to regular waves.....	1.1	1.2	4.3	6.1
By calculation from wave spectra and analytically computed responses to regular waves.....	1.1	1.3	5.0	5.8

^a Mean of 1/10 highest amplitudes.

motions measured directly in irregular waves, and those computed by equation (23) from two alternate frequency-response functions. One of these alternates was based on towing-tank tests in regular waves and the other on computations formulated by Korvin-Kroukovsky and Jacobs (1957).

Expression (23) can also be used in its transposed form

$$[T(\omega_e)]^2 = \Phi(\omega_e)/E(\omega_e) \quad (29)$$

which permits evaluation of the square of the absolute value of the frequency-response function from the recorded motions of a ship in irregular model-tank waves. This procedure can be considered as a possible alternate to series of tests in regular waves, and promises a certain saving of experimental effort. It was tried by Lewis, Numata, and Dalzell in long-crested irregular waves, with reasonable success. In this application a ship's responses are not completely described, as only the real part of the

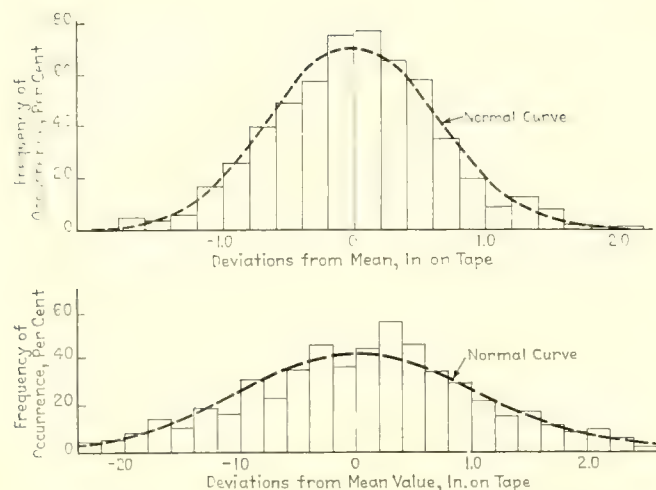


Fig. 13 Frequency of occurrence of deviations from mean values of points on wave record (upper) and on pitching record (lower) (from Lewis and Numata, 1956). Series 60-0.60-block-coefficient model 5-ft long at 2 fps in high irregular head waves

frequency-response function is evaluated and the phase relationships remain unknown. Complete determination of the amplitude and phase responses is made pos-

sible by cross-spectrum analysis which will be discussed shortly and by the method of Fuchs and MacCamy (1953) described in Section 3.2.

3.5 Analysis of Ship Motions Data—Single Parameter. In Section 1-8.1, the random properties of waves were discussed. Ship motions, caused by wave motions, are likewise random and never repeat. Fig. 12(a) is an example of wave and pitch motions recorded in the model tank. From the striking similarity between the wave and pitch records it is not difficult to conclude that both of these phenomena may be analyzed and interpreted in the same way. However, many observed wave and ship motions are quite dissimilar in appearance as evidenced by Fig. 12(b). In such cases, like treatment of ship and wave records is predicated on the hypothesis that a stationary random process (waves) exciting a linear system (ship) produces a stationary random process (ship motions). To further validate this hypothesis, some ship-motion records, made in irregular waves in a towing tank, were analyzed to determine whether points chosen at random were distributed according to the Gaussian law and whether the peak-to-peak oscillations in the ship motions were distributed according to the Rayleigh distribution, these properties having already been assigned to wave records in Chapter 1. The results appear in Figs. 13 and 14. It is fairly well es-

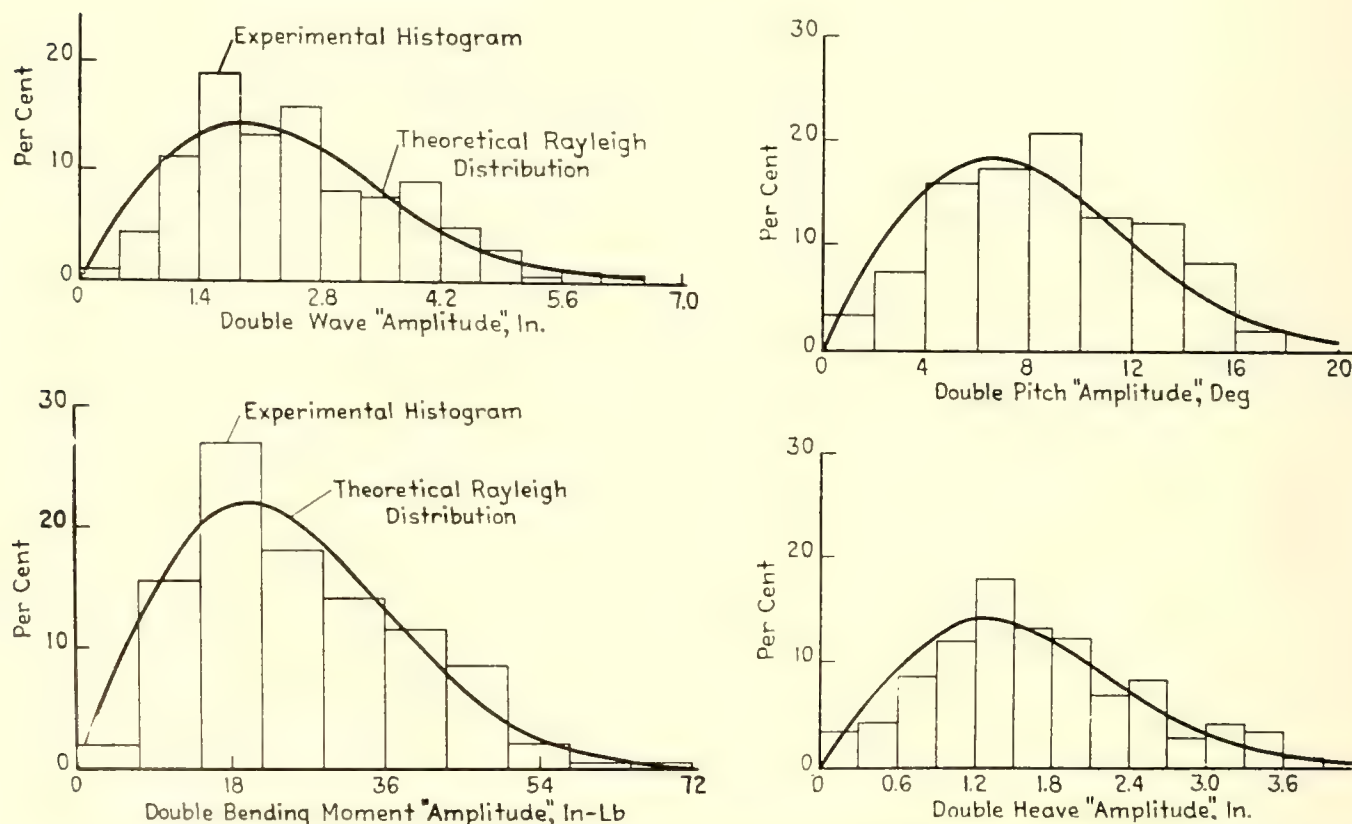


Fig. 14 Distribution of wave, pitching, heaving, and bending-moment amplitudes in towing-tank tests of Series 60-0.60-block-coefficient model at 2 fps in high irregular head seas (from Lewis, 1956)

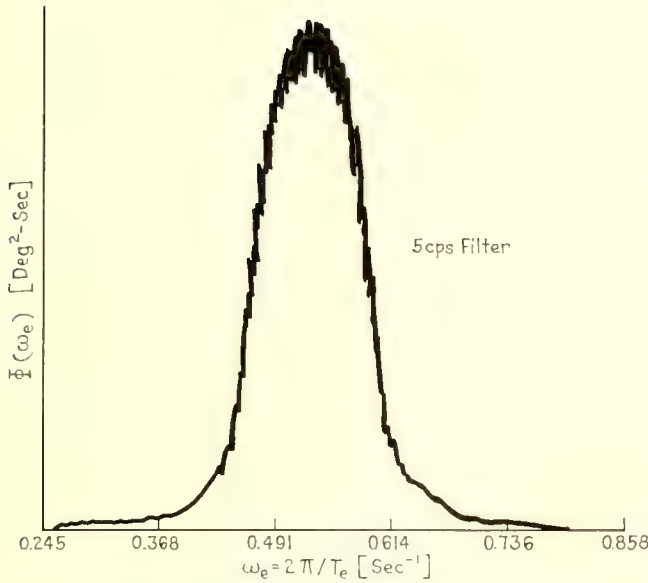


Fig. 15 Analog-filter computation of roll spectrum of a Liberty ship in a state of 5 sea (head seas, 4.5 knots)

tablished that ship motions, like waves, can be represented by a stationary random process so that ship-motion spectra may be interpreted in the same way as wave spectra.

The obvious starting place for evaluating ship motions is the energy spectrum $\Phi(\omega_e)$. If the linear superposition hypothesis as applied by St. Denis-Pierson is used, $\Phi(\omega_e)$ results directly from computation (model tests may be used to obtain hydrodynamic coefficients). Towing-tank tests of the type shown in Fig. 10 also result in $\Phi(\omega_e)$ directly. If the deterministic approach of Fuchs and MacCamy is used, a motion record $E(t)$ results which may be converted to $\Phi(\omega_e)$ by the numerical or analog methods described in Section 1-8. The same applies to motion measurements at sea and to motion measurements in irregular waves in model tanks. In any case, $\Phi(\omega_e)$ is obtainable and the statistics of a single ship-motion parameter derive from $\Phi(\omega_e)$ in accordance with the procedure outlined in Section 1-8.6.

Although not directly pertinent, it would be scarcely fitting to end a section dealing with ship-motions spectra without showing some typical examples. This obligation is fulfilled in Figs. 15 and 16. It should be remembered that it is incumbent upon the investigator to show that such spectra result from stationary, random, Gaussian processes, before the statistics derived from them can be interpreted properly.

3.6 Cross-Spectrum Analysis. Section 3.5 dealt with the analysis of single variables and consequently there was no question of phase relationships. However, true insight into ship-motion behavior can only be derived through a more intimate understanding of how the input spectrum (waves) and transfer function combine to produce a ship-response spectrum. In this case, one can study as before,

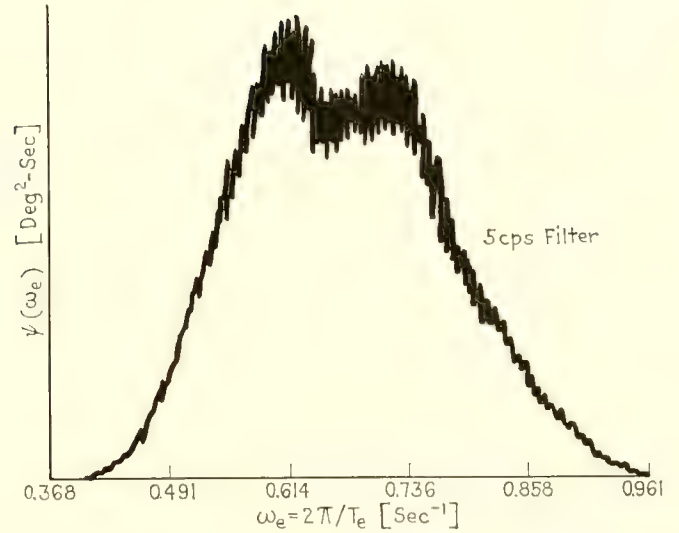


Fig. 16 Analog-filter computation of pitch spectrum of a liberty ship in a state 5 sea (head seas, 4.5 knots) (from Marks and Strausser, 1959)

the effect of the seaway upon the ship in eliciting a motion response. In addition, the phase of the response, relative to each frequency component of the wave spectrum, can be studied. This will depend on the point of wave measurement so that distances between observation points become important. One may also investigate the relationship say between heave and pitch, without regard to waves. The mechanism for providing information on phase as well as amplitude is the cross-spectrum.

Consider any two ship-motion parameters $y(t)$ and $z(t)$ recorded simultaneously. If, for example $y(t)$ represents the wave input and $z(t)$ represents the ship response to $y(t)$, then equation (23) suggests the necessary response amplitude operator (here the terminology of St. Denis and Pierson is more appropriate than the label "transfer function"). If, however, phase responses are desired, it is necessary to consider the response amplitude operator and the motion spectrum as complex quantities.

$$\Phi_{yz}(\omega_e) = c_{yz}(\omega_e) + iq_{yz}(\omega_e) \quad (30)$$

The real part of equation (30) is called the co-spectrum and indicates the energy associated with the in-phase components of the response, while the imaginary part is called the quadrature spectrum and indicates the 90-deg out-of-phase components of the response. The magnitude of the cross spectrum is the amplitude spectrum

$$\Phi(\omega_e) = \{[c_{yz}(\omega_e)]^2 + [q_{yz}(\omega_e)]^2\}^{1/2} \quad (31)$$

and the phase lag is

$$\epsilon(\omega_e) = \tan^{-1} \left[\frac{-q_{yz}(\omega_e)}{c_{yz}(\omega_e)} \right] \quad (32)$$

The ratio of the cross spectrum of any two seakeeping variables to the scalar wave spectrum together with equa-

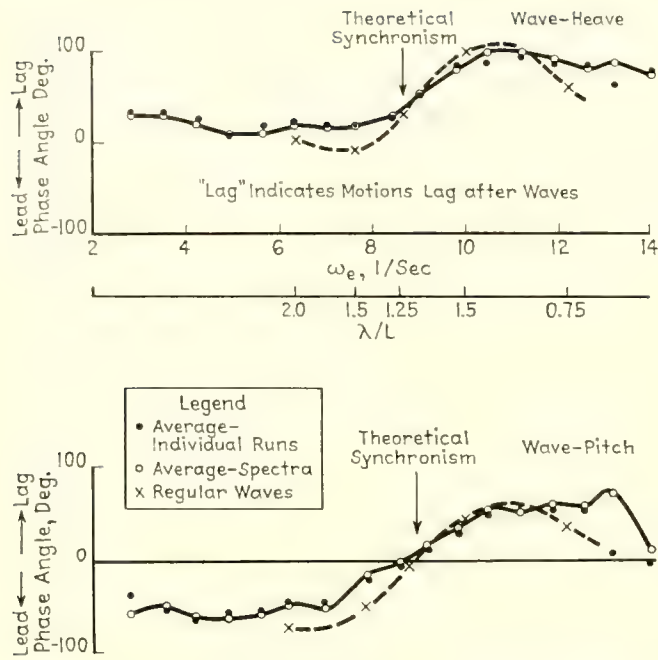


Fig. 17 Average motions phase angles compared with regular wave data (Model 1445, 2.9 fps broad spectrum) (from Dalzell and Yamanouchi, 1958)

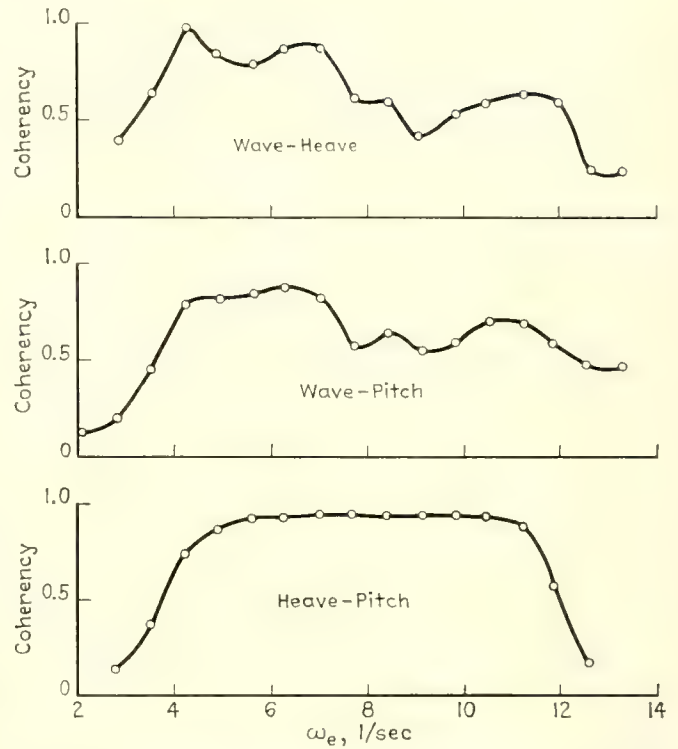


Fig. 18 Coherencies (Model 1445, 2.9 fps broad spectrum) (from Dalzell and Yamanouchi, 1958)

tion (30) results in the complex transfer function

$$T_i(\omega_e) = \frac{\Phi_{yz}(\omega_e)}{E(\omega_e)} = \frac{c_{yz}(\omega_e) + iq_{yz}(\omega_e)}{E(\omega_e)} \quad (33)$$

The kernel function $K_E(\tau)$ of Fuchs and MacCamy (16) requires the transfer function in the form

$$T_i(\omega_e) = \frac{\{[c_{yz}(\omega_e)]^2 + [q_{yz}(\omega_e)]^2\}^{1/2}}{E(\omega_e)} \quad (34)$$

while the St. Denis-Pierson transfer function is given by

$$[T_i(\omega_e)]^2 = \frac{[c_{yz}(\omega_e)]^2 + [q_{yz}(\omega_e)]^2}{[E(\omega_e)]^2} \quad (35)$$

If the wave spectrum $E(\omega_e)$ and the response spectrum $\Phi(\omega_e)$ are measured independently, a transfer function given by equation (29) results. From equations (29) and (35) a quantity called the coherency (γ_{yz}) is defined

$$\gamma_{yz} = \frac{[c_{yz}(\omega_e)]^2 + [q_{yz}(\omega_e)]^2}{E(\omega_e) \Phi(\omega_e)} \quad (36)$$

where $E(\omega_e)$ and $\Phi(\omega_e)$ are the energy spectra computed from $y(t)$ and $z(t)$, respectively. The coherency is a measure of the linear dependence of two trigonometric components of the same frequency in two spectra and determines whether the system being studied is indeed linear. At the suggestion of Pierson, Dalzell examined the effect on coherency of increasing the number of spectral estimates. The result was an increase in coherency between signals which had by their nature sug-

gested a strong linear relationship but which had showed low coherencies originally. These results are, as yet, unpublished.

Pierson (1957) has shown that theoretically $\gamma = 1$ for the long-crested case but that in the short-crested case complications arise because the same frequency of encounter can result in quite different phases in the response, depending on the relative angle with which the different wave trains encounter the ship. In general, when c is large and q is small, the same frequency components are in phase. When c is small and q is large, they are 90 deg out of phase. When $c = q$, the components are 45 deg out of phase.

Dalzell and Yamanouchi (1958) studied the phase relationships between waves at the midship section of a model and heave or pitch. The result for the average of six tests is shown in Fig. 17 where zero phase means maximum pitch up at the bow or maximum up-heave when the wave crest is at the midship section. Phase lag means that maximum motion occurred after the wave crest reached the midship section. Comparison between phase relationships obtained in regular and irregular wave experiments shows good agreement.

Dalzell and Yamanouchi (1958) also plotted the coherency (γ) between the parameters just discussed and these are shown in Fig. 18. If $\gamma \geq 0.85$ is used as the limiting criterion for a linear system, it is seen that heave-pitch fails at the very low and very high frequencies while wave-heave and wave-pitch fail in a large part of the fre-

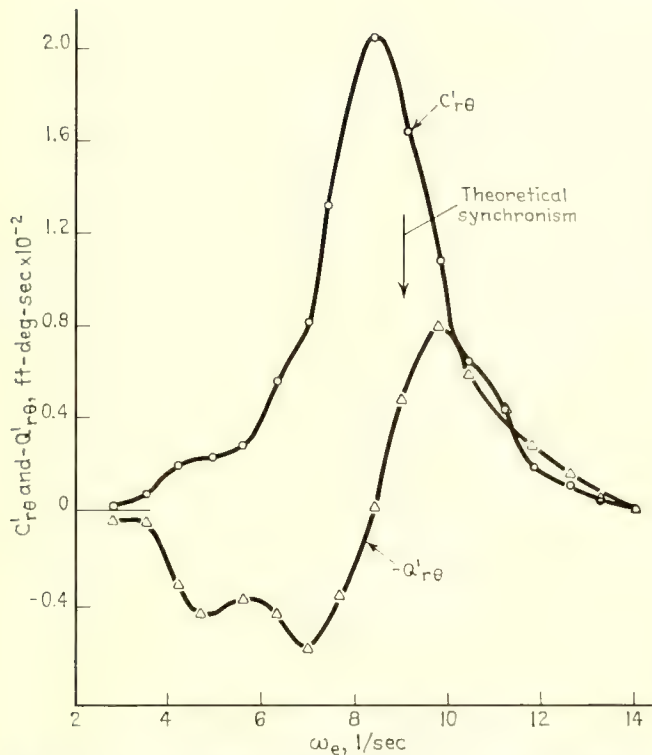


Fig. 19 Co- and quadrature spectra, wave and pitch, modified to reference phase relationships to wave amidships on model—average of 6 runs (from Dalzell and Yamanouchi, 1958)

quency range. Dalzell and Yamanouchi offer these possible reasons for $\gamma < 1$:

- “(a) Experimental effects such as the surging of the model.
- “(b) The possibility of short-crestedness resulting from tank side reflections.
- “(c) The shape of the tank section.
- “(d) Noise in the records and the effect of the smoothing process and other computational error.
- “(e) Possible minor nonlinear effects in responses.”

In other words the coherency for any frequency will be less than 1, if:

- (a) The ratio of the amplitude of that frequency component in $y(t)$ to the amplitude of the same frequency component in $z(t)$ is not constant throughout the record.
- (b) The phase relationship, at that frequency, between $y(t)$ and $z(t)$ is not constant throughout the record.
- (c) Noise is present in either $y(t)$ or $z(t)$ at that frequency.

The measure of coherence between two seakeeping parameters is perhaps the single most important contribution of cross-spectrum analysis. If linearity (through coherency) cannot be confirmed in model tanks, either the superposition theory for long-crested waves is in grave doubt, or the towing tank as a reliable test medium is questionable. Tests similar to those of Dalzell and Yamanouchi are being conducted independently at the Taylor Model Basin, where the tank section

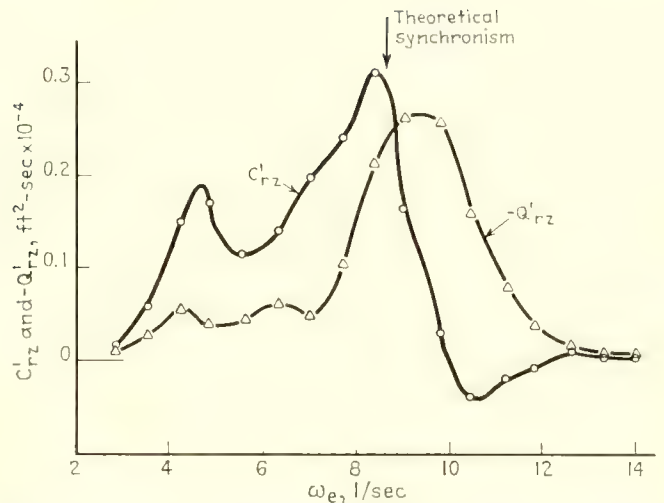


Fig. 20 Co- and quadrature spectra, wave and heave, modified to reference phase relationships to wave amidships on model—average of 6 runs (from Dalzell and Yamanouchi, 1958)

is wider and deeper, in order to eliminate the possible difficulties set forth in items (b) and (c).

This discussion of cross spectra would not be complete without some examples: Fig. 19 shows the co- and quadrature spectra for wave (amidships) and pitch of a model in irregular long-crested seas. The same frequency components are in phase for the lower frequencies; above $\omega = 10$, they are 45 deg out of phase. The wave-heave relationship; for the same model is shown in Fig. 20 where the same frequencies are generally in phase up to $\omega = 8.8$, equation (32), where they are 45 deg out of phase. The phase difference increases to 90 deg at $\omega = 10$.

3.61 Covariance-digital method. The popular digital method of cross-spectrum analysis is an extension of the autocovariance—Fourier transform method outlined in Sections 1-8.3 and 8.4. Consider any two seakeeping parameters $y(t)$ and $z(t)$; the covariance function is given by the convolution integral

$$R_{yz}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T y(t) z(t + \tau) dt \quad (37)$$

and the cross-spectral density function becomes

$$\Phi_{yz}(\omega_e) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{yz}(\tau) e^{-i\omega_e \tau} d\tau \quad (38)$$

Where $R_{yz}(\tau)$ is now an odd function and the cross spectrum is written in complex notation according to (30). When (38) is combined with (30) the resulting co- and quadrature spectra are

$$\begin{aligned} c_{yz}(\omega_e) &= \frac{1}{\pi} \int_0^{\infty} \left(\frac{R_{yz}(\tau) + R_{yz}(-\tau)}{2} \right) \cos \omega_e \tau d\tau \\ q_{yz}(\omega_e) &= \frac{1}{\pi} \int_0^{\infty} \left(\frac{R_{yz}(\tau) - R_{yz}(-\tau)}{2} \right) \sin \omega_e \tau d\tau \end{aligned} \quad (39)$$

For the digital process, where simultaneous equally

spaced points provide the data for analysis, the approximate covariance functions associated with the co- and quadrature spectra are given by

$$F_p = \frac{S_p + S_{-p}}{2} \quad (40)$$

$$G_p = \frac{-S_p + S_{-p}}{2}$$

F_p and G_p are the digital forms for the expressions inside the parentheses of equation (39).

where

$$S_p = \frac{2}{N-p} \sum_{q=1}^{N-p} y(t_q) Z(t_{q+p})$$

$$S_{-p} = \frac{2}{N-p} \sum_{q=1}^{N-p} y(t_{q+p}) Z(t_q) \quad (41)$$

The raw estimates of the cross spectrum are then given by

$$L(c_{yz}) = \frac{1}{m} \left(F_0 + 2 \sum_{p=1}^{m-1} F_p \cos \frac{\pi p h}{m} + F_m \cos \pi h \right) \quad (42)$$

$$L(q_{yz}) = \frac{1}{m} \left(G_0 + 2 \sum_{p=1}^{m-1} G_p \sin \frac{\pi p h}{m} + G_m \sin \pi h \right)$$

The rough estimates are smoothed according to equation (1-125).

$$U(c_{yz}, q_{yz})_0 = 0.5 L_0 + 0.5 L_1$$

$$U(c_{yz}, q_{yz})_h = 0.25 L_{h-1} + 0.5 L_h + 0.25 L_{h+1} \quad (43)$$

$$U(c_{yz}, q_{yz})_m = 0.5 L_{m-1} + 0.5 L_m$$

3.62 Analog methods. The analog-filter method of analysis in which a seakeeping parameter is recorded in the form of a fluctuating voltage signal on magnetic tape has been discussed in Section 1-8.5. The extension to cross-spectrum analysis of two signals is quite similar. Each signal is scanned simultaneously and separately by identical filters. The outputs of these filters are in one instance multiplied to yield the co-spectrum and at the same time the output of one filter is phase shifted 90 deg and then multiplied by the output of the other filter to yield the quadrature spectrum. This is analogous to the cosine and sine transformations in (39). The clearest description of this procedure was given by Smith (1955) and his paper is included as Appendix D. The difficulties associated with this method concern the matching of filters and the electronics associated with phase shifting.

Another (rather unique) method involves the use of one filter and the sum of the input signals as recorded as well as the sum of one signal and the derivative (or integral) of the other. Consider the simultaneous sum of the two signals $y(t)$ and $z(t)$. The covariance function of such a record is

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [y(t) + z(t)] [y(t + \tau) + z(t + \tau)] d\tau$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [y(t) y(t + \tau) + z(t) z(t + \tau) + z(t) y(t + \tau) + z(t + \tau) y(t)] dt \quad (44)$$

The Fourier transform of $R_{y+z}(\tau)$ results in

$$\Phi_{y+z}(\omega_c) = \Phi_y(\omega_c) + \Phi_z(\omega_c) + 2c_{yz}(\omega_c) \quad (45)$$

In a similar fashion, the Fourier transform of the covariance of the sum of one signal and the derivative (or integral) of the other is:

Differentiation

$$\Phi_{y+z}(\omega_c) = \Phi_y(\omega_c) + \omega_c^2 \Phi_z(\omega_c) + (2/\omega_c) q_{y+z}(\omega_c)$$

Integration

$$\Phi_{y+f_z}(\omega_c) = \Phi_y(\omega_c) + (1/\omega_c^2) \Phi_z(\omega_c) + (2/\omega_c) q_{y+z}(\omega_c) \quad (46)$$

If then, the spectra of the two signals are determined separately and subtracted from (45) and (46) the result is the co- and quadrature spectra, respectively (after their coefficients are cancelled). The price of eliminating matched filters and phase shifting is the additional time required to make the foregoing four analyses. Such a method is the basis of a cross-spectrum analyzer reported by Chadwick and Chang (1957). Tucker (1950, 1952) reported on a photoelectric correlation meter based on somewhat the same principle.

The literature abounds with papers on spectra and cross-spectra. To start the reader off, some current works will be cited that quite well cover the field of spectrum analysis. The reader can follow his own inclination from the many references given in each of these papers: Liepmann (1952); Press and Houbolt (1955); Press and Tukey (1956); Rosenblatt (1955); Goodman (1-1957); Pierson (1957).

Generalized harmonic analysis is treated well in the following books:

H. M. James, W. B. Nichols, and R. S. Phillips, "Theory of Servo-Mechanisms," McGraw-Hill Book Co., 1947.

H. S. Tsien, "Engineering Cybernetics," McGraw-Hill Book Co., 1954.

J. Halcombe Laning, Jr. and Richard H. Battin, "Random Processes in Automatic Control," McGraw-Hill Book Co., 1956.

3.7 Rolling of Ships in Natural Irregular Waves.

The only material on spectral analysis of ship rolling in irregular waves can be found in papers by Cartwright and Rydill (1957); Kato, Motora and Ishikawa (1957); and Voznessensky and Firsoff (1957).

Kato, Motora, and Ishikawa experimented with the rolling of a ship model in natural beam waves and wind. The model 2m (6.55 ft) long was placed about 100 ft offshore in water about 10 ft deep. The model was restrained by strings and springs to remain in a fixed position. Wave height, model roll angles, and wind-speed fluctuation were recorded. The model's natural rolling period was varied by changing ballast disposition. The response-amplitude operators (the real parts of the fre-

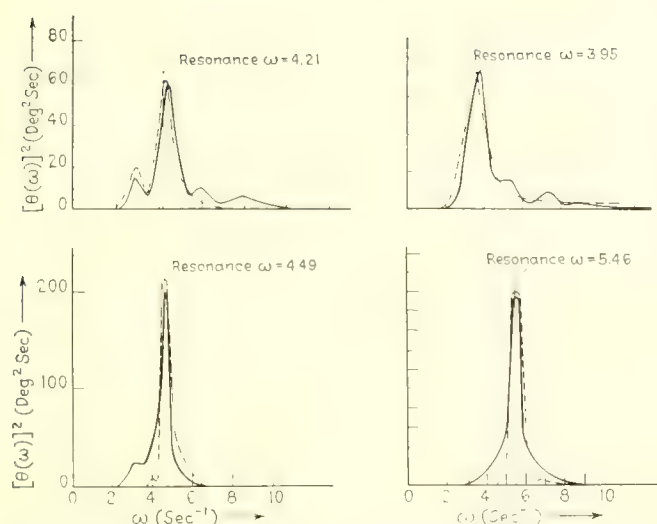


Fig. 21 Energy spectra of a ship model's rolling in natural irregular waves. Results of direct analysis of rolling records are shown by dotted lines. Results of wave-energy spectrum multiplied by model's frequency-response functions are shown by solid lines

quency-response function) were measured in a towing tank. Fig. 21 shows the comparison of the spectra obtained by analysis of the model's rolling with the spectra computed on the basis of wave spectra and frequency-response functions. The over-all agreement is very good, although there are certain differences in the forms of the spectra. In particular, the lower two diagrams indicate that the spectrum of actual motion is more narrowly concentrated around the natural frequency than has been indicated by the linear superposition theory. As far as the mean amplitude of the $1/3$ highest rolls is concerned, the agreement was excellent as shown by citation of the five cases in Table 2.

Table 2 Rolling Amplitude Obtained

Case	By actual measurement, deg	By computation, deg
1	12.4	13.5
2	12.7	11.6
3	18.9	20.6
4	10.9	11.3
5	17.8	18.3

Attention should be called to the fact that the excellent agreement shown in Table 2 was for the prediction of average ship rolling which is based on the spectral area (zero moment). Were the inverse problem attempted, that of evaluating ship-response functions from the wave and rolling records, the results would have been quite unsatisfactory. This is evident from a comparison of the ordinates of the solid and dotted lines at various specific frequencies. Heavy damping in heaving and pitching motions made the inverse problem practical as far as those modes were concerned. It appears to the author that the inadequacies of the superposition theory

make solution of this inverse problem impossible in rolling.

It should be emphasized, furthermore, that the foregoing experiments were limited to beam seas, zero forward speed, and artificially restrained model. The method of calculations used, which was based on St. Denis and Pierson (1953), cannot be expected to apply to a ship at sea with forward speed and in oblique wave directions. Under these conditions the rolling of a ship will be excited by:

- 1 Direct wave action.
- 2 Ship yawing caused by waves and wind.
- 3 Rudder action.
- 4 Rolling moment caused by wind.

Although rudder movements are made in response to wave and wind yawing disturbances, they are not uniquely connected with these. A certain indetermined response function is involved here. Furthermore, the rudder causes not only a yawing moment but also a lateral force. Analyses of ships' turning indicate the importance of considering this lateral force in the balance of forces and moments.

Kato, Motora, and Ishikawa provided some data on the rolling due to wind. They indicated that if wind and waves were not correlated, the rolling excited by each could be computed and the results of both added together. Under storm conditions, however, one should expect a certain correlation between waves and wind.¹⁶

If an attempt were made to derive frequency-response functions from wave and ship-rolling records alone, the motion resulting from several separate causes would be artificially attributed to one. It appears to be clear that no consistent results could be expected under these conditions. In this connection, attention should be called to the fact that Press and Tukey (1956, equations 45 and 46) indicated briefly a method of cross-spectral analysis for multiple disturbances. Within the author's knowledge, this method has not yet been applied to the analysis of a ship's rolling.

A certain confirmation of the difficulties with rolling (which may have been caused by transient and multiple responses) can be found in the paper by Cartwright and Rydill (1956). These authors applied cross-spectral analysis to the ship's pitching and rolling records in conjunction with wave records and compared the resultant responses with theoretically computed ones. In this comparison, the derived pitching amplitude responses either agreed with or showed a systematic deviation from theoretical curves with a reasonable scatter of data. The derived rolling amplitude responses, on the other hand, scattered widely and irregularly about theoretically computed curves and "agreement" could be estab-

¹⁶ The reader will find the wind-tunnel data on side force and heeling moments acting on several ships in steady side wind in the paper by Kinoshita and Okada (1957). Kato, Motora, and Ishikawa (1957) established the near-Gaussian distribution of wind pressure. Firsoff and Fedyaevsky (1957) developed theoretical formulas and furnished necessary empirical coefficients for calculating the impulsive lateral force and heeling moment caused by a sudden change of wind velocity and direction. The data on gust distribution in storms are, however, still lacking.

lished only in the broadest sense of a mean order of magnitude and of tendency to maximum at synchronism.

3.71 Work of Voznessensky and Firsoff. In the preceding sections, a particular ship's response to a specifically defined seaway was discussed. Voznessensky and Firsoff (1957) on the other hand, treated typical forms of seaway and the rolling responses of typical ships. Voznessensky and Firsoff's definition of the sea spectrum was discussed in Section 1-6.6. The form of scalar wave energy spectrum was defined by two parameters α and β as a function of the total spectrum area or, in other words, in terms of the significant wave height. The frequency-response function in rolling was taken to be that of a simple harmonic oscillator in terms of the damping coefficient and the ship's natural frequency in roll. To equation (23) was applied a corrective factor which took into account short-crestedness of the sea and also the reduction of the true frequency-response function in very short waves. Formulas and curves were given for estimating all factors needed for computations. Finally a ship's rolling amplitude in waves of a specified significant height was predicted. Application to six ships showed this prediction to hold within 25 per cent of the observed value. The main cause of the differences is the variability of the sea form (i.e., variability of parameters α and β) within a given significant wave-height specification.

The particular value of Voznessensky and Firsoff's (1957) paper appears to lie in its presentation of a very large amount of sea data obtained by instrumentation and in the classification of the data in terms of two parameters as a function of the significant wave height. Another valuable step is the introduction of a correction factor for transition from an idealized scalar spectrum to the true sea conditions.

The shortcomings of the method lie in failure to connect the sea state parameters α and β with wind condition and in the uniformity of the characteristics of all ships used in the analysis.

The latter remark can be amplified by a historical sketch. At the beginning of the steamship era, ships were built with excessive metacentric height and very short rolling period. Possibly an extreme example is the SS *Great Eastern* with its displacement of 22,500 tons and rolling period of 6 sec (Dugan, 1953; Wm. Froude, 1861). Ships of this type would roll heavily when hove-to head-on in a storm. Later, the advantages of a low metacentric height and a long rolling period (up to 20 sec) were demonstrated, both theoretically and by sea observations (Wm. Froude 1861; Möckel, 1941). Rolling of ships of this kind was treated by Manning (1942), and it was found that the most critical condition occurred in quartering seas. Lately, there has been a tendency to increase the metacentric height and shorten the rolling period. A rolling period of about 12 sec has become typical for ships of 60 to 70 ft beam. These ships roll most heavily in a sea from approximately beam direction. Voznessensky and Firsoff's treatment was limited to these conditions.

The loading of cargo ships, however, varies widely and

excessively short periods of rolling are frequently encountered (Patterson, 1955). Investigations of ship rolling should not be limited, therefore, to beam seas but should aim at universality.

4 Model Tests in Waves

Many towing tanks were equipped at an early date with devices for generating regular waves. Model experiments in waves were conducted by R. E. Froude (1905), Kent (1922, 1926), Kempf (1934, 1936) and many others. Since only long narrow tanks were available, tests were limited to head or following regular waves. Kent also experimented with an irregular sea generated by manually changing the wave-maker speed, without specifically defining the nature of the resultant irregularity.

For a long time no attempt was made to compare the test results of various tanks. The need for such a comparison was finally realized, and a program for action was devised at the Sixth International Towing Tank Conference in Washington in 1951. Results of the experiments in the various tanks on a model of the series 60, 0.60 block coefficient hull were collected and reported by Vedeler (1955a) at the Seventh International Conference on Ship Hydrodynamics in Norway, Denmark and Sweden. The data obtained in the different towing tanks for regular head waves demonstrated very poor agreement in amplitude of pitching motions and sometimes even in the frequency at which synchronism was obtained. Subsequently, a task group was formed under the auspices of the Seakeeping Characteristics Panel of Hydrodynamics Committee of The Society of Naval Architects and Marine Engineers in order to organize a new series of comparative tests in small tanks in the United States. Again a series 60, 0.60 block coefficient model was used, one 5 ft long. The results of these tests were reported by Abkowitz (1956a, 1957d) at the Eleventh American Towing Tank Conference and at the Eighth International Towing Tank Conference in Madrid. A satisfactory degree of agreement among the four participating tanks was obtained at higher speeds.¹⁷ The comparison included amplitudes of heaving as well as of pitching. The most important sources of previously encountered discrepancies were found to be the irregularity and uncertainty of the wave amplitude and the lack of precision in recording the data. The improved agreement, compared to the poor international one, resulted from improved wave generation, improved wave-height recording and because the results were referred to the wave height actually measured rather than to the nominal, specified wave. The recording apparatuses also were improved, and more painstaking care was used in conducting the experiments.

At lower speeds (roughly below 1.5 fps for a 5-ft model) the data appear to be affected by wave reflections from tank walls. Comparative tests in a conventional narrow tank and a newly available wide tank are planned.

¹⁷ At speeds above the synchronous one in wave length equal to ship length.

4.1 Towing Tanks and Equipment. Towing tanks first were built for the purpose of measuring resistance and propulsion characteristics. Since these are affected by model scale, tanks and models as large as possible were used. Models 16 ft long were considered the smallest acceptable. It was subsequently shown by Davidson (1936) that, by using artificial turbulence stimulation, satisfactory results also can be obtained in smaller tanks, with models 5 ft long, or even smaller depending on the type of ship. Since then a fairly large number of small towing tanks have been constructed and operated throughout the world. These tanks have been particularly active and useful during the recent upsurge of interest in the behavior of ships in waves. As early as 1861, W. Froude showed that wave-making was a primary cause of energy dissipation and damping in rolling of ships, thus indicating that viscous forces are secondary. As a result of all comparisons of test results with calculations based on the potential theory, it appears clear that potential hydrodynamic forces predominate strongly. This being the case, small models can be expected to give as good results as large ones in the investigation of ship motions, certainly as far as heaving and pitching are concerned since these depend primarily on the shape of the hull itself and not on appendages. The only direct experimental studies in this connection were made by Szebehely, Bledsoe, and Stefum (1956) and by Gerritsma (1957*b*). Their work appears to corroborate the foregoing statements, but is not extensive enough to be conclusive.

Probably the earliest data on model motions from small tanks were obtained at Newport News Shipbuilding and Dry Dock Company. Results were published by Hancock (1948) and Niedermair (1951). Recently a large amount of research in waves has been conducted in a 140-ft towing tank at the David Taylor Model Basin (Todd, 1954), a 100-ft tank at Stevens Institute of Technology (various papers by E. V. Lewis), a 100-ft tank at Massachusetts Institute of Technology (Abkowitz and Paulling, 1953), and a 200-ft tank at the University of California (Pauling, 1955).

Towing tanks generally can be divided into two classes, according to method used to tow a model. One is by carriage and the other by means of a long cord and a falling weight. Recent descriptions of the latter type were given by Todd (1954) and by Abkowitz and Paulling (1953). Model motions in such cases usually are obtained by analysis of a series of photographs. Abkowitz (1956*b*) described a method of recording pitching by using gyros in the model. Tanks using carriages have the advantage of a plane of reference. Motions are recorded either mechanically or more often by using electric pick-ups at the model and recording galvanometers on the carriage or on shore. The apparatus is designed to permit freedom of a model in surging, heaving and pitching, while restraining it in yaw and preventing it from rolling. Descriptions of this type of apparatus have been given by F. H. Todd (1954), Korvin-Kroukovsky (1954), Pauling (1956) and Gerritsma (1957*a*).

A self-propelled model may be left completely free to surge, but usually the model has to be towed so that a certain degree of restraint is present. In the apparatus described by Todd, Korvin-Kroukovsky, and Paulling only a short cord, two pulleys and a weight equal to model resistance were used, providing a minimum of inertial or frictional restraint. However, occasionally it is necessary to use light springs to limit excessive drifting of the model due to unbalance between towing weight and resistance. In systems using towing weights at the ends of the tank and a long towing cord, the masses involved are larger and the cord connecting them to the model has a certain degree of elasticity. Sibul (1956) and Reiss (1956) have shown that in certain cases partial, and particularly elastic, restraint of a model in surging may seriously affect the record of model motion. This happens, however, only at certain model speeds or frequencies of wave encounter.

Additional measuring instruments also may be installed in the model itself. E. V. Lewis (1954*a*, 1956*b*) described the apparatus and the procedure for measuring bending moments acting on a ship model in waves.

4.2 Tanks for Tests in Oblique Waves. Until recently only tests in head or following seas were possible because of the long and narrow tanks originally designed for resistance testing. During the past 4 years there has been some activity on the design and construction of towing tanks in which models can be tested in oblique waves. Five such facilities exist or are under construction at the time of writing. The first fully equipped one in operation is at Wageningen, Holland (van Lammeren and Vossers, 1955; van Lammeren, 1957). Seaplane and ship models had been tested earlier in oblique waves in an outdoor pool (Schulz, 1954), but the model guidance means and the recording system used at that time are not believed to have been of sufficient precision to permit valuable quantitative measurements.¹⁸ Next such a facility was constructed at the Davidson Laboratory (ETT) of Stevens Institute of Technology (E. V. Lewis, 1956*a*). Two very large installations are being built at the David Taylor Model Basin (Brownell, 1956; F. M. Todd 1957 NSMB Symp.) and at the Admiralty Experiment Works, Haslar, England.

4.3 Wave Generation. Many types of wave-making machines have been used in towing tanks with essentially equal success. Some of these were described by Kent (1922), Abkowitz and Paulling (1953), Caldwell (1954), Bisel (1954) and Allen (1957). Waves of unusually good regularity are produced by wave-makers of the pneumatic type (Todd 1954; Brownell, Asling and Marks, 1956; Gerritsma, 1957*a*). Until recently only (nominally) regular waves were produced, the degree of regularity depending on the design and the condition of the wave-maker, its control system, and the shape of the tank. These often left much to be desired. Interest in producing irregular waves in towing tanks arose

¹⁸ These have since been improved but apparently no new descriptions of the facilities have been published.

after the publication of a paper by St. Denis and Pierson (1953) on ship motions in irregular waves. At the same time, the definitions of sea-wave energy spectrum by Darbyshire and Neumann (see Chapter 1) indicated that irregular waves cannot be taken as mere disorderliness of wave motion, but must contain definite statistical characteristics. Artificially produced irregular waves must have the type of energy spectrum found at sea.

Working with a plunger type of wave-maker, E. V. Lewis (1955 and 1956b) produced irregular waves by varying the potential supplied to the driving motor from stroke to stroke by means of a mechanically driven step switch. The connections of 25 alternate potentials were arranged randomly and the resultant waves were found by analysis to resemble a partially developed Neumann spectrum (Lewis and Numata, 1956). Although the electric potential was varied step-wise, by the time the motor was accelerated or decelerated and by the time the waves were generated, the spectrum became continuous. When a fixed stroke of the plunger is used, the amplitudes of short waves become excessive. This is corrected by the use of light plastic foam floats, so proportioned as to act as filters attenuating short waves, while having little effect on the long ones. In E. V. Lewis' work, waves were produced of spectra similar to typical irregular sea spectra. Fuchs (1956), on the other hand, attempted to reproduce a specific wave pattern obtained from a sea record.

The waves just described have parallel crests normal to the tank length, and are irregular only in the sense of periods and amplitude variation. Such waves usually are referred to as "long-crested" and approximate the so-called regular swells in Nature, which in reality are never regular. The normal wind-produced sea is "short-crested" and is visualized as composed of wave components of many directions. The generation of such waves in rectangular or square tanks now becomes of interest. A brief summary of this process of wave generation was given by Marks (1956).

4.4 The Nature of Activity in Towing Tank Testing in Oblique Seas. In the foregoing paragraphs reference was made to descriptions of equipment used for model testing in oblique and irregular waves. These are now becoming available for commercial use and research. Literature on contemplated types of testing and on organization of research activities in these tanks however, is alarmingly meager. The only publications in this field appear to be those of Korvin-Kroukovsky and Lewis (1955a) and E. V. Lewis (1956d). Newton (1957) presented a formal discussion at the Eighth International Towing Tank Conference. Discussions of prospective testing activity are valuable in stimulating research and in eliminating pitfalls which are encountered in model testing in waves. Such discussions also will improve interpretation of test data. In this connection a determined effort should be made to advance the three-dimensional theory of ship motions, i.e., in six or seven (with rudder) degrees of freedom, in order to prevent errors and to assist in the

interpretation of results. There are many possibilities for valuable achievement.

The activity of oblique-wave towing tanks can be broadly divided into:

1 Tests in which the model behavior is assumed to represent directly the ship motions at sea.

2 Tests made in connection with the development and application of a ship motion theory.

4.41 Tests of practical nature; conditions of similarity. This heading is used for brevity to cover tests in which the model behavior is considered to represent directly the ship motions at sea. While this definition eliminates use of theoretical calculations beyond the usual reduction of test data, the theory of multimode ship motions must be kept in mind for the proper design of experiments and in the interpretation of the data. Because of the current lack of a sufficiently developed theory, "practical" tests probably will predominate in towing-tank activity in the near future, as has been emphasized by Newton (1957). For want of a developed theory also the following discussion must be limited to qualitative and intuitive aspects.

If the model behavior is to represent ship motions correctly, a complete similarity of ambient conditions must be provided. Lack of a workable theory makes it impossible to apply corrections for deviations of model test conditions from true ship conditions. In addition to the traditional conditions, the law of similarity applies to the following factors (in growing order of importance): (a) Mass distribution; (b) Rudder control; (c) Wave structure.

The proper scaling of weights and of moments of inertia has always been observed in towing tanks. In connection with tests in oblique waves it becomes necessary also to observe the proper orientation of the principal axes of inertia. The major practical difficulty here probably is the securing of sufficiently detailed information on the mass distribution in actual ships.

The powerful yaw-roll coupling of ships makes it necessary to observe similarity of rudder control used to maintain a model and a ship on course. In the theoretical literature the rudder-control function is usually described in the form $\delta = \delta(\int \psi dt, \psi, \dot{\psi}, \ddot{\psi})$, in which a limited number of the functions is chosen. It has been recommended (Dalman and Vossers, 1957) that such a control be used in model tests. While this may give a realistic control presentation for future ships, it is not realistic for most contemporary ships. It appears that the current practice at sea is to shift to manual control even in slightly rough seas because of imperfections of the automatic steering devices. It becomes important, therefore, to investigate empirically properties of commonly used manual steering and methods of reproducing it in a model. Conceivably this can be accomplished by a suitable application of statistical theory. Simultaneously obtained records of a ship's yawing and rudder motions are needed as basic material for such a study.

It is evident that towing-tank waves must correspond

closely to typical sea waves in Nature. These are invariably irregular and short-crested. The first requirement is to have available quantitative data on natural seas; it has been shown in Chapter 1 that such data are meager and uncertain. The directional wave spectra are required.¹⁹ There is evidently no such thing as a single typical sea. A towing tank must possess "a library" of typical seas. A convention as to the characteristics of these laboratory "seas" must be established to make possible a comparison of test results of different laboratories. It appears that such data on seas as are available at present have not been given sufficient attention in the construction of towing tanks. Several tanks have provided facilities for generation of waves in two mutually perpendicular directions, but have not considered such directional wave spectra as have been measured at sea to date.

Short-crestedness and irregularity of long-crested waves are both particularly important in determining a ship's rolling and yawing characteristics. It is well known that due to low damping in roll the ship's response has a sharp peak. In other words a ship acts as a narrow-pass filter in regard to its response to excitations in roll. In an irregular sea a ship responds primarily to the excitation near its natural frequency, and rolling has a narrow normal period distribution around the natural period of a ship. This is usually quite different from the period of pitch. In model tests made in *regular* oblique waves, an artificial condition is imposed on the model: i.e., it is forced to roll and pitch regularly with the same period. Because of the strong coupling influences on rolling of other modes of motion, there is apparently no practical way of estimating a ship's behavior in irregular short-crested seas from tests made in long-crested regular ones. Certain theoretical methods, to be described in the next section, are too laborious for practical use and do not give sufficiently complete information.

In connection with the foregoing remarks, the objectives of work in oblique seas should be stated. There is little doubt that irregular head seas produce the most critical conditions for ship behavior connected with pitching and heaving; i.e., vertical accelerations, shipping water, slamming and vertical bending moments. These can be determined generally in conventional narrow tanks. Wide tanks will provide more reliable data for below-synchronism speeds, but need no further discussion in this connection. It appears to the author that the research in oblique waves must be oriented primarily to an investigation of the rolling and steering characteristics of ships, and, connected with it, lateral bending moments. These motions are strongly influenced by both the cross couplings and the short-crestedness of the sea. Cross couplings define the dangerous pitch-yaw-roll motion in near-broaching condition, and short-crestedness, probably in conjunction with the yaw-roll coupling, defines a ship's rolling in bow seas.

4.42 Tests connected with theory development and application. The development of a theory for multi-mode ship motions is necessary:

- (a) To permit reliable interpretation of test data.
- (b) To permit generalization of the test data to ship forms and sea conditions other than those covered by laboratory tests.

A comparison of two ship models in identical wave conditions is one of the most useful types of practical tests. Any action taken to improve the inferior model is, however, based on intuition. Equations (12) show that a ship's behavior is governed by 108 derivatives, plus a complicated mass distribution effect, plus the exciting forces. Even if it were assumed that only a few of these discrete characteristics are significant, the effects of the significant ones are still too numerous for the human mind to trace empirically. The judgment of a naval architect would be facilitated and made more reliable were a sample of theoretically computed motion available as a guide. Provision of such a sample generally requires the theoretical calculations and their verification by a towing tank test. Test data on a typical ship²⁰ in long-crested oblique seas would be useful here. Tests should be made in regular and irregular seas. Additional tests in short-crested irregular seas are desirable in order to bring out the different effects of these three wave conditions. By using a linear theory, Rydill (1959) indicated that two most critical conditions as far as the rudder control is concerned occur with waves propagating towards a ship at 60 deg from the bow and 45 deg from the stern. The author suspects that a much smaller angle from the stern may prove to be most critical if the bow submersion were taken into account. The set of tests, just outlined, will provide most useful experimental material to stimulate theoretical activity and to verify its conclusions. In this particular test series, it is not important how typical of an actual sea the wave conditions are; the primary requirement is that towing-tank wave characteristics be completely and correctly reported.

Evaluation of the second, quasi-rational, theoretical approach suggested in Section 2.33(a) also appears to be desirable. This will require an additional series of tests in which a model will be restrained from rolling but left free in all other respects. Restraint in surging does not appear to be possible without interference with other motion modes. While rolling motion is restricted, the rolling moments can be measured. This will provide data for the design of stabilization devices and also will provide the verification of a theoretical evaluation of rolling moments with all coupling effects taken into account in the second project of Section 2.33(a).

(a) *Measurements of hydrodynamic forces and moments.* The exciting forces and moments exerted by waves, as well as the derivatives in equations (12), (13) and (14), can in principle be evaluated by a series of

¹⁹ Various methods of measuring directional spectra of sea waves were discussed in Chapter 1, and these are also applicable to towing tanks. The author considers Barber's method to be the most practical.

²⁰ A series of 60, 0.60 block coefficient model has recently been investigated with regard to heaving and pitching in head seas. It would be desirable to continue with the investigation of its properties in oblique waves.

model tests in regular long-crested waves using suitable restraints and oscillating devices. This procedure will be an elaboration of the one used for heaving and pitching motions by Haskind and Riman, Golovato, and Geritsma. Extending these measurements to all parameters involved in equations (12) would be a very large undertaking of questionable usefulness. It is the author's opinion that this work should be preceded by a step-by-step integration of equations (12) based on the best available theoretical estimates (however crude) of the parameters. Further theoretical and experimental work can then be directed towards a more complete evaluation of the limited number of parameters of greatest significance.

(b) *Tests connected with a statistical theory.* A statistical project was described by Marks (1957). In this project a ship's responses are to be measured in a series of wave lengths and wave directions at a series of forward speeds. Ship responses, considered independently in each motion mode and combined with the wave directional energy spectrum, will give the spectrum of ship motions in each particular mode. The average characteristics of each mode are then given by the Longuet-Higgins relationships. The procedure is an exact repetition of the one described in Section 3 for heaving and pitching motions in irregular long-crested head seas and used by Lewis and Numata (1956). It is made, however, more complicated by consideration of a directional wave spectrum which necessitates the use of the "frequency mapping."

A few questions arise in connection with this project. The necessity of making tests at a series of wave directions, in addition to a series of wave lengths and ship speeds, makes the experimental program so large that it is questionable whether it remains practical. It appears to the author that this program can be carried out once for the verification of the statistical theory, but that it is too cumbersome for either ship-design use or motion evaluations under specific weather conditions. The method, as described by Marks, may still be valuable for computational prediction of ship-behavior characteristics on the basis of theoretically evaluated responses to regular long-crested waves. Once the programming of computations is established, application to individual cases may not be laborious. The results of computations can be verified by comparison with the model test data described in Section 4.41, as well as with ship tests at sea.

The author feels, however, that scientific foundations of the procedure described need further investigation. In the case of the heaving-pitching motion, the superposition principle for ship motions is compatible with the linear superposition of velocity potentials on which all hydrodynamic forces depend. The nonlinearities appear to be of secondary importance in these motions. In six-component motions, defined by equation (12), the products of velocities are found on the left-hand sides. Is their superposition permissible? Apart from mathematical principles, the practical significance of this question depends on the motion mode under considera-

tion. In the pitching motion the hydrodynamic forces are expected to predominate strongly over the gyroscopic components of inertial forces. The procedure described by Marks can be expected, therefore, to give good results. In the rolling motion the hydrodynamic moments L are small and gyroscopic forces can be expected to exert large influence. In the yawing motion, sixth of equations (12), there is no restoring force in ψ and the factors $(I_y - I_x)$ and often p , are large. The applicability of Marks' independent consideration of motion modes needs, therefore, further investigation in application to rolling and yawing, which are the particular characteristics of interest in connection with oblique waves. An investigation of the yawing behavior of ship models at large roll amplitudes and short rolling periods is desirable in view of the important position of the rolling velocity $\dot{\phi} \equiv p$ in the sixth equation (12).

The non-constant behavior of coefficients is much more important in the case of yawing (and yaw-induced rolling) than in the case of pitching because the most significant derivatives $\partial N / \partial \psi$ and $\partial N / \partial v$ change their signs with bow emergence and submergence (and the opposite movements of the stern). This behavior of the coefficients indicates that linear theory, generally satisfactory for pitching, is probably inadequate for yawing. Yet, the evaluation of the coupled rolling and yawing characteristics represents the most important objective of model testing in oblique seas. An experimental evaluation of these aspects of ship-motion theory is needed and can be accomplished by applying Marks' recommended procedure to the model motions in oblique irregular long-crested waves at one or two headings, and one or two forward speeds. Such a limited test program and computations can be carried out easily.

5 Observations on Ships at Sea

The bulk of the quantitative information on ship behavior at sea has been obtained by theoretical methods and by towing-tank tests. The need of obtaining data by observations of actual ships at sea has been felt for a long time, but has been difficult to fill. The difficulty lies in the technical complexity of recording and interpreting the complex motions of a ship at sea, and also in the prodigious cost of comprehensive investigations. It is presumed that governmental naval research establishments, such as the David Taylor Model Basin in the United States, the Admiralty Experiment Works in England, and the Bassin d'Essais des Carènes in France have maintained contact with other naval activities and have conducted many trials. Most of their observations, however, have not been published and so are not available to naval architects in general. Independent laboratories have need for a close contact with the operation of ships at sea. Steps have been taken to fill this need even though only to a limited extent. Thus, J. L. Kent (1924, 1927a), of the National Physical Laboratory in England, made research voyages on six ships.

G. Kempf and Hans Hoppe (1926) of Hamburgische Schiffbau Versuchsanstalt, Hamburg, Germany, took observations on a passenger liner. Furthermore, a special division was established at the latter laboratory for the specific purpose of collecting data on ships at sea ("Sammelstelle für Fahrtergebnisse").

Observations aboard ships at sea are made for the purpose of establishing environmental conditions and evaluating a ship's response to these conditions. The first kind consists of quantitative descriptions of wind and waves. This subject was treated in Chapter 1 of the present monograph. Historically, however, sufficient oceanographic data had not been amassed in the time of Kent and Kempf. The first substantial organization of such data by Sverdrup and Munk (1946, 1947) took place some 20 years later, and was directed primarily at the evaluation of swells. Furthermore, the changeable nature of the sea requires that there be close co-ordination between investigations of wind and waves and of ship responses. Therefore, environmental conditions have formed an important part of all ship observations at sea.

The response of a ship to wind and waves can be considered under three headings:

- 1 Ship resistance and propulsion characteristics.
- 2 Ship motions.
- 3 Ship stresses, deflections and vibrations.

The first will be considered in Chapter 4, the second in the present chapter and the third partially in the present chapter, but more completely in Chapter 5. However, a complete separation in the discussion of all these aspects is not possible. In most tests at sea all are considered and usually reported together. Furthermore, taken together, all three aspects define the seagoing quality of a ship.

Treatment of sea observations is divided into:

- (a) A detailed investigation of wind, wave, ship motion, and stress phenomena.
- (b) A statistical tabulation of the data.

The establishment of a quantitative cause-effect relationship is sought in the first part. Ship motion is evaluated in terms of the wind and waves which cause it. Structural loading is evaluated in terms of water pressures and ship accelerations. A complete instant-by-instant history of ship behavior under certain given conditions is the aim of such detailed investigations.

In the second part, the aim is to establish, on a statistical basis, quantities which occur in Nature. A large number of simple quantitative observations of a single characteristic is collected and presented in the form of tables or plots. Data on one characteristic, for example pitch angles or stresses in a certain part of a ship, are collected independently of another. The only connection between recorded quantities of two characteristics is that they were recorded simultaneously. Direct causal relationships between any two characteristics cannot be obtained. Only a general correspondence of averages over a given time interval is established. For instance, the information may be given in a table show-

ing the number of times a certain high bending stress was reached in a given period of time during which the wind had a certain average or maximum velocity.

Statistical information is needed in order to establish the levels of severity of various sea and ship-behavior characteristics with which ship designers and mariners must cope. More detailed information is needed for design purposes, particularly for establishing the effect of a ship's form on its seakindliness. All characteristics of ship behavior at sea are so complex that useful deductions can be made only by frequent reference to theoretical work and towing-tank data. Such references are found, therefore, in most papers reporting sea observations.

The treatment of sea observations outlined in the foregoing is typical of past practice and applies to the abstracted data presented in the following text. The last two paragraphs above, in particular, refer to the "histo-grammatic" approach. Of the recently developed statistical approach, based on the properties of time series, which was described in Section 3, the only published applications to a ship at sea are in Cartwright and Rydill (1957) and Cartwright (1957, 1958). This approach occupies an intermediate position between the two described in this section. While it provides information on causal relationships among waves, ship form and ship motions, the results are given in the form of statistical averages rather than as a detailed history.

5.1 Observations Considering Sea Action on a Ship:

5.11 Observations of Kent, Kempf and Hoppe. J. L. Kent (1924, 1927a) made six ocean voyages on three passenger liners, one express cargo ship and two tankers. In 1924 he reported on the methods of measurement and the results for four ships under the following headings:

- 1 A general description of the observations taken and the manner in which they were obtained.
- 2 Description of the wind and waves encountered, and the effect of the former on the latter.
- 3 Analysis of the part played by the weather in reducing ship speed under various weather conditions.
- 4 Effect of the wind and ship form on steering of ships.
- 5 Summary of the principal factors causing loss of ship speed at sea.

The data on two passenger liners were reported in 1927 under the following headings:

- 1 Wind speed and wave dimensions.
- 2 Speed and power during voyages.
- 3 Fine weather propulsion data.
- 4 Rough weather propulsion data.
- 5 Propeller thrusts.
- 6 Weather producing propeller thrusts.
- 7 Fluctuation of torque on propeller shafts.
- 8 Pitching experienced on both ships.
- 9 Rolling experienced on both ships.
- 10 Comparison of rolling of the two ships.
- 11 Notes on pounding data.
- 12 Notes on steering data.

All of the observations were made by J. L. Kent alone.

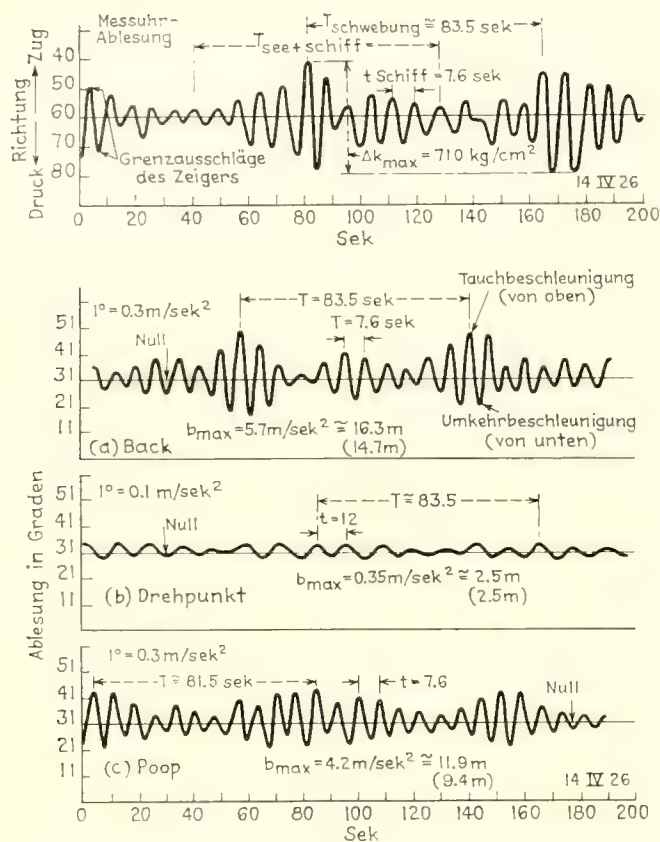


Fig. 22 Curves of vertical accelerations observed at bow, amidships, and at stern of SS Hamburg in waves reaching maximum heights of 6 m. Ship's double amplitude of pitching about 12 deg (from Kempf and Hoppe, 1926a). Uppermost part of figure is a typical strain diagram

Mostly standard ship equipment was used. Only three pieces of special portable instrumentation were introduced; namely, an anemometer, an apparatus for measuring roll angles by sighting on the horizon, and a long-period pendulum for measuring pitch angles. No description of the latter was given, but it apparently consisted of a suitable recording-pen device and a heavy flywheel having a slightly eccentric suspension. The pitch angle, therefore, was the only continuously recorded characteristic. Wave data were obtained visually, and periods of wave encounter were measured by a stop watch. The mean of about five consecutive waves was taken and the observations were repeated three or four times.

The ships were equipped with logs for speed measurements, and all but one were equipped with torque meters on the propeller shafts. The only ship without a torque meter was run by a reciprocating engine, the power of which was estimated from indicator cards.

Only a small part of the collected material was presented in Kent's papers. This is also true of practically all the other papers which will be discussed later. It is almost impossible to analyze and to report in a paper of practical length all of the material which can be col-

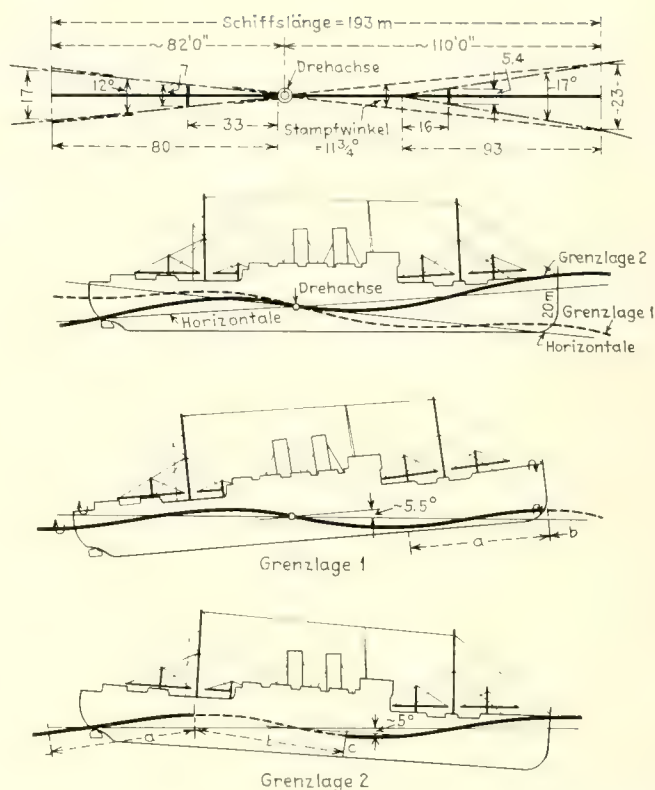


Fig. 23 Maximum pitching and heaving of SS Hamburg under wave conditions described in Fig. 22. These waves and ship motions caused a reduction of speed from 16.35 to 12 knots (from Kempf and Hoppe, 1926)

lected on a single voyage. In the two papers by Kent (1924, 1927a) tables of weather conditions, of ship motions, of rudder motions and of power consumption were given separately. Different record numbers often are used in compiling different tables, so that it is difficult for a reader to build a comprehensive picture of ship behavior, J. L. Kent's own interpretations and comments are probably the most valuable part of these papers. The experience gained by Kent on these six voyages was reflected in all subsequent papers listed under his name in the bibliography of this monograph. The latest published work (Kent, 1950) is a comprehensive summary based on these voyages, on many towing-tank tests in waves, and on the elementary theory of oscillations.

Kent's description of sea waves emphasized the irregularity of the sea, but his final data were given in terms of average wave length, height and period. Ship oscillations in pitch and in roll were shown to increase to maximum and then to decrease with a certain degree of regularity. Kent listed the number of individual oscillations between periodically occurring high motion amplitudes. This characteristic also was reported by Kempf and Hoppe (1926) who observed that the passenger liner *Hamburg* had its maxima of pitching amplitude at about 80-sec intervals. These "beats"

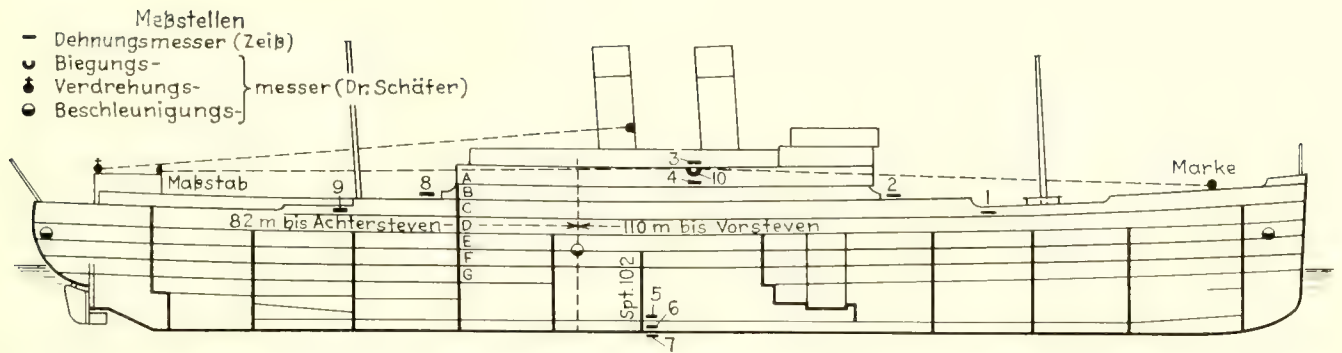


Fig. 24 Location of instruments on SS Hamburg; — strain gages; • accelerometers; o deflections (from Kempf and Hoppe 1926)

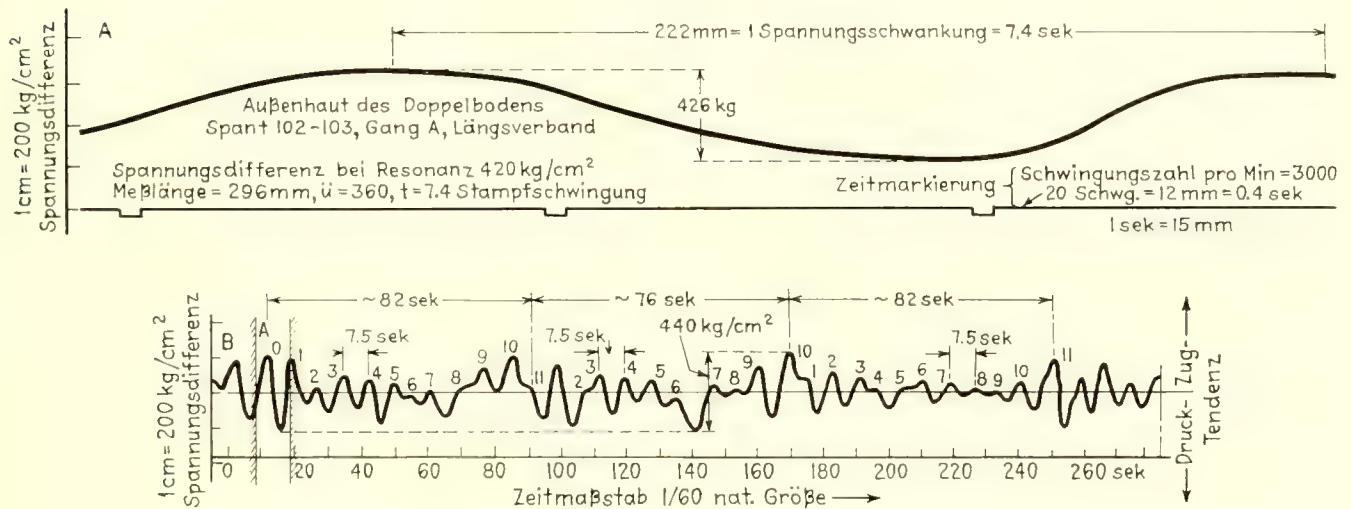


Fig. 25 Strain diagram for location 7 at keel (from Kempf and Hoppe, 1926)

had all the characteristics of a superposition of two oscillations—one at a regular wave frequency, the other at the ship's own frequency, such as was demonstrated by W. Froude (1861) for undamped rolling. Kempf and Kent gave this explanation for the "beats" observed in pitching. However, they considered the average wave as the regular one and neglected the heavy damping in pitching and heaving. In reality, as has been shown since, these "beats" occur in an irregular wave formation, and are explained by superposition of many wave trains.

In his work J. L. Kent also gave data on rudder motions and steering characteristics of ships under the influence of wind and waves. This is particularly valuable because very little data on this subject can be found in literature.

Kempf and Hoppe (1926) measured the SS *Hamburg's* speed by a log and visually estimated the average wave height and length encountered. The log consisted of a body towed by a cable from the end of a pole extending a sufficient lateral distance from the side of the

ship. The speed was obtained from the body drag on the basis of previous calibration. The average speed given by the log was compared with daily average speeds. The instantaneous readings of the log were affected, however, by wave impacts and by the ship's pitching. An apparatus for direct measurement of the ship's motions was not available but the pitching and heaving motions were defined by accelerometer readings at the bow, amidship and stern. The angle of pitch was computed from the acceleration and the observed period. Fig. 22 shows a sample accelerometer record, illustrating an apparent "beat" of 83.5 sec. Fig. 23 gives the extreme instantaneous positions of the ship in waves which were taken in the analysis to be regular. These conditions correspond approximately to synchronism between a ship's natural pitching frequency and the frequency of wave encounter.

Special efforts were made to observe ship deflections and strains. The location of the strain gages is shown in Fig. 24. Ship deflections were obtained by sighting targets at the bow and stern. Fig. 25 shows the varia-

tion of keel stresses during one cycle, and also during a period of about 270 sec. It can be observed that the irregularity of ship-stress variations is similar to the irregularity of accelerations. Kempf and Hoppe commented on the fact that simultaneous measurements of strains and accelerations are necessary for a complete analysis of ship stresses. However, the means for such simultaneous measurements were not available. Table

Table 3 Range of Stress in Longitudinal Members (from Kempf and Hoppe, 1926b)

Location	Maximum, entire voyage	Maximum at sea— Outward bound	Home bound	In port	Measured from — to 9.5, 1926
1	(750)	—	400	—	29.4
2	1000	(350)	525	750	14.4
3	(700)	—	475	—	26.4
4	(950)	800	600	—	29.4
5	(750)	—	300	—	25.4
6	850	500	375	500	13.4
7	600	(150)	275	350	15.4
8	925	625	450	650	12.4
9	700	(150)	400	450	16.4
10	9	9	6.5	—	12.4

All stresses (item 1 to 9) in kg/cm².

Item 10—deflections of ship ends in centimeters.

3 shows the maximum range of stress (hogging plus sagging) as measured by Kempf and Hoppe. The authors called attention to the fact that the stresses caused by different ship loadings while in port are of the same order of magnitude as the stresses caused by waves at sea. The waves met on this voyage of the SS *Hamburg* were 20 ft high and 425 ft long, corresponding to a wind strength of 9 on the Beaufort scale. Kempf and Hoppe expressed the opinion that these waves and the resultant ship stresses represented a degree of severity not likely to be greatly exceeded in the ship's normal service.

5.12 Voyage of the MS San Francisco. In 1934 Dr. Georg Schnadel organized an observational voyage on the well-instrumented MS *San Francisco*. Descriptions and results of the voyage were given by several participants as follows:

Schnadel (1936, 1936/37, 1938)—General Description and Ship Stresses.

Horn (1936)—Ship Motion and the Instrumentation Used in This Connection.

Weiss (1936)—Instrumentation for Measurement of Wave Contours on Sides of a Ship.

Weinblum and Block (1936)—Stereophotographic Measurements of Wave Contours.

Later the hydrodynamic properties of this ship were investigated by Kempf (1936) in a towing tank.

The MS *San Francisco* is a shelter deck cargo ship of 13,070 tons displacement, 130 m (426.5 ft) long BP, with a 18 m (59 ft) beam, a 6.9 m (22.6 ft) draft forward, and a 7.61 m (25 ft) draft aft. The block coefficient is 0.744. This ship was chosen because of its simple design. The strength deck is continuous through a large part of the ship's length, and only a light superstructure is located above it. The space between the

weather deck and the strength deck provided accessibility for installing strain gages on both sides of the strength deck. A two-stroke-cycle five-cylinder MAN engine delivered 4200 metric horsepower at 90 rpm and gave the ship a 13.2-knot speed in smooth water.

The instrumentation used for measuring the ship's motions consisted of a gyroscopic apparatus for recording pitching and rolling angles and several types of accelerometers for measuring vertical accelerations. The locations of these are shown in Fig. 26.

A wide variety of accelerometer types was employed to fill special needs. An attempt was made to obtain a specially designed long-period accelerometer for determining heaving motions. Accelerometers of lesser accuracy were available for this purpose. Also needed were accelerometers capable of recording for long periods of time at not readily accessible locations. A thorough discussion of the accelerometer theory was given by Horn (1936).

The wave profiles on both sides of the ship were determined by means of electric contacts (Weiss, 1936). The contacts were installed at six positions along the length of the ship as shown in Figure 27. At each position one contact was placed at the edge of the keel and 16 contacts located at equal vertical distances apart were installed on the starboard and port sides. Sea water in rising waves closed the contacts and lighted signal lamps. These lamps left traces on a film in a special recording apparatus. The wave profiles on both sides of the ship were constructed from the wave height taken at the longitudinal six locations. It also was possible to estimate the shape of a wave longer than the ship by extrapolation of the recorded profile.

The voyage was made from Hamburg to Vancouver and back, with both crossings taking the ship through the Panama Canal. On the outgoing voyage the ship was lightly loaded, but it was fully loaded on the homeward voyage. Throughout most of the voyage the weather was light, but two unusually severe storms were encountered in close succession during the homeward journey in the eastern North Atlantic. The first lasted from 11:15 am on December 11 to 2:15 am on December 12.²¹ The second from 11:40 am to 1:37 pm on December 14 and 4 pm on December 14 to 8:15 pm on December 15. Schnadel (1936) emphasized the fact that the first storm developed very rapidly—the highest waves were reached in 4 or 5 hr. The wind, described as of 9 to 11 strength on the Beaufort scale, reached 12 for a few hours. At the beginning of the storm the ship was traveling with the wind. At the height of the storm the ship hove-to with the bow against the wind and sea. The largest waves reached a height of 52.5 to 59 ft, as shown by the extrapolation of the wave profile at the ship's side and by stereophotographic measurements. By eye from the bridge the waves were seen to be at least 49 ft high.

Fig. 28 shows a sample of the ship's motion record in

²¹ This statement of the time is needed in connection with Tables 4 and 5.

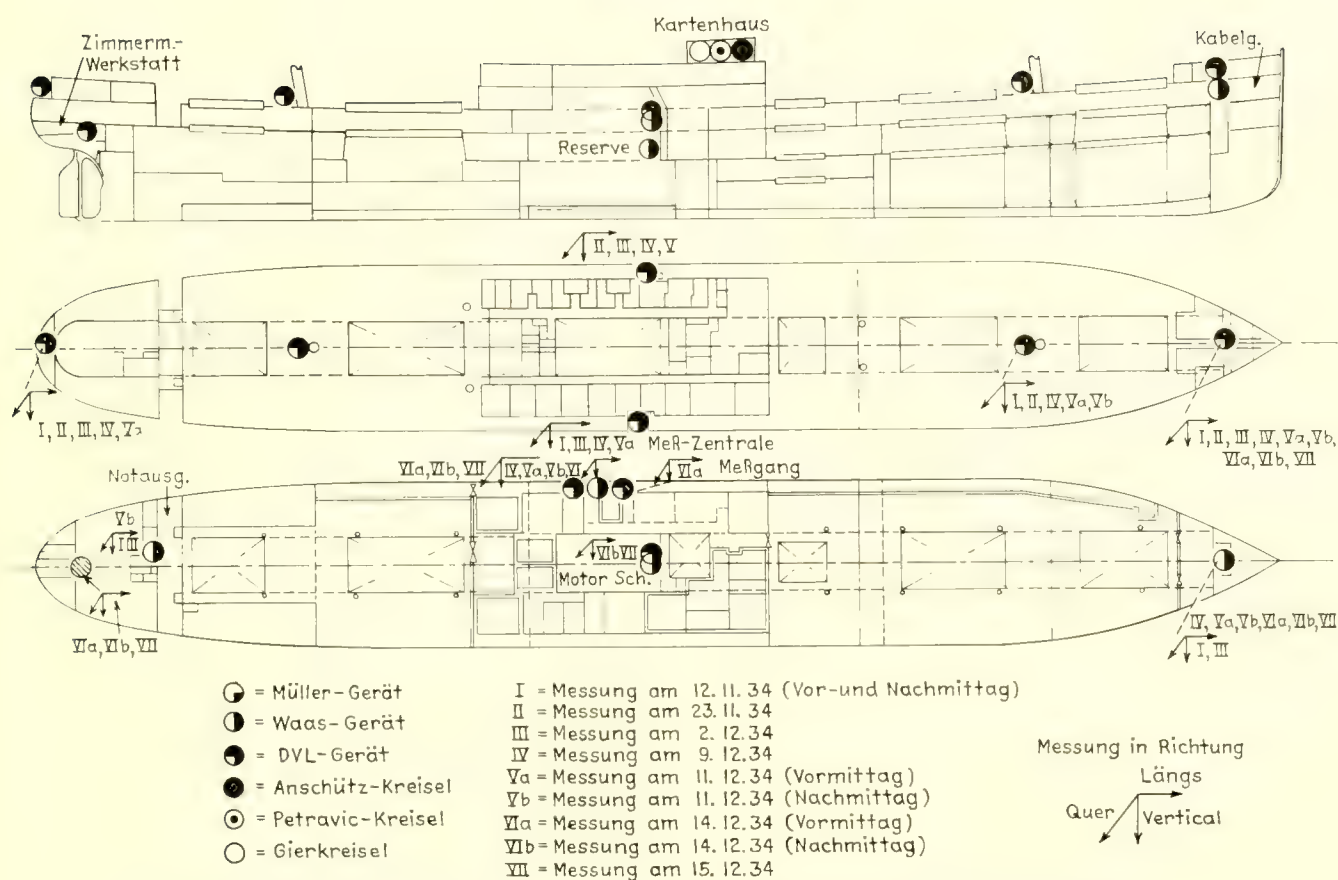


Fig. 26 Disposition of motion-measuring apparatus on board MS San Francisco (from Horn, 1936)

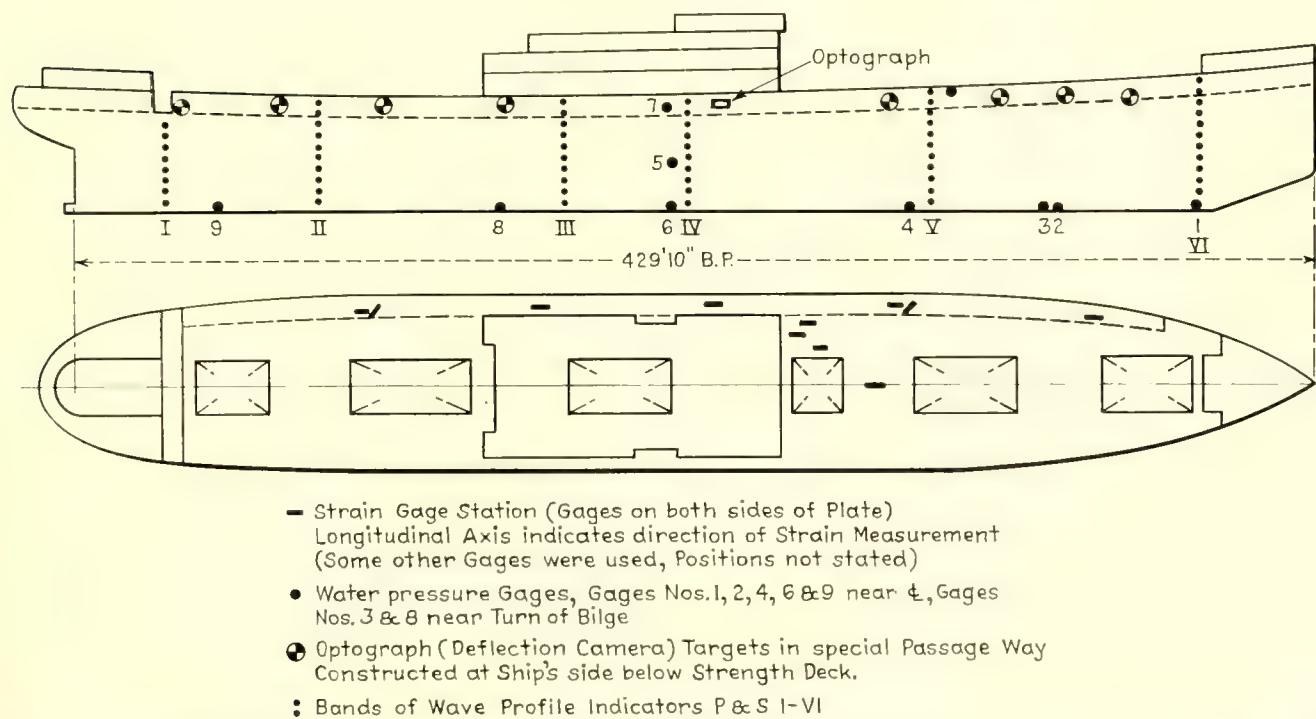


Fig. 27 Disposition of instruments for stress, deflection, wave profile, and water-pressure measurements on MS San Francisco (from Admiralty Ship Welding Committee, 1953)

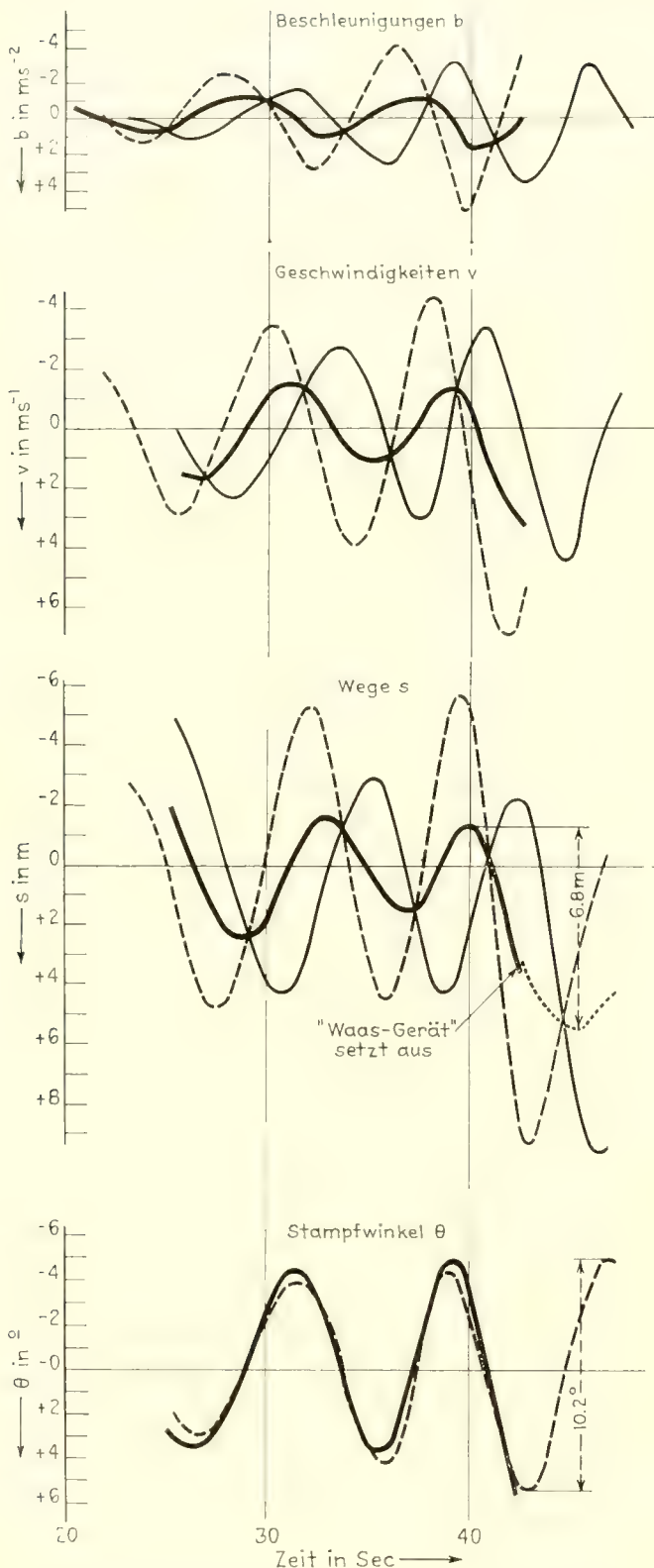


Fig. 28 Motion record of MS San Francisco in hove-to condition at 17 hr 34 min of December 11, 1934. Heaving motion is computed from heaving accelerations. Dotted line at the bow, light solid line at the stern, heavy solid line amidships. b —accelerations in m/sec^2 ; v —vertical velocity in m/sec ; s —heave in meters; θ —pitch angle in deg (from Horn, 1936)

the storm during a hove-to condition. A maximum double amplitude of pitch of 22 deg was reached in this condition. Fig. 29 shows the ship's motions in a moderate bow sea, and also when hove-to in a severe storm.

In his paper Horn (1936) gave many curves showing the distributions of amplitudes and frequencies of pitching and rolling. In discussing the characteristics of instruments he introduced the concept of irregular summation of harmonic disturbances. However, all analyses of wave and ship motions were based on the average harmonic wave.

The forces acting on the ship and its responses were obtained from several strain gages, a few water-pressure gages at the ship's bottom, and from measurements of hull deflections. The latter measurements were obtained by a method similar to the one used on the SS *Hamburg*, namely four targets forward and four aft of the midship plane were photographed simultaneously by a telescopic camera ("optograph"). The locations of these instruments are shown in Fig. 27. The instruments were of two kinds; those recording during short observational periods and those recording continuously, unattended. Table 4 shows sample data from the first group and Table 5 shows data from the second.

Data from the first group were subjected to a detailed analysis with reference to measured wave profiles, water pressures, and accelerations. It was found that measured stresses were considerably lower than those computed by conventional methods using the recorded wave profile. Schnadel introduced the concept of "effective wave height" (wirksame Wellenhöhen). This is the height of an imaginary quasi-static wave which would give the same stress or deflection as the actual measured wave. It is designated in Table 4 as H'_w when computed from stresses (aus σ) and H''_w when computed from deflections (aus f). Other notations in the tables are

Stress—Spannung σ
 Wave hollow (sagging condition)—Tal
 Wave crest (hogging condition)—Berg
 Tension—(+) Zug
 Compression—(−) Druck
 Bending moment—Biegemoment M
 Deflection—Durchbiegung f
 Slam—Stoss
 Schnadel arrived at the following conclusions:

(a) "The waves met by a ship at sea exceed previously assumed heights and steepnesses. To the effective wave height of some 5.5 m (18 feet) corresponds an actual height of 9 to 10 m (29.5 to 32.8 feet) by 130 m (426.5 feet) in length.

(b) "For the hogging stress of a ship, the observed wave heights must be reduced. The reduction is stronger than is given by the so-called Smith effect. The reduction probably can be attributed to the disturbance of

Table 4 MS San Francisco (from Schnadel, 1936)

Tafel II. Statistik*).

Optographen-Rolle 14^{II} ergibt Werte für f , $H''_{w'}$, Ritzapparat M 46, Spt. 89/90 ergibt Werte für σ , M , $H''_{w'}$, $H''_{w'}$ und $H''_{w'}$. Wirksame Wellenhöhen, aus Spannung bzw. Durchbiegung errechnet.

Zeit	Spannung σ		Biege- Moment M 1000 mt	Durchbiegung f		Pe- riode sec	Länge m	Wellendaten				Bemer- kungen
	+ Zug - Druck kg/cm ²	$\frac{\sigma_{Tal}}{\sigma_{Berg}}$		+ Tal - Berg cm	$\frac{f_{Tal}}{f_{Berg}}$			$H'_{w'}$ aus σ m	$\frac{H'_{w'}(Tal)}{H'_{w'}(Berg)}$	$H'_{w'}$ aus f m	$\frac{H'_{w'}(Tal)}{H'_{w'}(Berg)}$	
11. XII. 11 ^h 15'	- 477 + 387	1,23	+ 20,0 - 16,2	+ 5,3 - 4,1	1,29	10,5	175	3,8 3,2	1,19	3,8 3,3	1,15	
	- 845 + 540 - 566	1,57	+ 35,3 - 22,5 + 23,7	+ 8,9 - 5,1 + 5,5	1,75	9,0	126	6,6 4,5 4,5	1,47	6,2 4,3 4,0	1,40	Stoß!
						10,5	175					
11 ^h 24'	- 370 + 270	1,37	+ 15,5 - 11,3	+ 4,0 - 2,9	1,38	15,0	350	3,0 2,2	1,37	2,9 2,3	1,26	
	- 567 + 378	1,50	+ 23,7 - 15,8	+ 6,5 - 3,8	1,71	7,9	100	4,5 3,2	1,40	4,5 3,2	1,41	
11 ^h 39'	- 640 + 360 - 405	1,50	+ 22,6 - 15,0 + 16,9	+ 5,3 - 3,6 + 4,9	1,47	11,0	190	4,3 3,0 3,3	1,43	3,9 3,1 3,5	1,26	
						9,8	150					
11 ^h 40'	- 360 + 387 - 405	0,93	+ 15,0 - 16,2 + 16,9	+ 3,3 - 4,4 + 5,1	0,75	9,0	126	2,9 3,3 3,3	0,85	2,5 3,6 3,6	0,70	
						13,8	300					
11 ^h 40/1'	- 560 + 500 - 675	1,12	+ 23,4 - 20,9 + 28,2	+ 6,8 - 6,0 + 8,3	1,13	11,3	200	4,5 4,2 5,4	1,07	4,7 5,0 5,5	0,94	
						11,3	200					Stoß!
18 ^h 02'	- 675 + 450 - 566	1,50	+ 28,2 - 18,8 + 23,7	+ 8,0 - 5,4 + 6,2	1,48	10,8	180	5,4 3,8 4,5	1,42	5,5 4,4 4,4	1,25	Stoß!
	- 270	2,10	- 11,3	- 3,7	1,68	11,5	205	2,3	1,95	2,9	1,52	Stoß!
	- 450 + 432 - 765 + 450	1,04	+ 18,8 - 18,1 + 32,0 - 18,8	+ 6,8 - 4,4 + 8,5 - 4,9	1,55 1,73	13,8 10,8	300 180	3,7 3,6 6,1 3,8	1,03 1,60	4,6 3,7 5,8 4,1	1,24 1,41	Stoß! Stoß!
14. XII. 18 ^h 51'	- 500 + 405 - 450 + 216 - 380 + 380 - 405 + 270 - 400	1,24 2,08 1,00 1,50	+ 20,9 - 16,9 + 18,9 - 9,0 + 15,9 - 15,9 + 16,9 - 11,3 + 16,7	+ 5,8 - 3,0 + 5,1 - 1,7 + 4,7 - 2,9 + 4,8 - 2,2 + 4,3	1,93 3,00 1,62 2,18	9,8 9,8 11,3 13,3 8,5	150 150 200 275 110	4,0 3,3 3,6 1,8 3,0 3,1 3,3 2,2 3,2	1,21 2,00 0,97 1,50	4,1 2,7 3,6 1,4 3,3 2,6 3,4 1,8 3,1	1,52 2,57 1,27 1,89	
18 ^h 52' 21 ^h 07' 23 ^h 56'	- 690 + 315 - 315	2,19	+ 28,8 - 13,2 + 13,2	+ 7,3 - 2,8 + 2,8	2,61	9,7	150	5,4 2,5 2,5	2,16	5,1 2,3 2,2	2,22	
	- 270 + 360	0,75	+ 11,3 - 15,1	+ 4,0 - 3,0	1,33	8,6	115	2,2 3,0	0,73	2,9 2,4	1,21	
	620 + 378	1,64	+ 25,9 - 15,8	+ 8,7 - 3,1	2,81	9,1	130	5,0 3,1	1,61	5,8 2,5	2,32	Stoß!
	- 270 + 360 - 270	0,75 0,50	+ 11,3 - 15,1 + 11,3	+ 3,4 - 3,0 + 3,2	1,13 0,60	11,3 12,0	200 225	2,2 3,0 2,2	0,73 0,48	2,4 2,4 2,2	1,00 0,49	
	+ 540 - 910 + 412	1,97	- 22,6 + 33,8 - 17,2	- 5,3 + 9,0 - 3,0	3,00	9,0	126	4,6 6,4 3,4	1,88	4,5 6,3 2,4	2,62	Stoß!
15. XII 12 ^h 22'	- 425 + 360 540	1,18	+ 17,8 - 15,0 + 22,6	+ 5,1 - 4,6 + 6,6	1,11	10,0	156	3,4 3,0 4,3	1,13	3,6 3,7 4,5	0,97	
						8,8	121					Stoß!

*) Die Tafelwerte enthalten den Anteil der Vibrationen an der Beanspruchung nicht.

Table 5 MS San Francisco (from Schnadel, 1936)

Tafel III. Statistik.
Maximale Sturmwerte*).

Meßstelle M 46

Zeit und Datum	Spannung σ			Biegemoment M		Wirksame Wellenhöhen		Bemerkungen
	σ_{Berg} kg/cm ²	σ_{Tal} kg/cm ²	σ_{Tal} σ_{Berg}	Berg 1000 mt	Tal 1000 mt	Berg m	Tal m	
Nacht vom 10. XII. zum 11. XII.	+ 500	- 460	0,92	- 20,9	+ 18,85	4,2	3,6	
11. XII. 12 ^h 15 bis 14 ^h 13 . . .	+ 520	- 720	1,38	- 21,8	+ 30,10	4,4	5,7	Stoß!
„ 14 ^h 30 bis 17 ^h 30 . . .	+ 505	- 720	1,43	- 21,1	+ 30,1	4,3	5,7	Ermittelt aus M 40
„ 18 ^h 40 bis 23 ^h 00 . . .	+ 650	- 920	—	- 30,1	+ 39,5	5,6	7,2	
11. XII. 23 ^h 20 bis 12. XII. 14 ^h 07	+ 600	- 920	1,42	- 27,2	+ 38,40	5,3	7,2	Stoß!
		(- 695)	(1,07)		(+ 29,10)		(5,5)	
14. XII. 12 ^h 00 bis 13 ^h 30 . . .	+ 435	- 560	1,29	- 18,2	+ 23,4	3,6	4,5	
„ 14 ^h 30 bis 15 ^h 37 . . .	+ 540	- 560	1,04	- 22,6	+ 23,4	4,6	4,5	
„ 16 ^h 49 bis 18 ^h 32 . . .	+ 640	- 720	1,12	- 26,8	+ 30,1	5,6	5,7	
„ 18 ^h 59 bis 21 ^h 00 . . .	+ 450	- 765	1,70	- 18,8	+ 32,0	3,8	6,1	Stoß!
		(- 630)	(1,40)		(+ 26,4)		(5,0)	
„ 21 ^h 25 bis 23 ^h 50 . . .	+ 585	- 765	1,31	- 24,5	+ 32,9	5,0	6,1	
14. XII. 0 ^h 20 bis 15. XII. 9 ^h 15	+ 560	- 720	1,29	- 23,4	+ 30,1	4,8	5,7	
15. XII. 9 ^h 35 bis 11 ^h 30 . . .	+ 560	- 810	1,45	- 23,4	+ 33,9	4,8	6,4	Stoß!
		(- 660)	(1,18)		(+ 27,6)		(5,2)	
„ 12 ^h 30 bis 13 ^h 30 . . .	+ 450	- 675	1,50	- 18,8	+ 28,2	3,8	5,3	Stoß!
		(- 630)	(1,40)		(+ 26,4)		(5,0)	
„ 14 ^h 00 bis 17 ^h 00 . . .	+ 540	- 720	1,33	- 22,6	+ 30,1	4,6	5,7	Stoß!
		(- 650)	(1,20)		(+ 27,2)		(5,1)	
„ 18 ^h 00 bis 20 ^h 01 . . .	+ 470	- 540	1,15	- 19,7	+ 22,6	4,0	4,3	
„ 21 ^h 01 bis 22 ^h 16 . . .	+ 450	- 450	1,0	- 18,8	18,8	3,8	3,6	

Anm.: Bei den eingeklammerten Werten ist der Anteil des Stoßes abgezogen.

*) Die Tafelwerte enthalten den Anteil der Vibration.

orbital velocities of water particles by the ship's body.²²

(c) "In sagging conditions the stresses are larger than in hogging because of the impact of waves against a ship's bottom.

(d) "The calculations of the longitudinal strength of ships must be modified. For the real stresses of ships at sea, higher wave heights must be taken for sagging than for hogging conditions. The relationships can be taken from the foregoing research for ships of low engine power. For faster large ships these must be increased.

(e) "The usual methods of calculation of bending moments, bending stresses and deflections from moments and compressive forces are satisfactory for practical purposes. The wave length equal to ship length must be chosen for calculations as before."

Slamming was observed during the storm. The slamming accelerations were low because the ship was in the hove-to condition.

²² Unexpectedly low bending stresses were found later by E. V. Lewis (1956b) in towing-tank tests. This phenomenon can be readily explained on the basis of the work of Korvin-Kroukovsky and Jacobs (1957), and Jacobs (5-1958). It was shown that the Smith effect is approximately doubled with inclusion of the body-wave interference effect.

Stereophotographic measurements by Weinblum and Block (1936) had no direct connection with the ship's motions and stresses. They recorded the sea surface contours a short distance in front of the ship. They vividly demonstrated the irregularity of the storm sea and confirmed the magnitude of the wave heights obtained by extrapolation of the profile measured at the ship's sides. However, it is virtually impossible to apply these data to the quantitative study of wave properties by modern statistical methods because stereophotographs give samplings of small sea areas at isolated instants.

5.13 Admiralty Ship Welding Committee—SS Ocean Vulcan. Observations aboard the SS *Ocean Vulcan* were made as a part of a broad program initiated by the Admiralty Ship Welding Committee (England). Under this program the structural responses of riveted and welded ships were compared. The entire work was described in the following reports.²³

²³ These reports are obtainable from H.M. Stationary Office, P.O. Box 569, S.E. 1, London, England, or in the United States of America from British Information Services, 30 Rockefeller Plaza, New York 20, N. Y.

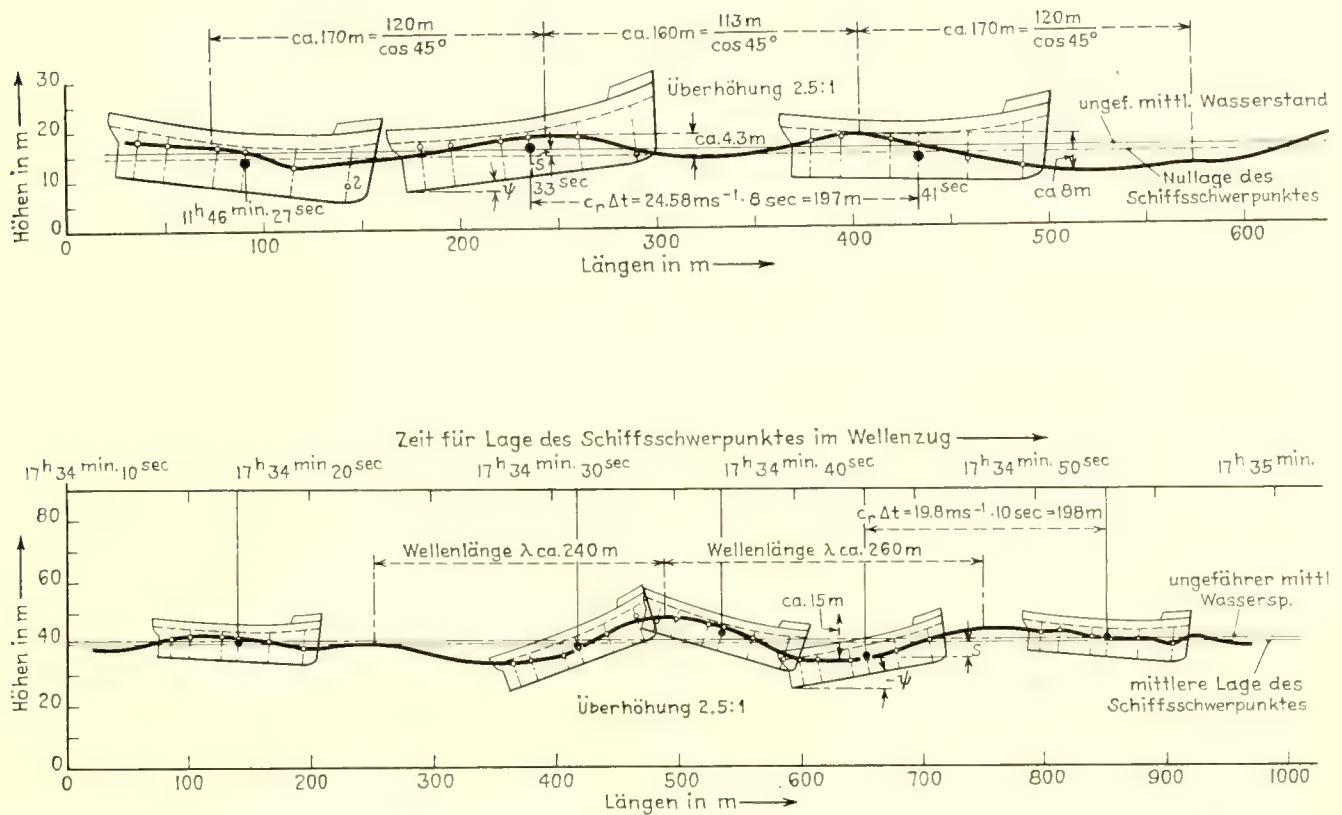


Fig. 29 Motions of MS San Francisco. Upper, at beginning of storm, wind and waves 45 deg off bow. Ship at 10.13 knots. Lower, hove-to during storm at zero speed. Vertical scale enlarged 2.5 times (from Horn, 1936)

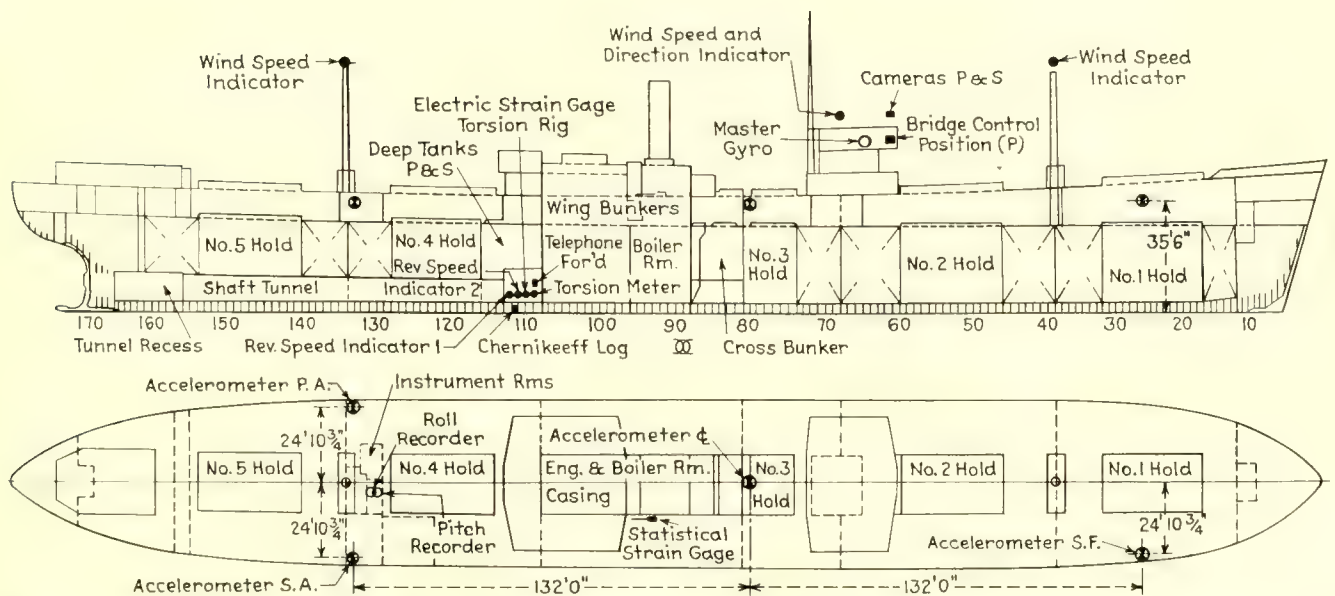


Fig. 30 Disposition of instruments, accelerometers, and miscellaneous equipment (from Admiralty Ship Welding Committee 1953)

Table 6 Summary of Instruments Used on MS *Ocean Vulcan* (from Admiralty Ship Welding Committee, 1953)

	EFFECT TO BE MEASURED	INSTRUMENTS FITTED
Instruments simultaneously recording on the Main Panel	Normal water pressure	63 pressure gauges
	Wave profiles at ship's side	700 wave profile indicators arranged on 12 transverse sections
	Accelerations	4 accelerometers, each capable of measuring accelerations in three directions
	Angles of roll and pitch	Gyroscopic roll and pitch recorders
	Angles of yaw	Repeater from ship's gyro-compass
	Wind force and direction	3 anemometers
	Ship's speed	Chernikeeff log (also ship's Walker log)
	Engine revolutions . .	Tachometer on main shaft
	Shaft horse power . .	Ford torsionmeter, also strain gauges on shaft
Miscellaneous Instruments	Stresses on transverse section near amidships	20 complete groups of strain gauges giving a heart of plate stress
	Survey of wave heights	One pair of cameras for stereophotography
	Stresses and accelerations at particular places	Two portable strain gauges and one portable accelerometer
	Statistical data on magnitude and frequency occurrence of large waves	An automatic statistical gauge (fitted after sixth voyage)
	Slamming phenomena . .	Visual observations of pressure gauges, stress gauges, accelerometers, etc.

stresses will be for the most part deferred until Chapter 5. In this chapter attention is concentrated on ship motions.

The SS *Ocean Vulcan* is a largely welded ship built in 1924 in the United States for the British government. The design is based on a small standard British 10,000-ton tramp design, of which the riveted SS *Clan Alpine* is typical. These ships are close to American Liberty ships. The principal dimensions are:

Length BP, ft-in.	416-0
Breadth molded, ft-in.	56-10 ³ / ₄
Draft molded (Summer), ft-in.	26-10 ¹ / ₈
Displacement (Summer), tons	13752
Engine: triple-expansion reciprocating, ihp	2500

The ship was instrumented extensively. A list of instruments is given in Table 6 and the arrangement of the instruments is shown in Figs. 30 and 31. The disposition of the wave-profile indicators and the pressure gages along the contours of the ship's sections is shown in Figs. 32, 33 and 34.

Most of the relationships between stresses and imposed loads were to be obtained by static tests in smooth water. The sea trials of the SS *Ocean Vulcan* were conducted for the primary purpose of *evaluating loads* imposed on the ship structure by waves. The loads were evaluated from records of pressure gages, wave-profile indicators, and accelerometers. However, a ship section at frame 77¹/₂ was instrumented with strain gages disposed as shown in Fig. 35. Fig. 36 shows the comparison of bending moments deduced from strain gages with those deduced from pressure gages, accelerometers, and so on.

The instrumentation was designed for recording water pressures and relatively long-period acceleration cycles, disregarding such rapidly occurring phenomena as slamming. Under these conditions correct accelerometer readings were obtained by using pick-ups designed to have a short resonant period (80 cps).

Cameras for stereophotographic measurements were installed. The shutters of the camera were synchronized electrically, while the film-winding mechanism was power-driven and set to make exposures automatically at about 2-sec intervals. The cameras were in exposed positions on the flying bridge and were difficult to maintain in working order despite the protection provided. It was not considered worth while to take records under mild or moderate sea conditions since the analysis of such cases would be difficult owing to the absence of identifiable features in the sea. During severe weather the stereophotography was hindered by rain and spray collecting on the lenses. On the whole the results were unsatisfactory although some successful records were obtained. These did not represent maximum sea conditions. The remarks made in reference to Weinblum and Block's (1936) stereophotographic data (Section 5.12) also apply to those for the SS *Ocean Vulcan*.

Most sea conditions were obtained by visual observations of the wave length, height, and direction. Wave

R.1 Hogging and Sagging Tests on All-Welded Tanker MV *Neverita*.

R.2 Hogging and Sagging Tests on Riveted Tanker MV *Newcombia*.

R.6 *Ocean Vulcan* Static Experiments.

R.7 *Clan Alpine* Static Experiments.

R.8 SS *Ocean Vulcan* sea Trials.

R.9 }
R.10 } Detailed Analysis of *Ocean Vulcan* Sea Trials
R.11 }

R.12 Structural Trials on SS *Ocean Vulcan* and SS *Clan Alpine*. Discussion and results.

Report No. 8, prepared by F. B. Bull (1953), is particularly relevant to the subject of the present monograph. Since the report is of recent origin and is readily available, only a brief description will be given here. Observations pertaining to the structural loading and

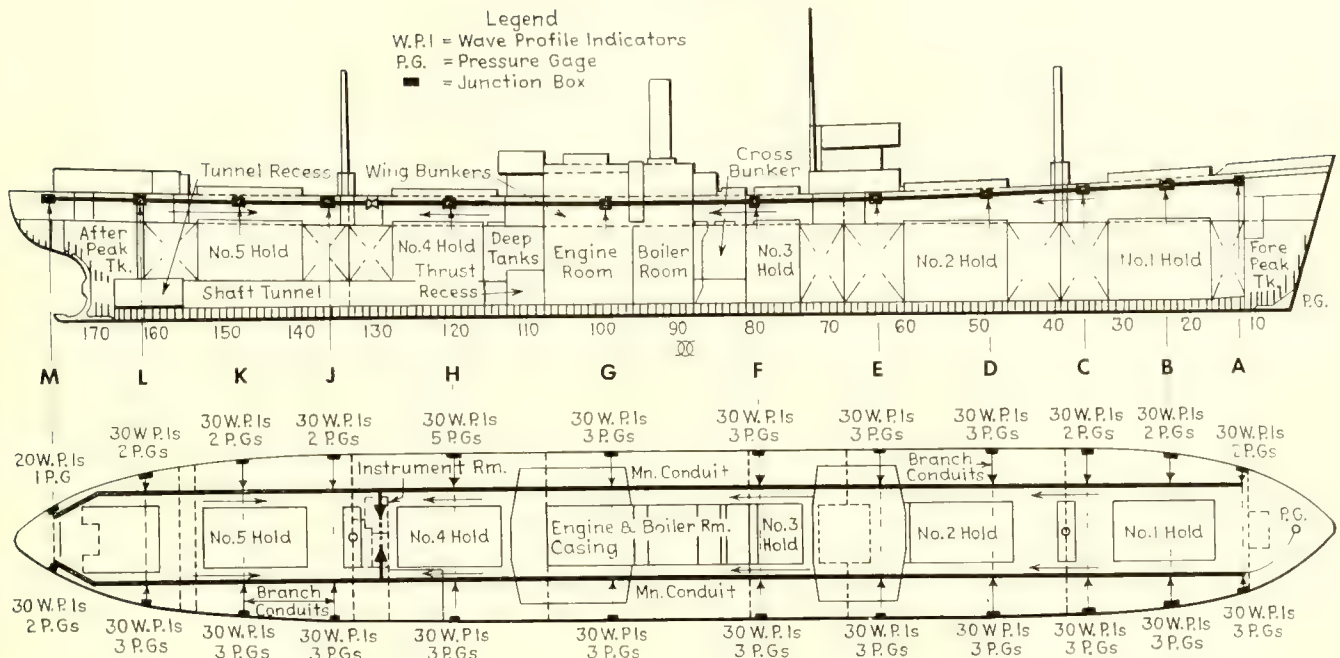


Fig. 31 Disposition of wave-profile indicators and pressure gages (from Admiralty Ship Welding Committee, 1953)

lengths also were estimated from the periods of encounter using relationships valid for trochoidal waves. "Forecast" estimates of swells made by the Admiralty Meteorological Department also were taken into account. Wave profiles recorded on the ship's sides were used for the detailed analysis of bending moments.

When planning the sea trials, the Admiralty Ship Welding Committee decided that the trials should extend over a minimum period of 12 months. Eight round-trip voyages were made during the period from December 29, 1945 to June 2, 1947. This period totalled 521 days, during which 290 days were spent at sea, 203 days of which (i.e., 39 per cent of the total) were in the open ocean. A study of eight cargo ships, chosen at random, was made. On an average they were found to be in the open ocean 35 per cent of the time. The voyages were made between English ports and the east coast of North America (Montreal, New York, and Boston), except one voyage to Galveston, Tex. The conditions were mostly mild, but the sixth voyage was much rougher than any of the others. However, even on this voyage a wind force of 8 to 9 on the Beaufort scale occurred only on one day.

Ship motions and ship stresses are affected mostly by the ratio of the wave height to its length, generally referred to as "steepness." Fig. 37 shows the wave height plotted versus the wave length taken from data obtained by several observers. Fig. 38 shows the data collected on voyages of the SS *Ocean Vulcan*. The solid line on the latter figure corresponds to the dotted one in Fig. 37.

A summary of the ship-motion data was given in

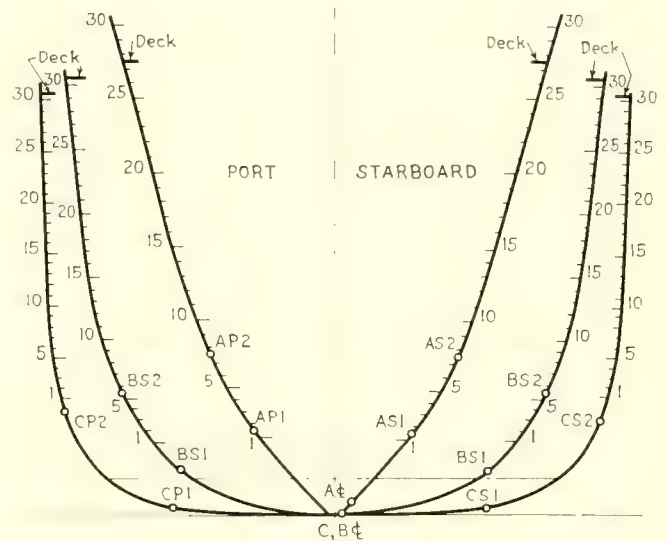


Fig. 32 Locations of wave-profile indicators (—) and of pressure gages (⊗) along contours of ship sections (from Admiralty Ship Welding Committee, 1953)

Admiralty Ship Welding Committee Report No. 8 as charts of amplitude and period distribution. The motions were generally mild. The greatest amplitude of pitching observed during the trials was ± 7.6 deg. The greatest angle of roll recorded throughout the trial was 20 deg. The observations were made by photographically recording the instrument panel at intervals of about 0.4 sec so that continuous curves of pitching

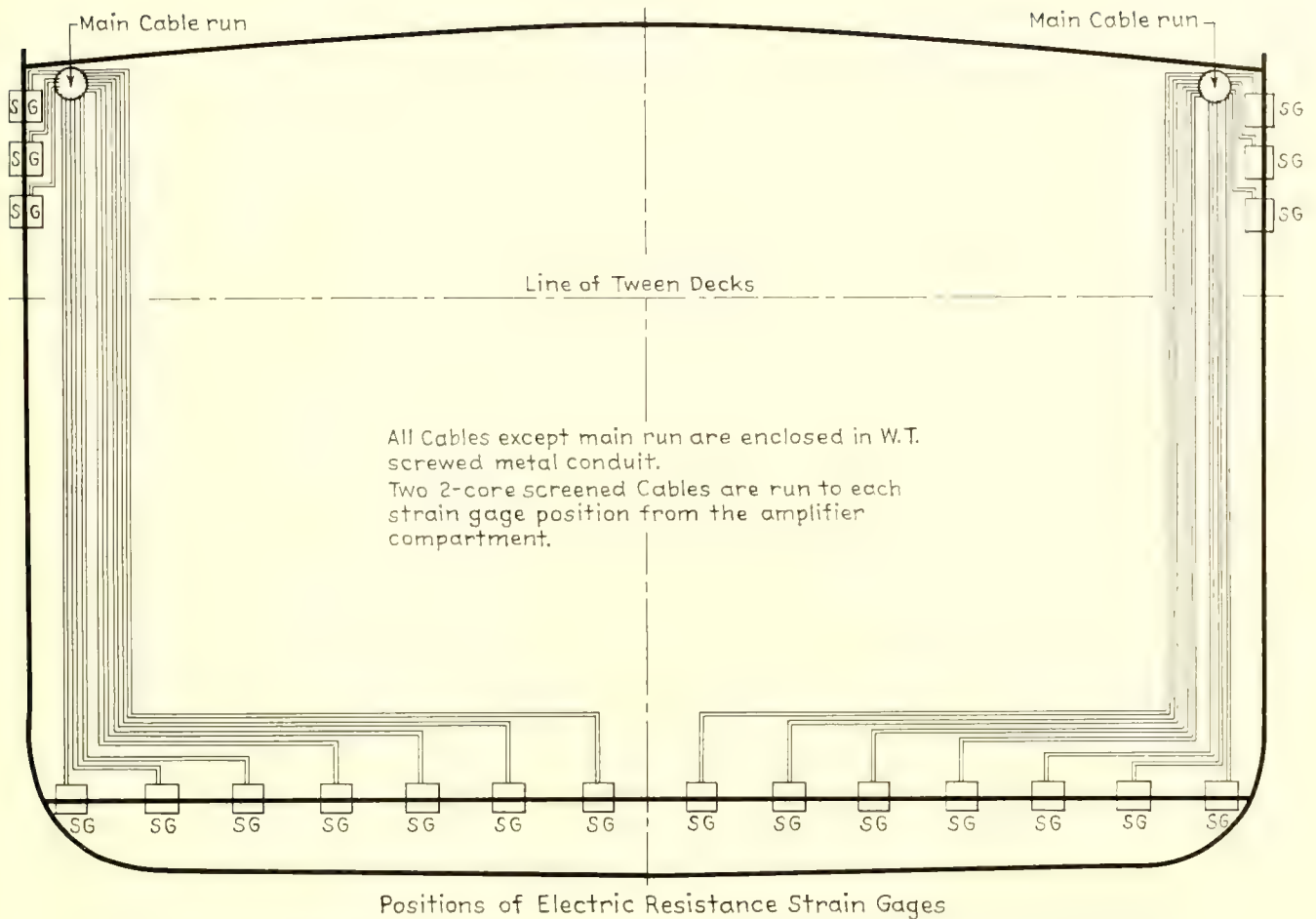


Fig. 35 Positions of electric resistance strain gages (from Admiralty Ship Welding Committee, 1953)

seas. A wind of strength 6 is indicated. Waves were observed to be 10 ft high and 200 ft long. The average daily ship speed was reduced to 5.5 knots. The ship's motions appeared to correspond to synchronism, the phase lag being about 90 deg and the highest angle of pitch occurring when the wave crest was amidship. Fig. 41 is particularly interesting in that it shows the ship's position $\frac{1}{2}$ sec before and after a slam.

The first digits in the designations in Fig. 41 are the voyage numbers. The letter following designates an eastward or westward crossing and the numbers after the letters are the consecutive observation numbers during each crossing.

The SS *Ocean Vulcan* trials were not designed to include detailed measurements of slamming phenomena. Certain general observations were made, however, on the conditions tending to produce slamming. Slamming was encountered only on the west-bound crossings during which the ship was in a ballast condition with the forward drafts ranging from 8-ft-2 to 10-ft-1. Slamming was experienced on 31 days, or nearly one day in every three in which the ship was in a ballast condition on the open ocean. It is estimated that the ship ex-

perienced between 2000 and 4000 slams during the 17-month period of the trials.

Quoting from Report No. 8: "The period of the hull vibration set up by slamming was measured by a stop watch on several occasions. It was found to be constant at 100 cycles per minute, which corresponds to the frequency of the two-node vertical vibration for the ballast condition. The vibration following a heavy slam could still be clearly identified by those on board 30 seconds after impact."

Report No. 8 stated that slamming caused a greater increase in the sagging stresses amidship than in the hogging stresses. This confirms the findings of Schnadel (1936, 1937/38). Further discussion of stresses will be deferred to Chapter 5. It should be mentioned, however, that the relatively small increase in bending stress as a result of slamming on the MS *San Francisco* and SS *Ocean Vulcan* can be attributed to the low power and low speed of these ships. Larger stress increases can be expected in faster ships.²⁴

²⁴ See Section 5.16—Warnsinck and St. Denis (1957), and also Jasper and Birmingham (1958).

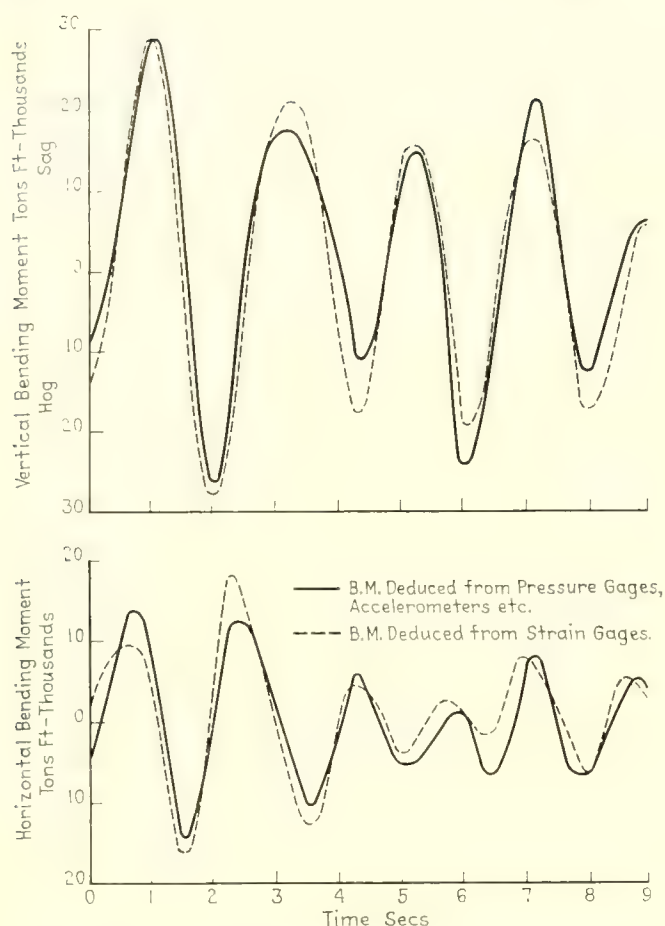


Fig. 36 Curves showing agreement between computed bending moments (from Admiralty Ship Welding Committee, 1953)

5.14 W. Möckel's observations on fishing trawlers.

Captain Walter Möckel (1953) made several voyages in fishing trawlers and reported on the comparative performance of four ships engaged in the same service in the same waters. A particularly valuable part of his report can be found in the correlation of the opinions of the crews with the data on accelerations. The latter were either measured or deduced from amplitudes and periods. This information is reproduced in some detail in Table 7.

A model of the most seakindly of the four ships—the *SS Stralsund*—was tested later in a towing tank (Numata and Lewis, 1957; Korvin-Kroukovsky and Jacobs, 1957). The speed for synchronism with waves in head seas was found to be low which is typical for full ships of a high displacement-length ratio. However, the trawling operations were conducted at a speed well below synchronism with waves equal to the ship's length. The high speed used in traveling to and from fishing areas was well above synchronism; the ship was in a "super critical" condition, as defined by E. V. Lewis (1955). The ship was thus in favorable circumstances during all phases of its service.

Table 7 Behavior of Fishing Trawlers at Sea (from Möckel 1953)

Ship	Maximum angle, deg	Maximum acceleration		Crew's views
		Rolling		
A	20.6	2.25 ft/sec ²	0.687 m/sec ²	Good Too hard pitch Too violent roll Very good
B	17.0	1.92 ft/sec ²	0.586 m/sec ²	
C	29.0	4.81 ft/sec ²	1.470 m/sec ²	
D	28.0	2.83 ft/sec ²	0.862 m/sec ²	
Ship	Maximum angle, deg	Maximum acceleration		Crew's views
		Pitching		
A	18	12.6 ft/sec ²	3.86 m/sec ²	Good Too hard pitch Too violent roll Very good
B	26	14.4 ft/sec ²	4.38 m/sec ²	
C	16	12.1 ft/sec ²	3.68 m/sec ²	
D	20	9.6 ft/sec ²	2.92 m/sec ²	

Another outstanding feature of Möckel's (1953) report is the vivid description of a near-foundering of a trawler in a following sea. While traveling at high speed, the trawler was flooded by the breaking of a high wave over the stern. The flooding of the deck well was so severe that the ship lost its freeboard and stability and, for half an hour, was on the verge of capsizing. After drastic reduction of speed, it continued traveling without further difficulty.

In looking for explanations of the phenomenon, one should recollect the statement of Kent (1924) that breaking waves in the open ocean are very rare. With Möckel's description and Kent's statement in mind, the present author paid particular attention to these phenomena during a few sea voyages he had the opportunity to make. During these voyages only relatively small breaking waves were observed at a distance from the ship, thus confirming Kent's statement. On the other hand, ocean waves were observed to break violently after interfering with a ship's own waves and particularly after encountering a ship's wake. An occurrence similar to Möckel's was observed when traveling at high speed in a following storm sea on a modified Liberty-type ship. A large wave broke over the stern, reaching the height of the stern's superstructure (gun-mount deck). It had the appearance of a large surf breaker. The author believes, therefore, that in Möckel's case two causes of the trouble were present; namely, existence of a large stern wave and the slow rate at which the ocean waves overtook the ship. A large stern wave can be assumed to exist in the wake of a ship of the full lines of a trawler. Conversely, the favorable conditions resulting from reduced speed may be attributed to two causes; that is, elimination of the stern wave and the increased rate at which the ocean waves overtook the ship.

5.15 Voyage of the *SS Nissei Maru*. An observational voyage by the cargo ship *SS Nissei Maru* was described by the Experiment Tank Committee of Japan (1954). The first half of the publication described the

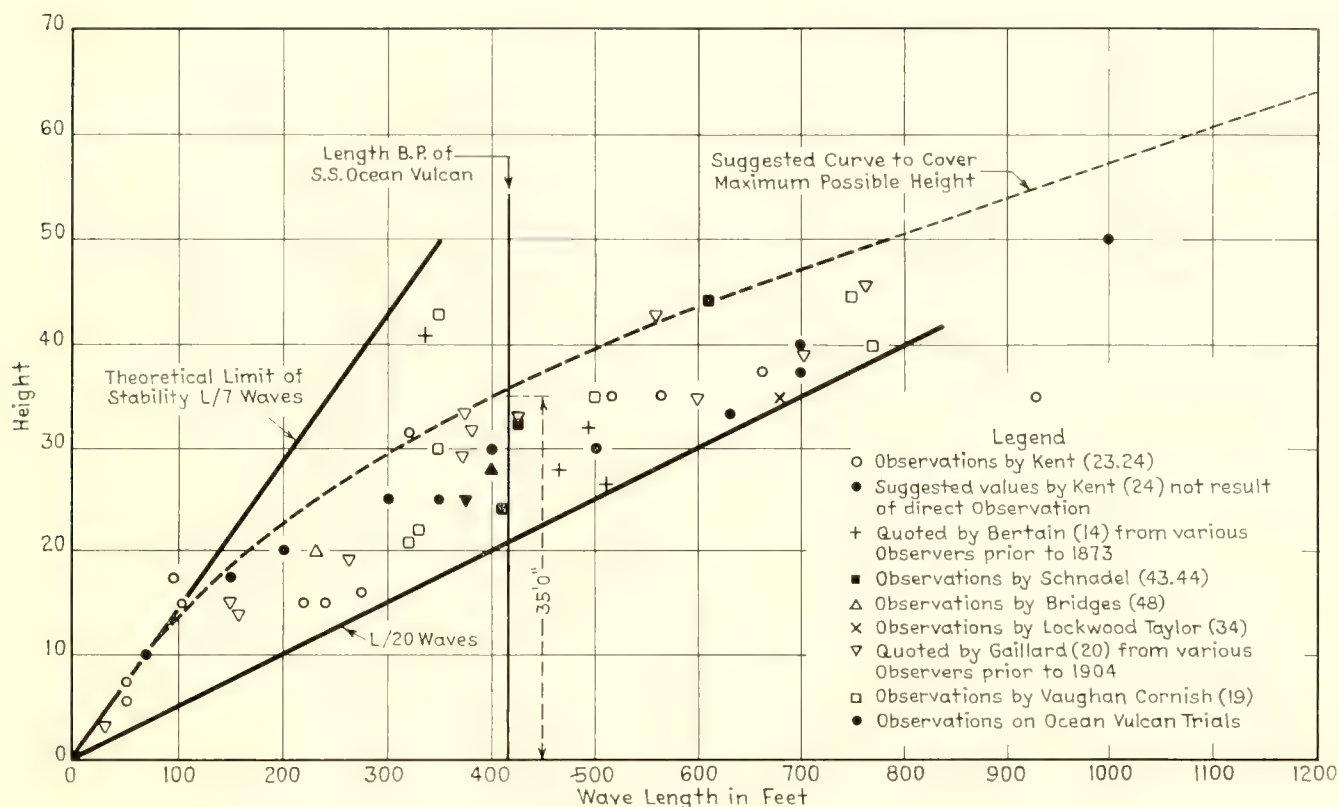


Fig. 37 Maximum wave heights for various wave lengths (from Admiralty Ship Welding Committee, 1953)

ship, its instrumentation and sea-observation results. The second half reported on wind-tunnel and towing-tank results of tests made on a model of the SS *Nissei Maru*. The lines and principal dimensions of the ship are shown in Fig. 42. The ship, completed in December 1951, is driven by a turbine with double-reduction gearing. The maximum continuous output is 4000 shp at 105 rpm, while the normal service output is 3400 shp at 99 rpm. A single four-bladed propeller 17.2 ft diam propels the vessel.

The extensive instrumentation of the ship is given in Table 8. There were two or three instruments for measuring each characteristic of ship behavior. This was a wise precaution because on previous voyages, such as Schnadel's (1934) for instance, some important records were lost because of instrument failure in storm conditions.

The observational voyage totalled 129 days, from December 26, 1951 to May 3, 1952. This was the maiden voyage of the ship from Yokohama to Vancouver, Honolulu, Singapore, Bombay, Singapore and back to Yokohama, a distance of 21,700 nautical miles. Through most of the voyage the weather was mild, but the passage from Vancouver to Honolulu was very rough. Quoting from the report:

"Fully loaded with wheat, she sailed for India via Yokohama at 17:30 on January 16, 1952, following a

south-west course along the boundary line in Load Line Rule, turning dead west at point $34^{\circ}55'N$, $149^{\circ}0'W$, at 5:20 on January 22nd, when she encountered heavy storms every day, and terrific wind, with waves growing in height. The voyage was so strenuous that the vessel exhausted her fuel reserve and was obliged to abandon the intention of proceeding direct to Yokohama and to touch at Honolulu for bunker supplies.

"The ship turned toward Honolulu at $34^{\circ}29'N$, $170^{\circ}25'W$, at 1:30 on January 28th, arriving at Honolulu at 9:15 on January 31st.

"Regular measurements of the ship's sea performance were made three times daily at 9, 12 and 15 o'clock having regard to the light necessary for filming sea conditions. After leaving Honolulu, regular measurements were made only once a day at noon, as some members of the committee left the ship at Honolulu, and very few recording papers remained, and the sea conditions were thought comparatively calm."

The observational periods were of 3 min duration. In addition progressive (step-wise) tests were conducted six times during the voyage for measuring propulsive characteristics.

The nature of wave measurements is indicated by the following quotation: "When the measurements were taken, attention was concentrated on the irregularity of the ocean waves throughout the whole voyage. The

Table 8 Items Measured and Instruments Used on SS Nissei Maru (from Experiment Tank Committee of Japan, 1954)

Items measured		Instruments used
Speed of Ship	Through Water	Shiba Log Sal Log Walker Log
	Over Ground	Sextant, Loran
Speed of Revolution of Propeller Shaft		Electric Tachometer Hitachi Revolution Recorder Togino Revolution Recorder
Torque		Hopkinson-Thring Torsionmeter Hitachi Torsionmeter Togino Torsionmeter
Sea and Wave		Stereo Camera, 2 Sets (Forward and Side) 16mm Cine Camera 35mm Camera (Ordinary & Colour Film)
Wind	Direction and Speed	Koshin Vane
	Speed	Robinson Cup
Oscillation	Rolling	} Sperry Recorder
	Pitching	
	Yawing	} Air-Gyro Recorder
	Heaving	
Helm Angle		Electric Helm Angle Recorder
Stress of Hull		Wire Strain Gauge Mechanical Strain Gauge (Giken Type and Leuner Type)

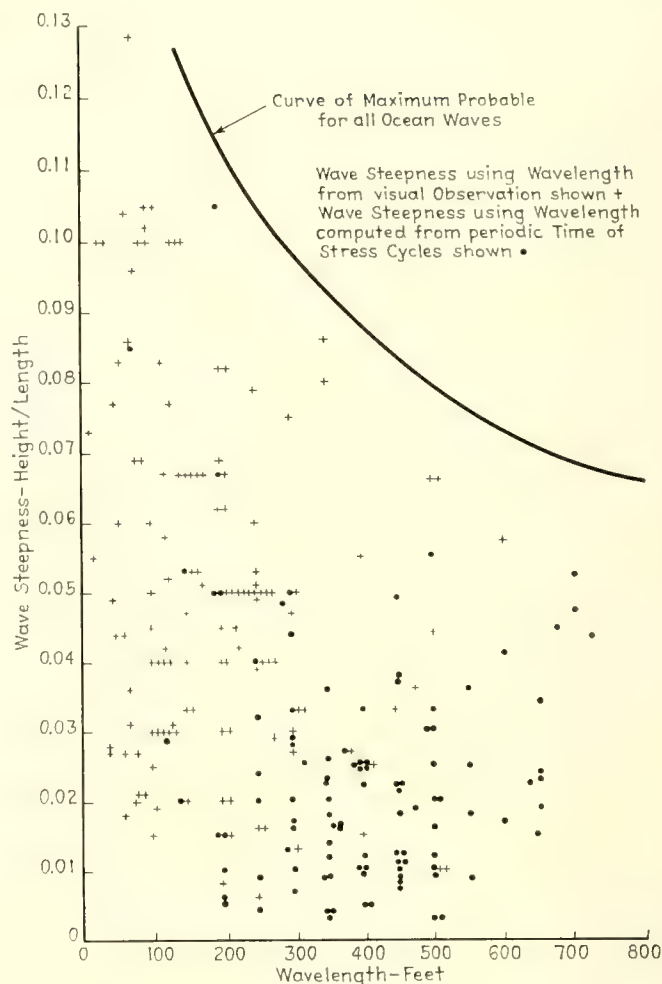
waves observed were far more irregular as to both time and locality than was previously expected, and no wave trains were observed which encountered the ship in regular periods and amplitudes. The condition of the sea surface around the ship seemed to be very confused, and the size and profile of the waves seemed to be remarkably different as to time and locality.

"Therefore, it was thought difficult to secure a standard sea condition affecting the sea performance of the ship, by measuring the waves in a certain location and at a certain instant, even though they were measured as accurately as possible. *Measuring waves by a stereo-camera has a weak point in connection with said features.*²⁵

"As to the observations of waves, though high accuracy could not be expected, observers made efforts to get an average value considering the irregularities of the waves as to both time and locality."

At the date of publication the analysis of the stereo-photographic data had not been completed, and only a few samples of these were given. As in the case of Weinblum and Block (1936) and the Admiralty Ship Welding Committee Report (1953), the data suffered from the smallness of the ocean area covered. Hardly more than one long wave length is included. It appears that the value of the stereo-photographs may be limited to showing the nature of small waves overlaying large ones and to demonstrating the non-symmetry of waves. For instance, they confirmed the visually observed fact

²⁵ Italics by the author.

**Fig. 38** Steepness of waves observed during trials (from Admiralty Ship Welding Committee, 1953)

that the slope of a wave is steeper on the lee-side than on the weather side.

The observed relationships between wave heights and lengths are shown in Fig. 43. It appears that an envelop curve could be fitted similar in form to the dotted line in Fig. 37. The wave heights observed on the SS *Nissei Maru* are, however, about 60 per cent as high as those observed on the SS *Ocean Vulcan*.

Fig. 44 shows a comparison of observed wave periods with the periods expected on the basis of the trochoidal-wave theory. For any observed period the sea wave is shown to be shorter than expected from the simple theory.²⁶

The results of each 3-min observation were summarized on charts, two of which were given in the report. These are reproduced here in Figs. 45 and 46. The headings of the figures, Exp. No. 18 and Exp. No. 77, designate observation numbers. The data of 154 observation periods

²⁶ This is in agreement with theoretical deductions of W. Pierson given in Section 1-8.6.

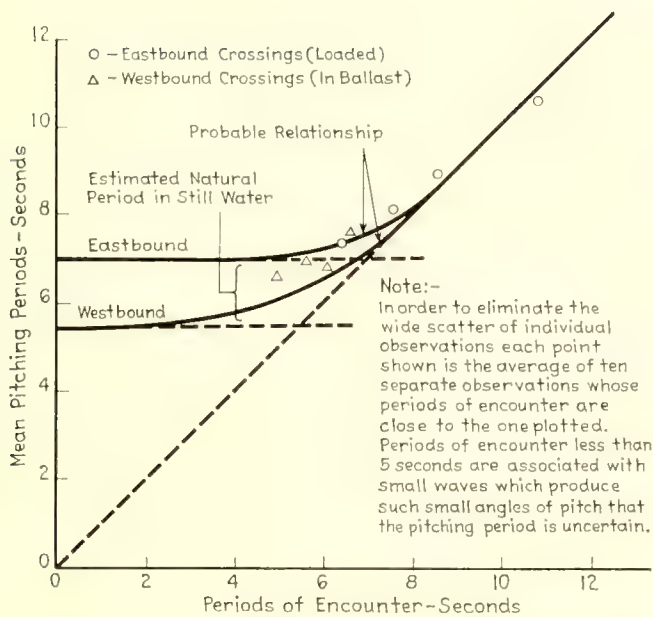


Fig. 39 Relation between pitching period and period of encounter (from Admiralty Ship Welding Committee, 1953)

were summarized in tables. An abstract from these tables is given in Table 9 for five observation periods. Observations 18 and 77 correspond to Figs. 45 and 46.

Fig. 45 and observation No. 18 in Table 9 correspond to a mild quartering sea. They disclose an interesting

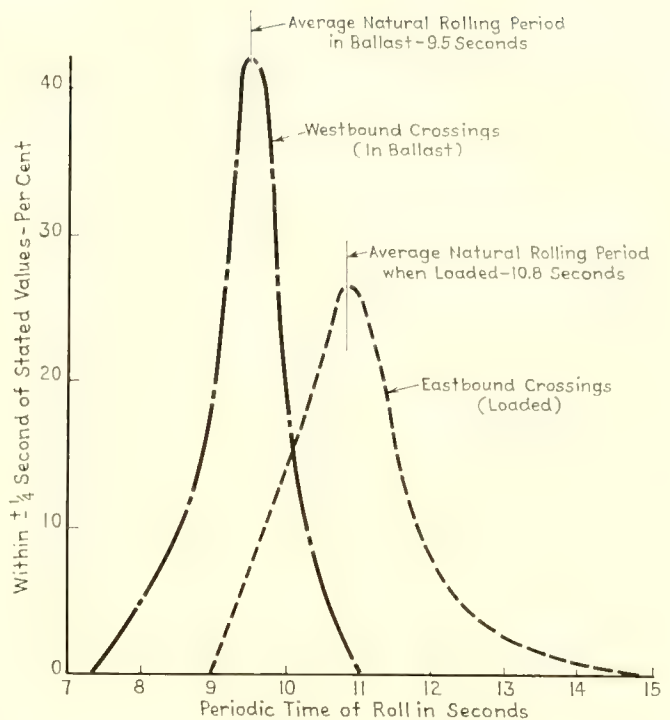


Fig. 40 Distribution of rolling periods over a typical period of one year. Note: Values on which these curves are based are taken from six crossings in each direction. There are inevitable slight differences in GM between crossings, which will result in some variations in mean of natural period of roll for individual crossings (from Admiralty Ship Welding Committee, 1953)

Table 9 Data on the Voyage of the SS Nissei Maru

(Abstracted from Experiment Tank Committee of Japan, 1954)						
Observation number	18	69	74	77	79	82
Displacement, tons	6440	13,450				
Course, degrees	62	232	265	250	270	270
Wind absolute, Beaufort	W5	S/E6	W/N7	W7	WNW5	W/N6
Wind absolute, direction deg.	267	164	286	276	296	283
Wind absolute, velocity m/s.	12.0	12.4	15.2	18.1	10.8	13.1
Wind relative direction, deg.	P/130	P50	S12	S20	S20	S10
Wind relative velocity, m/s.	6.5	15	19.5	21	15	16
Wave length, m.	30	55	100	150	120	140
Wave height, m.	1.5	3	5	6	5	5
Wave direction, deg.	P/140	P40	S5	S20	S10	S5
Ship speed, knots	13.47	9.6	8.73	6.10	8.73	5.61
RPM	92.3	89.7	87.1	78.8	85.5	69.0
SHP	2379	3069	3389	2861	3028	2042
Torque (mean), tm.	18.56	24.52	27.86	26.02	25.37	21.20
Torque (fluctuation ampl), per cent	39	34	27	45	40	55
Pitching (maximum), deg.	—	3.5	4.8	6.0	5.2	8.5
Pitching (mean), deg.	—	1.7	3.1	3.0	3.4	2.4
Pitching period, sec.	—	7.5	8.2	8.6	8.6	6.7
Rolling (maximum), deg.	—	2.5	6.5	10.1	8.3	5.0
Rolling mean, deg.	—	1.3	2.8	6.1	5.7	1.8
Rolling period, sec.	—	12.8	12.9	12.9	12.0	10
Yawing (maximum), deg.	5.0	2.0	4.9	5.4	3.8	—
Yawing (mean), deg.	2.2	—	—	—	—	—
Yawing period, sec.	—	6.2	9.5	9.0	10.0	—
Helm angle (maximum), deg.	14.0	17.5	17.5	10.4	6.9	18.1
Stress ^a (maximum), kg/cm ²	113.5	265.0	303.0	416.0	416.0	378.0
Stress (mean), kg/cm ²	74.4	136.0	144.0	182.0	160.0	166.5

^a Near the edge of the deck, just aft of No. 3 cargo hatch and in front of the superstructure.

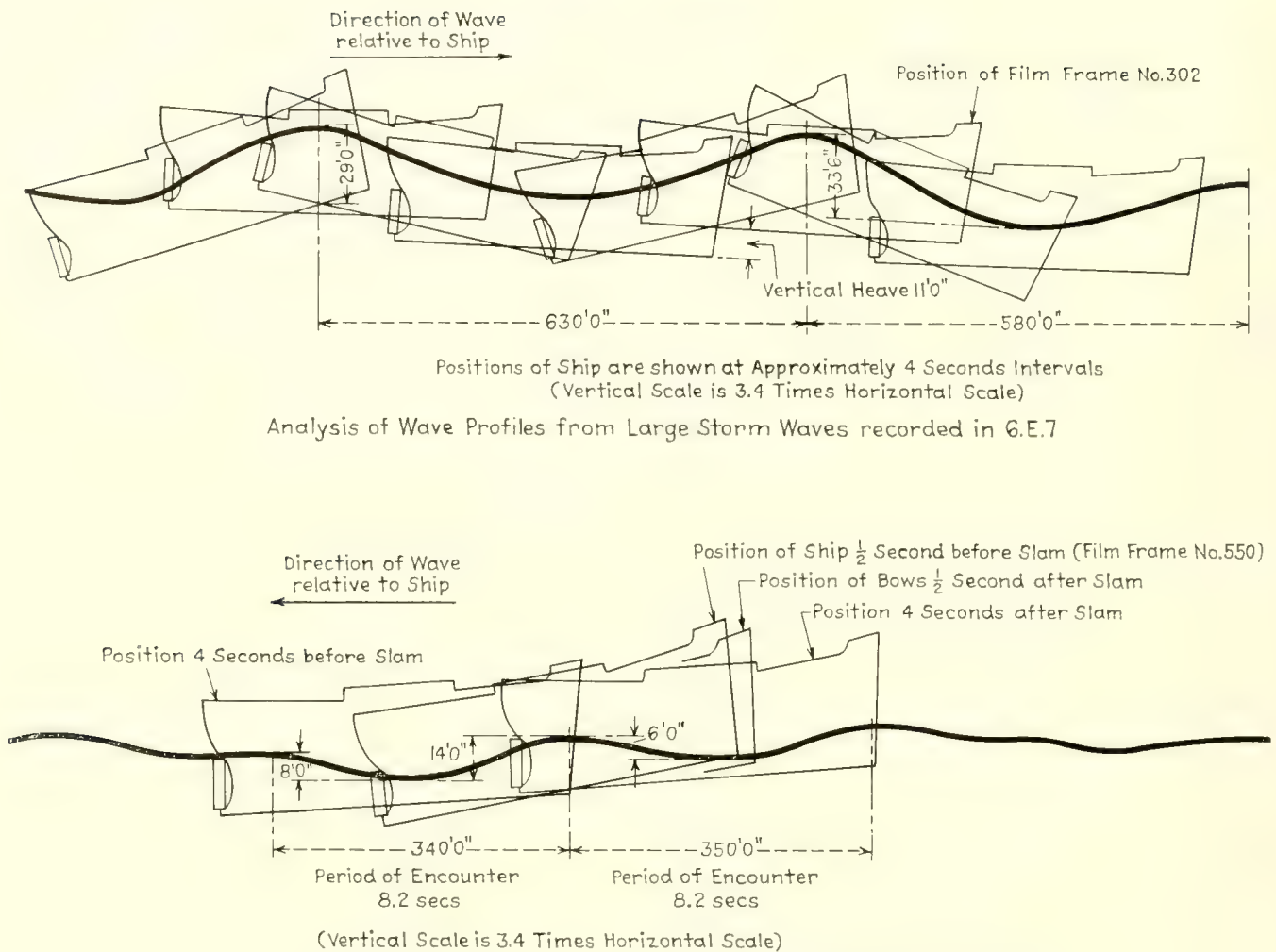


Fig. 41 Analyses of wave profiles (from Admiralty Ship Welding Committee, 1953)

feature in that large speed and torque variation are observed with oscillation periods of 15 to 20 sec and RPM cycles as long as 60 sec. A similar periodicity is observed in the helm angle. This results from the fact that the celerity of 100-ft-long waves is very close to a ship speed of 13.47 knots. This is a good illustration of large variations in speed and propeller torque which can be caused by mild disturbances of long periods.

Observations 69 to 82 in Table 9 are taken from the rough-weather passage made in predominantly head seas during the Vancouver to Honolulu voyage. Observation No. 77 corresponds to Fig. 46. The report also contained a photograph showing water being shipped during this observation period. Other photographs showed that slamming occurred and water was shipped 1 hr after observational period No. 74, and also before No. 79. Data for observation No. 82 are included in Table 9 because they correspond to the maximum reduction of engine power under storm conditions.

Fig. 47 shows the observed relationship between the

ship's period of pitching oscillation and frequency of encounter with waves. It confirms the data given in Fig. 39 for the SS *Ocean Vulcan* in that, at short periods of encounter, a ship pitches in its natural period and, at long periods of encounter, it pitches with the period of encounter. A ship rolls predominantly with its natural period, regardless of the period of encounter. This observation in irregular sea waves agrees with the theoretical expectation for rolling of a ship with low damping and a sharply peaked form of the response curve.

In regular waves a ship model rolls most in beam seas and does not roll at all in head or following seas. This is shown by the curves in Fig. 48 which depict towing-tank tests. An actual ship at sea rolls in response to wave components of varying directions. Fig. 48 shows that in this case a large amount of rolling was observed in head and following seas.

Almost all observers on ships at sea, whose work has been reviewed in this monograph, have commented on

Length bet' Perpendiculars—128.00 m
 Breadth moulded—17.50 m
 Depth moulded—10.40 m

Designed Full Load Condition, Draught moulded—8.25 m
 Displacement (inc. skin)—13,870 tons
 Block Coefficient—0.728
 Prismatic Coefficient—0.738
 Midship Coefficient—0.987
 Long. Position of C. of Buoyancy
 0.42 m Forward of Midship

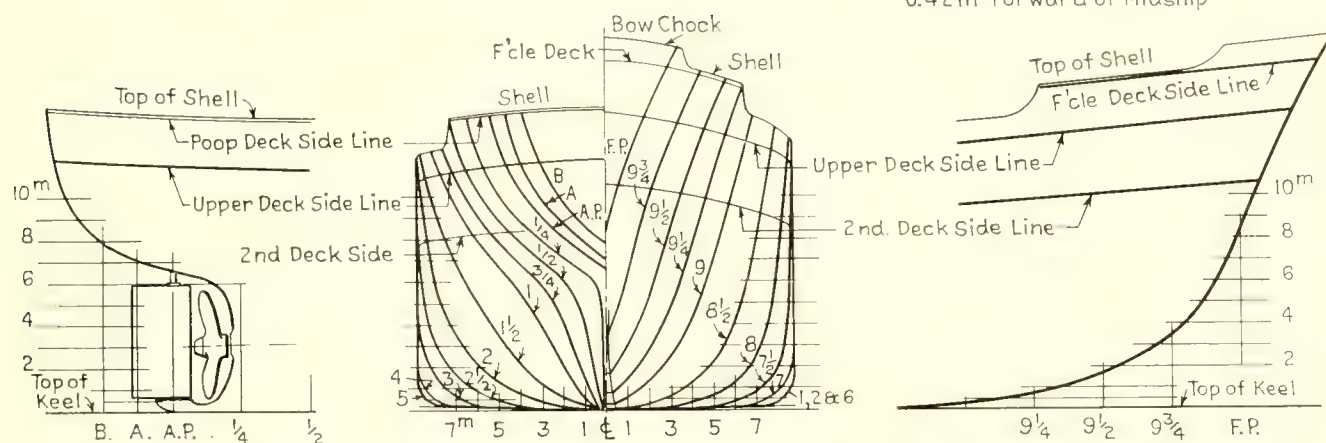


Fig. 42 Body plan, stem and stern contour of the Nissei Maru (from Experiment Tank Committee of Japan, 1954)

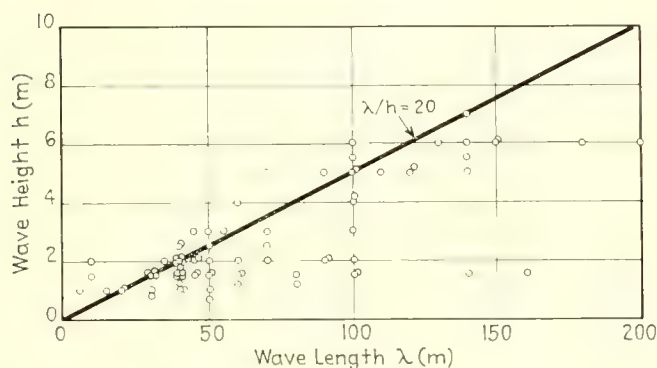


Fig. 43 Relation between wave height and wave length observed (from Experiment Tank Committee of Japan, 1954)

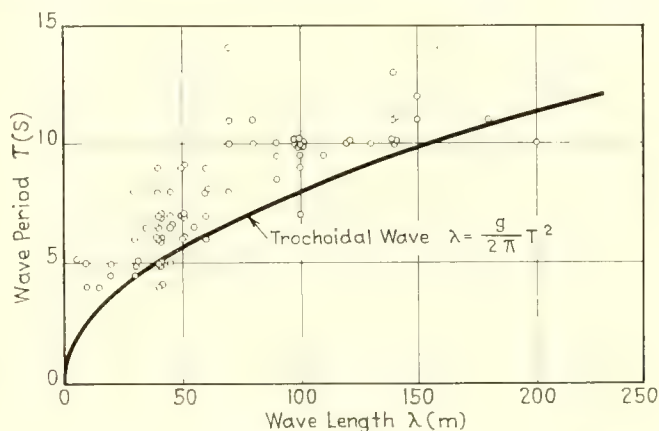


Fig. 44 Relation between wave period and wave length observed (from Experiment Tank Committee of Japan, 1954)

the irregularity of the sea and its effects. The observers on the SS *Nissei Maru*, however, stressed this aspect more than it has been done in the past and also gave various graphs and discussions comparing ship behavior in irregular sea waves with model behavior in regular towing-tank waves. However, they had no practical means of measuring sea irregularity and establishing the quantitative relationship between irregular waves and ship motions, which was done by Cartwright (1957, 1958) and Cartwright and Rydill (1957).

5.16 Additional data on sea voyages. Reviews of observations at sea, particularly of ship stresses can be found in the work of Roop (1932) and in Admiralty Ship Welding Committee Report No. 8 (1953).

Williams (1952) gave data on the distribution of rolling amplitudes and periods measured at sea on British cruisers. He pointed out that, on the average, rolling

records conform to Rayleigh's distribution law although individual records vary. In the appendix to his paper, Williams gave the derivation²⁷ of Rayleigh's law for the frequency distribution of wave heights of randomly varying sea waves. He also gave some data on damping in roll as determined by the artificial rolling of ships.

Captain Patterson (1955) described many cases of ship behavior and handling during severe storm conditions. Cartwright (1957, 1958) and Cartwright and Rydill (1957) opened a new era in the field of quantitative ship measurements at sea. For the first time a ship-borne wave-height recorder was used simultaneously with the accelerometers and gyros recording ship motions.

Warnsinck and St. Denis (1957) described comparative

²⁷ The derivations were made by N. F. Barber.

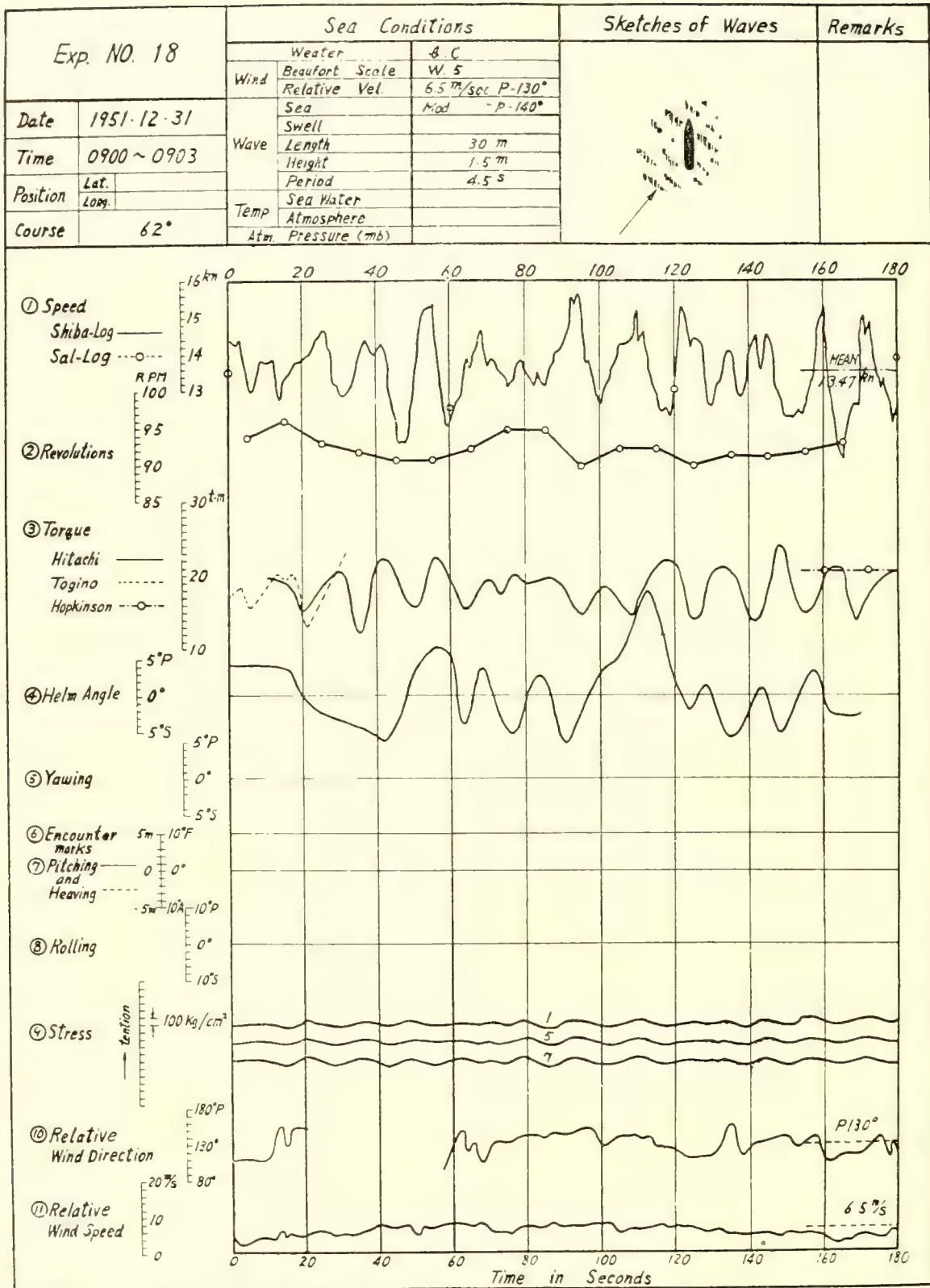


Fig. 45 An example of consolidated graph of simultaneously measured data (from Experiment Tank Committee of Japan, 1954)

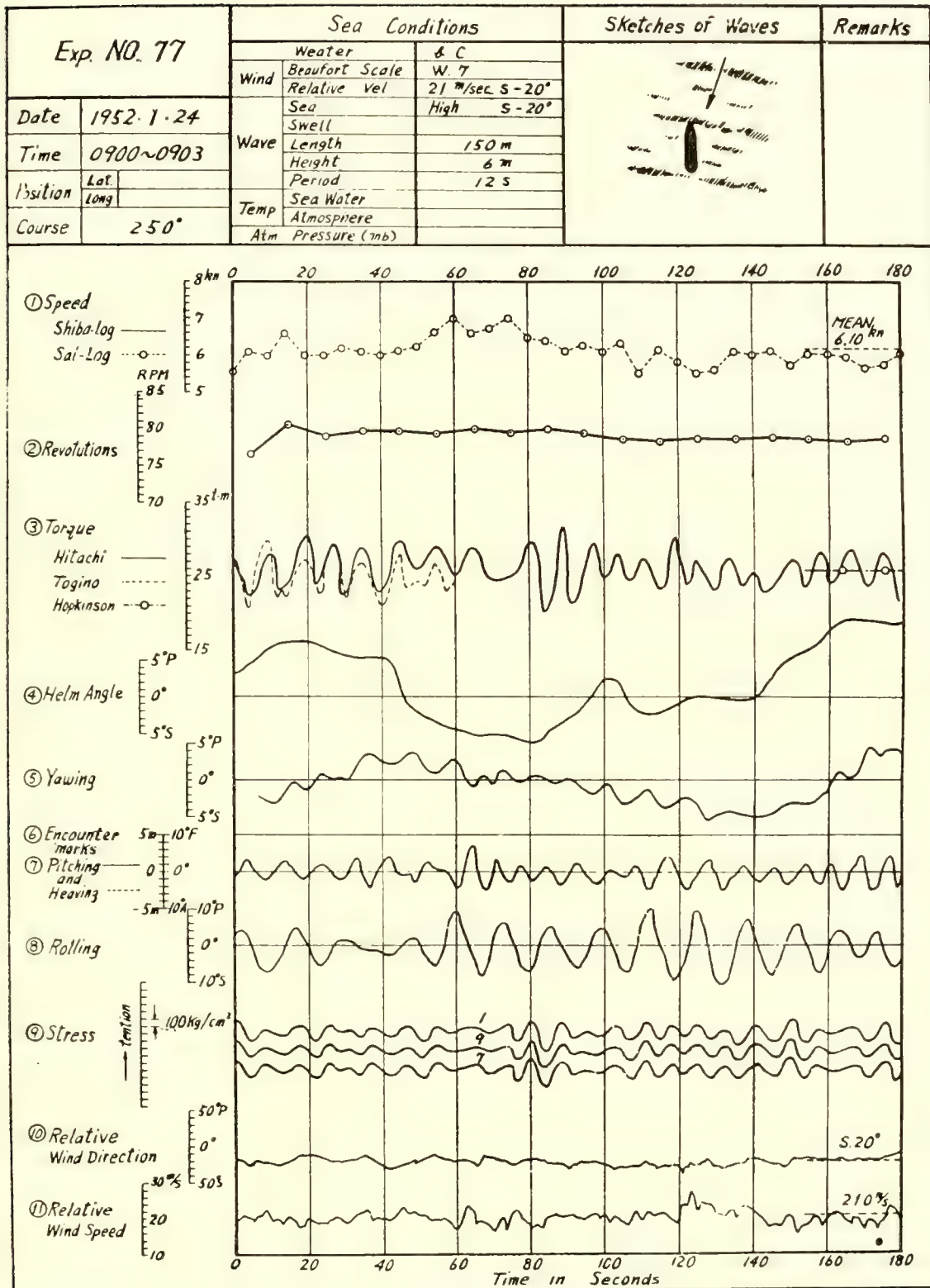


Fig. 46 An example of consolidated graph of simultaneously measured data (from experiment Tank Committee of Japan, 1954)

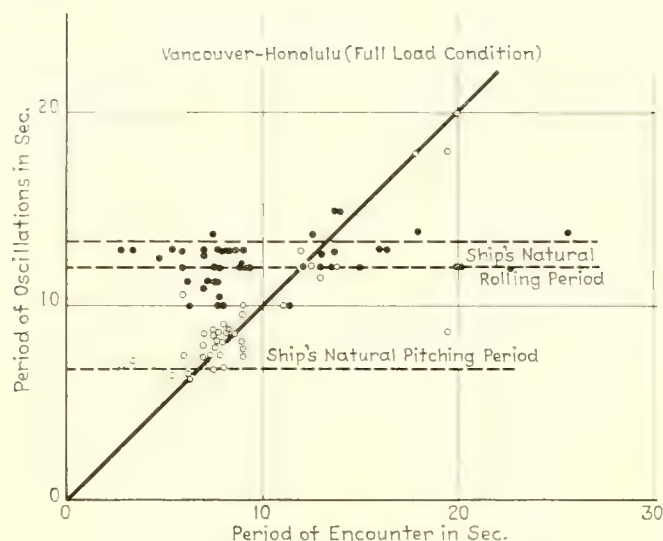


Fig. 47 Relation between period of encounter of waves and period of oscillations of ship (from Experiment Tank Committee of Japan, 1954)

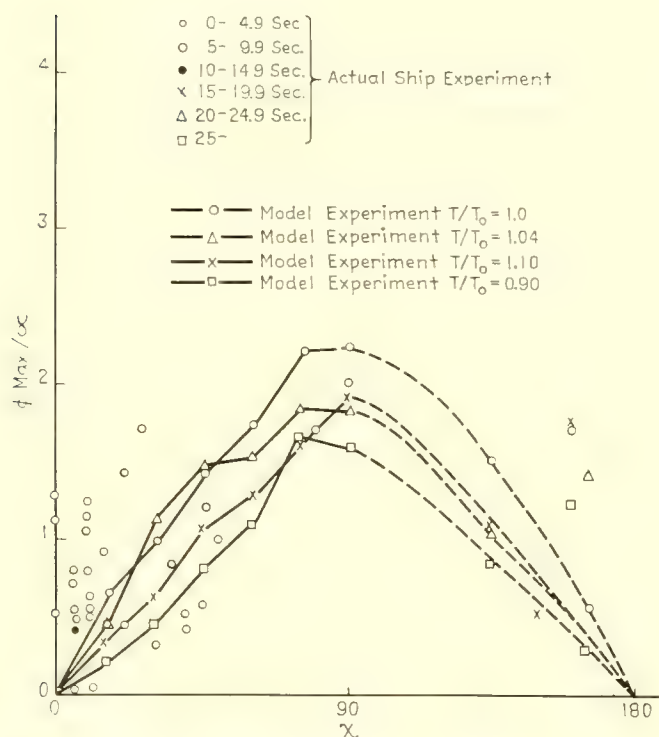


Fig. 48 Effect of irregularity of waves on ship's rolling angle (from Experiment Tank Committee of Japan, 1954)

sea tests of three destroyers of the Royal Netherlands Navy. The instrumentation used is shown in Fig. 49. "Verimeter," not otherwise described, is apparently a gyroscopic instrument used for recording pitching and rolling motions. The destroyers, sailing abreast, went through a series of maneuvers in different sea conditions.

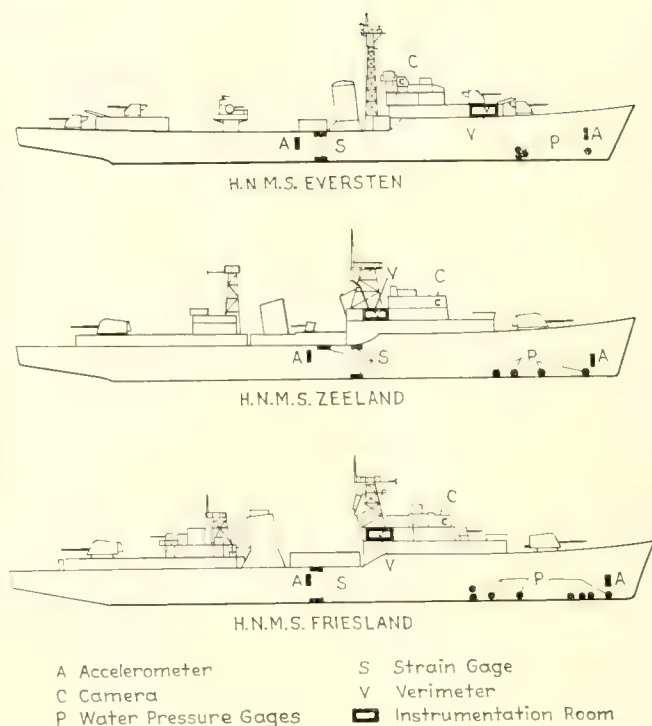


Fig. 49 Locations of instruments (from Warnsinck and St. Denis, 1957 NSMB Symposium)

A novel feature of these tests was the taking of a rapid sequence of photographs of each ship from the other ships in the formation. The instant of each photograph was identified on the motion record.

A sample of a 15-sec record with a sequence of 20 photographs was presented in the paper. A reproduction of it will be found in Fig. 5-40 on page 289. These data and certain additional data on slamming shed considerable light on the nature of the slamming process. This will be discussed in connection with stresses in Chapter 5. However, it can be mentioned here that the added stresses caused by slamming occasionally doubled the normal bending stress.

The ships were not equipped with shipborne wave recorders, so that correlation of ship motions with waves was not possible.

Figs. 50 and 51 give the distribution of roll and pitch angles with the relative direction of the waves. Examination of Fig. 50 confirms the well-known fact that fast ships roll the most in quartering seas. Fig. 51 is interesting since it shows that, at 17 knots, essentially the same amount of pitching was experienced in a range of wind directions contained within a 90-deg sector from head to beam seas. The reason for this evidently lies in the short-crestedness of the natural irregular waves.

5.2 Statistical Observations at Sea. A large amount of statistical data on sea waves and on ship motions has been collected by the "Sammelstelle für Fahrtergebnisse der Hamburgischen Schiffbau Versuchsanstalt." Some results were published by Hinterthan (1937), Hebecker

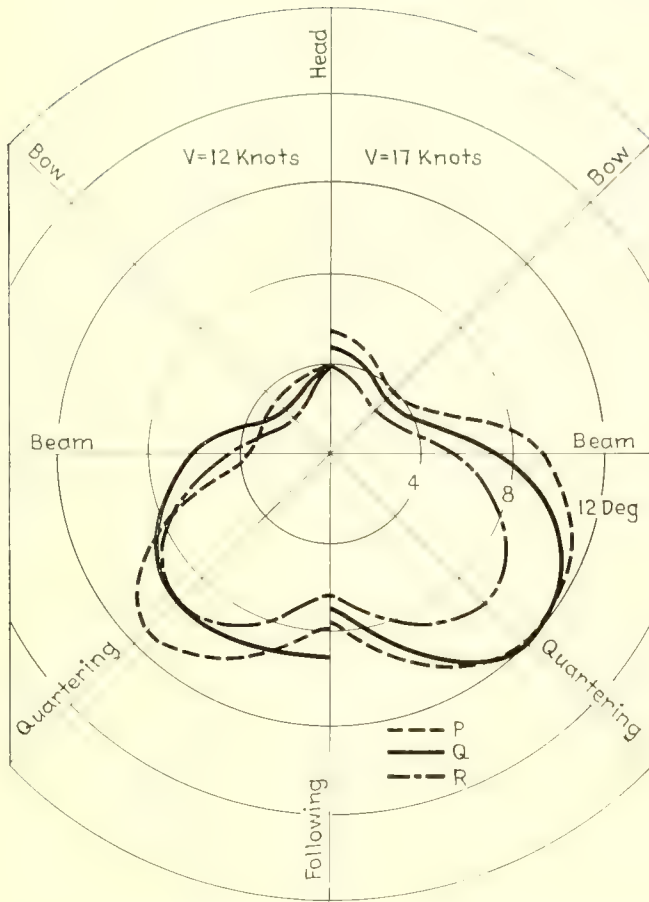


Fig. 50 Average of one-tenth highest roll amplitudes of three destroyers (designated P, Q, R) (from Warnsinck and St. Denis, 1957 NSMB Symposium)

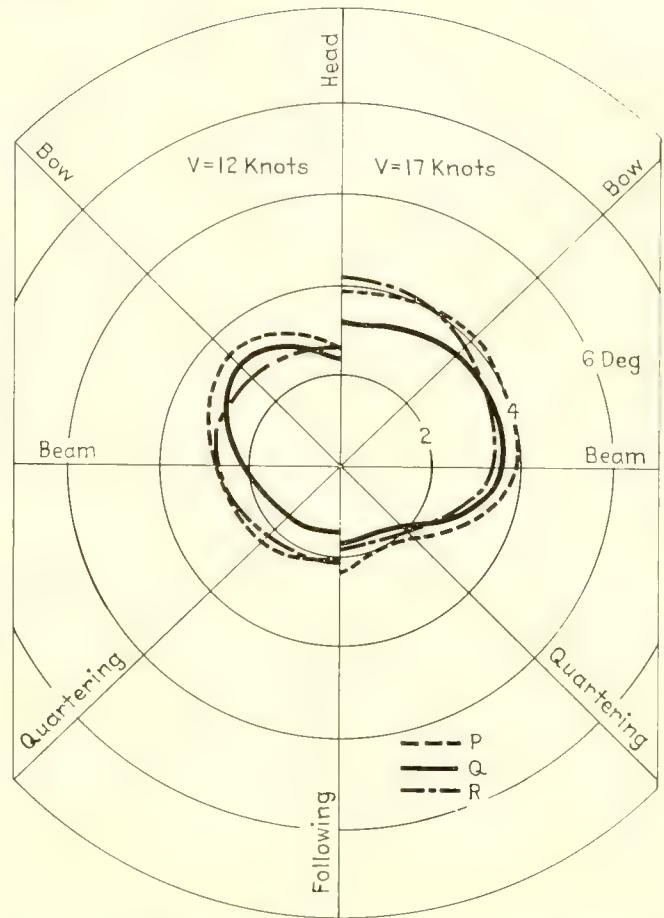


Fig. 51 Average of one-tenth highest pitch amplitudes of three destroyers in the rough sea (from Warnsinck and St. Denis, 1957 NSMB Symposium)

(1940), Langmaack (1-1941), and Möckel (1941, 1944, 1953). Observations have been made on many ships when opportunities presented themselves, but there has been no permanently installed instrumentation. Wave observations have been made visually with the help of a stop watch. Anemometers have been used to determine wind velocities. It has also been found that wind appraisals by ship officers have been very accurate. Great ingenuity has been displayed in designing simple, portable instruments for recording maxima of roll and other data.

Results of other extensive statistical observations have been published by the personnel of the David Taylor Model Basin. A number of publications is listed in the Bibliography at the end of this chapter under the names of Jasper and Greenspon with several other co-authors. A comprehensive summary of the statistical results was published by Jasper (1956).

Jasper (1956) gave a general introduction to the subject of statistics and cited several references for the statistical theory. He described three forms of statistical analysis of observed data:

- 1 A normal distribution curve, such as shown in Fig. 13.
- 2 A Rayleigh distribution curve, such as shown in Fig. 14.
- 3 A log-normal distribution curve; i.e., the normal distribution of the logarithm of a variable.

The first two forms are used for analyses of data obtained under constant environmental conditions; i.e., for data of one observational run. The last is used for the study of data obtained under different conditions over a long period.

Jasper (1956) plotted numerous curves of data on pitching, rolling, and bending stresses obtained from observations on seven ships over an extended period. The distributions of ocean wave heights were based on visual observations taken from records of the U. S. Weather Bureau and also on Darbyshire's (1-1955) measurements. In the summary he stated:

"The analysis of the voluminous data accumulated during the past several years has indicated that the pitch, roll and heave motions of ships as well as the hull girder

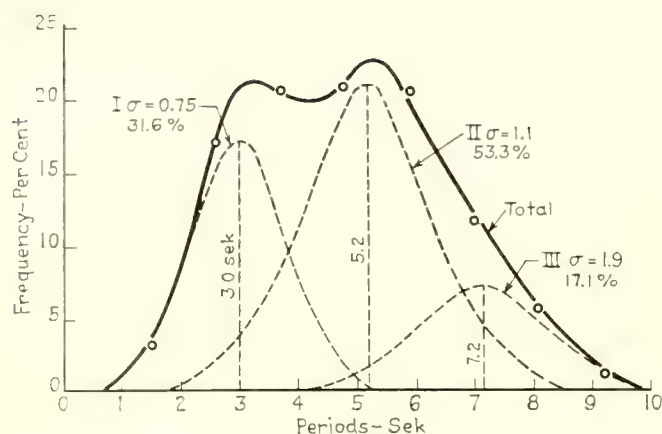


Fig. 52 Distribution of wave periods in a sea composed of three wave systems (from Langmaack, 1-1941)

stresses follow the same general distribution pattern as do the heights of ocean waves.”

In a discussion of Jasper’s paper, E. V. Lewis showed that the same conclusion was reached in tests in irregular waves in a towing tank. This is shown in Fig. 14.

The usefulness of this information was described by Jasper (1956) in the following two passages. The first one is taken from the introduction:

“There are many applications in which a knowledge of the frequency distribution of hull motions, stresses and the heights of ocean waves can be used to advantage, for example:

- 1 Prediction of the most probable amplitudes of roll and pitch motion of ships under given environmental conditions.
- 2 Estimation of the extreme values of ship responses or of wave heights encountered over a given period of time.
- 3 Statistical estimation of the capacity for which shipboard stabilization equipment must be designed.
- 4 Estimation of the endurance strength of the ship structure.”

The second passage is taken from the summary:

“There are many practical applications of the frequency distribution patterns. In the determination of the capacity of ship and shipboard stabilization equipment, it is necessary to have a reliable estimate of the probability of exceeding given angles of roll and pitch. The design of aircraft landing gear, rocket launchers and fire-control apparatus requires a knowledge of the ship motions expected in service. The ability to land planes on a carrier in a given sea can be predicted on the basis of frequency distributions such as given herein.”

Jasper (1956) stressed the similarities in the observations as shown by the Rayleigh distribution curves. Langmaack (1941), on the other hand, made ingenious use of the fact that normal distribution curves have their peaks at different periods for different wave systems. Fig. 52 shows how an apparently complex distribution

of wave periods at sea can be considered as the superposition of three distinct wave systems.

Representation of the sea statistically by a single parameter, as is done in Rayleigh’s distribution, appears to be undesirable. North Atlantic waves for instance, are almost always composed of a wind sea and one or more swells. Distinctions between these are reported rather imperfectly in ship logs. Yet pitching, rolling and yawing are affected differently by these different wave systems. While it becomes practically impossible to see swells in a heavy wind sea they are often distinguishable in the records obtained by a ship-borne wave recorder and also can be identified by photographing a radar screen (Roll, 1952).

5.3 Concluding Remarks on Ship Observations at Sea. Only sea condition and ship-motion data were considered in this section. Questions on powering are deferred to Chapter 4, and those on stresses to Chapter 5.

Reports of the early sea voyages of Kent and Kempf gave a picture of the irregularity of seas and ship motions in them. They also gave an idea of the magnitude of ship motion to be expected. Subsequent observations enlarged this picture without making it clear, however. Each voyage on a different ship in different weather conditions produced different data. There was no way to correlate the results of the different observations. Practically the only definite conclusion was that a certain low metacentric height and a long natural period of rolling are desirable for seakindliness. In this respect sea observations have confirmed Froude’s (1861) theoretical conclusions. The best summary of information on sea kindliness of ships was given by Kent (1950, 1958).

The primary reason for the failure to provide tangible, practical and usable information was the lack of a means for measuring sea waves. The second reason has been the lack of means for quantitative treatment of sea irregularity. Although the irregularity of sea waves was acknowledged, attempts to correlate ship behavior with observed waves invariably were based on an assumed regular average wave. Furthermore, observations were usually made on isolated voyages. It was uncertain what sort of weather would be met on these voyages. The only exception was in the case of the SS *Ocean Vulcan* on which observations were made over an extended period.

The situation has been changed recently by two simultaneous developments: Introduction of a ship-borne wave recorder and development of the irregular sea theory. The ship-borne wave recorder (Tucker, 1-1952, 1956) now makes it possible to obtain a continuous record of wave height from a ship.²⁸ The theory permits the correlation of ship motions with the waves causing the

²⁸ Tucker’s wave recorder gives the best results on a small ship in hove-to condition. The recording high-frequency wave components is uncertain when the recorder is used on a large ship. The nature of the errors introduced by the speed of ship is not yet established. It appears to consist of an introduction of spurious high-frequency wave components. Certain suggestions for the improvement of the recorder were given in Chapter 2.

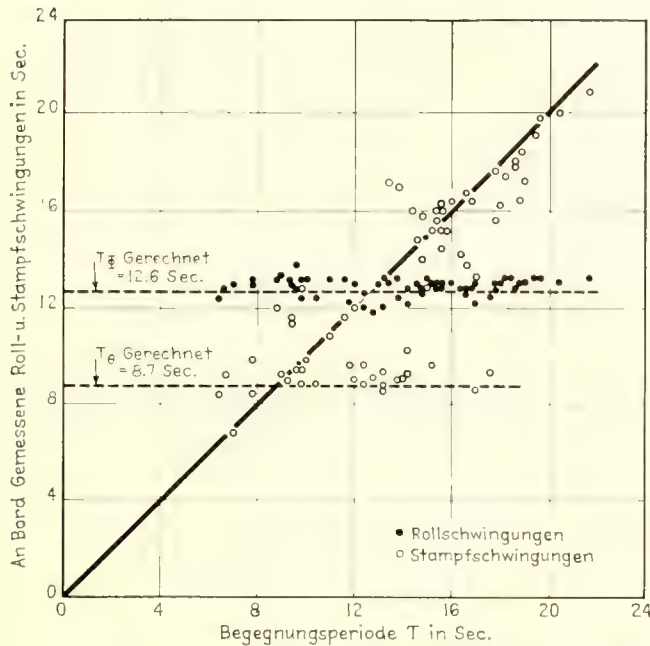


Fig. 53 Periods of rolling, T_ϕ , and pitching (Stampfschwingungen) T_θ , as measured on ships at sea plotted versus periods of encounter (Begegnungsperiode) (from Möckel, 1941)

movements.²⁹ More precisely, the theory permits abstraction of inherent characteristics of a ship from the irregular variations of ship motions which occur in irregular waves.

Thus far use of this equipment and process has been made only by Cartwright (1957, 1958) and Cartwright and Rydill (1957). Theirs was a pilot application and emphasis was placed on a description of the method rather than on the seagoing properties of the ship. Recently a series of tests using ship-borne wave recorders has been conducted jointly by the Maritime Administration and the David Taylor Model Basin. Two modified Liberty-type ships were instrumented and placed in regular commercial service in the North Atlantic during the winter. Information on the observed data is not yet available.

Analysis of data obtained on a ship at sea poses a very difficult problem. It is clear that only a small part of the collected material has been published. Writers describing observations on the *MS San Francisco*, for instance, repeatedly mentioned their intention to publish results of further analysis. However, these results have not been published.³⁰ It is very important that the method of analysis be foreseen and planned in the early stages of an observational project. The type of records obtained on a ship must be such that they can be processed by available analyzing equipment. The mass of data to be handled makes it a foregone conclusion

²⁹ The present state of the theoretical development permits this only under certain limited conditions.

³⁰ Except an analysis of single wave by Schnadel (1937/38).

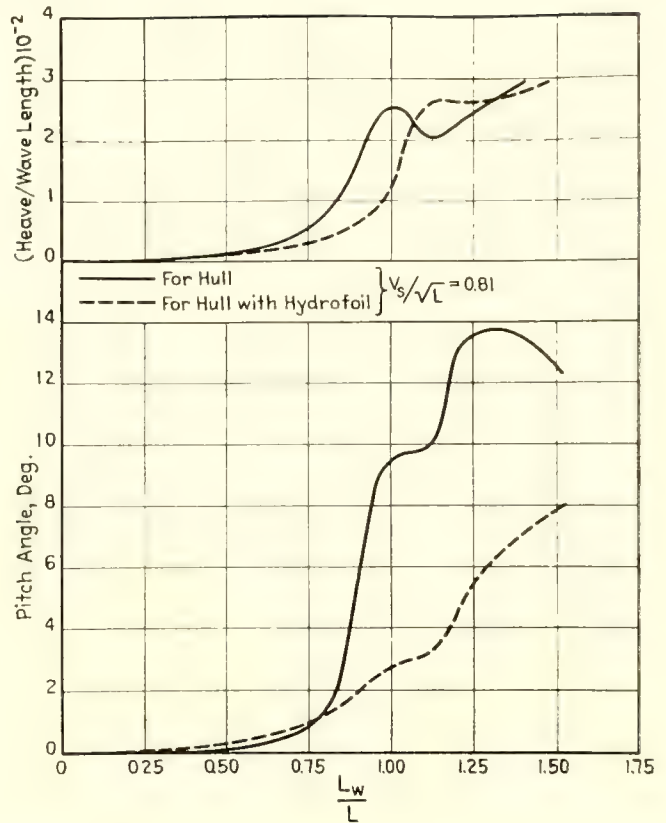


Fig. 54 Reduction of ship-model motions by bow hydrofoils (from Abkowitz, 1955)

that some type of rapid electronic equipment will have to be used.

The foregoing comments apply to recorded quantitative data. A valuable feature of the earlier observations made by Kent (1924, 1927), Möckel (1953) and Patterson (1955) is their vivid personal descriptions of sea and ship behavior. This valuable feature is usually lost as instrumentation becomes more elaborate. The recording means are often such that it is difficult to obtain information immediately, while the results of the analysis only become available many months later. The author considers it extremely important that advanced instrumentation be supplemented by simple easily read instruments. With the help of these, observers on board ships should appraise and describe the physical conditions of the sea and ship.

It is clearly impossible to formulate standards of performance which a satisfactory seakindly ship should meet on the basis of the recorded figures alone. The need is to define quantitatively the performance characteristics which correspond to mariners' appraisals of a ship's seakindliness. The subject is timely. Both towing-tank technique and ship-motion theory are rapidly approaching the state where it will be possible in the design stage to predict a ship's responses to the sea. But what is the optimum set of many (possibly

conflicting) ship characteristics? To the author's knowledge, the only work aimed at giving the answer is Möckel's (1953). This work applies to fishing trawlers. It is clear that there can be no universal specification for good sea behavior. A separate one must be formulated for each class of ship and linked to the states of sea it is likely to meet in service.

6 Ship Stabilization

6.1 Rolling Stabilization. General reviews of the many devices for controlling ship rolling have been made by von den Steinen (1955) and by Chadwick (1955-Part 1). Only three of these devices progressed beyond the initial trials and were used on a number of ships. These were stabilizing gyros, stabilizing water tanks and stabilizing fins. The use of a gyroscope for stabilizing a large passenger liner was described by De Santis and Russo (1936). This device has since been abandoned.

The theory and use of stabilizing tanks have been treated extensively in German engineering literature and also have received some attention in the United States. Some of the pertinent references are listed in the Bibliography under the following names: Baumann (1938), Feld (1937), Föppl (1934*a, b, c*; 1937), Frahm (1911*a, b*), Hahnkamm (1936), Horn (1911, 1955, 1957), Herrmann (1934), Hort (1937), Minorsky (1935, 1941, 1947) and Weinblum (1938). The latest and most thorough exposition of the theory of activated stabilizing tanks was presented by Chadwick and Klotter (1955 *a, b*). They determined the criteria for the proper proportioning of the tanks and tabulated data on several ships indicating satisfactory and faulty designs. Their papers included a critical commentary on the work of others. Also they clarified the apparent confusion in the existing literature regarding the mathematical form of the cross-coupling coefficients. The reason for the differences was shown to be the use of different co-ordinate axes. This question was discussed previously by Klotter (1934). The latter reference is pertinent to the entire problem of coupled ship motions and is not in any way limited to stabilization.

Roll stabilization by activated fins was described by Allan (1945), Volpich (1955), Wallace (1955) and Chadwick (1955-Part 2).³¹ Quoting from the latter work: "Fins are of some interest, having a virtual monopoly in the field at present. They are unexcelled where stabilization is wanted at speed, and thus are the systems of choice in a wide variety of passenger and military applications. The principles of operation have been proved

in installations of all sizes ranging from cross-channel steamers up to and including *Queen Elizabeth*. For this we have to thank, of course, the Denny-Brown Company."

Both tanks and fins have proved to be very effective in reducing ship rolling. The degree of reduction depends, of course, on the capacity of the installation. This is often rated by the angle of heel which a stabilizing device can generate in a ship moving in smooth water. A 2-degree angle is definitely too small, but, nevertheless, provides a satisfactory stabilization if conditions are not too severe. An angle of 4 deg is satisfactory and a 6-deg angle is probably the largest practical angle. Chadwick (1955) showed that the cost and weight of an installation grow rapidly with increase of the specified angle. Chadwick showed by computation that a device of a practical capacity will reduce an uncontrolled roll of 5 deg to $\frac{1}{10}$ of its value; i.e., will practically eliminate rolling. The latest description of the stabilizing-fins installation and the data on their performance were published by Flipse (1957). A 20,000-ton ship, traveling at 20 knots and controlled by the fins, rolled only about 2 deg (average double amplitude) in a very rough beam sea in wind force 8. From time to time the fins reached the limits of their travel of 25 deg. Flipse emphasized that the list of a ship in side wind should be controlled by the disposition of the water ballast in order to preserve the full fin capacity for controlling rolling oscillations.

Fins are the most effective device for controlling rolling of fast passenger and naval ships. They have, however, two drawbacks; namely, vulnerability to damage and rapid loss of effectiveness with reduction in speed. A ship forced to reduce its speed or to heave-to in adverse weather may lose most of its stabilization.³² This is not important for ships designed according to the recommendations of Froude (1861) and Möckel (1941); i.e., with small metacentric heights and long natural periods of rolling. However, natural periods of rolling have been drastically reduced in recent years because of damage-stability considerations. Modern ships, therefore, may develop a severe rolling even in bow seas under storm conditions. Such rolling is shown in Fig. 48 for the SS *Nissei Maru*. Patterson described vividly the occurrence of heavy rolling in head seas. This took place on a cargo ship which was loaded so as to have a very short rolling period.

Stabilizing tanks retain their effectiveness at low speeds. While they appear to have been abandoned on passenger ships at present, they are very useful on lightships and weather-observation ships. The author feels, however, that they may come into wider use again because of their simplicity, their lack of vulnerability and their effectiveness at reduced speeds. They appear to be particularly suitable in connection with ship-damage considerations. A low metacentric height, needed for naturally good rolling behavior, may be secured by

³¹ Only relatively recent references are mentioned here, and no attempt is made at presenting the history of the development. Reviewers of an early draft of this monograph mentioned a patent by Mr. Wilson in 1898, a manually controlled design by Motora in 1925, a proposal by a Russian naval officer submitted to Messrs. Denny in 1931, and a patent by Sir William Wallace in 1936. Practical success was eventually achieved on introduction of the "A.R.L. Continuous Control" developed by Mr. J. Bell of the Admiralty Research Laboratory of Teddington, England.

³² It has been shown in Chapter 2 that natural damping in roll also becomes smaller with decrease in speed.

locating the stabilizing tanks high in the ship's structure. Necessary damage stability may be secured at the same time by quickly draining the tanks in emergency.

The need for roll stabilization is increased by demands for damage stability. Competitive commercial considerations should further increase the use of stabilizing devices (at present activated fins) on passenger ships. Naval architects, shipowners and ship operators may become accustomed to these added complications. It is possible that consideration will be given to the stabilization of cargo ships. Activated tanks (located high) may prove to be the best choice for these applications.

6.2 Pitching Stabilization. Pitching motion of ships also can be reduced by the use of fins. The motions are reduced to a lesser extent than in rolling because damping in pitching and heaving is already large. Nevertheless, theoretical and model research indicate that a significant reduction of the amplitude of pitching can be achieved.

Quoting from Abkowitz (1955): "Antipitching fins of hydrofoil shape located at the bow have been investigated at the MIT towing tank. Models of three different type vessels (a) a modern merchant form, (b) an aircraft carrier, and (c) a destroyer, with and without antipitching fins, were tested for seakeeping in a series of wave lengths wherein the ratio of wave height to wave length was held constant at $\frac{1}{30}$. The hydrofoil was located at the bow at the keel depth and measured 4 in. along the chord. The models were of the following lengths:

Model 1	Series 60, block 0.60, ft.	5
Model 2	aircraft carrier, ft.	6.25
Model 3	destroyer, ft.	5.5

"A constant towing force was used in the series of waves and represents approximately the ship at constant EHP. . . . At resonance, the pitch angle (double amplitude) with hydrofoil is reduced to about $\frac{1}{3}$ its value without hydrofoils for the merchant vessel, to about $\frac{1}{2}$ for the aircraft carrier and destroyer. The speed loss is also reduced appreciably for the merchant ship with antipitching fins. The heave is also reduced for all three ship types."

Abkowitz (1955) presented curves of the data for three ships. The data for Series 60 are reproduced here in Fig. 54. Abkowitz (1957a) furthermore plotted curves showing a good agreement between theoretically computed and measured data for Series 60 with and without fins.

Pournaras (1956) also presented towing-tank data (obtained at DTMB) for a Series 60 hull with and without fins. The model was 10 ft long and was tested in waves 2.5 in. high at λ/L ratios of 0.75, 1, 1.25, and 1.5. Tests also were made in these wave lengths at a constant wave length-to-height ratio of 30. A large reduction of pitching motion was demonstrated. Quoting a part of the summary:

"The pitch reduction attributable to the fins considerably improved dryness of the model in head seas. The practical speed range, as restricted by motions of undesirable magnitude, is also extended. Forefoot and forebody emergence occurring during the test without fins were not observed when the fins were installed."

The fins just discussed were fixed. In an unpublished theoretical study, E. V. Lewis and W. R. Jacobs³³ found that no advantage can be expected from controlling bow fins. This is because of the vectorial addition of water orbital velocity, bow vertical velocity and ship forward velocity. This addition indicates that fin angles of attack approach the stalling angle. On the other hand, a much smaller variation of the relative direction of water flow is found at the stern. Therefore, fixed fins at the stern are less effective, but their usefulness can be increased by controlled movements. Spens (1958) presented an advance discussion on experiments with oscillating stern fins.

The fact that the stalling angle of bow fins is approached in waves indicates the need for certain precautions in model testing and in fin design. Eddies shed as a consequence of fin stall may cause fin and hull vibrations. Since the hull areas at the bow are predominantly vertical, the vibratory forces can be expected to act in a horizontal direction. Presumably they can be disclosed by placing an accelerometer, oriented to record lateral accelerations, at the bow. It is essential that these tests be conducted in severe sea conditions, as mild conditions may not disclose a potential danger. It is known that stalling occurs at a smaller angle of attack at low Reynolds numbers than at high ones. Therefore, models may indicate lesser effectiveness than can be expected on a full scale. Earlier stalling of model fins may be caused by a laminar separation³⁴ followed by less violent eddy-making. It may be desirable, therefore, to equip the models with antistalling devices which are not intended for full-size use. Such measures are employed, for instance, in towing-tank tests of seaplane models. A model wing equipped with an antistalling leading-edge slot has been found to represent the action of an unslotted full-size wing. Abkowitz (1958) achieved good results by applying suction on a small stabilizing fin model. This practice can be recommended for small skegs and rudders as well.

In the case of controllable stern fins, theoretical and model research is needed in order to determine the optimum phasing of fin motions with respect to ship motions.

Attention also should be given to the shape of appendages in order to increase their fin action. Abkowitz (1955) indicated that bossings may be significant in reducing pitching. Kort nozzles may not only improve the propulsion in heavy weather conditions, but may also reduce ship motions.

³³ Davidson Laboratory, Stevens Institute of Technology.

³⁴ Spens (1958) indicated that this separation, known to occur in a steady fluid flow, may not necessarily occur in the case of an oscillating hydrofoil.

7 Concluding Remarks—General

Chapters 2 and 3 have been written with these aims:

1 To provide naval architects with tools for designing more seakindly ships.

2 To provide data on loads imposed on a ship's structure.

Ship motions in irregular seas are very complex and cannot be understood on the basis of visual observation alone. Designers of seakindly ships have attained considerable success by their intuitive ability alone. Nevertheless, this is an uncertain process as is manifested by the widely different qualities of ships in service. While a designer's intuition cannot be replaced, it should be exercised within the framework of hard facts. Intuition which is not guided by facts degenerates into superstition.

Thus far observations at sea have provided only qualitative material.³⁵ The necessary quantitative knowledge has been obtained from model tests and theoretical considerations. Sea conditions can be simplified in model testing and certain sea and ship characteristics can be segregated for more detailed investigation. However, even in a model, the dynamics of motions is complex and cannot be understood fully. As a consequence it becomes necessary to resort to theoretical considerations. Such successful investigators as W. Froude, for example, employed all three methods; sea observations, model testing, and theoretical considerations.

A sufficiently developed theory indicates an inexorable logic of events: given a certain set of conditions, certain results must inevitably follow. In the field of naval architecture two sets of conditions are present:

1 The state of the sea.

2 A multitude of characteristics resulting from ship form and mass distribution.

The first set of conditions is not under a designer's control, but he must have information about it. This was the subject of Chapter 1. The second set of conditions is under a designer's control in that he can choose a ship's form and can control its mass distribution to a limited extent. Once these are selected, a certain ship behavior will result. The relationship between a ship's form as laid out on a drawing board and its hydrodynamic characteristics is not obvious. However, it can be understood in the light of a suitable theory. Once the quantitative relationship between the geometrical form and its hydrodynamic and dynamic characteristics is established, the designer must use his intuition in selecting an advantageous and practical combination of various features.

Ship-motion theory has been developing slowly partly because it involves both hydrodynamics and the dynamics of rigid bodies. Developments during the past three decades of dynamics theory in the field of aeronautical engineering can be applied to ships. The glaring

need at the moment is for further development of hydrodynamic theory.

The foregoing remarks explain what may appear to be a tendency toward the theoretical considerations in Chapters 2 and 3. These remarks may be needed in reference to the first objective; namely, the seakindliness of ships. Application of ship-motion theory to the second objective (information on structural loading) requires no apology. Here definite quantitative information is evidently required. Generally speaking, this information can be obtained only by the co-ordinated use of theory, towing-tank experiments and observations at sea.

7.1 Applications of Ship Motion Theory. The "theory" invariably takes the form of a set of coupled differential equations, the coefficients of which depend on a ship's form and on its mass distribution. In the simplest realistic form of ship motion (coupled heaving and pitching in regular waves), there are two equations containing a total of twelve coefficients on the left-hand sides. The effects produced by variations of so many quantities are scarcely self-evident. It also appears impossible to evaluate these effects by model tests because each change of ship form affects simultaneously and in a different manner several coefficients. In theoretical calculations it is possible to vary each coefficient separately in order to investigate its effects. This requires making a systematic series of calculations similar in nature to the systematic series of model tests.

Until recently the theory of ship motions was too crude and neglected too many factors to warrant such a series of calculations. In the past few years, however, it has been improved greatly. In the case of heaving and pitching motions in head seas it has reached a practical degree of development. Up to now the work has concentrated entirely on development of the theory and on demonstration of its validity. Today a designer can use it in order to predict the seakindliness of a new vessel in head seas in comparison with a prototype of known performance. For research intended to give a broader understanding of ship motions, the series of calculations mentioned earlier is needed.

The manual calculations which have been used in the development of the ship-motion theory are awkward and time-consuming. It is recommended, therefore, that a suitable programming of the calculations be made for available digital computing machines. It is suggested, however, that excessive automatism be guarded against. The computational procedure should be broken up into steps of moderate sizes and the results of each step should be inspected. It should be kept in mind that the theory is still new and developing. Therefore, it should be possible to make changes in different parts of the programming without upsetting the entire process. In particular, it is suggested that the computation of coefficients be made a separate preliminary step prior to solving the equations. By a judicious breakdown of the entire programming into several steps, it should be

³⁵ With the exception of the pilot project of Cartwright and Rydill (1957).

possible to use available small-capacity computing machines.

It is recommended that the foregoing programming first be made for the solution of linearized equations with constant coefficients for immediate practical application. A research program also is recommended for solution of equations with variable coefficients (step-by-step integration).

The foregoing discussion referred particularly to heaving-pitching motions in regular waves. The results should be generalized in order to represent ship motions in a typical irregular sea. This can be done by statistical methods and a suitable computational procedure.

It should be remembered that in oblique seas the entire six-mode motion (seven degrees of freedom including rudder movements) must be analyzed in evaluating a ship's trajectory. However it can be assumed that analyses of more limited groups, outlined in Sections 2.33(a) and 2.33(b), will provide useful results. The author doubts that a solution of equations of motion with constant coefficients is realistic in connection with roll-pitch-yaw motions of a ship in quartering seas, and suggests a step-by-step integration of equations with variable coefficients. The coefficients of the equations can be found in the way used by Korvin-Kroukovsky and Jacobs (1957) for pitching and heaving. Evaluation of some of the coefficients is less certain at present in the case of unsymmetric motions; nevertheless, the available material is believed to be sufficient to justify the project. It should be understood that some of the coefficients may be evaluated on the basis of model test data, and that the first attempt necessarily will be semi-empirical.

The use of digital computing machines was visualized for the solution of the differential equations. However, analog computers capable of analyzing motions with many degrees of freedom have been developed for controlled aerial missiles. Setting up a similar facility for the study of seven-degrees-of-freedom ship motions in irregular seas is recommended.

7.2 Observations on Ships at Sea. For many years investigators of ship motions thought in terms of simple harmonic or simple trochoidal waves, as representing the average effect of irregular sea conditions. The irregularity of the sea was well known qualitatively, but the significance of this irregularity for ship motions was not sufficiently realized. The technique of quantitative treatment of irregular sea and ship motions is barely 7 years old and further development is expected. Nevertheless, it can now be considered as established. Its broad application, however, is handicapped (a) by difficulty in following the current literature on the subject and (b) by the scarcity of electromechanical analyzing equipment.

Regarding the first item, it should be realized that knowledge of the effects of irregular seas cannot be limited to a few specialists. It is essential that all towing-tank technicians, naval architects and mariners have sufficient understanding of it. These men are fully

occupied in their professions and cannot be asked to study books on statistics in which only a small percentage of the contents applies to the present problem. The author considers it essential, therefore, that simple texts be prepared in order to present the irregular sea theory and its application with a minimum of specialized statistical terminology.

The analyses needed in connection with irregular seas can be made on digital computing machines or on electronic filtering machines. The use of digital computing machines requires reading the record tapes, and doing this manually is too tedious to permit widespread use of the analysis. The digital computers must be supplemented by curve-reading machines for rapid conversion of tape records into typed tables. Or, better yet, the tabulating device should be made a part of the recording equipment. At best, however, the use of digital machines leaves a wide time gap between obtaining the information and getting the results analyzed. This procedure can be considered, therefore, only as a stop-gap, pending the development of special analyzers.

An analyzer operating on the principle of electronic frequency filtering may give the results of an analysis in a few minutes after the record is taken. The author considers this feature vital in efficient towing-tank operations and in observations at sea. The investigator should obtain the analyzed data while the observed behavior of a ship or a model is fresh in his mind. This should facilitate greatly the interpretation of events and should speed up final conclusions. It also will help to decide the next step in towing-tank experiments. At sea it may help to determine the success of a certain test maneuver. The development of simple portable electromechanical analyzers, capable of spectral and cross-spectral analyses, is recommended. It is desirable that they also give the autocorrelation functions, which are necessary for certain analyses and are particularly valuable in indicating the presence of harmonic (nonrandom) oscillations.

The need for development of such equipment in towing tanks will become acute as the use of irregular waves widens. The author has expressed his belief, Section 4.41, that the use of irregular waves is necessary for model testing in oblique waves.

Observations on ships at sea are now entering a new era with the availability of ship-borne wave recorders and development of the statistical theory of irregular waves. These two must be considered together. At present, Tucker's (1-1952b, 1956) recorder appears to be the most practical device. However, the wave indicators used on the *MS San Francisco* and *SS Ocean Vulcan* would have been capable of giving the same valuable information if the measurements were made and analyzed in the light of the new theory.

In addition to a wind-sea, swells almost always are present in the North Atlantic. Ship motions probably are affected more frequently by swells than by the wind-sea, and, therefore, both should be observed and recorded. However, it becomes impossible to distinguish

visually a swell when the wind exceeds certain strength (say, force 5 on the Beaufort scale). Yet a swell often can be distinguished by its period on the wave record. However, the observer on board usually does not have enough time and facilities to read and analyze the recorder tape. The best solution would be to record on a magnetic tape, provided a portable analyzer is on hand. Should this prove impossible, a tabulating attachment to the tape recorder would provide a typed table which an observer can scan for a rough estimate of longer periods.

A ship-borne wave recorder will give the sea spectrum at a moving point. No method for evaluating the sea directional spectrum from a moving ship is available at present, and, as a result, all analyses have to depend on an assumed directional distribution within each identifiable system.³⁶ The predominating direction of each, for instance the wind-sea and one or two swells, can be visually estimated during daylight hours. This estimate is practically impossible at night, and in a strong wind is difficult for swells even in the day time. Roll (1952) described a method of evaluating the direction, wave length, and velocity of propagation of a swell by repeatedly photographing the radar screen. This arrangement is recommended on ships instrumented for extensive observations. Without it night observations are not recommended.

It is necessary to continue to look for ways of recording directional wave spectra from a moving ship. Stereophotography is an extremely tedious method which is suitable only for short waves. It appears impossible to include a sufficient number of long waves in a photograph in order to make a statistically valid record.

In most of the previous sea tests, observation periods of 2 or 3 min duration were used. Such recordings are of little use in view of the properties of irregular seas. Kent and Kempf commented on the "beat" cycles of ship pitching with periods from 30 to 80 sec. The author observed, however, a longer cycle containing a number of such beats. These appeared to have half-hour periods. One becomes aware of these long cycles when trying to photograph outstanding events (for example, spectacular shipping of water), or trying to record slamming.

A 12-min observational run is definitely too short. This has been demonstrated by the variations of significant wave heights determined by Cartwright's test (also Tucker, 1-1957). A record length of 300 waves has occasionally been recommended. Generally, a broad wind-sea spectrum may be satisfactorily evaluated by a shorter record, and a narrow swell spectrum requires a longer one. It appears to the author that test runs of an hour's duration may be satisfactory under average conditions. Trial programs must be limited therefore to a small number of runs.

In the past the usual equipment on extensively instrumented observational ships consisted of gyros for

pitching and rolling and accelerometers for heaving. Not having gyroscopic instruments, Schnadel (1936) and Horn (1936) estimated pitch angles from accelerometer readings. Cartwright (1957, NSMB Symposium) presented a comprehensive spectral analysis based entirely on accelerometer readings. On the SS *Nissei Maru* the yawing-angle record was obtained from the ship's gyro compass. However, this is not satisfactory because compass repeaters usually operate in steps of about half a degree. A continuous record is necessary.

The author believes that in the future all six modes of ship motion and the rudder angle must be recorded simultaneously along with the waves. Four groups of accelerometer installations are visualized for the motions—at the bow, at the stern, and two not far from amidships. Of the latter two, one should be located very low in the engine room and the other as high as possible on the superstructure. The accelerometers are to measure vertical and lateral accelerations at each station. Surging accelerations also should be measured at the two stations amidships. All accelerometers should be duplicated to guard against loss of records caused by failures. If it is preferred to use gyros for recording angles, the electrical output of their sensing pick-ups must be identical with that of accelerometers to permit interchangeability of electronic equipment.

The central recording station should consist of a battery of identical amplifiers (including a reserve stand-by) and devices for adding and subtracting readings in order to obtain records of translational and rotational motions. All recorders should be identical and spare recorders must be provided. It must be possible to switch inputs instantly to a spare amplifier and a spare recorder. It is extremely annoying to lose a record of a valuable event in a storm because of failure of some delicate recorder detail or, worse yet, because of the need to change the recording paper.

Attention should be given to shortening the warm-up period for electronic equipment. It appears to be an ironical law of Nature that many interesting events occur unexpectedly outside of the scheduled recording periods. These could be recorded if the equipment could be put into operation instantly. The only alternative is to have the electronic equipment sufficiently rugged to stay in operation continuously for the duration of an ocean crossing; i.e., up to 3 weeks.

Recording seven modes of motion (including rudder movements) and using recording periods of an hour or more make it a foregone conclusion that magnetic tapes will be used. In addition to the timing signal, two to four channels per tape may be expected, so that several recorders will be needed. The wave record must be contained on each tape. As a result, it will be possible to evaluate by means of a cross-spectral analysis the amplitude and phase of each ship motion in relation to the wave. If the analyzing equipment is provided, analysis of an hour's recording probably can be completed and presented in the form of several spectrum plots within an hour. Thus, observers on board can appraise a ship's

³⁶ In special sea tests the directional sea spectrum can be obtained from the information on wave slopes telemetered from a suitably instrumented buoy.

behavior on the basis of both visual interpretations and spectral charts. Once the spectral charts are completed, it is no longer necessary to retain the original recording tapes and they can be used over and over again. Most of the results of the voyage then will be available immediately upon return to the home port.

Experience has shown that a large bulk of the recorded data is of little direct interest. Continuous recordings of an hour's duration are necessary, however, in order to derive the spectrum. Only relatively short portions of some of the records must be kept and transcribed into motion curves for visual examination. These may include, for instance, some large motions during which water is shipped or slamming occurs.

7.3 Summary. Ship motions were considered in Chapter 3 from three points of view:

- 1 Theory of motions.
- 2 Model testing.
- 3 Observations at sea.

Each of the subjects was considered in detail and suggestions for further research activity were made.

It was shown that, while theoretical research on heaving-pitching motions has been progressing satisfactorily, the treatment of rolling and yawing is in a deplorable state. The linear theory of ship heaving and pitching has reached a state of practical usefulness. Its further improvement will consist of more reliable evaluations of coefficients of the differential equations of motions. This formed the subject of Chapter 2.

In model tests the emphasis has been placed on experiments in oblique irregular waves. Tests in oblique regular waves are believed to be misleading in predicting rolling and yawing motions. Attention has been called to the apparent lack of planning and definition of methods and objectives in model testing in oblique irregular waves. The basic theory of such testing also needs further development. The theory of superposition has been proved true (within practical limits) for heaving-pitching motions in which the simple cause-effect relationship is evident. Its application is not clear when several causes (due to several modes of motion) exist, and when a ship's response depends on dynamic (involving products of velocities) as well as hydrodynamic characteristics.

The introduction of irregular seas and oblique waves into towing tanks leads to greatly increased reliance on theoretical considerations. Steps will have to be taken to provide the necessary training for personnel.

The object of observations on ships at sea are:

(a) To indicate the conditions occurring in service. These will be shown by the spectra of sea and ship motions.

(b) To derive the hydrodynamic properties of ships. These will be obtained from cross-spectral analyses of waves and ship motions.³⁷

³⁷ This will be possible only under certain favorable conditions. It does not appear that it will be possible to derive hydrodynamic characteristics in the general case of a ship moving in short-crested irregular seas.

(c) To obtain detailed information on ship and sea motions at certain short intervals of time; for instance, at the time of slamming or of maximum bending moment. The principal use of this information will be in structural analysis.

(d) To collect information on sea conditions.

It is recommended that in observations on ships all six components of ship motion and also rudder motions be recorded. Observations must take cognizance of the properties of irregular seas and ship motions. A ship-borne wave recorder must be used, and it is recommended that recordings be of an hour's duration. A minimum number of maneuvers and test runs should be specified to permit long observation runs.

Effort should be made to use one type of pick-up, amplifier, and recorder. All pick-ups should be in duplicate, and circuits should be so arranged that spare amplifiers and recorders can be switched on instantly.

It is presumed that magnetic-tape recordings will be used. Portable analyzers, based on the principle of frequency filtering, should be provided aboard instrumented observation ships. The analysis should be made immediately after recording. Observers should make visual appraisals of sea and ship conditions aided by analyzed data which will be presented in the form of spectra. Only records of certain specific interesting events need be preserved on the original tapes so that they can be transcribed into curves of motions.

Only ship motions have been discussed in this chapter. The effect of motions on speed and powering will be the subject of Chapter 4. The effect of sea and ship motions on stresses will be discussed in Chapter 5.

8 Condensed List of Suggested Research Topics³⁸

1 A Step-by-Step Integration is suggested of the coupled heave and pitch differential equations of motions, Section 2, with variable coefficients. The coefficients are to be evaluated using the actual sectional wetted beams and drafts at a given instant. This project is needed in order to evaluate the error involved in the usual assumption of constant coefficients. A small error is expected in the case of normal cargo ship forms. This error is expected to increase with increased inclination of ship sides, and, as it now appears, invalidates the application of theory of small motions to sailing yacht forms. The error is also expected to be large in shallow-draft craft, Section 2.13.

2 Step-by-Step Integration, as outlined in the foregoing, is suggested with the particular objective of evaluating the effect of nonlinear damping described, Section 2-3.22.

3 Summary of Data should be prepared for use in evaluating ship motions by means of equations (2). The collection of reference data and the computation of

³⁸ Approximately in the order of presentation in the body of the Monograph. These suggestions are confined to ship motions. Suggestions for the evaluation of the *coefficients* of the equations of motion are given at the end of Chapter 2.

coefficients consume the largest part of the time spent in evaluating the pitching and heaving motions of a ship.

4 Programming Digital Machine Computations of the coefficients of equations (2) is desirable. In the case of section shapes obtained by transformation of a circle (Lewis, 1929; Prohaska, 1947), the machine can solve directly the expressions resulting from such a transformation. The results of more complicated computational methods (of Haskind, Ursell, Hanaoka for example, suggested in Chapter 2) can be approximated by simple expressions using a curve-fitting technique.

5 Towing-Tank Test to evaluate coupled heaving-pitching motion of a ship will be needed for verifying the results of projects 1 and 2.

6 Tests in Wide Tanks are needed in order to establish the amplitudes and phase lags of coupled heaving-pitching motions at low speeds. Previous tests have shown considerable uncertainty in this speed range. This uncertainty is usually attributed to wave reflections from towing-tank walls. The difficulty may conceivably also be caused by the erratic behavior of damping coefficients b and B at low speed, Section 2-3.24. One of the aims of the experimental program suggested here is to separate these two effects by eliminating the tank-wall reflections.

7 Solutions for Transient Response of the coupled heaving-pitching equations of motion, Section 2, are suggested in order to provide the material needed in projects 20 and 21 on irregular ship motions under conditions in which the superposition principle is inapplicable (nonlinearities and variable coefficients).

8 Attenuation of an Incident Wave as it progresses along a ship was discussed by Grim (1957, second part). Further development and experimental verification of the subject is recommended. The object is to establish a correction to the usual strip theory in which each element of a ship's length is assumed to be acted upon by an undistorted wave.

9 Changes of a Ship's Mean Trim during pitching and heaving oscillations are known to occur and may be significant in defining a ship's seakindness. A ship which increases its mean trim in head seas is evidently a drier ship than one exhibiting a decrease in mean trim. Linearized theories so far have treated only oscillations about a mean and have not taken up the changes of mean trim from the still-water trim. Formulations to correct this defect are recommended.

It is presumed that in calculating the trajectory of a nonlinear motion by a step-by-step integration the change of the ship's attitude will automatically be shown.

10 Interpretation of Computations and Model Data should be made specifically for the appraisal of a ship's seakindness. The computation and model test data are often presented as tables or curves-of-motion amplitudes. Such quantitative data can be used directly only in a few specialized naval problems. They do not describe the seakindness of a cargo ship, for instance, as a mariner would appraise it. It is necessary to estab-

lish which properties are desired in a ship and how these properties are indicated by the results of theory or of model tests. The amplitude and phase data must be supplemented by information on the frequency and severity of shipping of water and of slamming. Methods are needed for estimating this "severity" quantitatively. Geometric superposition of wave and ship motions does not necessarily define the shipping of water, since, on the one hand, the water surface rises at a submerging flared bow, and, on the other hand, the deflection of water by the bow delays the shipping of water. It is suggested that a project be established to investigate these various effects, with the ultimate aim of preparing a specification for model tests and test reports.

11 Rolling, Heaving and Side-Sway motions of a ship at zero speed in regular beam seas should be investigated. By formally setting a system of three coupled differential equations, a procedure analogous to that used by Korvin-Kroukovsky and Jacobs (1957) for the heaving-pitching motion can be followed. The supporting material for the evaluation of coefficients can be found in Ursell (1949a), Grim (1956, 1957a and b) and Landweber and de Macagno (1957).

It is suggested that the series 60, 0.60 block coefficient model be used for the first project. The information obtained, added to the already available data on behavior in head seas, will be valuable in the subsequent multimode motion investigations. Attention should be paid to the wave lengths (frequencies) which are significant in practice.

12 Rolling, Heaving and Side-Sway motions of non-circular cylindrical and prismatic bodies should also be investigated theoretically in connection with project 11 in order to verify the theoretical procedure in a simpler case and to provide the data for a three-dimensional correction.

13 Model Experiments in regular waves (a) on cylindrical and prismatic bodies, and (b) on ship models are needed in connection with projects 11 and 12. The experiments on ship models should be conducted in wide tanks or ponds in order to avoid interference of the wall-reflected waves.

14 Experiments on Model Rolling in Long-Crested Irregular Waves are recommended as a supplement to project 11. The objective is to provide information on the applicability of irregular-wave theories to the rolling of ships, which are characterized by low damping. The results also are expected to contradict certain conclusions in regard to nonlinear rolling, Section 2.22, which were based on excessive attention to regular waves.

15 Transient Response of a Ship to an Excitation in Roll should be derived on the basis of the differential equations of motion formed under project 11. This is the preliminary step to project 16 which follows.

16 Theory of a Ship's Response to Irregular Waves based on a succession of transient responses to excitation in roll should be developed. This would be a verification and elaboration of the brief calculations made by Kriloff (1896, 1898) in which it was demonstrated that a

ship's rolling in its natural period persisted in even slightly irregular waves.

17 Ships' Rolling When Anchored should be investigated theoretically and experimentally. Dangerously large rolling has been observed in a case which has been attributed to the regularity of swells (Saunders' discussion of Korvin-Kroukovsky, 1957). However, swell records invariably disclose a large degree of irregularity. It is conceivable that mooring-cable forces, resulting from several ship motions incited by waves, may act as sufficient excitation for rolling.

18 Investigation of Environmental Conditions can be suggested as a companion project to 17. It should be investigated to what extent the usual swell may become more regular in the process of entering a shallow-water harbor and possibly refracting after approaching a sea shore obliquely. To what extent can the severe rolling, attributed to an alleged swell regularity, be relieved by changing the anchorage point to a greater water depth?

19 Rolling and Yawing Moments Caused by Oblique Waves should be determined theoretically and measured experimentally. Theoretical evaluation may follow the pattern used for heaving-pitching motion by Korvin-Kroukovsky and Jacobs (1957) with the supporting material of Ursell (1949a), Grim (1956, 1957) and Landweber and de Macagno (1957). Presently available material permits making these calculations only for the zero-speed case. Completion of certain projects listed in Chapter 2 may permit extension to cases involving forward speed.

The yawing moment caused by waves depends on a ship's trim. Calculations at several trim angles are therefore needed.

In towing-tank tests, the forces and moments acting on a restrained model are to be measured. Vertical and lateral forces and rolling and yawing moments are involved. The effect of the wave direction, model's forward speed and model's angle of trim are to be determined. Several wave heights should be used, since there is the probability of a strong nonlinear effect at certain trim angles.

It is customary to express the wave forces and moments as amplitudes in a harmonic variation. This is the form in which the data are used in the now accepted theory of linear superposition. There is a possibility, however, that this will not be suitable in the analysis of yawing and rolling in which nonlinear characteristics may be decisive. This may make it necessary to use superposition of transient responses to a series of successive impulses. To provide the material for such an eventuality, detailed information on the pattern of cyclic variations of all quantities must be reported.

20 Theoretical Evaluation of Four-Mode Model Motions in long-crested regular waves is suggested, as described under the second project in Section 2.33a. Five degrees of freedom are involved; namely heaving, sidesway, yawing, pitching, and rudder movements. The effect of surging is disregarded, and restraint in roll is assumed. After evaluation of all variables, except the

roll angle ϕ , the rolling moment is computed. As a subsequent step, the rolling motion is computed using the fourth of equations (12).

It is suggested that the initial activity be centered about the series 60, 0.60 block coefficient model. Waves approaching a ship from 60 deg off the bow, and 45 and 10 deg off the stern are suggested as likely to be critical for the rudder control, Rydill (1959); also Section 2.33a.

In Section 2.33a the author expressed an opinion that nonlinear behavior is decisive in yawing and rolling motions. Step-by-step integration with variable coefficients in the equations of motion is therefore visualized under this project.

21 Towing-Tank Tests in Regular Oblique Waves are suggested under conditions matching those of project 20. The model is to be self-propelled and free in every respect, except for the restraint in roll, with provision for measuring the rolling moment.

A well-defined rudder-control function must be chosen, suitable for both projects 20 and 21. The optimum rudder-control function will differ in bow and quartering seas. Papers by Schiff and Gimprich (1949) and Rydill (1959) can be used as a guide in choosing a suitable function. Motions of the rudder must be recorded.

Alternative tests are suggested in which the model will be left free in rolling as well as in all other modes. The comparison of the two series of tests will indicate the error resulting from neglect of the effect of rolling on other modes, which was implied in project 20.

There are large differences in the motion characteristics of ships, and tests on one model cannot arbitrarily be assumed to apply to another. The particular value of this project lies, therefore, in confirming the calculations of project 20. The calculational procedure, once confirmed, can be used subsequently for predicting characteristics of other ship forms of comparable prismatic coefficients. A separate confirmation is needed for ships with large prismatic coefficient.

22 Measurement of the Derivatives $\partial Y/\partial \psi$ and $\partial N/\partial \psi$ at various angles of trim and ship speeds is recommended in order to provide the supporting material for project 20. The effect of the trim on the rudder effectiveness $N_\delta = N_\delta(\theta)$ should also be measured. The angles of trim in these tests should be chosen to give the stern emergence corresponding to that found experimentally in project 21.

23 Results of Project 20 should be examined in an attempt to derive simple relationships between direct roll response to waves and the roll response resulting from various couplings, particularly that of yaw-roll. It is hoped to derive the optimum proportions of bilge keels and skegs to minimize the angle of roll, Section 2.33.

The presence of nonlinear effects should be remembered. Transient roll response may be excited at instants of positive $\partial N/\partial \psi$ (at bow submersion) or unusually large negative $\partial N/\partial \psi$ (at bow emersion and stern submersion) during a pitching cycle in waves. Once excited, lightly damped rolling oscillations in a

ship's natural period persist and are superposed on rolling resulting from other causes.

24 Rudder-Control Functions $\delta = \delta(\int \psi dt, \psi, \dot{\psi}, \ddot{\psi})$ have been examined in the past solely with regard to minimizing yawing excursion. The yaw-roll coupling and the resultant effect on rolling have not been considered. It is suggested that the rudder-control function be examined with regard to the side-swaying and rolling, as well as yawing characteristics of ships. A more seakindly ship can conceivably be obtained by permitting greater yawing excursions, while minimizing lateral accelerations caused by side-swaying and rolling. Greater safety (in regards to the shifting of cargo) may thereby be secured, in addition to greater comfort of passengers and crew.

25 Mathematical Representation of Manual Rudder-Control Functions is needed for proper representation in model tests in oblique seas. It also may be useful as a target in the design of automatic steering systems. It is believed possible to derive such a mathematical representation by a statistical study of the simultaneous records of a ship's yawing and rudder motions.

The control functions usually discussed involve one or more functions of ψ (see under project 24). It appears to the author that an important characteristic of manual control may be almost continual shifting of emphasis from one function of ψ to another. This shifting may well be connected with changes of sign of $\partial N / \partial \psi$ caused by the pitching of the ship and its phase relationship to the waves.

26 A Theoretical Investigation of Steering Characteristics under the Combined Action of Waves and Wind would work toward improving the steering characteristics of ships and would help in the interpretation of model tests. The steering characteristics of a ship depend on wind-caused forces and moments as well as on the waves. Erroneous conclusions may be derived from towing-tank tests, if the wind effects (not present in these tests) are not kept in mind. In most cases these effects will have to be allowed for intuitively. This process will be made more reliable if a theoretically derived example is available.

27 For Experiments Taking Wind Effects Into Account, the wind-caused force and the center of pressure can be measured in a wind tunnel. These will depend on the relative mean wind direction but will not vary appreciably with the normal yawing of a ship in waves. The wind effect can be included, therefore, in towing-tank tests in oblique waves as a constant force applied in a specified direction at a specified point on the model. It also can be taken as a function of the pitch angle. The wind gustiness is neglected in the method proposed here.

28 Uniform Instrumentation should be developed for fully instrumented ships. The output of all sensing elements should be such as to permit the use of identical amplifiers and recorders. Duplication of all sensing elements and provision for instant switching to a spare amplifier or recorder are desirable. Records should

include the wave height, all six modes of ship motions, rudder motions and possibly wind strength and direction. All records must be connected by a timing code. The development of transistorized equipment will simplify installation by making it independent of a ship's electrical system, and also can be expected to shorten the time needed to put equipment in operation for each test run.

Magnetic-tape recorders should be supplemented by visual recorders and/or tabulating devices. The visual records in this case are not intended for precise analysis and should present information in compact form for scanning and rough analysis by observers.

29 Portable Instrumentation for recording ship motions should be developed. Rugged and compact instruments have been used in missiles and research rockets (for instance Kenimer, 1955). The rapidly progressing transistor technique will also permit the development of compact multichannel equipment. Six ship-motion components can be sensed by entirely self-contained equipment using accelerometers and possibly small gyros. Additional recording channels and external jacks should be provided for connection to ship's instruments, particularly to wave-height gage, when these are available. An hour capacity of continuous recording is suggested.

The need for this equipment is indicated by the difficulty in finding suitable weather conditions on trips made by a few instrumented ships. For instance, only mild weather was encountered by the SS *Ocean Vulcan* in a time interval of many months. Reaching a satisfactory understanding of ship conditions at sea requires the collection of a much greater amount of data than is feasible by using a few fully instrumented ships. The use of self-contained portable equipment will permit more observations on ships of all kinds.

30 Ship-Borne Wave Recorder, such as is now available, cannot be considered portable equipment. It is, however, a relatively simple instrument and its installation is recommended on as many ships as possible as a part of the general investigation of ocean waves. In particular its installation is recommended on all weather ships. It is suggested that external jacks be incorporated in its construction to permit ready connection to other instruments, particularly to the portable equipment listed under project 29.

31 Tucker's Shipborne Wave Recorder is the most practical instrument available to date. However, in its original design form it is intended for weather ships in hove-to condition, and its performance can probably be improved (for use on larger ships) with additional development. A hydrodynamic problem recommended as project 23 of Chapter 1, was directed at evaluation of ship-wave interference effects on the pressure recorded by the gage. Two additional projects can be recommended here; namely, improvement of the electronic integration of accelerations and investigation of the most favorable location for the pressure and acceleration-sensing assembly. The objective of the first is to eliminate the influence of wave frequency on the inte-

grated result and to eliminate the phase shift. This conceivably can be accomplished by application of the modern servomechanism theory.

The pressure gage and accelerometer unit have habitually been installed amidships. In this location the accelerations are least, but hydrodynamic effects are at their maximum. On cargo type and passenger ships, the rectangular ship section with sharp turn of the bilge and the presence of bilge keels make the evaluation of hydrodynamic effects on the pressure-gage readings highly uncertain. The effect of the ship-made wave at forward ship speed is also large. It appears to the author that locating the sensing unit a short distance ahead of the front edge of bilge keels is more favorable. A ship's cross section in this locality, more rounded and unbroken by bilge keels, is more amenable to the evaluation of hydrodynamic effects. In this location the accelerations will be greater and more reliance has to be placed on the methods of computing vertical displacements from accelerations. A project is recommended for comprehensive investigation of the problems just outlined.

32 Development of a Portable Wave Recorder is desirable for use in connection with the portable recording instrumentation listed as project 29. An instrument which would be a combination of acceleration integrater and echo-ranging altimeter is visualized. Such an instrument can be clamped to the railing at the end of the flying bridge or carried on a short outrigger.

33 Development of a Portable Analyzer is recommended. This is intended for use with either the portable motion recorder (project 29) or the complete recording system (project 28). The analyzer must be capable of spectral and cross-spectral analyses. It is suggested that the recording and analyzing equipment be standardized for a 1-hr duration of recording.

A portable analyzer, based on the filtering principle, can conceivably be developed as a self-contained compact unit (a "suitcase") by the proper use of transistorized circuits.

The author has repeatedly emphasized his belief that the most rapid progress in understanding ship motions will result from a combination of the personal impressions of ship observers and on-the-spot analyses of recorded data. With a portable analyzer the data taken on the voyage would furthermore, be immediately useful upon return to the home port. Experience with elaborately instrumented ships has been unsatisfactory because many months have been needed for the analyses of data and in the end only a small part of the gathered material has been utilized.

34 Portable Strain Gage can well be used in connection with recording ship motions on ships not extensively instrumented for strain measurements. Such gages were used in the past for auxilliary measurements in connection with extensive permanent instrumentation. The gage suggested here is for use with the portable continuous recording equipment of project 29. It will give indication of the stress variations connected with wave

and ship motions and it will indicate stresses and vibrations caused by slamming. Since a portable gage will be applied to one side of the deck plate, it is suggested that it be made with a long base (say 5 ft) to minimize the errors caused by local plate deflections. Such a gage can be installed in a protected location along the passageway on the strength deck.

35 Measurements of Directional Sea Spectra by free-floating buoys can be recommended during special ship tests conducted in the vicinity of such buoys. Dorrestein (1957) described buoys used for scalar spectrum measurements. Vosnessensky and Firsoff (1957) based their spectrum derivations on the results of data obtained from buoys. The author understands that a disposable buoy for measurement of scalar wave spectra was developed by DTMB. A buoy has apparently been developed in England which will permit evaluation of the directional spectra from wave slopes.

A summary giving all available information on the subject would be helpful in stimulating research.

36 Methods of Measuring Directional Sea Spectra from a ship at sea should be developed. As a starting point, the author can suggest installation of four Tucker-type ship-borne wave recorders (improved in project 31) along the length of a ship. Analysis of the recorded data can follow the pattern based on Barber's work which was described in project 38 of Chapter 1. The use of a ship of low displacement/length ratio and low prismatic coefficient is suggested in this connection. If a special weather ship were to be so equipped, the use of roll-stabilizing tanks and omission of bilge keels is recommended. With a ship hove-to in head seas, a reasonable estimate of the directional distribution of waves can be expected, at least for waves $\frac{1}{2}$ to 2 times as long as the ship. Other, possibly simpler, approaches to this problem must be sought.

37 Further Development of Pitch-Stabilizing Fins is needed. This must follow from theoretical investigation, towing-tank model testing and observations on ships at sea. The efficacy of bow fins has been established. Further activity is needed in the development of fin shapes to minimize stalling effects, in the evaluation and, if possible, elimination of bow vibration and in the development of structural design criteria.

The available data on stern fins are less complete and further investigations are needed. In this case particular attention should be directed to the development of suitable control functions.

Nomenclature

Symbols listed on pages 150 and 151 in connection with Chapter 2 apply to Chapter 3 as well. NACA designations (page 151) are particularly important in connection with Section 2.3 on six-component ship motions in waves. Additional symbols are as follows:

- h = righting arm
- H = wave height (double amplitude)
- T_0 = natural period of a ship's oscillation
- T_e = period of wave encounter
- δ = rudder angle

$[\delta, \tau, \sigma, \epsilon]$ = phase angles
 γ = coherency
 χ = relative heading of ship to waves
 S_p, S_{-p} = contributions to the covariance function, equation (40)

i, j = subscripts denoting mapping regions, equations (26) and (27)

Symbols used once and defined locally are not included in the above list.

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Code of Abbreviations

ASME—American Society of Mechanical Engineers.
 ASNE—Journal of the American Society of Naval Engineers.
 ATMA—Association Technique Maritime et Aéronautique, Paris.
 ATTC—American Towing Tank Conference.
 DTMB—David Taylor Model Basin.
 ETT or DL—Stevens Institute of Technology, Experimental Towing Tank (now Davidson Laboratory).
 Hansa—Die "HANSA" - Zeitschrift für Schiffahrt-Schiffbau-Hafen.
 IAeSc—Institute of Aeronautical Sciences, New York.
 IEES—Transactions of the Institution of Engineers and Shipbuilders in Scotland.
 INA—Transactions of the Royal Institution of Naval Architects, London.
 Int. Shipb. Pr.—International Shipbuilding Progress.
 JSTG—Jahrbuch der Schiffbautechnischen Gesellschaft, Hamburg.
 J. Ship Res.—Journal of Ship Research, SNAME.
 J. Aero/Sp. Sc.—Journal of Aero/Space Sciences.
 J. Appl. Ph.—Journal of Applied Physics.
 JZK—Journal of Zōsen Kyōkai (The Society of Naval Architects of Japan, Tokyo).
 NACA—National Advisory Committee for Aeronautics.
 NEC—Transactions of the North East Coast Institution of Engineers and Shipbuilders.
 NSMB—Proceedings, Symposium on the Behavior of Ships in a Seaway, September 7–10, 1957. Netherlands Ship Model Basin, Wageningen, Holland. Final two-volume edition.
 Phil. Mag.—Philosophical Magazine.
 Phil. Trans. R. Soc. A.—Philosophical Transactions of the Royal Society of London, Series A.
 Proc. R. Soc. A.—Proceedings of the Royal Society, Series A, London.
 Proc. Am. Soc. C. E.—Proceedings of the American Society of Civil Engineers.
 Proc. IRE—Proceedings of the Institute of Radio Engineers.
 Schiffstechnik—Schiffstechnik-Forschungshefte für Schiffbau und Schiffmaschinenbau.
 Ships and Waves—Proceedings of the first conference on ships and waves held in October 1954 at Hoboken, N. J. Published by Council on Wave Research (University of California) and SNAME.
 SNAME—Transactions of The Society of Naval Architects and Marine Engineers, New York.
 WRH—Werft-Reederei-Hafen.
 ZAMM—Zeitschrift für Angewandte Mathematik und Mechanik.

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- 1 "The Propagation of Groups of Waves in Dispersive Media, with Application to Waves on Water produced by a Travelling Disturbance," *Proc. Roy. Soc. A*, **81**, pp. 398-430. 1908.
- 2 "The Wave-Making Resistance of Ships: a Theoretical and Practical Analysis," *Proc. Roy. Soc. A*, **82**, pp. 276-300. 1909.
- 3 "Ship Resistance: a Numerical Analysis of the Distribution of Effective Horse-Power," *Proc. Univ. Durh. Phil. Soc.*, **3**, pp. 215-20. 1910.
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- 5 "The Displacement of the Particles in a Case of Fluid Motion," *Proc. Univ. Durh. Phil. Soc.*, **4**, pp. 62-74. 1911.
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- 7 "The Initial Wave Resistance of a Moving Surface Pressure," *Proc. Roy. Soc. A*, **93**, pp. 240-53. 1917.
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Resistance, Propulsion and Speed of Ships in Waves

1 Introduction

The resistance, propulsion and speed of ships will be considered only inasmuch as they are affected by wind and waves. The evaluation of these characteristics was the main objective of Kent's work (see numerous references to Kent in Chapter 3). In particular, Kent (3-1936/37; 3-1957)¹ can be considered as a summary. Kempf (3-1936) also presented a thorough treatment of this subject. The data given by these investigators apply, however, to ships built between 1913 and 1934. These ships were characterized by high block coefficients and low horsepower per ton of displacement and they lost the speed in head seas so rapidly that severe motions did not develop. The loss of speed in waves at the full engine power was, therefore, the primary objective of investigation. These data are still valid for tankers and bulk cargo carriers the speeds of which are limited primarily by the available power. In other types of ships the violence of motions makes it necessary to reduce the engine power in high waves. Therefore, ship motions, and not the available power, limit the attainable speed.

The speed loss of a general cargo ship in waves is shown in Fig. 1. Quoting from Lewis (1955a): "Speed reduction in heavy weather results from two types of influence. The first is the direct effect of the added resistance to forward motion caused by the action of wind and waves. The second may be termed indirect and refers to the necessity of voluntary reduction of power—and hence speed—to reduce the violence of the secondary motions.

"...We have recently analyzed some log data for typical cargo ships in winter North Atlantic service—north of the British Isles to Scandinavia—undoubtedly the most severe of the important trade routes of the world. The graph shows first the expected trend of ship speed with increasing average wave height, if full power were maintained and only the added resistance effects were taken into account. . . . The other steeper line shows the actual trend of speed found by plotting average daily speeds from log records. . . ."

Fig. 1 discloses that apparently engine power had to be reduced in waves over 5 ft in height. This low height is explained by the high power (8500 hp) and smooth water speed of Victory ships for which the analysis was made.

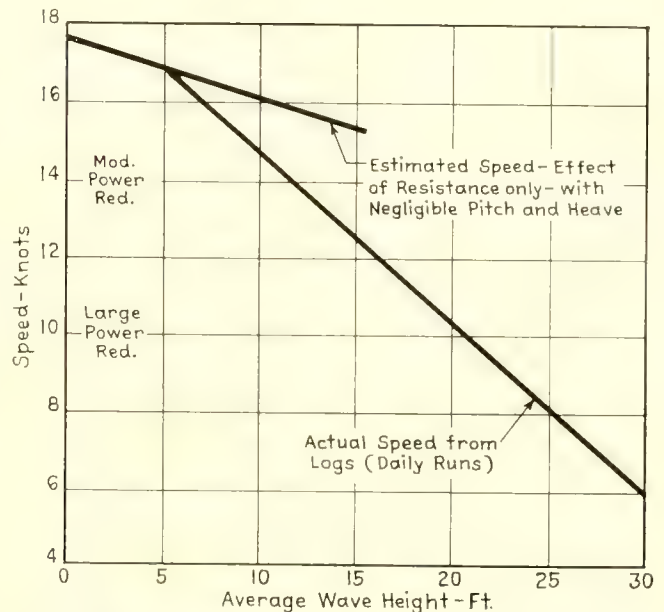


Fig. 1 Effect of head seas on ship speed, Victory Ship, No. Atl., West B. (from Lewis, 1955a)

Aertssen (1957 NSMB Symp.) considers that 5000 hp would have been normal for a ship of the size of Victories. The maximum speed in smooth water then would have been lower and a further reduction of power would have been required only in higher waves. The basic conclusion would remain, however, unchanged; namely, the speed of a ship at sea must be considered under two conditions:

- 1 In a moderate sea in which ship motions are not critical and the speed is determined by the available power.

- 2 In a higher sea in which the violence of ship motions limits the speed which can be maintained safely.

These two conditions will be separately considered in the following.

2 Ship Speed and Power in a Moderate Sea

The resistance of a ship in a moderate sea can be further subdivided into:

- (a) Waves so short that there is no appreciable

¹ The number 3 preceding the year indicates that the reference is made to the Bibliography at the end of Chapter 3. All other references appear in the Bibliography at the end of this chapter.

pitching, and the resistance is caused by waves themselves.

(b) Longer waves in which there is appreciable pitching and heaving. The resistance in this case is only partially due to waves themselves and it is caused to a large extent by ship motions.

2.1 Resistance Caused by Waves in Absence of Heaving and Pitching. The first case, (a), was investigated theoretically by Kreitner (1939) and Havelock (3-1940). Kreitner expressed the force R , exerted by waves on a motionless hull as

$$R = g\rho B(h/2)^2 \sin \alpha \quad (1)$$

where α is the mean angle of entrance between WL and center plane, B is the beam, h is the wave height, and ρ is the specific gravity of water. By expressing angle α in terms of length/beam ratio of a ship and introducing the concept of "additional resistance per ton of displacement," \bar{R} , he arrived at the expression

$$\bar{R} = \frac{B}{H} \left[0.8 \frac{h}{L} \right]^2 \quad (2)$$

where H is a ship's draft and L is ship's length.

Havelock (3-1940) derived the expression:

$$R = \frac{1}{2} g\rho a^2 B \overline{\sin^2 \alpha} \quad (3)$$

where the last factor is the mean value of $\sin \alpha^*$ with respect to the beam. This expression was derived for a body of infinite draft with the extreme assumption of reflection around the front half and a smooth water around the rest of a body. Therefore, Havelock considered that actual ship resistance caused by reflection of waves should not exceed that given by the equation (3).

Jacobs and Lewis (1953) applied Kreitner's formula (1) to several recently tested ship models. They obtained an excellent agreement after making certain empirical adjustments in the original formula. The revised formula is

$$R = 0.174 g\rho(h/2)^{5/3} (\overline{\sin \alpha})^{1/3} \quad (4)$$

2.2 Added Resistance Caused by a Combined Action of Waves and Ship Motions. Havelock (3-1942) investigated the increase of a ship's resistance caused by the combined action of waves and of heaving and pitching motions.² It has been assumed that added resistance due to wave reflection, outlined in the previous section, is small as compared to the resistance caused by the variation of buoyancy distribution. This assumption was verified by the results of calculation. Froude-Kriloff hypothesis was adhered to; i.e., the pressures acting on a ship were taken to be those existing in the wave structure when the ship was not there. The

following wave properties, taken from Table 1 of Appendix A,³ are relevant:

Velocity potential

$$\phi = (ag/\omega)e^{kz} \sin(\omega t + kx) \quad (5)$$

Pressure (exclusive of hydrostatic)

$$p = ag\rho e^{kz} \cos(\omega t + kx) \quad (6)$$

Pressure gradient in x -direction

$$\partial p/\partial x = ag\rho k e^{kz} \sin(\omega t + kx) \quad (7)$$

Pressure gradient in z -direction

$$\partial p/\partial z = ag\rho k e^{kz} \cos(\omega t + kx) \quad (8)$$

Assuming the ship to be restrained in its equilibrium position, the instantaneous resistance R_0 can be evaluated by the integration of pressures p over the ship's wetted surface S_0 or by integration of pressure gradient over the displaced volume ∇ :

$$\begin{aligned} R_0 &= \iint p l dS_0 = \iiint \frac{\partial p}{\partial x} d\nabla \\ &= -ag\rho \iiint e^{kz} \sin(\omega t + kx) d\nabla \end{aligned} \quad (9)$$

l and n designate directional cosines of normals to the elements of wetted area dS in x and z -directions. The force calculated from the foregoing expression is purely periodic with mean value zero (Havelock, 3-1937).

Suppose now the ship to be in slightly displaced position due to a heave⁴ ζ and pitch angle θ . The increment of the volume added by each surface element is

$$d\nabla = [n\zeta + nx\theta - l\theta(z - h')]dS_0 \quad (10)$$

where h' is the height of the instantaneous center of rotation above the origin 0. The added resistance can be calculated by applying equation (9) to the added volume, so that total wave resistance becomes

$$R = R_0 - ag\rho k \iint e^{kz} \sin(\omega t + kx) [n\zeta + nx\theta - l\theta(z - h')]dS_0 \quad (11)$$

Expression (11) can be used directly for the computation of the instantaneous resistance caused by waves when the data on ship motions ζ and θ are available. The mean resistance is obtained by integration of the instantaneous resistance over the oscillation cycle. In order to investigate the nature of the added resistance, Havelock endeavored to connect it with the wave-caused heaving force and pitching moment, and with the resultant ship motions.

On basis of equations (6) and (9) the wave-caused buoyant force and pitching moment are

$$F = -ag\rho \iint e^{kz} \cos(\omega t + kx) n dS_0 \quad (12)$$

* Havelock defined α as: "... α being the angle which the tangent at any point makes with the fore-and-aft-central axis."

² This derivation was repeated by Havelock (1945) and by St. Denis (3-1951).

³ Except that symbol z is used for vertical displacement instead of y , g/ω used instead of c , and ω instead of kc . Opposite direction of the wave travel is taken.

⁴ The use of symbol ζ for the heaving motion of a ship is limited to the present section.

$$M = ag\rho \iint e^{kz} \cos(\omega t + kx) [nx - l(z - h')] dS_0 \quad (13)$$

Equation (11) can now be rewritten in terms of F and M as

$$R = R_0 - \frac{k}{\omega} \zeta \frac{\partial F}{\partial t} - \frac{k}{\omega} \theta \frac{\partial M}{\partial t} \quad (14)$$

In periodic waves the exciting force F and moment M take the form

$$\begin{aligned} Z &= Z_0 \sin(\omega t + \sigma) \\ M &= M_0 \sin(\omega t + \tau) \end{aligned} \quad (15)$$

and from uncoupled differential equations of motions, given in Chapter 2, it follows that ship motions can be expressed as

$$\begin{aligned} \zeta &= KF_0 \sin(\omega t + \sigma + \delta) \\ \theta &= K'M_0 \sin(\omega t + \tau + \epsilon) \end{aligned} \quad (16)$$

where K and K' are magnification factors and δ and ϵ are phase-lag angles for forced oscillations.

Using equations (15) and (16) in (14) and taking mean values of the quadratic terms, Havelock arrived at the expression for the mean backward force

$$R = \frac{1}{2}kKF_0^2 \sin \delta + \frac{1}{2}kK'M_0^2 \sin \epsilon \quad (17)$$

or

$$R = \frac{1}{2}kF_0\zeta_0 \sin \delta + \frac{1}{2}k_1M_0\theta_0 \sin \epsilon \quad (18)$$

The amplitudes of the exciting functions F_0 and M_0 and the amplitudes of motions ζ_0 and θ_0 are essentially proportional to wave heights. Equations (17) and (18) show, therefore, that the resistance added by waves and wave-caused ship oscillations is *proportional to the square of the wave height*. These equations show furthermore that the resistance depends on the phase lag angles δ and ϵ . At very low frequencies ω the resistance practically vanishes. At the synchronous frequency, $\sin \delta$ and $\sin \epsilon$ approach unity and the resistance is at its maximum.

The nature of the derivation, based on the Froude-Kriloff hypothesis, excluded the hydrodynamic effects of ship's speed. In the later work of Korvin-Kroukovsky and Jacobs (1957) it was shown that ship speed has relatively weak effect on the magnitude of the exciting functions F_0 and M_0 . Therefore, the resistance, based on equations (17) and (18), should be nearly independent of the speed *per se*, but should be strongly dependent on the frequency of encounter ω_e , which governs the phase-lag angles. This is qualitatively confirmed by the towing-tank tests, for instance, those shown in Fig. 5.

Equations (17) and (18) were derived in order to demonstrate the dependence of the resistance on the wave-caused exciting forces and on ship motions. Havelock (3-1942) warned against using them for actual computations because of the uncertainties involved in the use of differential equations of motions. He particularly emphasized the uncertainty of estimating damping

forces. It also should be remembered that the derivation was based only on the $\partial\phi/\partial t$ term of the complete Bernoulli equation

$$p = p_0 - g\rho z + \rho\partial\phi/\partial t \quad (19)$$

Havelock mentioned that "...the usual approximate equations for the motion of the ship are obtained by taking into account also the hydrostatic buoyancy and moment arising from the term $g\rho z$ in (19)." However, he has not further discussed this. Korvin-Kroukovsky and Jacobs (1957) showed that heaving and pitching amplitudes and particularly phase relationships are strongly affected by cross-coupling of heaving and pitching motions. By virtue of equations (17) and (18), the resistance in waves also should be affected by cross-coupling.

Hanaoka (3-1957 NSMB Symp.), using advanced mathematical methods, Section 2-6, has derived the expressions for the resistance of a Mitchell-type ship in waves.⁵ In agreement with Havelock's work, the added resistance caused by waves and ship motions is shown to be proportional to the square of the wave height and to sines of the phase-lag angles. The calculations were made for an idealized ship identified as Weinblum's (1932) "form 1097." The principal dimensions of the ship are as follows: Beam/length, 0.10; draft/length, 0.04; midship-section and block coefficients, 0.75. Model resistance in kilograms is shown plotted versus (wave length/ship length) ratio in Fig. 2. The data are given for three ship speeds expressed in terms of Froude numbers. Excellent agreement between calculated and experimentally measured resistance is demonstrated. However, this agreement is obtained by using an idealized ship form. The experience of Emerson (1954) and of Korvin-Kroukovsky and Jacobs (1954), in calculating ship resistance in smooth water, indicated that it is more difficult to reach agreement in applying calculations to ships of normal commercial form. Furthermore, the calculations become tedious in this case.

Rather small variation of the wave resistance with wave length is shown in Fig. 2. Changes of ship's course, which modify the apparent wave length λ , can be expected, therefore, to have rather small effect on the resistance. This confirms the earlier statement of Kreitner (1939): "Therefore, the course must be altered by about 40 deg or more in order to get a tangible reduction of the additional resistance, whereas the character of the ship's motion will be completely changed with a much smaller deviation. Thus the loss of sea speed is somewhat independent of the course; hence improvement lies not with navigation but with design."

In summary, a simple workable expression for the resistance of a ship pitching and heaving in waves is not yet available. The efforts to derive it are recommended. The primary use of such an expression will be in application to ships of high block coefficient and low horsepower

⁵ For additional theoretical work see Haskind (3-1946), Eggers (3-1960), Inui and Maruo (1957), and Maruo (1960).

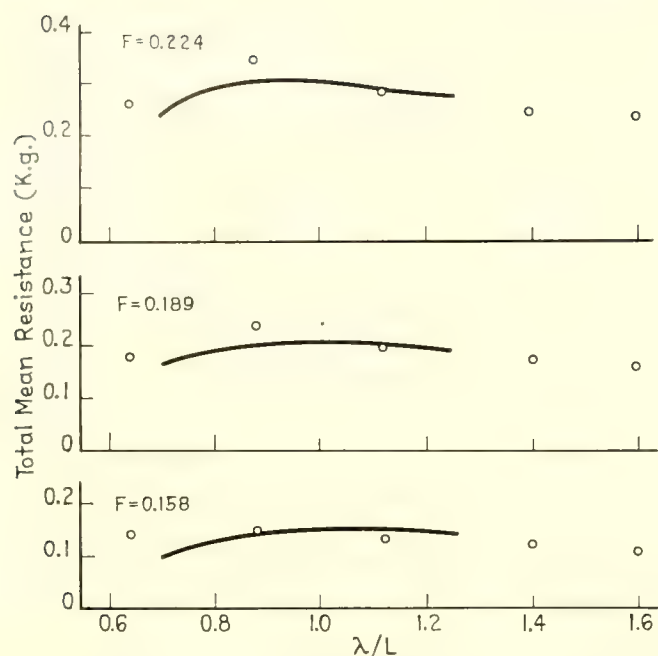


Fig. 2 Comparison of calculated and experimentally measured resistance of Weinblum's (1932) No. 1097 ship model in waves at three Froude numbers, F (from Hanaoka, 1957 NSMB Symp.)

per ton of displacement. Tankers and bulk cargo carriers fall into this category.

Nevertheless, certain broad conclusions can be made on the basis of the data just outlined. Over a predominating part of its time at sea a large and long ship will encounter waves not exceeding three quarters of the ship's length. The pitching and heaving motions will be small and the increase of resistance will be governed by the formulas (1), (3) and (4). The small mean angle of entrance, α , is indicated as beneficial, and will be obtained by the use of U-type sections. Large tankers and large fast liners, such as SS *United States*, should have, therefore, U-sections at the bow and waterplane lines as fine as the block coefficient permits.

As an extreme opposite, small cargo ships can be considered. Over a large proportion of their operating time these ships will meet the waves of the length equal to ship's length. Furthermore, these waves can be expected to be steep, i.e., to have small λ/h ratios. Pitching and heaving motions are expected to be large and they will cause large increase of resistance. The use of a pronounced V-form of bow sections or of Maier form can be expected to decrease the amplitude of motions and thereby to reduce the resistance. This observation gives the rational explanation to the widespread use of V-forms in the small ships engaged in the trade between English, Scandinavian, and North European ports. Kreitner (1939) has already called attention to these characteristics of the bow forms.

A modern general cargo ship represents a case intermediate between two extreme ones just considered. A

wide variety of bow forms is, therefore, found in practice. A moderate to pronounced V-form appears to be prevailing in Europe while the current tendency in the United States is towards U-form, as exemplified by DTMB Series 60 (F. H. Todd, 3-1953). The towing-tank experiments of Kempf (1935) and of Lewis and Numata (3-1956) showed that a pronounced V-form is advantageous for ships of this type in adverse weather. The large mean entrance angle in case of V-sections, however, increases the resistance in fair weather. The choice between U and V-sections hinges, therefore, on whether the emphasis is placed on the fair or on the rough-weather operations. In this connection it is of interest to observe that Kempf at an early date established a practice to test ship models in waves while the recent development of Series 60 at the David Taylor Model Basin was based entirely on smooth water tests.

2.3 Towing-Tank Tests for Resistance in Waves.

Towing-tank tests for resistance in waves are conducted for a wide range of ship speeds and wave sizes. These latter often exceed (relatively to the model) the waves found at sea. The subject of model testing is placed, however, under Section 2 (on resistance in moderate seas) because only in moderate seas the resistance determines a ship's speed. In higher seas a ship's speed is governed by considerations of safety of a ship and its cargo, and not by the resistance. This subject will be treated in Section 3.

2.31 Methods of testing and presenting test results.

The resistance and speed of a ship in waves are usually represented in one of two forms corresponding to two methods of towing-tank testing. The first one corresponds to the early practice of Kent and Kempf who made tests in large towing tanks. The mean speed of a model was controlled by the speed of a towing carriage. Kempf introduced the method of towing a model by a weight and a towing cord which were installed on the main carriage. The model was towed, therefore, with a constant force and it was free to surge about the mean speed of the main carriage. This system was adopted later by the Davidson Laboratory of the Stevens Institute of Technology (3-E. V. Lewis, B. V. Korvin-Kroukovsky), by the Delft Shipbuilding Laboratory (3-Gerritsma) and by University of California (3-Paulling).⁶ The results of tests are expressed by curves of the resistance versus model speed with wave size as the parameter. An example of such a plot is shown in Fig. 3, taken from Gerritsma (3-1957, NSMB Symp.).

The second method is employed in the towing tanks not equipped with a towing carriage. It is used by Newport News Shipbuilding and Dry Dock Company (Hancock, 3-1948b), Massachusetts Institute of Technology (Abkowitz and Paulling, 3-1953) and at the 140-ft tank of the David Taylor Model Basin (F. H. Todd, 3-1954). In this method the model is towed by a long cord and a falling weight at the end of a tank. The test results are given in a plot of model speed versus wave length.

⁶ Listed in the bibliography at the end of Chapter 3.

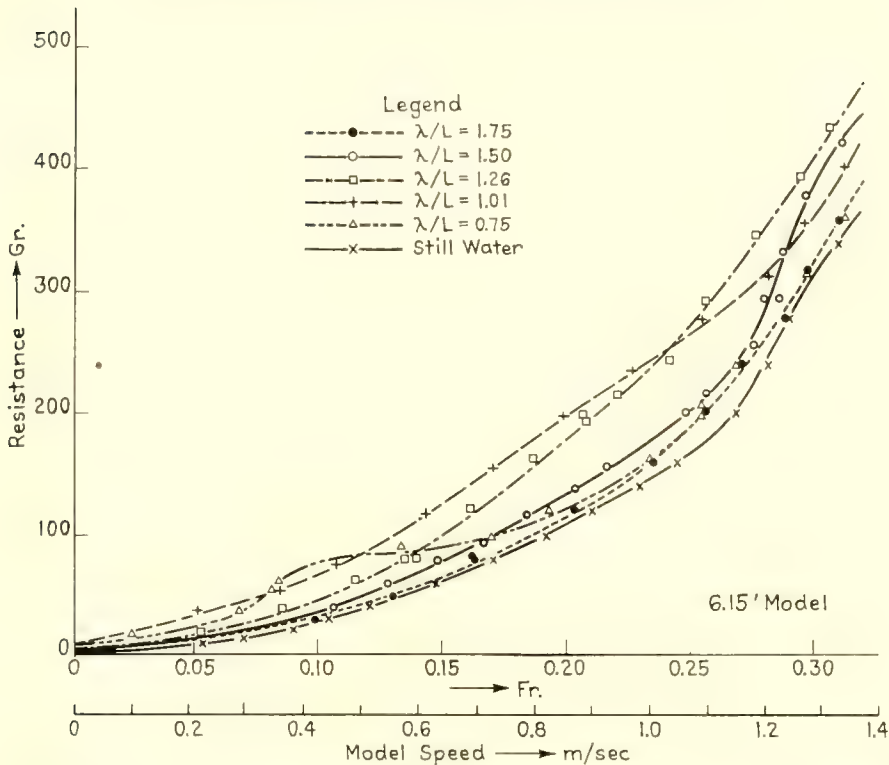


Fig. 3 Resistance of Series 60, 0.60 block coefficient model 6.15 ft long in regular waves 1.53 in. high (from Gerritsma, 1957 NSMB Symp.)

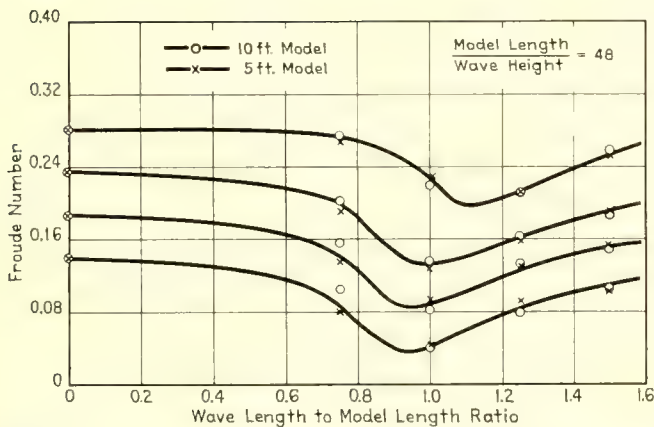


Fig. 4 Speed reduction in waves of constant height at constant tow force—5 and 10 ft models (from Szebehely, Bledsoe and Stefun, 3-1956)

Several curves result from the use of several towing weights. An example of such a plot is given in Fig. 4. This figure, as well as Fig. 3, gives the data on DTMB Series 60, 0.60 block coefficient hull. One of these two forms of plotting can be converted to the other by a suitable interpolation.

2.32 Typical data on resistance in waves. The uppermost curve in Fig. 4 corresponds to the towing

weight which gave the model speed in smooth water defined by the Froude number of 0.28. For a *Mariner* type ship this corresponds approximately to the trial speed of 21 knots. The wave conditions giving the maximum reduction of speed are similar to a swell 12 ft high. It may be assumed that in such a swell the normal propeller RPM can still be maintained. The towing-tank test shows that ship's speed has dropped to about $F = 0.20$ or 15 knots. This is 71 per cent of the smooth-water speed. Were propeller RPM maintained corresponding to the third curve from the top, i.e., to $F = 0.185$, the smooth water speed of 15 knots would have dropped to 6.7 knots. This is only 45 per cent of the smooth water speed. This example demonstrates the well-known fact that ships of low horsepower per ton of displacement (i.e., slow ships) lose speed in waves more rapidly than high-powered, fast ships. In this example, however, high and low powers were applied to the same hull form. In practice low-powered slow ship would have fuller lines and the loss of speed in waves would be further increased.

The increase of the resistance in waves can be found in Fig. 3. At the Froude number of 0.20 corresponding to ship's speed of 15 knots, the model resistance is shown to be about 110 grams in smooth water and it is increased to 200 grams in waves of $\lambda/L = 1.01$. This is an increase of 82 per cent. The increase of resistance is about

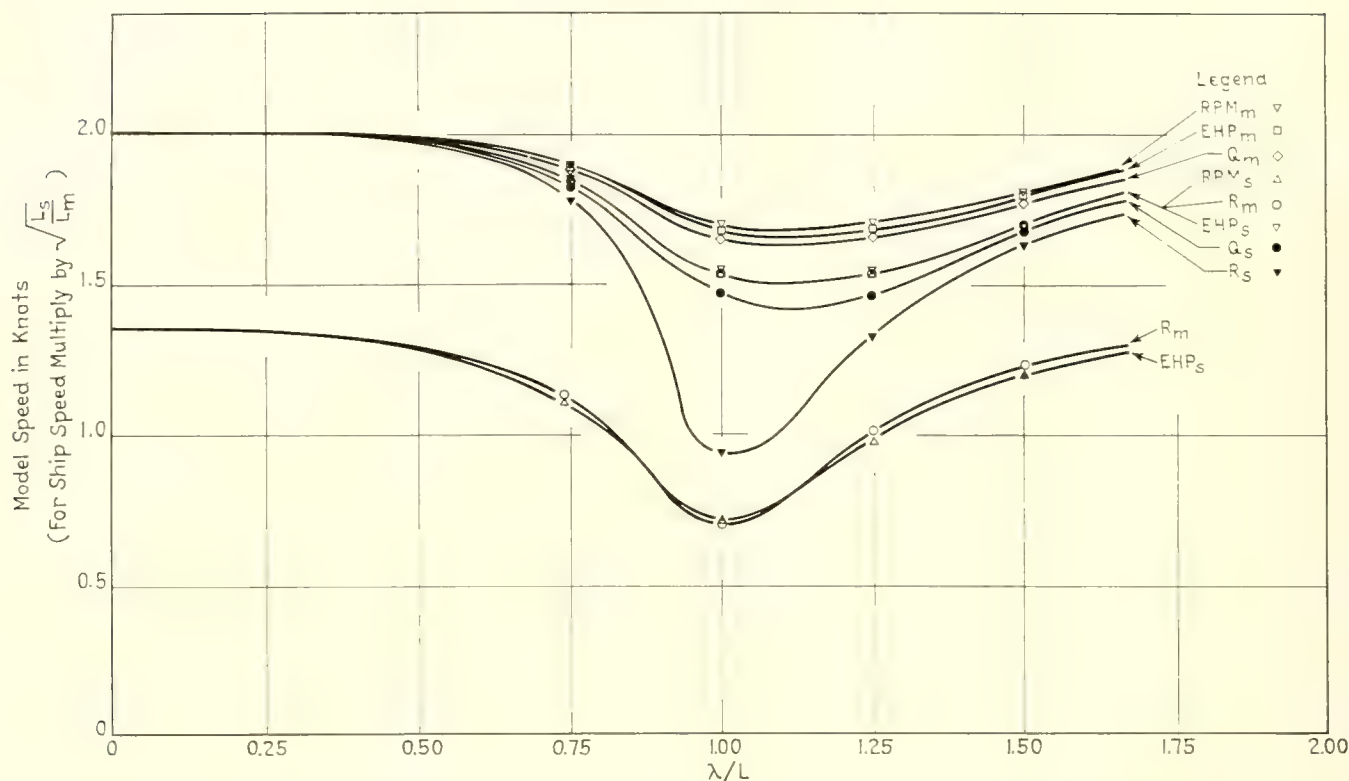


Fig. 5 Model speed in waves under various operating conditions (from Abkowitz, 3-1957a)

160 per cent at the lower power corresponding to the speed of 6.7 knots in waves.

2.33 Ship-model correlation. Three aspects are to be considered under the heading of ship-model correlation:

- (a) Scale effect.
- (b) Effect of tank walls.
- (c) Similarity law for ship and model speeds.

The scale effect and the effect of tank walls usually occur simultaneously and are difficult to separate. Szebehely, Bledsoe and Stefun (3-1956) compared test results under three conditions; namely, a 5 ft model in a 140 ft towing tank, and 10-ft and 20-ft models in the large 1800-ft-long tank. The comparison of only 5 and 10-ft models was presented by numerous graphs and the data on the 20-ft model were not yet available. The models used in this series of tests were small in relation to the sizes of towing tanks and it may be assumed that there was no tank wall effect. The loss of speed in waves was found to be identical for two models in waves of constant height equal to $1/48$ of the model length. The loss of speed of the 5-ft model was, however, greater than that of the 10-ft model when tested in waves of constant (wave length)/(wave height) ratio of 30.

Gerritsma (3-1957 NSMB Symp.) tested three series 60, 0.60 block coefficient models in a tank 14 ft wide and 317 ft long. The models were 6.15, 8 and 10 ft long. In this case, therefore, both the tank-wall effect and the

scale effects were potentially present. Neither of these has affected motions in waves and excellent agreement was found among three models. However, important differences in resistance of three models in waves were found. These appear to be caused by the tank-wall interference and primarily are manifested by the occurrence of humps in the resistance curves at waves of $\lambda/L = 1$. The size of this hump increased with the size of the model.

Taken together, the results of the two investigations just outlined are reassuring as to the motions, but leave the problem open as to the resistance in waves. Further investigations are necessary. In particular, tests for resistance in waves should be made as soon as the large rectangular tanks (maneuvering tanks) will be available. The tank-wall effect evidently will not be present in these.

The similarity law (for lack of a better term) for model's speed loss in waves was discussed by Abkowitz (3-1957a and 3-1957c, NSMB Symp.). Abkowitz made a theoretical analysis of the speed loss of models and ships under several assumptions as to the ship's and model's condition. These assumptions are:

- 1 A constant towing force applied to the model.
- 2 A constant resistance of the ship (approximately constant propeller thrust).
- 3 Constant effective horsepower for the ship.
- 4 Constant RPM for the model.
- 5 Constant RPM for the ship.

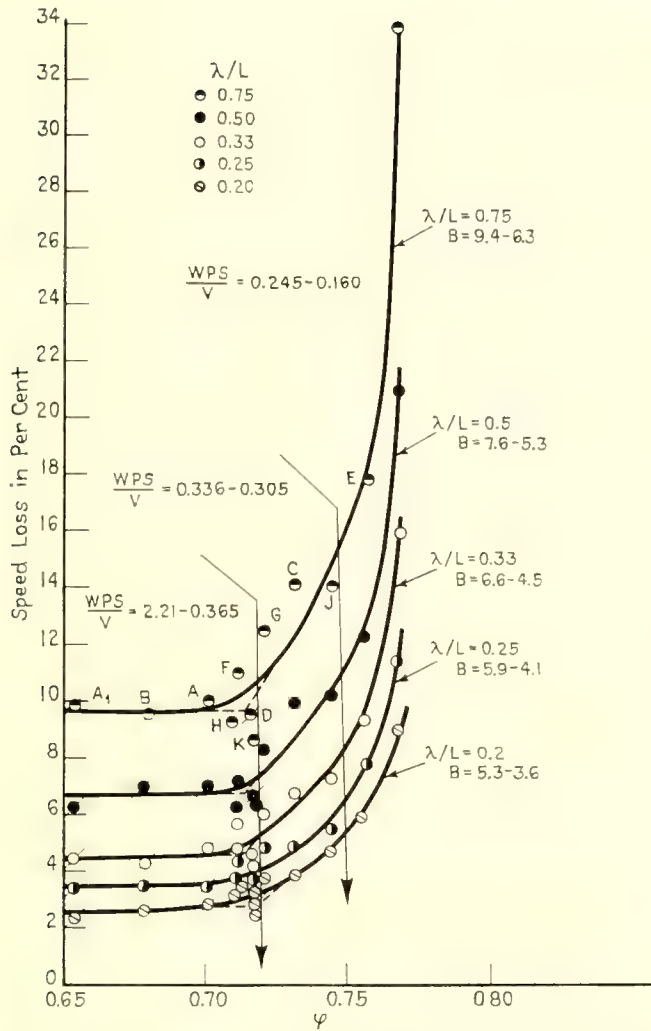


Fig. 6 Loss of ships' speed in North Atlantic as a function of block coefficient with ratio (wave length) to (ship length) as parameter. Three ranges of horsepower per ton of displacement values are indicated (from Möckel, 1944)

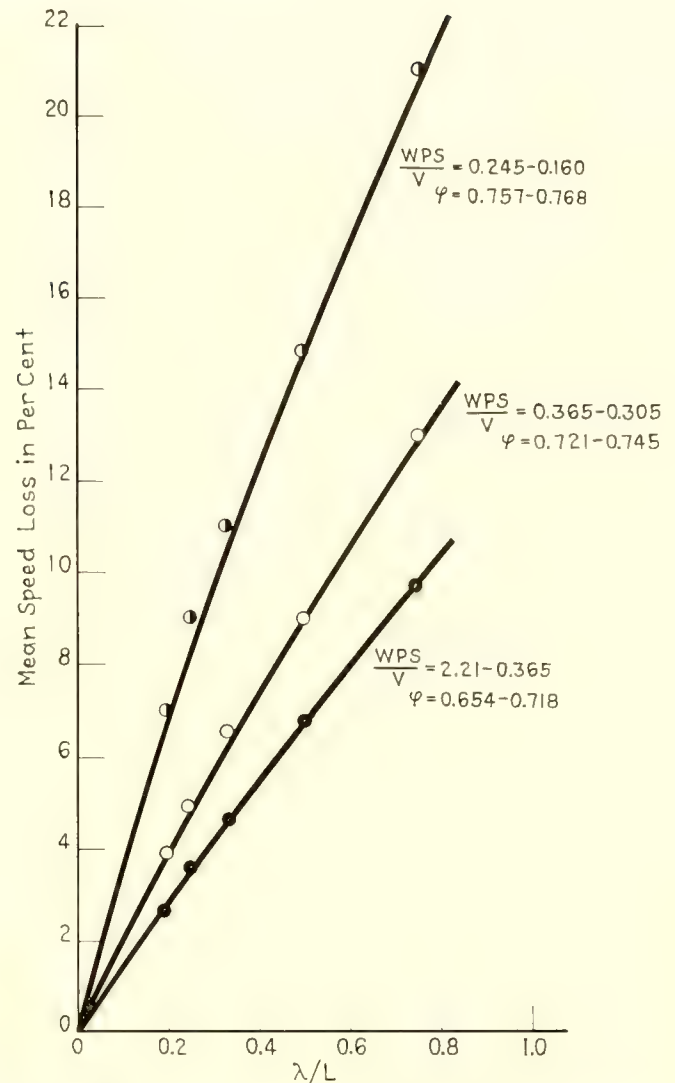


Fig. 7 Mean loss of ships' speed at different values of horsepower per ton of displacement plotted versus (wave length)/(ship length) ratio (from Möckel, 1944)

6 Constant torque for the ship.

The results of this investigation are summarized in Fig. 5. The speed loss of a model, towed with a constant force (curve R_m), is found to be identical with the speed loss of a ship at constant RPM. This latter is the usual operating condition of a ship, the engines of which are governor controlled.

2.4 Propeller Efficiency. The increase of the shaft horsepower due to waves is larger than the increase of the resistance shown in Fig. 3. The increased propeller thrust and decreased ship speed lead to a rapid increase of the propeller thrust-loading coefficient. This in turn leads to a pronounced reduction of the propeller efficiency. This reduction can be evaluated readily from the usual propeller-data curves. However, it has been necessary to assume that thrust deduction and wake fraction are not affected by waves. Kempf (3-1935, 1936) vividly demonstrated large losses of the propeller

efficiency in waves and showed that these occur primarily as a result of increased resistance and decreased ship speed. Vertical velocity of a ship's stern in pitching and heaving has only a small effect on the efficiency.

The losses of propeller efficiency are also shown in the work of Kent. They are, however, not so conspicuous in this case because Kent used rather mild sea conditions in his work.

2.5 Estimate of Ship Resistance and Speed Loss in Operating Conditions. The discussion of the ship resistance in the previous sections was limited to evaluating the effect of the regular long-crested waves. The estimation of the speed loss in operating conditions requires the consideration of:

- (a) Sea irregularity.
- (b) Other causes of speed loss.

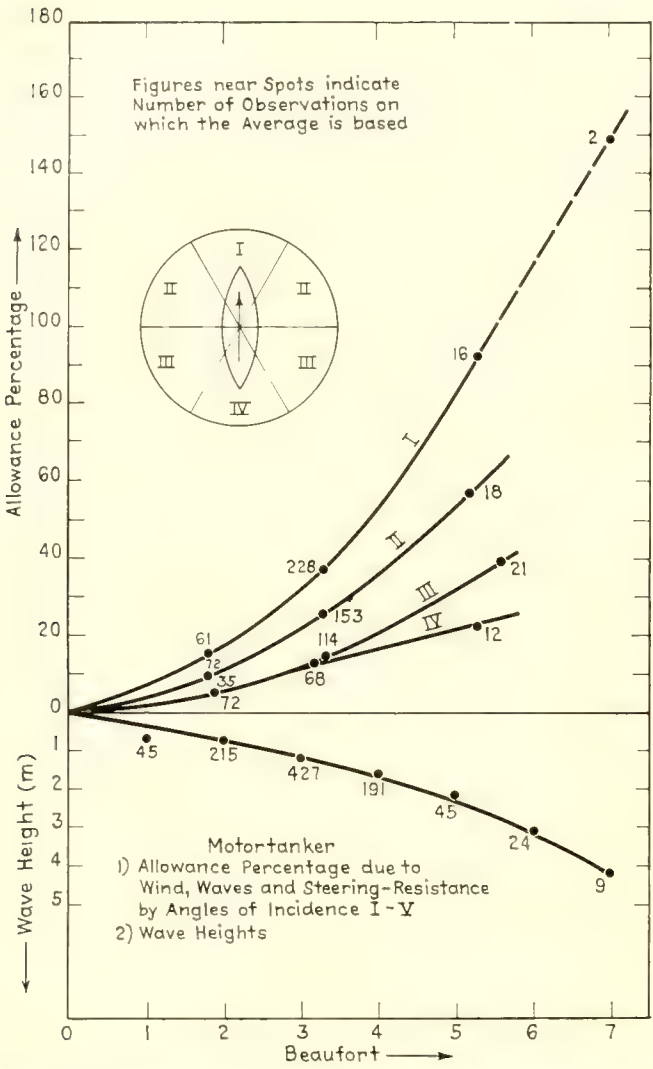


Fig. 8 Increase of shaft horsepower in rough weather (from Bonebakker, 1954)

Havelock and Hanaoka, Section 2.2 showed that ship resistance is proportional to the square of the wave height. This fact precludes the application of the linear superposition theory, Section 3-3, to resistance estimates.

Since the superposition of the spectral components is not possible, Pershin and Voznessensky (3-1957) solved the problem by summing the effects of apparent waves. This solution was made possible by adopting rather crude assumptions, such, for instance, as neglecting the effect of phase-lag angles, which were found to be important by Havelock and Hanaoka. The authors justified these assumptions by showing that the part of the resistance affected by assumptions formed a rather small percentage of the total resistance at sea.

The total resistance was defined by Pershin and Voznessensky as composed of:

- 1 Change in effective resistance due to ship motions and waves.

- 2 Change of effective resistance due to wind acting on ship's hull and superstructures.
- 3 Change in propeller working conditions.
- 4 Surface current induced by wind.
- 5 Horizontal displacement of water participating in water motion.
- 6 Yawing.

2.6 Observations on Ships at Sea. Macdonald and Telfer (3-1938) gave a general review of seakindliness of ships, which was based on seagoing experience. They mentioned that a ship can be expected to face ordinary heavy weather characterized by the apparent propeller slip up to 30 per cent. They expressed the indicated horsepower, needed in this connection, as $16\sqrt{\Delta}$, where Δ is the displacement in tons.

A good summary of ship resistance and powering in heavy weather will be found in Kent (3-1958).⁷ Table 1,

Table 1 Percentages of Total Ship Resistance With Weather Head On

Wind speed, knots	30	40	50	60	70
Hull resistance in smooth water	56	46	36	28	21
Extra hull resistance in rough water	33	36	37	36	34
Wind resistance	10	16	24	32	39
Rudder resistance	1	2	3	4	6

taken from Chapter 15 of that book, shows the distribution of the resistance for a low-powered ship in head seas.

2.61 Möckel's summary. Möckel (1944) summarized the results of observations at sea collected by Kent (3-1936/37) and by the "Sammelstelle für Fahrtergebnisse der Hamburgischen Schiffbau-Versuchsanstalt." Ship data, which satisfied the following conditions, were chosen:

- 1 The ship was fully loaded.
- 2 The relative direction of waves (wind-sea plus swell) was within 30 deg off the bow.
- 3 The swell was not higher than scale 4 when wind did not exceed strength 5 on the Beaufort scale. The swell exceeded scale number 4 when the wind exceeded Beaufort 5. These conditions were defined on the basis of the frequency distribution of meteorological observations.

The wind strength on the Beaufort scale was chosen as the parameter, to which all influences of weather on ships were referred.

The results of the investigation are summarized in Figs. 6 and 7. These plots demonstrate vividly that losses of a ship's speed increase rapidly with increase of a ship's fullness and decrease of the engine power.

Figs. 6 and 7 describe ship's operations in moderate sea, as indicated by the λ/L ratio not exceeding 0.75. Under these conditions the ships having block coefficients in excess of 0.72 lose speed rapidly, while the ships with lesser block coefficients are affected to a lesser extent. It should be emphasized that with λ/L ratio not exceeding 0.75 there can be very little pitching and that the

⁷ Based on observations at sea and with emphasis on low-powered ships.

Table 2 Particulars of Two Ships Investigated by Bonebakker (1954)

	Motor tanker	Passenger steamer
Hull		
<i>L</i> (b.pp)	148.74 m = 487' 8"	128.013 m = 420' 0"
<i>L</i> (on lwl)	152.31	128.587 m
<i>B</i>	22.25	17.932 m
Draught	8.57	7.315 m
Displacement	21,111 m ³	11,906 m ³
Block coefficient ..	0.744	0.706 (lwl)
Propeller		
<i>D</i> (diameter)	5.75 m	5.658 m
<i>P</i> (pitch)	4.68	5.692 m
<i>P</i> (0.7 <i>R</i>)	4.415	5.180 m
<i>P</i> (hub)	3.25	—
<i>P</i> (virtual)	4.80	5.914 m
Number of blades ..	4	4
<i>F_a/F</i>	0.40	0.481
Speed, designed ..	12.5 knots	13.5 knots
<i>V</i> /√ <i>Lf_t</i>	0.565	0.660
1. Model regression equation	$d.h.p./(.0.1N)^3 = 0.050 s_a + 4.50$	$d.h.p./(.0.1N)^3 = 0.084 s_a + 6.60$
2. Ship trial regression equation	$d.h.p./(.0.1N)^3 = 0.0562 s_a + 3.88$	
3. Mean service regression equation	$d.h.p./(.0.1N)^3 = 0.0519 s_a + 4.31$	$d.h.p./(.0.1N)^3 = 0.0756 s_a + 6.838$

speed loss indicated is primarily due to direct action of waves. A fine and powerful ship is shown to lose 10 per cent in speed under these conditions. The λ/L ratio of 1.0 was used earlier in an estimate of speed loss of a *Mariner* which was made on the basis of the towing-tank for series 60, 0.60 block coefficient. Pitching and heaving motions reach large amplitudes at this λ/L ratio and the speed loss of 29 per cent was indicated.

Figs. 6 and 7 are of particular interest in connection with full and low-powered ships. Under now prevailing design trends this applies to tankers and bulk cargo carriers. Modern general cargo, passenger, and naval ships are relatively little affected by short waves to which these figures are limited.

2.62 Bonebakker's data. Möckel's data were collected from numerous ship observations and were presented directly in the form of speed loss versus wind strength. It was impossible to obtain detailed data on ships' powering. Bonebakker, (1954) on the other hand, made a detailed investigation of the power consumed by two ships under different weather conditions. The properties of two ships are shown in Table 2. The increase of the power caused by waves is plotted in Figs. 8 and 9. It will be observed that "passenger steamer" is a relatively small and very slow ship in comparison with a modern general cargo ship.

Nine hundred and ninety sets of observations at sea

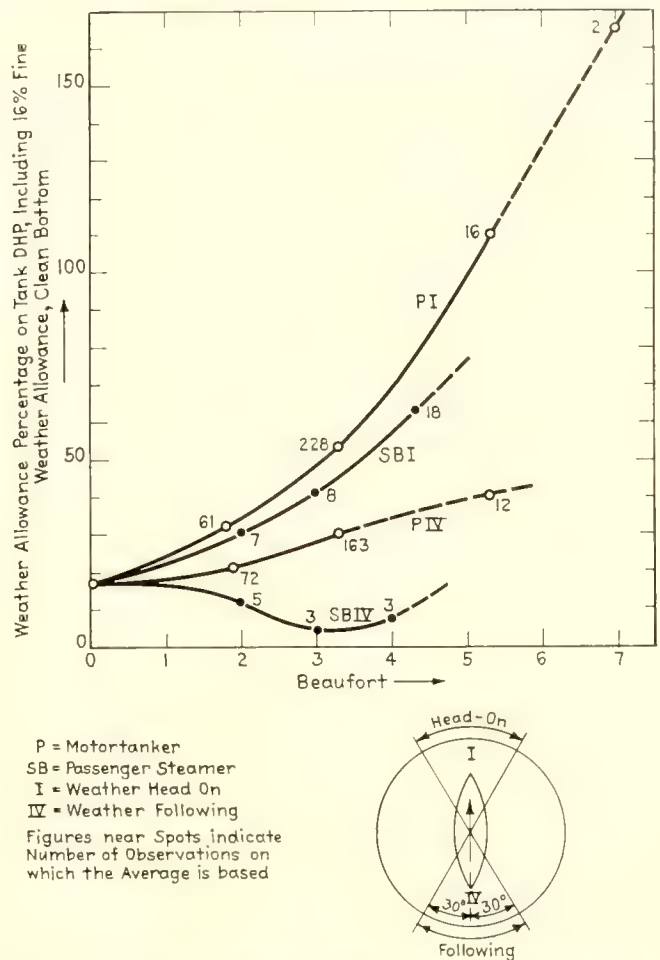


Fig. 9 Increase of shaft horsepower in rough weather (from Bonebakker, 1954)

were made on the tanker, but only eleven power records were taken. It is a rather tedious process to obtain the shaft horsepower of a ship at sea and ship's records often do not contain sufficiently complete and accurate data for this purpose. On the other hand, engineer's log usually gives the data on the daily average apparent slip s_a , defined as

$$s_a = \frac{(\text{propeller pitch}) \times \text{total number of revolutions}}{\text{distance by observation}} - 1 \quad (20)$$

In a previous work Bonebakker (1951) established an equation connecting horsepower with the apparent slip,

$$(\text{DHP})/(.0.1 N)^3 = a s_a + b \quad (21)$$

where DHP is the delivered horsepower and is assumed to be 0.97 of the shaft horsepower. The coefficients a and b usually have different values for a model and a ship and they also depend on the degree of fouling of a ship's bottom. Once the coefficients a and b are evaluated from a few detailed power records, equation (21) can be

used to evaluate the mean horsepower in all other observations.⁸ The assumption is made here that wake fraction and thrust deduction remain unchanged in waves. Bonebakker defined s_a on the basis of the effective propeller pitch; that is the propeller advance at zero thrust.

The use of ship's log data is often criticized because of the alleged low accuracy. In this connection it is of interest to quote from Burrill (1951):⁹ "The analysis of service or trial data is to me the correct means of assessing propulsive performance, and my experience has been that if sufficient information is available and is carefully analyzed, a great deal can be learned from these ship data. Those who would reject ship data on the ground that they are not sufficiently accurate are, I think, rejecting the criterion by which they are ultimately judged."

Quoting from Bonebakker (1954): "First of all, the relation between wind forces and wave height was investigated. In Table 3 frequencies are tabulated, the abscissa being wind forces, the ordinates wave heights in meters. In the majority of cases the directions of wind and waves coincided; so we may speak of the direction of the weather. The direction of the weather relative to the ship's course is grouped according to Fig. 8."

The discussion will be limited here to head seas; i.e., Condition I or the uppermost curve of Fig. 8. Each point on this curve is a mean of an indicated number of observations. It shows the increase of the delivered horsepower over the horsepower used at the same speed in smooth water. The speed, however, is reduced from that in smooth water because of the lack of engine power to overcome the added resistance in waves. The data incorporated in Fig. 8 are given in Table 4.

The tanker, to which Table 4 applies, had a block coefficient of 0.744 and horsepower per ton of only 0.16. Nevertheless, it was not practical to maintain the full power beyond the wind force of 5. At the mean wind force of 5.3 the RPM were reduced by 4.9 and the power by 8 per cent. It was possible to maintain the full power in two observations at the wind force 7 only because the ship resistance was increased by the bottom fouling (39 weeks out of dock). The combined action of waves and of increased fouling kept the ship at a safe speed of 9 knots; i.e. at 75 per cent of the initially low speed of 11.9 knots.

The information given by Bonebakker (1954) on the slow passenger ship is limited to Fig. 9 and material for further discussion is not available.

Clements (1956-57) extended Bonebakker's work by explicitly expressing the effect of the bottom fouling, the effect of the weather, and by basing the analysis on the voyage data obtained from eight ships of different commercial types. He assumed the relationship of the form

⁸ The reader is referred to Brard and Jourdain (3-1953) and Retali and Birdel (3-1955) for description of another method of estimating shaft horsepower on basis of propeller properties.

⁹ Discussion of Bonebakker's (1951) paper, Trans. NECI, 1951, pages D115. References to figures and tables were changed to correspond to the present monograph.

Table 3 Relation Between Wind Force and Wave Height (from Bonebakker, 1954)

Beaufort degr. wave height, m	0	1	2	3	4	5	6	7	fre- quencies
<0.25	66	17	3						86
0.25<0.75	6	31	110	81	1				229
0.75<1.25	8	14	105	218	71	2			418
1.25<1.75	4	2	38	209	114	26			393
1.75<2.25	2	3	18	54	77	27	1		182
2.25<2.75	8	2	10	16	33	18	3		90
2.75<3.25				12	14	15	18	1	60
3.25<3.75				2	5	8	6	3	24
3.75<4.25				1		6	2	6	15
4.25<4.75					2	1		4	7
Grand total	94	69	284	593	317	103	30	14	1,504
Percentage	6	4	19	40	21	7	2	1	100
Selected observations	66	45	215	427	191	45	24	10	1,023
Percentage	6	4	21	42	19	4	2	1	100
Mean wave	0.44	0.62	0.98	1.30	1.72	2.35	3.08	3.96	1,504
Height, m	0.00	0.65	0.75	1.24	1.70	2.20	3.13	4.20	1,023

Selected observations are

$$\text{SHP}/N^3 = aT_d + bW + c \quad (22)$$

where T_d is time out of dock in weeks, and W the weather factor (to be defined later). The linear assumption for the increase of SHP/N^3 with the time out of dock is the best approximation to cover most eventualities. In case of tankers having short periods in port, the variation of SHP/N^3 is well defined and the term in T_d^2 can be added.

E. V. Telfer pointed out in the discussion of the Clements' paper that the greater the fouling the less is the propeller efficiency and hence the greater must be the b -value for any particular weather. He suggested that a more rational basic equation than (22) would be the following:

$$\text{SHP}/N^3 = aT_d + (b + \beta T_d)W + c \quad (23)$$

Telfer also pointed out that his own publications¹⁰ can be considered as precursory to the work of Bonebakker and Clements.

The "weather factor," W , is an empirically evaluated function relating the increase of ship resistance to the Beaufort wind force. A few sentences can be quoted from Clements (1957) in order to outline the general approach: "It was also considered after the analysis of published data . . . , that the estimation of prevailing wind conditions was far more uniform and accurate from man to man and from ship to ship than the corresponding

¹⁰ See entries under Telfer in the bibliography at the end of this chapter.

Table 4

<i>n</i>	<i>B</i>	<i>D</i>	<i>N</i>	<i>V</i>	DHP		Difference	
					Ship	Tow. tank	DHP	Per cent
69	0	19.4	87.2	11.9	3.275	2.580	695	27
61	1.8	21.4	85.3	11.3	3.155	2.210	945	43
228	3.3	13.7	84.9	10.9	3.189	1.995	1194	60
16	5.3	22.7	80.0	9.4	2.857	1.290	1567	121
2	7	39.0	82.3	9.0	3.258	1.154	2104	182

n = number of observations *N* = average RPM
B = mean wind force—Beaufort scale *V* = speed in knots
D = weeks out of drydock DHP = delivered horsepower

Table 5 Particulars of Ships Investigated by Aertssen (1959)

Ship's Name Type of vessel...	<i>Jadotville/Baudouinville</i> Single-screw steam-turbine passenger-cargo vessel	<i>Elisabeth</i> Single-screw steam-turbine tanker
Length BP, ft.....	511.8	628.3
Breadth molded, ft.....	69.9	82.7
Depth molded to upper deck, ft.....	40.4	44.5
Depth molded to second deck, ft.....	31.8	..
Mean draft molded, ft.....	27.4	33.2
Displacement (plated), tons.....	18970	38000
Block coefficient, δ	0.672	0.770
Prismatic coefficient, φ	0.690	0.776
Half-angle entrance, deg.....	10	24.5
Deadweight, tons.....	10200	29200
Power of engine, cv. } 1 cv = 0.986 hp..... }	{ 9500 ^a in service 12500 maximum	13000
Revolutions per minute.....	100-110	120
Nominal sea speed, knots.....	16.5	16
Screw particulars:		
1 Diameter, ft.....	19.7	20.4
Mean pitch, ft.....	18.2	14.8
Developed area, sq ft.....	156	170
Number of blades.....	4	4 ^b
2 Diameter, ft.....		19.5
Mean pitch, ft.....		14.5
Developed area, sq ft.....		184
Number of blades...		5 ^b

^a In service.^b The four-bladed propeller was used for the tank tests and for the measured-mile trials; the five-bladed propeller was used during the trials of May and June 1956.

assessment of the sea conditions. This is an important point when selecting a weather scale by which the performance of a number of vessels of different types are to be compared" . . . "When analysing service performance data statistically taking into account the separate effects of wind and sea, a high degree of correlation was found to exist between the wind and sea scales. . . ." "This led to the conclusion, which has often been suggested, that the performance of a vessel may be determined from a single weather scale, the value of this scale being determined from the relative wind force and direction" . . . "If this scale is then fixed, the statistical analysis determines the combined effect of wind and sea on performance, in this instance being taken as the percentage increase in power over still-air, calm-sea conditions to maintain a given speed" . . . "Considering first the case where the wind and sea are from ahead, analysis of the . . . showed that increase in power was roughly proportional to wave height. Analysis of the weather data for these

two vessels and other data indicated that the average wave height, for a given Beaufort number, agreed reasonably well with that quoted by Russel and Macmillan.¹¹ It was also noted, as shown in Table—¹² that a good approximation to these wave heights could be obtained by squaring the sea disturbance number¹³ (given in the Table—¹²) and dividing by two. Thus, since the increase of power due to sea condition is proportional to wave height, which can be related to sea-disturbance number, which can in turn be related to Beaufort number, we can relate increase of power due to sea condition to Beaufort number." Clements demonstrated by a plot that the derived relationship checks well with the data reported by six ships for winds up to Beaufort 6. Continuing with quotations: "...for winds from ahead a convenient

¹¹ Reference IV of Chapter 1.¹² This table is not reproduced¹³ I.e., Admiralty sea scale number.

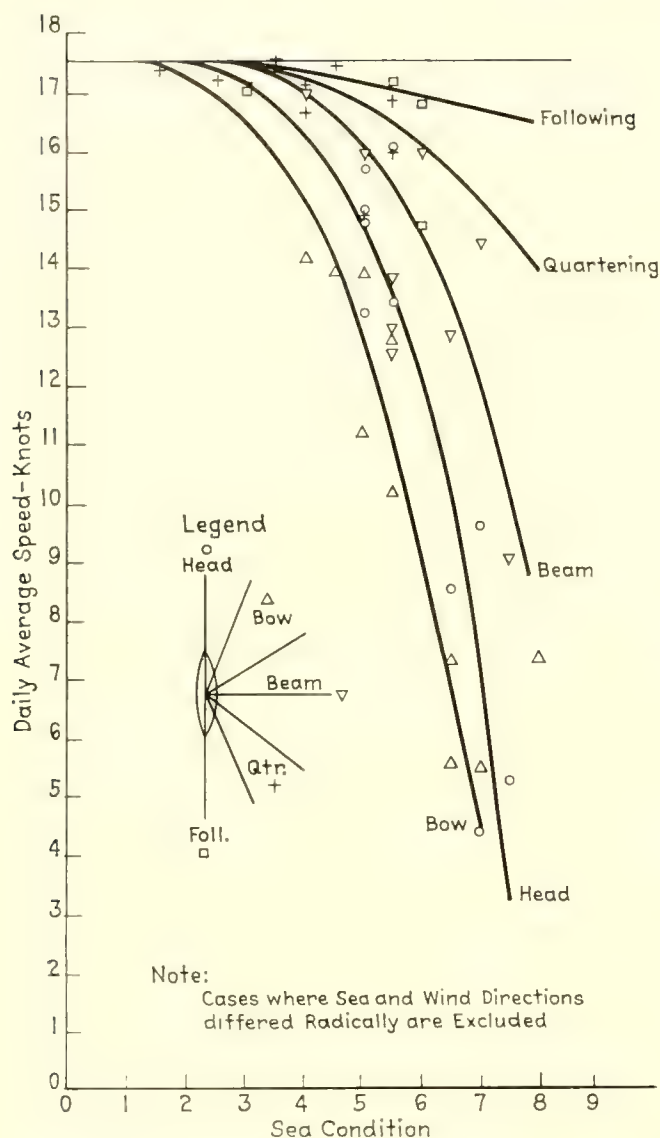


Fig. 10 Reduction of speed of Victory-type ships in a rough sea (from Lewis and Morrison, 1954)

weather scale can be obtained by $\frac{1}{2}$ (sea disturbance number)² and plotting this against the corresponding Beaufort wind force. The effects of winds on the beam are taken to be one half, and following winds one fifth this amount for any given Beaufort wind force, this again being established from BSRA¹⁴ data."

The effect of the weather factor on shaft horsepower is then estimated using the coefficient b from statistical analysis. The relationship derived by Clements is,

$$\frac{SHP_2}{SHP_1} = \frac{Y_1 + bW}{Y_1} \left[\frac{Y_{100} - Y_1}{Y_{100} - Y_1 - bW} \right]^3 \quad (24)$$

where the symbol Y denotes the ratio shp/N^3 , and subscripts denote:

- 1—Mean voyage data under calm weather conditions
- 2—Any given condition of weather
- 100—condition when apparent slip is 100 per cent
- Shaft horsepower is related to apparent slip by equation (21).

Additional data on the increase of the ship resistance due to weather will be found in Aertssen (1959). Aertssen investigated the propulsive characteristics of two ships, the particulars of which are shown in Table 5. The relationship between the delivered horsepower and ship speed is shown in Fig. 15 for different wind velocities up to Beaufort 6. Quoting Aertssen: "A mean value of the increase of power due to weather effect for two winter voyages of the *Jadotville* between Antwerp and the Canary Islands is 11 and 19 per cent, and for the whole voyage of the *Elisabeth* 15 per cent. Thus 15 per cent is the mean power allowance for weather effect for both ships on their usual routes." Attention should be called to the fact that these data refer to the route of predominantly fair weather and to the ships of large length.

3 Ship Speeds in Storm Seas

When wind strength and wave height exceed a certain limit, the maximum speed of a ship depends on its motions and not on the resistance and the available horsepower. Excessive pitching and heaving may cause slamming and shipping of water. The latter may damage the deck gear and the deck cargo which is often carried on fully loaded ships. Excessive accelerations may cause shifting of the cargo or, on liners, they cause excessive discomfort to passengers and difficulties in hotel service. It becomes necessary to reduce the propeller RPM and ship speed in order to ease its motions. Weather conditions, at which the reduction of power must begin, vary with the type of a ship and its power. On the tanker, investigated by Bonebakker, this occurred at wind strength of 5 on Beaufort scale. Aertssen (1957, NSMB Symp.) showed that a modern general cargo ship can be expected to maintain the full power up to wind strength of 6.

The characteristics of a *Victory* type ship are shown in Fig. 1. This simplified figure was constructed on the basis of a preliminary analysis of log books. The results of a more complete analysis are shown in Fig. 10. Explanation of this figure can best be given in words of Lewis and Morrison (1954, 1955): "In this study daily average data were used from three round voyages during winter months. In order to eliminate extraneous effects, figures were excluded in any of the following cases:

- 1 Speed reduced due to other causes than wind and sea (i.e., fog, machinery trouble, etc.).
- 2 Mean drafts other than full load (i.e. less than 26 feet).
- 3 Wind or sea changed more than 90° during the day.
- 4 Average wind differed more than 55° from the sea direction. (The wind force and sea condition never

¹⁴ British Shipbuilders Research Association.

differed more than 2 points on the wind and sea scales for the data used.)

"The direction of wind and sea with respect to the ship's heading was determined approximately from:

- (1) log data on wind and sea compass directions,
- (2) estimated heading of ship on the basis of track normally followed. . .

"A plot was made of average daily speed versus sea condition, as shown in the accompanying graph, Fig. 10.¹⁵ A different symbol was used for each sea direction, and lines drawn to show the trend of speed for each case. The definition of sea condition used is as follows (U. S. Hydrographic Office modified Douglas Sea Scale):

Code figure	Approximate height of sea	Description
0	0	Calm
1	less than 1 ft	Smooth
2	1-3 ft	Slight
3	3-5 ft	Moderate
4	5-8 ft	Rough
5	8-12 ft	Very rough
6	12-20 ft	High
7	20-40 ft	Very high
8	40 ft and over	Mountainous

"It may be seen that some very clear trends of speed in relation to sea condition emerge from this plot. The records show full power to have been used on only a few of the high-speed runs. In all other cases, power (as estimated from fuel consumption) was reduced by varying amounts, in some cases the power being as low as 30% of normal. This indicated the predominant importance of ship motions in a rough weather service such as this. Apparently the principal reason for loss of speed was the voluntary reduction of power to ease the motions of the vessel."

3.1 Speed Loss Versus Ship Form. Considerable clarification of the relationship between a ship's proportions and its behavior in head irregular seas was furnished by semi-theoretical investigation of E. V. Lewis (1955b). This is based on three postulates:

- (a) Only waves equal or longer than a ship provide sufficient excitation for large motions.
- (b) Prohibitively large ship motions occur only when the ship's natural period of pitching and the period of wave encounter are nearly equal.

- (c) In a stormy irregular sea the wave components of all lengths are present and a ship, therefore, can always find a component with which it can fall into synchronism.

It follows from the foregoing that violent ship motions can be expected if the synchronous wave component has the wave length equal to or longer than ship's length. The periods of encounter depend on the wave celerity (in turn dependent on wave length) and on a ship's speed. The natural pitching period is approximately proportional to the square root of a ship's length. Fig. 11 was drawn on this basis with speed-length ratios as

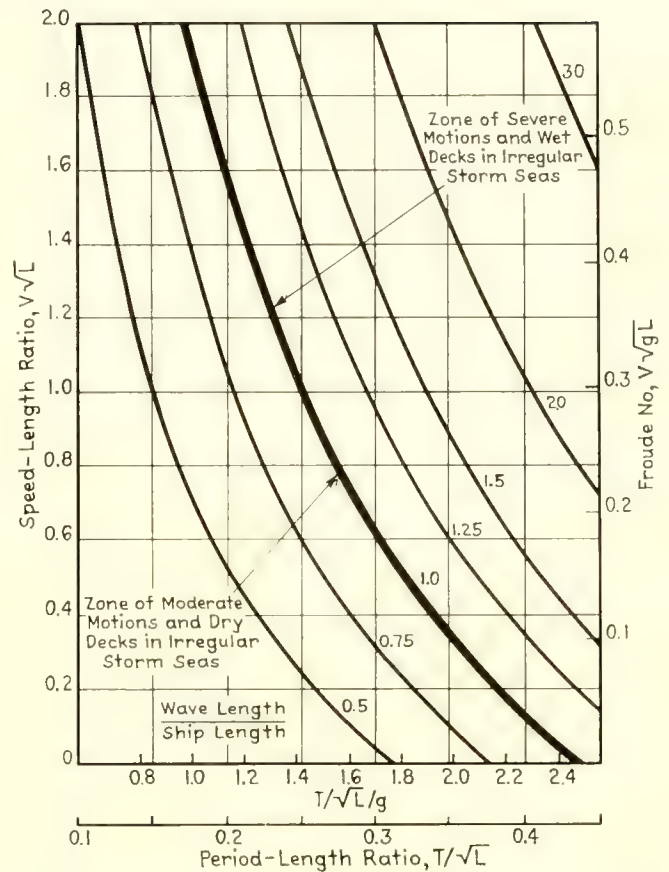


Fig. 11 Theoretical ship speed-length ratios for synchronous oscillation in regular head seas of different lengths—defining typical zones of severe and moderate motions in irregular seas (from Lewis, 1955b)

ordinates and T/\sqrt{L} ratios as abscissae, where T is the natural pitching period and L is the ship's length. The parametric lines are the λ/L ratios for the wave components of synchronous frequency of encounter. The heavy line, $\lambda/L = 1.0$ has been interpreted as the demarcation between the zones of moderate and severe ship motions. A ship can always be placed in a moderate region by sufficient reduction of its speed.

Lewis next established an empirical relationship between the period ratio, T/\sqrt{L} , and the displacement/length ratio, $\Delta/(L/100)^3$ for typical ships. By means of this relationship he converted Fig. 11 into the form shown in Fig. 12. The solid curve in this figure corresponds to $\lambda/L = 1.00$. This curve is now assumed to be the demarcation line between regions of a satisfactory and unsatisfactory behavior of ships under storm conditions. A number of points above this line were plotted on the basis of available data on smooth-water speeds of ships. A few points (mostly towing-tank models) below the curve represent the speeds at which ship's motions under storm conditions were mild. Four heavy vertical arrows indicate the reliably reported reductions of speed on ships at sea. These observed data agree well with Lewis' curve.

¹⁵ Figure number of quotation was changed to refer to the present monograph.

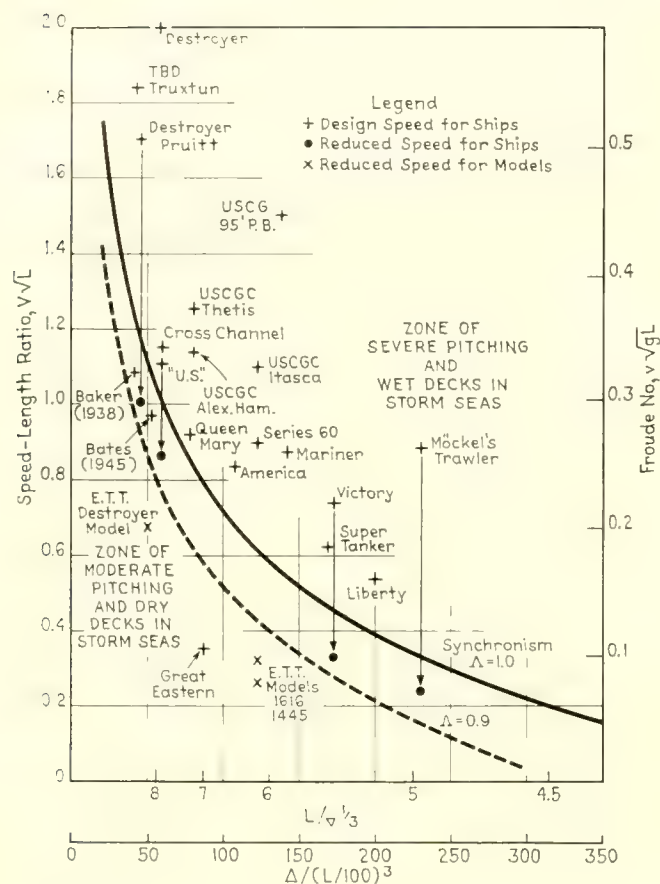


Fig. 12 Trend of speed-length ratios for synchronous pitching of typical ships in regular head seas of length equal to ship length—defining zones of serious and moderate pitching in irregular storm seas (from E. V. Lewis, 1955b)

As an example of the practical use of Fig. 12, Lewis (1956) investigated changes in proportions of a *Victory* type ship. A certain increase of the length and a decrease of the beam and draft were assumed, and the resultant improvement in service was investigated. The change is shown by the arrow in Fig. 13, and the revised design is designated as "Proposed Trend." The new data on the *Mariner*, with commonly used reduced draft (Allen and Sullivan, 1954) were added. The frequency of occurrence of different types of weather was considered and the final changes of revenue were evaluated. The increase of revenue was shown to range from 12.8 per cent at a high cost of ship construction with cargo at \$20 per ton, to 60.3 per cent at a low construction cost and cargo at \$30 per ton.

Aertssen (1957, 1959) further extended E. V. Lewis' work by including in the chart sea observation data, which were obtained on several recently built Belgian ships. On this basis he drew several demarcation lines corresponding to different wind strengths on the Beaufort scale. The resultant chart is shown in Fig. 14.

In a 1959 paper Aertssen presented a detailed analysis of the power and speed of a passenger ship *Jadotville*

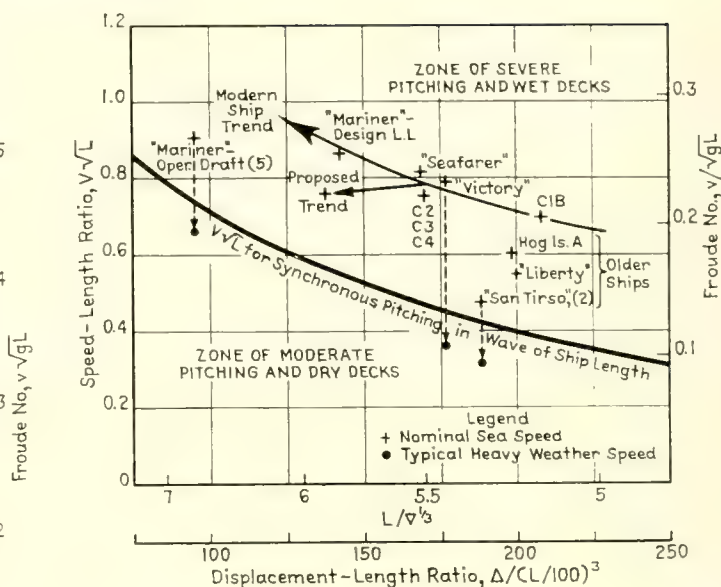


Fig. 13 Sea speed of cargo ships in relation to zones of severe and moderate pitching in irregular head seas (from Lewis, 1956)

and a tanker *Elisabeth*. The particulars of these ships are given in Table 5. The plot of the DHP versus ship speed for SS *Jadotville* is presented in Fig. 15. This figure shows the DHP required to maintain a given ship's speed in different weather. The latter is defined by Beaufort numbers and the relative wind direction (see the inserted sketch). A novel and important feature of this plot is the "limit of speed" line. This indicates the maximum safe ship speed and engine power in any wind. A higher speed is prevented by the necessity to reduce the engine power in order to avoid dangerous ship motions. It will be observed that in bow wind of Beaufort 7 only about 7300 dhp can be used, although as much as 12,500 is available. At Beaufort 6 a small reduction from the service power of 9500 is indicated. The available excess engine power over 9500 was used only in winds below Beaufort 6.

The foregoing discussion was limited to ships at full load. The permissible speed of ships in light condition appears to be limited primarily by slamming. The investigation of the Admiralty Ship Welding Committee (3-1953) on the SS *Ocean Vulcan* showed that slamming occurred on one out of each 3 days of ship at sea and in light condition. In the economic circumstances prevailing over the past several years, many North Atlantic westward voyages are made by ships at small drafts, and large reduction of speed is frequently caused by slamming. Therefore, it appears to be desirable to make a separate investigation of the speed loss of ships in ballast condition, following the pattern set up by Lewis and Aertssen.

In the author's experience, slamming appears to result more often from encountering the swell than from the wind sea. In this case the wind strength is not a satis-

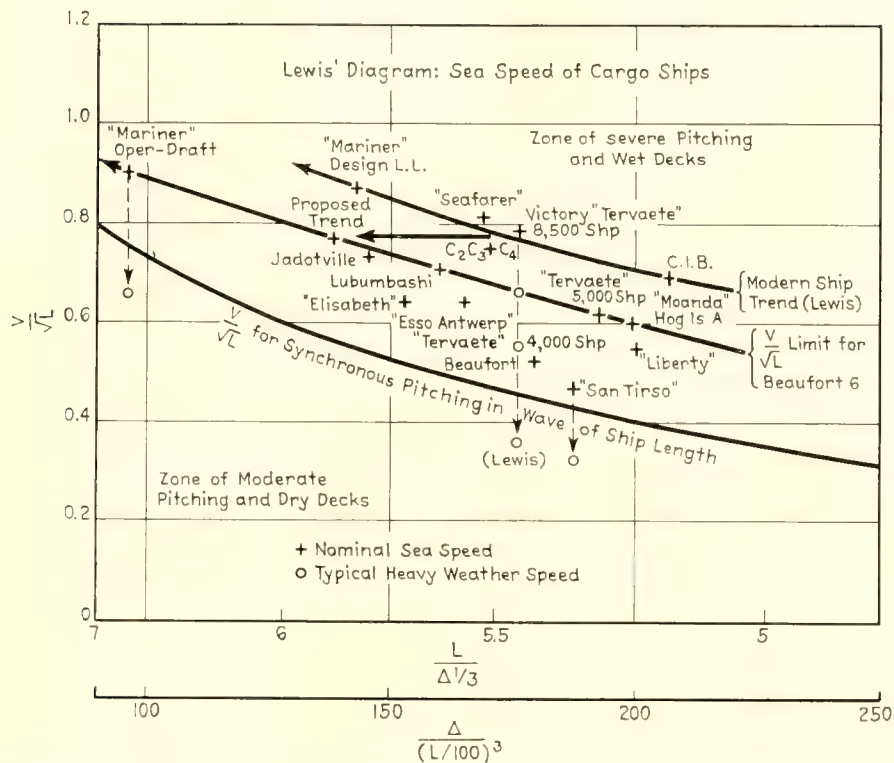


Fig. 14 Aertssen's modification of Lewis' chart for ship speeds in a rough irregular sea (from Aertssen, 1959 NSMB Symp.)

factory parameter to which to refer the performance of a ship. The use of the wave dimensions appears to be preferable. The existence of several separate scales for wave description is, however, confusing and it is suggested that wave sizes in feet be used. Ships' log data appear to be unsatisfactory in this connection because it is often not clear whether a wind-wave or a swell code numbers are used.

4 Concluding Remarks

The subject of resistance increase and speed loss of ships in waves is subdivided into

- (a) Speed of ships as limited by the available power.
- (b) Speed of ships as limited by the severity of motions, regardless of the power.

In the past work, that of Kent, Kempf, and Möckel, only the first of these subdivisions received attention. The ships considered by these investigators were of full form and low power. In a rough sea they rapidly lost the speed, and dangerous amplitudes of motion did not develop. These conditions still exist at present in the case of tankers and bulk cargo carriers. The tanker investigated by Bonebakker, with 0.16 hp per ton, had to reduce slightly the engine power in head seas when wind exceeded Beaufort 5. This occurred, however, on only 16 observations out of 990. The primary objective of research in this case is to evaluate the reserve power

needed to maintain the normal ship speed in face of the moderately adverse weather.

The second consideration, (b), applies to all modern ships except tankers and bulk cargo carriers. The lines of these ships are sufficiently fine and the power is sufficiently high to permit reaching dangerous speed in waves. In adverse weather it becomes necessary to reduce the propeller RPM and the engine power. The level of weather severity at which this is done depends on the qualities of a ship and on the judgment of her master. With clean decks and without deck cargo a fully loaded ship may be driven faster. The danger of damage to the deck cargo is one of the main causes of speed reduction. A dry ship will certainly be driven faster than a wet one.

The type of the chart developed by E. V. Lewis, as shown in Figs. 12 and 13, appears to be an excellent instrument for defining the permissible ship speed and for judging relative seakindliness of ships. In its present state of completion it permits only a rough judgment of the first objective and none of the second. It must be developed further by inclusion of a large number of reliable sea observations. The initial steps in this direction were taken by Aertssen, but much more remains to be done. It is suggested that log-book abstracts for a number of ships be plotted as this was done by Lewis and Morrison in Fig. 10. It is preferable to use the wind strength for abscissa because the wave-height observa-

tions are less reliable and because several wave-height codes are in use. By interpolation, the points can be placed on the blank chart similar to Fig. 12. A separate sheet for each wind strength can be used and a mean curve can be drawn through the points corresponding to a certain wind strength for all ships. This will indicate the mean speed-length ratio used by ships at a particular wind strength. The relative positions of the plotted points above or below the mean curve will indicate the relative seakindliness of ships. The objective of the research in this case is to develop ship forms and proportions, which would permit the maintenance of higher speed in adverse weather without dangers connected with excessive ship motions. The necessary reserve power can be evaluated after the maximum safe speed is established in various degrees of severity of adverse weather.

The published research on the ship speed in adverse weather has been limited to full or nearly full-load condition. A similar research for ships in ballast is suggested. In this case the permissible ship speed appears to be limited by slamming.

5 Suggested Research Topics

1 The Project on Evaluation of a Ship's Resistance in Waves by a Strip Theory is recommended. This is visualized as following Havelock's approach, Section 2.2, but using a more complete evaluation of hydrodynamic pressures. In this evaluation cognizance should be taken of the ship-wave interaction and of the coupling of heaving and pitching oscillations; the coupling has a strong effect on phase relationships which primarily govern the resistance. The evaluation of hydrodynamic pressures used by Korvin-Kroukovsky and Jacobs (3-1957) may be useful in this connection.

2 The Evaluation of the Resistance Caused by Waves and Ship Motions of Large Amplitude is suggested. The solution of the problem in Project 1 was based on infinitely small ship motions so that pressure integrations were made over the still-water wetted area. The integration over true instantaneous wetted areas may be expected to yield greater resistance in case of ships of pronounced V-form. The problem may be solved theoretically by expressing the pressure and wetted-area fluctuations by a series of harmonics. It also can be solved numerically by a suitable calculation procedure based on a series of instantaneous ship-wave positions. Harmonic ship oscillations can be assumed.

3 A Search for Methods of Extending the Resistance Data obtained in regular waves to the irregular ones is recommended. The work of Pershin and Voznessensky (3-1957) appears to be the only attempt made to date, Section 2.5. The linear superposition, so successful in the analysis of ship motions, is not applicable in the present case, since the resistance appears to be proportional to the square of the wave height.

4 Model Resistance Tests on Geosim Series in Wide Tanks are needed. Test data obtained in narrow tanks

appear to be of questionable validity because of simultaneously occurring scale effect and tank-wall interference.

5 Theoretical and Semi-Empirical Evaluation of the Resistance Added by Wave Reflection, Section 2.1, is still in a rudimentary stage and further development is needed. This component of the resistance can be expected to be significant for large tankers and bulk cargo ships. These ships have a large angle of entrance and, because of their length, pitch but little in moderate weather.

6 The Evaluation of the Total Ship Resistance at Sea due to all causes requires further attention. The primary references in this connection are Kent (3-1951, 3-1957) and Pershin and Voznessensky (3-1957). These publications contain also further references on the subject. It appears that the problem can be solved best by proper blending of theoretical and empirical methods.

7 Further Statistical Observations on ship resistance and powering at sea are needed. A general pattern of such research has been well formulated by Bonebakker, Clements, and Aertssen, and the primary need is in a greater volume of data.

8 An Investigation of Statistical Averaging of the data of Projects 1 to 6 is needed in order to provide typical ship-voyage data. This is a necessary prerequisite to correlating theoretical and towing tank data with sea observations of project (7).

9 Detailed Investigations of Ship Powering under specific weather conditions are needed. This will provide a verification of the data obtained under Project 6. Past work of Kent and others suffered from inability to provide reliable quantitative data on sea waves. It should be emphasized that wave conditions and ship motions must be recorded simultaneously with power measurements.

10 Detailed Investigations of a Ship's Powering in head swell and light wind is needed for a relatively simple correlation with theoretical and experimental work under Projects 1, 2, 3 and 5. In this case the resistance caused by waves can be assumed to predominate strongly over that caused by all other influences.

11 Application of Bonebakker's (1951, 1954) Method of Power Evaluation to Model Tests in oblique waves is suggested. Head waves and towed models were visualized in Project 4. The towing is not practical in oblique waves since it would impose restraints and affect model motions. The resistance and powering of a free self-propelled model can be estimated, however, on the basis of the apparent propeller slip, following Telfer, Bonebakker, and Clements. The case of oblique waves is important because the components of resistance due to rolling, yawing and rudder motions are added to the wave resistance.

12 Log Analysis Data, patterned after Lewis and Morrison (1954, 1955), are needed for a large number of ships and operating conditions. The immediate objective of this research is to provide a sufficiently large number of points for Lewis' type plot, Figs. 12, 13 and 14, in order to permit:

(a) More reliable evaluation of the demarcation lines between moderate and severe ship motions.

(b) Establishment of demarcation lines for each wind speed and direction.

(c) Comparison of seagoing qualities of various ships at similar displacement/length ratios.

An examination of the engineer's log is also suggested in order to establish the power reduction corresponding to each demarcation line. Investigators' attention is called to the fact that the weather log usually provides more information on wind and waves than the deck log. In general, all three logs should be examined: the deck log, the weather log and the engineer's log.

The research suggested in the foregoing has a dual ultimate purpose:

(a) To indicate favorable ship forms and proportions.

(b) To establish the engine power commensurate with the speed that safely can be maintained in adverse weather.

Nomenclature

The symbols listed at the end of Chapter 2 apply to equations connected with waves and ship motions. Additional symbols used in Chapter 4 are as follows:

DHP	= delivered horsepower
K, K'	= amplification factors in heaving and pitching motions, Section 2.2
l, m, n	= directional cosines along x, y , and z -axes
N	= propeller RPM
SHP	= shaft horsepower
s_a	= average apparent propeller slip
T_d	= time out of dock
W	= weather factor
Δ	= ship displacement (weight)
∇	= ship displacement (volume)

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1 For the code of Abbreviations see page 218 of Chapter 3.

2 All references listed under names of Kent and Kempf in Chapter 3 are relevant to the subject of Chapter 4.

3 References preceded by 3 before the year will be found in the Bibliography at the end of Chapter 3.

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Loads Acting on a Ship and the Elastic Response of a Ship

1 Introduction

A most important factor in the design of a ship's structure is the bending moment acting on the midship section. In the design process, this is obtained by drawing a curve of load distribution along the length of a ship and integrating it twice. The details of this procedure can be found in a number of existing textbooks and will not be discussed further in this monograph. Bending moments also have been measured by means of suitable dispositions of strain gages on models in towing tanks and on ships at sea. Results of such measurements indicate a wide range of possible stresses which depend on a wave size and form, properties of a ship's sections and a ship's form, speed, and weight distribution. These data can be systematized and better understood if considered in the light of a suitable theory. Therefore, a theory of bending moments acting on a ship will, be presented first and will be followed by model-test and sea-observation data. A complete separation of these three domains of activity is, however, not practical and frequent cross references will be made.

1.1 Conventional Static Method of Bending Moment Calculation. The method of evaluation of the load-distribution curve, which is in general use at present, was introduced by Reed (1872). It consists of placing a ship on an imaginary wave equal in length to the ship's length and $1/20$ of this length in height. The wave is imagined as stationary and all effects of orbital water velocities and of wave celerity are neglected. A ship poised on such a wave is also considered as stationary. In fact, the entire complex system of a moving ship among moving waves is replaced by a static model. The loading curve is then calculated considering the difference between the weight and buoyancy at each section of a ship. Two critical conditions are evaluated. These are known as "hogging" and "sagging." In the first, the wave crest is located at the midship section and the resultant bending moment causes tensile stresses in the deck structure and compressive stresses in a ship's bottom structure. In the second condition, the wave trough is placed at the midship section so that the deck is under compression and the bottom is in tension.

The inadequacy of the foregoing static model to represent true ship stresses is subsequently compensated

by supplementary empirical rules. If the computed bending moment were a true moment, it would be sufficient to equate it to the product of the section modulus and the allowable strength of the material used. Instead, the choice of the section modulus is based on various supplementary rules which have been developed empirically by classification societies and various government agencies.

1.2 Attempts at Improvement of the Static Method. Several attempts were made to supplement the fictitious static-wave method by introducing certain concepts based on the true laws of nature. The most important of these was due to W. E. Smith (3-1883).¹ Smith called attention to the fact that orbital velocities of water in waves modify the hydrostatic-pressure gradient. As a result of accelerations of water particles, the water appears to be lighter than normal at wave crests and heavier than normal at wave troughs. When these properties are taken into account in otherwise conventional computations, a decrease of the bending moment is found to occur. Conversely, neglect of the "Smith effect" will result in overestimation of the bending moment.

Smith calculated the relative values for three sample ships as shown in Table 1.

The modification of the pressure gradient with depth (the Smith correction) follows an exponential law²

Table 1 Range of Stress From Maximum Hogging to Maximum Sagging Calculated on the Basis of:

	Buoyancy with Smith's correction, per cent	Buoyancy simply proportioned to volume displaced, per cent
Ship 1.....	100	170
Ship 2.....	100	165
Ship 3.....	100	155

and depends on a ship's draft and section coefficients. Use of this correction has become a universal practice

¹ An Arabic number preceding the year indicates the chapter at the end of which the complete reference will be found. References without such preceding numbers will be found in the Bibliography at the end of this chapter.

² See Table 1 of Appendix A.

with writers on ship motions and ship bending moments. Quoting from a recent paper by Radoslavljević (1957a): "... the amplitudes of forced oscillations taking into account Smith Effect can be over 50% smaller in heaving and over 30% smaller in pitching, in relation to the same amplitudes when the distribution of buoyancy in the disturbed sea is hydrostatic."

W. E. Smith included the effect of orbital water velocities in the wave, but neglected the wave celerity and ship motions. Except for the adjustment in the effective weight of water, the calculations are still based on statics. The definition "static calculation" is often used, therefore, in the contemporary technical literature³ in reference to calculations which include the Smith effect.

T. C. Read (3-1890) called attention to the fact that the effective ship weight is also modified by heaving accelerations. It is increased in wave troughs and decreased on wave crests. Read showed that there is a corresponding increase of the sagging moment from 8.4 to 24 per cent and decrease of hogging moment from 0 to 7.7 per cent. These figures refer to two examples given by Read. The larger figures correspond to a larger bending moment and a fine vessel and the smaller figures to a full vessel and a smaller bending moment. The excess of sagging over hogging bending moment was later confirmed qualitatively by sea observations on the *MS San Francisco* (Schnadel, 3-1936) and *SS Ocean Vulcan* (3-Adm. Ship Weld. Comm., 1953, 1954). It also was confirmed by model tests in towing tanks (Sato, 1951; Ochi, 1956, 1957; Lewis and Dalzell, 1958).

1.3 Dynamics of Ship Motions. The significance and magnitude of Smith and Read effects were further demonstrated by Alexander (1911) and Robb (1918) without introducing new concepts. Kriloff (3-1896, 1898a and b), Horn (1910), Hazen and Nims (3-1940) and Bull (3-Adm. Ship. Weld. Comm., 1953) considered ship motions and inertial forces more completely. The differential equations of a ship's heaving and pitching were formed and investigated. The common weak points in this activity were the failure to consider the coupling between heaving and pitching motions, the failure to consider the modifications of the wave pressure gradient (Smith effect) by interference of a ship's hull, and the inadequacy of the available hydrodynamic data. Attention was concentrated on the dynamics of ship motion while the necessary hydrodynamic development was neglected.

An advanced theory of coupled ship motions was developed by Haskind (3-1947), but it did not extend to the calculation of bending moments and was not complete in regard to the forces caused by waves. Recently, Hanaoka (3-1957a, b) extended a similar theory and presented a sample of bending moment calculations. It was pointed out in Chapter 2 that such advanced mathematical methods are valuable in guiding simpler approaches. However, they cannot be used in engineering

³ For example, Admiralty Ship Welding Committee Reports Nos. 8 and 12 (1953, 1954), and Hanaoka (1957).

problems because of excessive mathematical complexity and because their application is limited to certain idealized ship forms. In the present monograph, therefore, attention will be concentrated on a simpler engineering approach.

A solution of the coupled differential equations of pitching and heaving motions was developed in a simple form in the course of the past few years (Korvin-Kroukovsky and Lewis, 3-1955). This was followed by an evaluation of hydrodynamic-force coefficients which included the interference effects between a ship and waves (Korvin-Kroukovsky and Jacobs, 3-1957). The material has now been collected for the formulation of a simple rational theory of bending moments. The word "rational" is used here to indicate a theory formulated on the basis of and compatible with the physical laws of nature and free of empirical assumptions. The word rational should not be confused with exact. The process of application of physical laws to an engineering problem necessarily involves various approximations. These approximations, however, are compatible with dynamic and hydrodynamic concepts and are not based on empiricism.

1.4 Rate of Load Application to a Ship's Structure.

In the discussion of bending moments it is necessary to distinguish between slowly and rapidly varying hydrodynamic loads. The slowly varying loads are imposed by waves which have a period of encounter from 5 to 15 sec in the case of a normal cargo ship. These periods are from 8 to 25 times longer than the period of the two-node hull vibration. A ship's structure responds to such loads as though they were static loads. At any instant the stress is equal to the bending moment divided by the effective section modulus and a ship's elastic properties need not be considered. The historical outline, presented earlier, refers to bending moments caused by such slowly varying loads and this subject will be discussed in the next three sections of this chapter.

Slamming is a well-known example of rapidly applied loads which cause vibration of a ship's structure. Less well known is the nature of wave shocks felt in fast ships and ships with excessive bow flare. These loads are applied with such rapidity that stresses cannot be ascertained by static considerations and it is necessary to consider a ship's elastic properties. These loads will be discussed later following the exposition of the slowly acting loads.

2 Rational Theory of Bending Moments

The weight-distribution curve and the bending moment acting on a ship floating in still water are readily evaluated by conventional methods and will not be considered here.

With increasing speed of a ship a certain sinkage occurs. This sinkage will affect the bending moment because the water-suction forces are not distributed in proportion to displacement. Furthermore the formation of a long wave is observed at the sides of a fast ship. This wave

leads to the development of a sagging moment which increases with a ship's speed. None of these time-independent moments will be considered here. Their existence, however, should be remembered and appropriate measurements in still water should be included in model tests and ship observations in waves. Only the time-dependent loads added by waves and by a ship's heaving and pitching will be considered.

The loading, p , per foot of a ship's length is equal to the algebraic sum of acceleration forces and water pressures:

$$p = \bar{m}\ddot{z}_0 + dF/d\xi \quad (1)$$

where p is the time-dependent vertical force per foot of ship's length, \bar{m} the mass of the ship's structure, equipment, and load apportioned to this length and $dF/d\xi$ the vertical force due to water pressures per unit length. The symbol z_0 is used for the heaving displacement of an element $d\xi$ of a ship's length. When the heaving motion of an entire ship is considered, the sum of all water pressures is balanced by the inertia force and the sum of all forces is equal to zero. At a particular ship's section of unit length, however, the inertial and water pressures are usually not balanced, $p \neq 0$, and the necessary balance of forces results from the application of shear forces at the ends of the section. The double integration of these shear forces gives the bending moment.

The inertial force, $\bar{m}\ddot{z}_0$, depends on the value of the acceleration, \ddot{z}_0 . The force, $dF/d\xi$, of water pressures depends on the acceleration, velocity and displacement of the ship and water. *The solution of the equations of motion is, therefore, a necessary prerequisite to the evaluation of the loading curve and a ship's bending moment.* For the structural analysis the load per foot, p , is to be evaluated at a certain instant; i.e., with certain specific values of z , \dot{z} , and \ddot{z} , as well as wave elevation η .

2.1 Linear Theory. The linear theory of a ship's heaving and pitching motions in waves and of the resulting bending moments was originally developed by Kriloff (3-1896, 1898*a* and *b*) and was further elaborated by Horn (1910). In both cases the heaving and pitching motions were considered independently; i.e., their coupling was neglected. Horn's work is particularly important in demonstrating the significance of phase relationships (between ship and wave motions) in the summation of water pressure and ship inertia loads.

The phase relationships are strongly affected by the coupling of heaving and pitching motions, and the consideration of this coupling is therefore important. The differential equations of coupled pitch-heave motion were formulated and a method of evaluating the coefficients of these equations was developed by Korvin-Kroukovsky and Jacobs (3-1957). The application of this material to the evaluation of bending moments will be outlined briefly. The equations of motion are

$$a\ddot{z} + b\dot{z} + cz + d\ddot{\theta} + e\dot{\theta} + g\theta = \bar{F}e^{i\omega t} \quad (2)$$

$$A\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{z} + E\dot{z} + Gz = \bar{M}e^{i\omega t}$$

The nomenclature used in these equations and the evaluation of coefficients are given in Appendix C.

In the process of derivation of equations (1) and (2), it was shown that the total hydrodynamic force per unit length, $dF/d\xi$, can be considered as the sum of two forces, $(dF/d\xi)_b + (dF/d\xi)_w$. The first of these forces is developed when a ship oscillates in smooth water and the second when waves encounter a restrained (non-oscillating) ship. Corresponding to this subdivision, all terms of the left-hand sides of equations (2) represent forces and moments obtained by different forms of integration of $(dF/d\xi)_b$, with respect to a ship's length. The terms on the right-hand sides of the equations result from integrations of the force exerted by waves on a restrained ship, $(dF/d\xi)_w$. These latter terms are usually referred to as "exciting functions."

The forces caused by ship's oscillations in smooth water will be considered first. An isolated section of a ship's length, $d\xi$, has only a simple heaving motion, z_0 , which results from the contributions of a ship's heaving, z , and pitching, θ , so that

$$z_0 = z + \xi\theta \quad (3)$$

The total force and moment acting on a ship, oscillating in smooth water, are obtained by the integration of the sectional forces $(dF/d\xi)_b$, which result from the foregoing composite motion. This integration leads to 12 terms. The coefficients of these terms are divided into four groups, (a,b,c) , (A,B,C) , (d,e,g) , and (D,E,G) . The first two groups define pure heaving and pure pitching motions and the last two define the effect of the pitching on heaving and vice versa.

Application of the foregoing material to the evaluation of the loading curve requires a regrouping of terms. Instead of referring forces, acting on the entire ship, to isolated heaving and pitching motions, they are now to be expressed as acting on an isolated section, $d\xi$, in a composite vertical motion of the section, z_0 . Equation (3), however, indicates only the displacement z_0 . It is necessary to derive the corresponding velocity \dot{z}_0 and the acceleration \ddot{z}_0 . In doing so, one must remember the dual nature of the co-ordinate ξ . It is a constant defining the distance from the ship's center of gravity of a mass \bar{m} , apportioned to a certain ship section. On the other hand, it is a time-dependent co-ordinate when it refers to a stationary slice of water $d\xi$, and its time derivative in this case is $d\xi/dt = -V$.

Since only the simple vertical motion of an isolated section $d\xi$ is now to be considered, the integrands of the coefficients a , b , and c will suffice. These are obtained by adding integrands of coefficients listed in groups of equations (34) and (41) of Appendix C. Certain coefficients vanish in the process of integration with respect to ship length and are omitted in the final summary of equations (42), which, therefore, is not to be used in this connection. Use of this material now permits evaluation of three components of the force caused by the ship's motion, $(dF/d\xi)_b$, as follows:

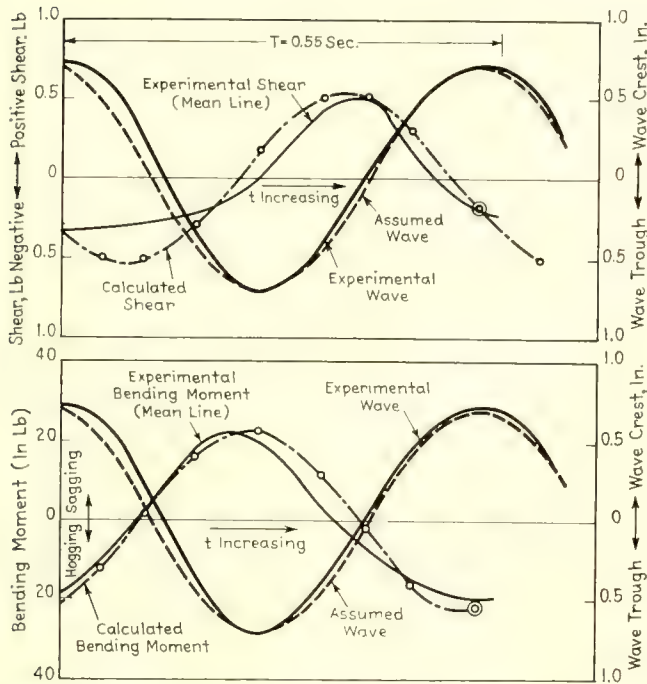


Fig. 1 (top) and Fig. 2 (bottom) Experimental towing tank calculations and experimental data for amidships of Model 1723 (destroyer). 5.71-ft model at 5 fps; wave length = model length; wave height = $1/15$ model length

Force component caused by

$$\left\{ \begin{array}{l} \text{displacement} = \rho g B(z + \xi \theta) \\ \text{velocity} = [N(\xi) - V \rho d(Sk_2 k_4)/d\xi] (\dot{z} + \xi \dot{\theta} - V\theta) \\ \text{acceleration} = \ddot{m}(\ddot{z} + \xi \ddot{\theta}) + \rho(\ddot{z} + \xi \ddot{\theta} - 2V\dot{\theta})(Sk_2 k_4) \end{array} \right. \quad (4)$$

Next, the forces exerted by waves on a restrained ship will be considered. The sectional force due to wave action, $(dF/d\xi)_w$ is given directly by equation (25) of Appendix C. It may be convenient to separate the first term of K_1 coefficient from the others and to write the force in the form of two components:

$$(dF/d\xi)_{w1} = \rho g B \left(h \sin \frac{2\pi x}{\lambda} \right) \quad (5)$$

$(dF/d\xi)_{w2}$ = remaining terms of equation (25) of Appendix C

The first of these components is simply the change of the displacement caused by the wave rise as it is used in conventional static calculations. This force (with sign changed) can be added to the first of equations (4) in order to get the total force due to the change of displacement. The remaining part of the first term of equation (25) of Appendix C represents the Smith effect modified by a ship's interference with waves. The second term of equation (25) is a further correction for a ship's speed.

Equation (25) of Appendix C was derived from the

potential solution which does not account for the dissipation of energy in the form of surface waves. The force component caused by the vertical water motion in waves, $v_w N(\xi)$, must be added, therefore, to the force resulting from ship motions, $(\ddot{z} + \xi \ddot{\theta} - V\dot{\theta})N(\xi)$, in equation (4). In the analysis of ship motions in Appendix C, the force $v_w N(\xi)$ was neglected because it became insignificant after integration with respect to ship's length. However, it gives a significant contribution to the loading of a ship's section, $d\xi$, and should be included in the analysis of bending moments.

As an example, the results of bending-moment calculations are shown in Table 2 and Figs. 1 and 2.⁴ These data refer to the model 1723 (destroyer, the body plan of which is shown in Fig. 31 and the particulars are given in Table 6). The data on motions of this model in waves were given by Korvin-Kroukovsky and Jacobs (3-1957).

Fig. 2 shows an excellent agreement between experimental and calculated bending moments. Table 2 shows in detail the conditions existing at a particular instant indicated by the double circles in Figs. 1 and 2. This instant corresponds to the hogging condition with the bending moment nearly at its maximum. The data at the bottom of the table indicate the dynamic conditions existing at this instant. The wave crest is amidships but at the same time the pitch angle is at its maximum as this is indicated by $\theta = 0$ and by the high value of the pitching acceleration, $\ddot{\theta}$. The small value of the phase lag between heaving and pitching, 55 deg, brings about simultaneously high values of heaving and pitching accelerations. This brings about, in turn, a large inertial force due to ship's masses and the added water masses. At station 5 for instance, the inertial forces, $2.00 + 1.12$, are greater than the displacement force of -2.81 lb. Most of the remaining force of -0.51 lb is the result of the velocity-dependent (damping) force computed on the basis of $N(\xi)$. Table 2 demonstrates that the net loading at any ship section is a relatively small difference between several large components of different signs. The final shear distribution and the bending moment are therefore very sensitive to the accuracy of calculation of the individual components.

The large part played by inertial and by velocity-dependent forces also indicates the sensitivity of the loading curve and of bending-moment curves to phase relationships.

For an additional example of bending-moment calculations, the reader is referred to Section 4.12. In this section T-2 tanker model data are analyzed. A more complete description of the procedures outlined in the foregoing and in the T-2 tanker analysis will be found in Jacobs (1958).

2.2 Nonlinear Theory. The destroyer model in low waves, $(\lambda/h = 48)$, cited in the foregoing example, appears to be amenable to both motion and bending-

⁴ This material is taken from an unpublished work of Winnifred R. Jacobs (1957) of the Davidson Laboratory, Stevens Institute of Technology.

Table 2 Computed Shear Forces and Bending Moments for 5.71-Ft Model 1723 Destroyer at 5 fps, in Waves of Model Length and $L/48$ in Height

(For condition when wave crest is at Midsection)										
Stations	1	3	5	7	9	11	13	15	17	19
Loads, lb/ft										
Ship mass inertia.....	0.90	1.25	2.00	1.97	1.14	0.71	0.14	-0.50	-0.76	-0.63
Added mass inertia....	0.16	0.60	1.12	1.40	1.24	0.76	0.16	-0.35	-0.68	-0.60
Damping.....	-0.12	-0.43	-0.82	-0.83	-0.55	-0.25	-0.10	-0.06	-0.14	-0.29
Added displacement...	-1.27	-2.84	-2.81	-1.58	0	0.94	1.08	0.72	0.32	0.35
Modified Smith effect..	0.04	0.12	0	-0.34	-0.65	-0.66	-0.39	0	0.26	0.25
Total load, lb/ft.....	-0.29	-1.30	-0.51	0.62	1.18	1.50	0.89	-0.19	-1.00	-0.92
Shear, lb.....	-0.09	-0.54	-1.07	-1.03	-0.51	0.26	0.94	1.13	0.79	0.25
Bending moment, lb-ft.....	-0.02	-0.21	-0.67	-1.27	-1.61	-1.78	-1.43	-0.84	-0.30	0

Dynamic Conditions at Instant Considered in above Analysis

$$\begin{aligned}\ddot{z} &= -4.73 & \ddot{\theta} &= -5.88 \\ \dot{z} &= 0.59 & \dot{\theta} &= 0 \\ z &= 0.036 & \theta &= 0.045\end{aligned}$$

Phase lag of heaving after pitching, 55 deg

NOTE: The Model was divided into 10 segments. There were 21 equidistant stations and the loads were computed at the odd stations.

moment linearized analyses. Greater discrepancies between calculated and measured bending moments appeared in the analysis of the T-2 tanker model in higher waves ($\lambda/h \cong 20$). These discrepancies can be attributed to two primary causes; i.e., the uncertainties in the damping data and the non-constancy of the hydrodynamic coefficients of equations of motions. The examples analyzed by Korvin-Kroukovsky and Jacobs (3-1957) showed that the use of linearized equations of motions leads to satisfactory results in case of ships of usual form, but fails for such extreme forms as the sailing yachts. A strong effect of nonlinearity on ship-motion phases also was demonstrated by the experiments of Akita and Ochi (3-1955) in case of an unusually shallow-draft, flat-bottomed hull.

A complete analysis of motions and bending moments of ships of unusual form, or of normal ships in high waves, requires a step-by-step integration of equations of motions with time-dependent coefficients. Such integrations were performed (on basis of uncoupled equations) by Horn (1910) and by Hazen and Nims (3-1940). The results indicated that time variation of the bending moment in harmonic waves distinctly deviates from a sinusoidal form. Horn emphasized that, because of this deviation, it is necessary to calculate the bending moments for a series of small time intervals.

The work of Korvin-Kroukovsky and Jacobs (3-1957) and of Gerritsma (3-1957*c* and *d*) indicated that non-constancy of coefficients apparently affected predicted motions of ships of usual form only to a small degree. The predominant discrepancies in this case were traced to the error in estimating mean values of damping coefficients. This can be explained rationally by observing that motions are obtained by double integration of the forces acting on a ship. Variations of instantaneous

forces are smoothed out in the process of this integration and only the total work of the forces is of significance. This observation will often permit an important short cut in the nonlinear evaluation of bending moments: relative ship-wave positions, velocities, accelerations and phase relationships can be assumed to remain as indicated by analytical solution of the coupled linearized differential equations of motions. Subsequently, the loading curve can be computed for isolated instances, using the coefficients based on the instantaneous conditions, (i.e., instantaneous waterline positions) at each ship section. The analysis of this type, but extended to include slamming vibrations, was attempted by Dalzell (1959) with a reasonable success. This analysis will be discussed in Section 5.55 in connection with slamming.

3 Bending Moment—Observations on Ships at Sea

The content of this section is limited to the quantitative correlation of a ship's bending moment with the wave size. The paper by G. Schnadel (1937/38) and the reports of the Admiralty Ship Welding Committee appear to be the only available sources of this information.

3.1 MS San Francisco. Schnadel's paper contains the analysis of measurements taken aboard the MS *San Francisco* over the period of a few seconds. During this period, the ship passed through the hogging and sagging conditions on an unusually large and steep wave. The general description and data on the voyage of the MS *San Francisco* will be found in Section 3-5.12.⁵

⁵ Reference to sections, equations, and figures in preceding chapters will be designated by chapter number and section, equation, or figure number; reference above, for example, is to chapter 3, section 5.12.

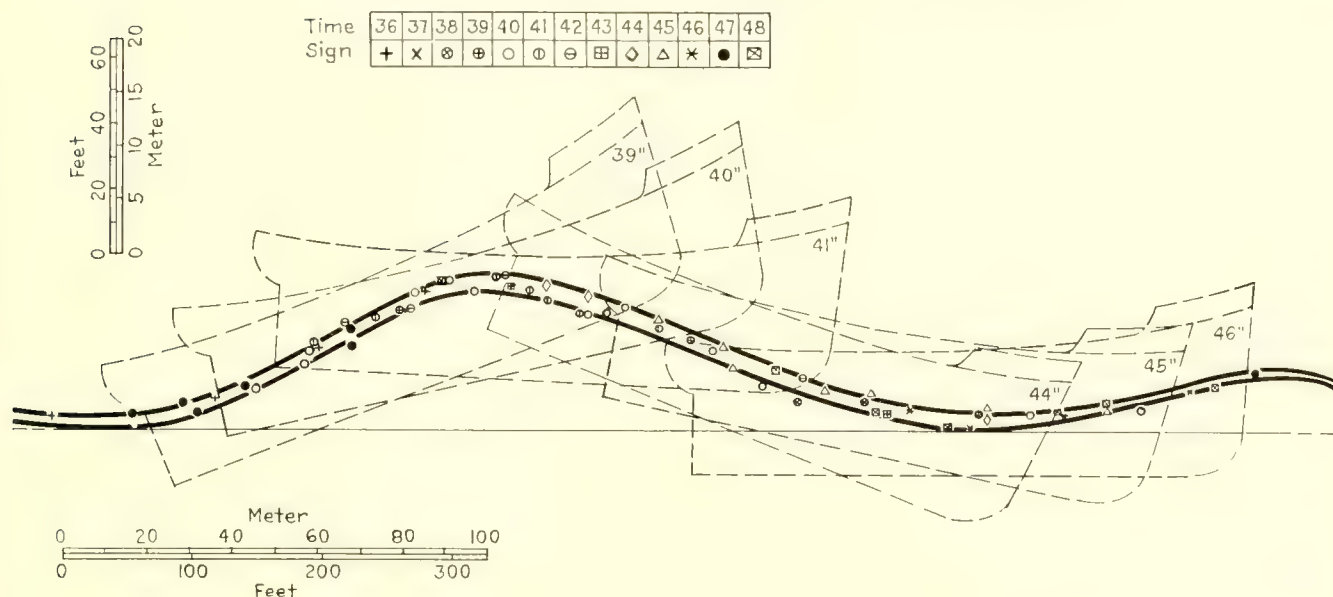


Fig. 3 Complete wave measurement (from Schnadel, 1937-1938)

The wave profile on the ship's side was recorded continuously on a running strip of photographic paper. For a wave longer than the ship, this record gave at any instant only a part of the complete wave profile. Since ship motions had been recorded at the same time and had been interconnected by time signals, it was possible to piece together several sections of the record and to obtain the entire wave profile. The results of this process are shown in Fig. 3. This diagram covers the period from the 36th to the 46th second following 11 hr 40 min AM on December 11, 1934.

The wave profile, defined by several successive observations, may have contained errors caused by the change of wave form with time. Schnadel assumed this error to be small as far as the main wave profile was concerned. However, the main wave profile was covered by smaller waves. These changed their forms rapidly and affected the readings of individual gages. The range of the uncertainty, caused by this action, was shown by the spacing of two profiles drawn in Fig. 3. The wave length was found to be 186m (610 ft), mean wave height from trough to crest, 14m (46 ft) and the ratio of length to height was 13.3. This is exceptional steepness for a wave of such a size.

Figs. 4 and 5 show ship attitudes, the wave profiles, A, and the pressure head profiles B and C at hogging and sagging conditions. The vertical scale is 2.5 times the horizontal one. The pressure head profiles D and E resulted from the correction of pressure heads to the condition of a free wave. This process can be best explained by the following quotation from the original paper:

"The acceleration forces of heaving and pitching cause an acceleration of the added mass of water which is in-

cluded in the measurement of the diaphragms.⁶ If we intend to compare the measurement of the water pressure with the normal theory of waves, we have to take into account the acceleration pressure of the water. For the acceleration of the added mass the formulas of Lewis⁷ and Lockwood Taylor⁸ are used. A simple investigation shows that the water pressure is increased by the added mass in the case of hogging and reduced in the case of sagging. Therefore, we have to subtract the acceleration forces on the crest and to sum up in the trough.

"The pressure found by this operation contains the influence of the Smith effect and the influence of the ship's hull in waves. If we calculate the influence of the Smith effect, we are able to determine the influence of the hull. For a wave 186m length and 14m height, we get the influence of the Smith effect on the crest with 75% of the height of water column. Our measurements show that the pressure in the wave crest is reduced to 60%."

The following passage is also useful in clarifying the questions as to water pressures: "The investigations of Gerstner and Hagen show that the water pressure in an undisturbed wave does not agree with the height of the corresponding water column. Smith⁹ has drawn the conclusion that the calculated wave height should be taken much less than the observed wave height. The investigations of Smith are only partially correct, as they neglect the disturbance of the wave by the ship

⁶ I.e., of the bottom pressure gages.

⁷ F. M. Lewis (3-1929).

⁸ Lockwood Taylor, 1930.

⁹ W. E. Smith, Trans. INA, 1883, p. 135

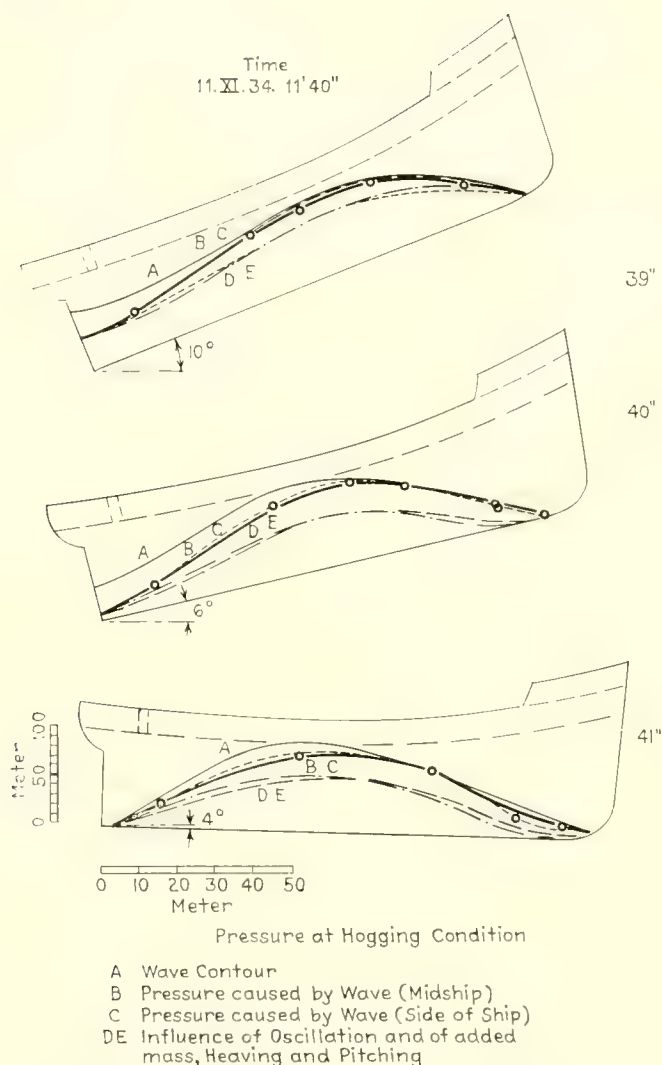


Fig. 4 Pressure in hogging condition (from Schnadel, 1937-1938)

itself and the influence of the added mass of water in the case of heaving and pitching."

The measurements of water pressures and of heaving and pitching accelerations provided the necessary data for the construction of the load-distribution curves. These are shown in Fig. 6 for the hogging conditions. By the usual integration the diagrams of shear forces and of bending moments, shown in Fig. 7, were obtained. Schnadel commented on these as follows:

"For the accurate knowledge of the ship's stress in a rough water, it is important to separate the influences of dynamic and static forces. The dynamic forces contain two different parts: the forces resulting from the acceleration at the hull and the acceleration of the added mass. The load curve of the first part results from the difference of the acceleration forces on the ship's mass and the change of the buoyancy caused by the heaving oscillation. For the hogging ship the bending moment is reduced by the dynamic forces.

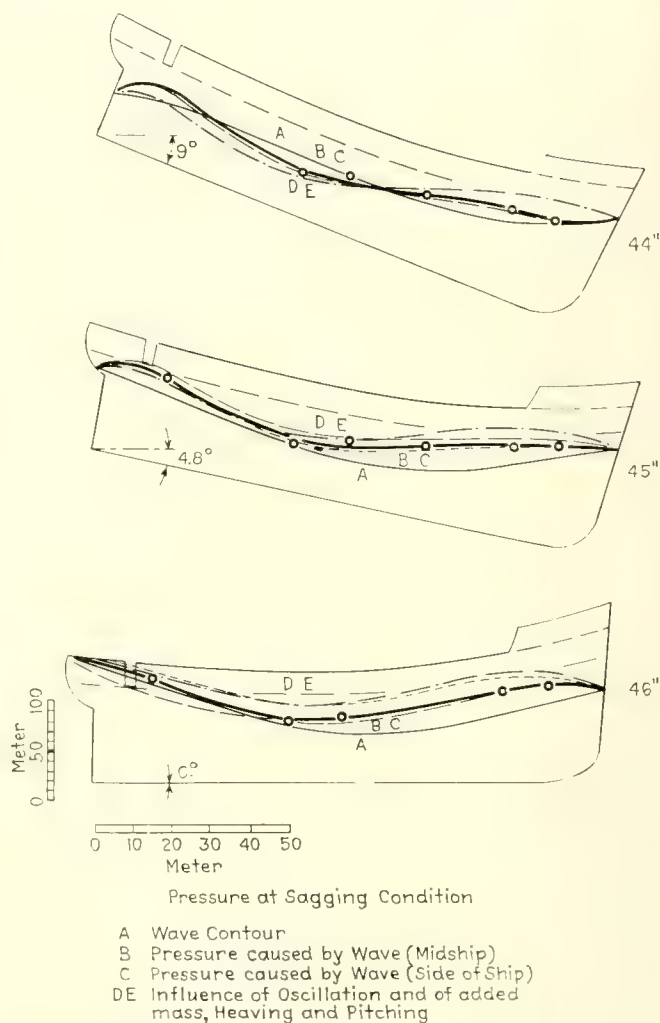


Fig. 5 Pressure in sagging condition (from Schnadel, 1937-1938)

"Calculating the second part, we observe that the forces of the added mass carry a part of the ship's weight instead of the corresponding buoyancy. The load diagram is therefore the difference between the forces of the added mass and the buoyancy curve of the corresponding diving. This load causes, in general, a bending moment which increases the static moments.

"Consider Fig. 7, which shows the static and dynamic hogging moments; it is to be seen that the influence calculated as described is very different at the midship section and at the ends. The results for the midship section are as follows:

Static hogging moment as influenced by sea.....	+30,000 metre-tons
Dynamic moment as influenced by sea (ship hove to).....	- 7,500 metre-tons
Resultant moment.....	+22,500 metre-tons
This moment is measured from the moment of still water condition.....	+23,000 metre-tons

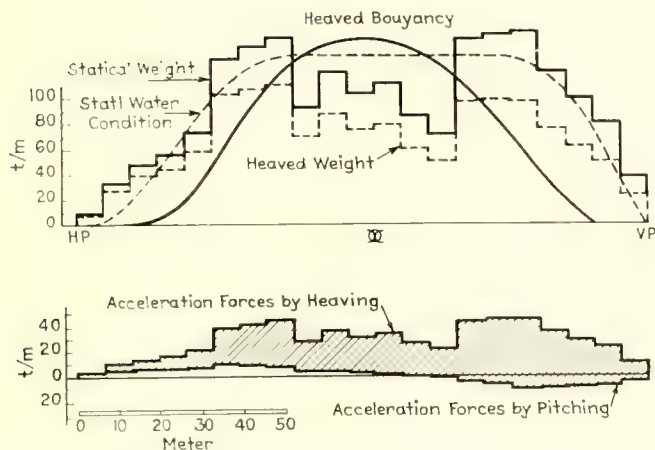


Fig. 6 Acceleration forces by heaving and pitching. Hogging: time 11. xii. 34, 11h. 40'. 40" (from Schnadel, 1937-1938)

The real hogging moment in our case is therefore..... +45,500 metre-tons

"The static moment of 30,000 mt is nearly the greatest influenced by sea. It corresponds to an effective wave height of 5.5 m or $L/24$, if the wave length is equal to the ship's length. Considering the dynamic force too, we get an effective wave height of 4.2 m for the ship hove to, if we call 'effective' wave height the height of wave to be taken for the normal strength calculation to get the same moments and assume that the wave length is equal to the ship's length."

Fig. 8 shows the good agreement which was obtained between bending moments calculated by the foregoing procedure and those deduced from strain-gage measurements. Fig. 9 shows a similarly good agreement between calculated and measured ship deflections. Schnadel's final conclusions were quoted in Section 3-5.12.

The work of Schnadel on the MS *San Francisco* is important in describing how the bending moment is composed of contributions of buoyancy and of a ship's and ambient water's accelerations. He brought out the significance of a ship's interference with waves. These data, observed for the first time on a ship at sea, were later confirmed by the model tests and by theoretical computations. The computations were described in the preceding section and the model tests will be covered in the following one.

The word "confirmed" is used in a broad and qualitative sense. The data obtained on the MS *San Francisco* scarcely can be considered accurate enough for a detailed comparison with calculations and model tests. The primary sources of possible inaccuracies are:

- 1 Only one wave was analyzed.
- 2 The determination of the wave profile from several consecutive records introduces an uncertainty. The uncertainty indicated by the spacing of two curves in Fig. 3 also can introduce an appreciable error.
- 3 The total number of pressure gages is too small to

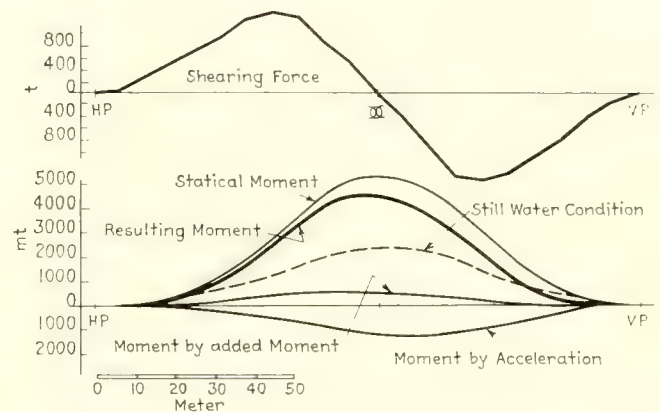


Fig. 7 Shearing force and bending moment. Hogging: time 11. xii. 34, 11h. 40'. 40" (from Schnadel, 1937-1938)

give an accurate determination of the total pressure force.

4 The arrangement for time synchronization of data did not permit sufficient precision.

5 The relationships used in the analysis were based on the properties of regular waves. Peculiarities of the recorded wave were not considered.

An outstanding characteristic of the MS *San Francisco* voyage is the fact that observations were made by an outstanding group of qualified men. These men, Schnadel, Horn, Weinblum, and Weiss, had developed the necessary instrumentation and they used it to collect aboard the ship. They had witnessed the conditions which they subsequently analyzed. As a result, an unusually large amount of valuable information was obtained.

The strain gages were of a mechanical type and did not permit centralization. They had, however, the advantage of a fixed zero point so that hogging and sagging stresses were clearly defined. In the more advanced type of electric strain gages, which have been used since by other investigators, the zero calibration has been uncertain. Only the total hogging-sagging stress ranges were, therefore, reported.

3.2 SS Ocean Vulcan. The general type of instrumentation used on the SS *Ocean Vulcan*¹⁰ was similar to the MS *San Francisco*, but a much larger number of instruments was used. The general description of instrumentation was given in Section 3-5.13. The instrumentation was of a more advanced type than on the MS *San Francisco* and all records were taken at a central recording location. It had, however, two important drawbacks. The electrical strain gages permitted the hogging-sagging range to be measured, but were uncertain in the distribution of stress between hogging and sagging. The records were read from the photographs of gages taken at about 0.4-sec intervals and no continuous curves of data were obtained.

¹⁰ Admiralty Ship Welding Committee Reports Nos. 8 and 12 (see bibliography at end of chapter 3).

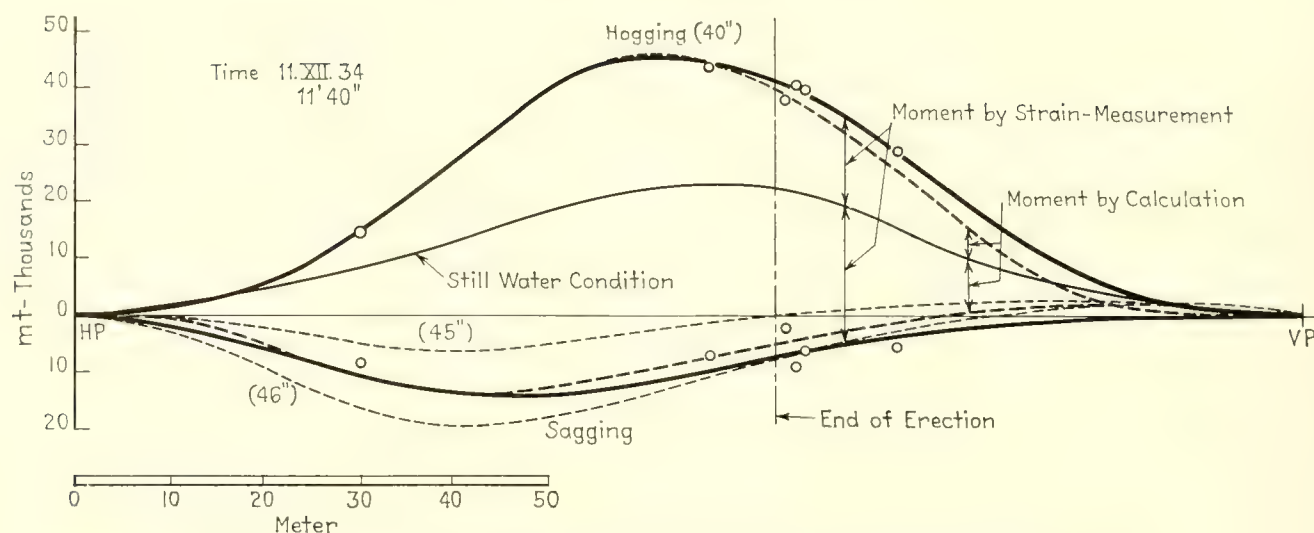


Fig. 8 Bending moment by strain measurement and by calculation (from Schnadel, 1937-1938)

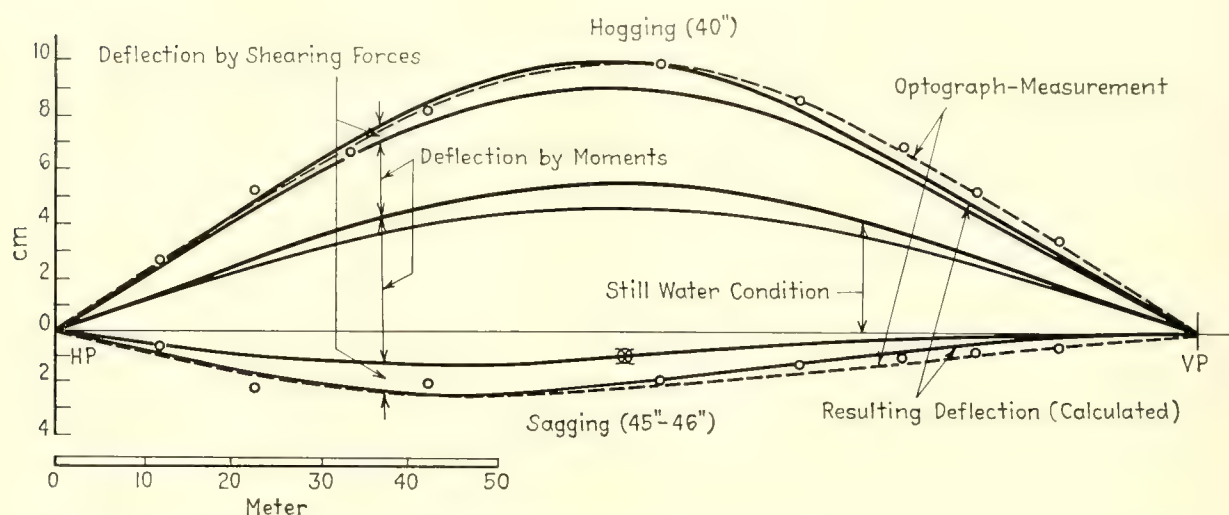


Fig. 9 Deflections by optograph measurement and by moments and shearing forces. Time: 11. xii. 34, 11h. 40' (from Schnadel, 1937-1938)

The *Ocean Vulcan* reports contain a large amount of miscellaneous useful information, but the complete wave profile is shown for only two cases, Fig. 3-41. In most cases, the water profiles recorded on the ship's sides were not used to define the ambient sea waves. The instrumentation, which was used for this purpose, was similar to Schnadel's. The larger number of observation stations could have been expected to give a greater accuracy. However, an opinion was expressed that Schnadel's method of evaluating ocean-wave profiles had not been reliable and apparently the *Ocean Vulcan* data were not used for this purpose. Ship attitudes and wave profiles on the ship's sides were presented in the final reports only in the form of crude sketches which are not suitable for further analysis. The observed and calculated stresses were referred to the visual wave ob-

servations and to weather forecasts of the Admiralty.

The bending moments were calculated for a number of instances on the basis of accelerometer and water-pressure readings as this had been done by Schnadel. Results of these calculations were compared to the results of strain-gage readings. A satisfactory agreement was shown to exist. An example of this comparison is shown in Fig. 3-36.

A sample of calculations of bending moments is presented in Table 3.¹¹ The instant to which this table refers, is indicated by the film frame No. 300. This is 0.96 sec earlier than the position labelled "position at film No. 302" on the upper part of Fig. 3-41. The reader's attention is called to the fact that in this case

¹¹ Reproduced from Table 29, page 116 of Admiralty Ship Welding Committee report no. 8.

Table 3 Vertical Bending Moment Sheet (from Adm. Ship Weld. Comm. R. 8, 1953)

FRAME No. 300		ALL FORCES ARE IN TONS										RECORD 6E7	
SECTION	A	B	C	D	E	F	G	H	J	K	L	M	
Hydrostatic Forces	167	244	256	254	107	337	333	12	-221	-303	-186	25	
Acceleration Forces	9	11	23	48	77	86	-87	-138	-119	-88	-38	10	
Nett Loads on Section	158	233	233	206	184	423	420	150	-102	215	-148	15	
Σ Loads = Shear Force	0	158	391	624	830	-646	-223	197	347	245	30	118	-133
Correction Factor	0	08	15	21	30	39	49	61	71	80	87	95	100
Correction	0	11	20	28	40	52	65	81	94	-106	-116	-126	-133
Corrected Shear Force	0	147	371	596	790	594	158	278	441	351	146	8	0
Average Shear Force over Section	73	259	483	693	692	376	60	360	396	248	77	4	
Length of Section (Ft.)	36.1	26.5	29.5	34.6	38.7	44.7	50.2	45.6	35.0	32.6	31.6	24.0	
Average Shear Force \times Length (A)	2635	6864	14249	23978	26780	16807	3012	16416	13860	8085	2433	96	
End Thrust B.M. Correction (B)	925	595	270	0	0	0	0	-10	340	445	525	60	
Line (A) — Line (B)	1710	6269	13979	23978	26780	16807	3012	16426	13520	7640	1908	156	
Uncorrected B.M.	0	1710	7979	21958	-45936	72716	89523	86511	70085	56565	48925	47017	-46861
Correction Factor	0	08	15	21	30	39	49	61	71	80	87	95	100
Correction	0	-3749	-7029	-9841	-14058	-18276	-22962	-28585	33271	37489	40769	44518	-46861
Corrected B.M.	0	2039	950	2117	31878	54440	66561	57926	36814	-19076	-8156	-2499	0

the ship is in the following sea and that the frequency of wave encounter is, therefore, low. The conditions approach the static ones and the dynamic forces are not as important as in two head-sea cases discussed earlier; i.e., the Lewis and Dalzell destroyer model and Schnadel's MS *San Francisco*. Nevertheless, the acceleration forces shown on the second line of Table 3 are important.

The "hydrostatic forces" are believed to be obtained from the pressure-gage readings and, therefore, include the Smith effect and the hull-wave interference. In the theoretical section of Report No. 8, the authors emphasize the necessity of taking Smith effect into account but consider hull-wave interaction to be unimportant. The present author was not able to follow their logic in this connection and finds this conclusion to be in disagreement with Schnadel's work, with the conclusion of the theory outlined in Section 2 and with the model test data outlined in Section 4 which follows.

The curves of shear forces and of bending moments are shown in Fig. 10 for various instances defined by the film frame numbers. The interval between consecutive film frames on this record is 0.48 sec. The insert figure shows a good agreement between the bending moments computed from the pressure and acceleration measurements (solid curve) and computed from strain-gage readings (dotted curve). Examination of Table 3 shows, however, that this agreement was secured only after drastic corrections of the closing errors in shear-force and bending-moment diagrams. The failure of these diagrams to close indicates the failure to account correctly for all forces acting on the ship. The authors of

the report, however, presented a discussion expressing the opinion that no significant errors in bending moments had been caused by these discrepancies.

In the summary tables of Report R-8, the range of hogging to sagging stress during the observation 6E7, just discussed, is given as 150,000 tons-feet in the vertical direction and 50,000 tons-feet in the lateral direction. The wind NNW force 7 to 8 and sea WNW-7 were recorded in the ship's log. The trial party recorded the wind as WNW, force 8 to 9. Maxima of the ship's pitching amplitudes were ± 7.6 deg and rolling amplitude ± 10 deg. The mean period of pitching was 12.1 sec. Average ship's speed was 10.6 knots.

4 Bending Moment—Ship Model Experiments

Measurements of bending moments on ship models in towing tanks were made by E. V. Lewis, Lewis and Dalzell, Sato, Akita and Ochi, and K. Ochi.¹² In the present section the discussion will be limited to the slow-acting loads occurring in the course of normal pitching and heaving in head seas. Slamming and other rapidly occurring loads will be discussed as a separate matter in Section 5.

4.1 E. V. Lewis, T-2 Tanker. E. V. Lewis (1954) conducted a series of towing-tank tests on the model of T2-SE-A1 tanker.¹³ The test arrangement is shown

¹² Hanaoka (1957) also refers to bending moments measured on an idealized ship model. Furthermore, attention should be called to two translations by Ochi (3-1958b and c).

¹³ These tests were sponsored by the Hull Structure Committee of SNAME through its S-3 panel.

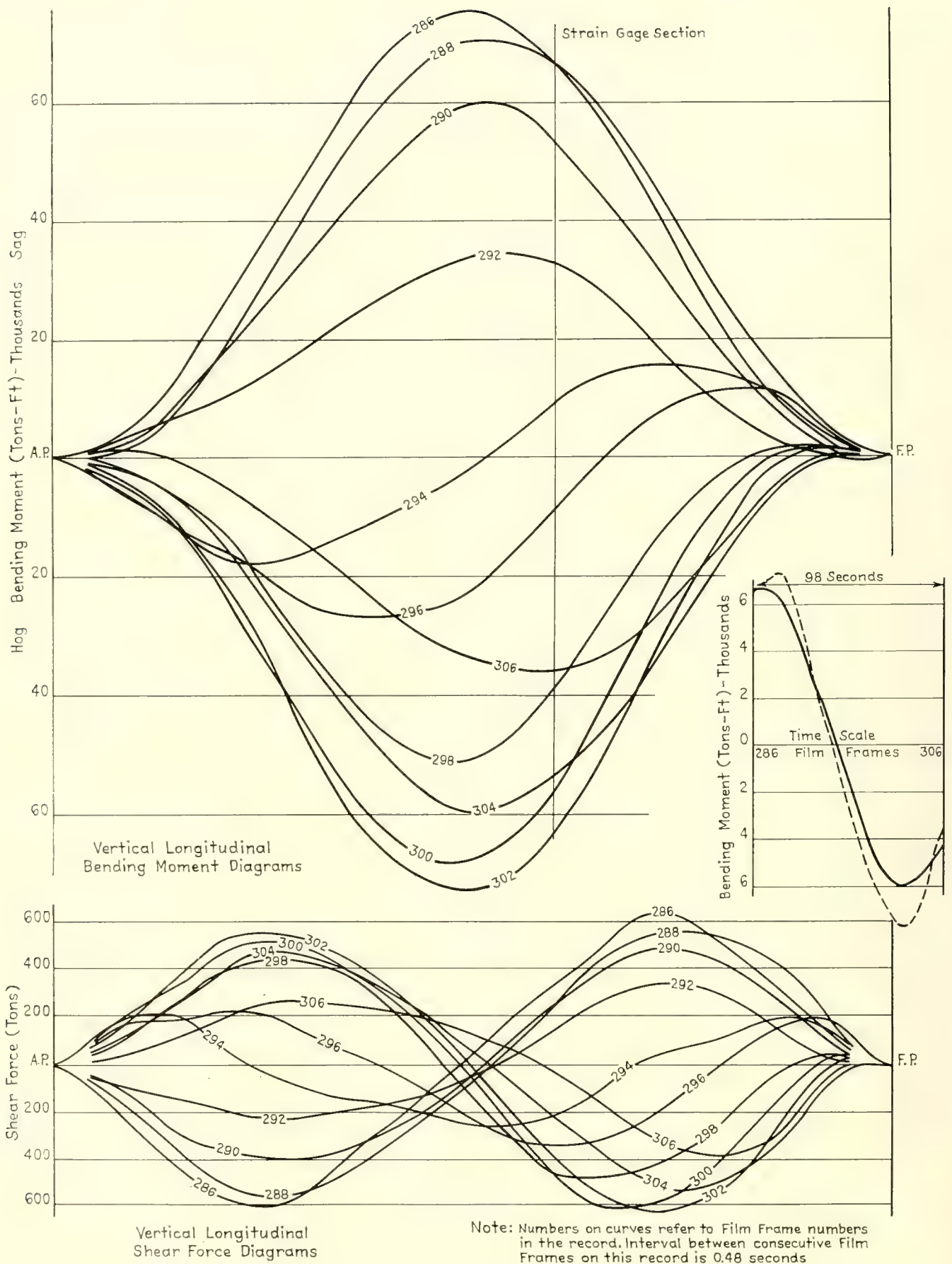


Fig. 10 Samples of vertical bending moments and shearing forces (from Adm. Ship Weld. Comm. R.8, 1953)

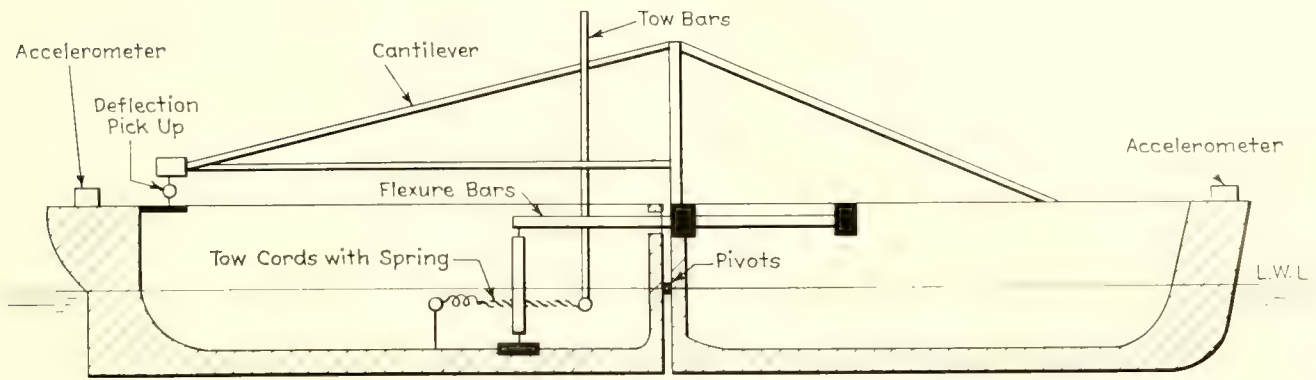


Fig. 11 Diagram of model setup for determining bending moments in waves (from E. V. Lewis, 1954)

in Fig. 11. A wooden model, 4.79 ft long, had been divided at the midsection and was jointed by a hinge at a neutral axis of the ship's midsection. The alignment of two halves was maintained by metal flexure bars. A deflection transducer had been located at the stern and was operated by a cantilever truss from the forward part of the model. The relationship between deflections and bending moments was established by calibration. Hull deflections, bow and stern accelerations, and wave profiles were recorded. The deflections of the model had been sufficiently small so that it was permissible to consider the model as a rigid body with regard to slowly applied loads. The rigidity of the flexure bars was such that the natural vibration period of the model corresponded to a two-node frequency of the ship. This correspondence between the model and the ship was expected to give a faithful representation of slamming phenomena. All data were transmitted from a model to a shore station and were recorded by a galvanometer on the same photographic tape. In the early tests the pitching and heaving motions were obtained from the photographs of the targets installed at the bow and stern of the model. In later tests the motions transducers were used and the motions were recorded by the galvanometer together with other data.

4.11 Experimental data. Fig. 12 shows the bending moments for the model at zero speed as well as the bending moments computed by the standard static procedure, with and without Smith effect. The qualitative agreement with Schnadel's sea measurements is evident: at $L/20$ wave height, the measured hogging bending moment is equal to half of the one computed by the conventional procedure. It is about 74 per cent of the static moment when the latter is computed with Smith effect.

In order to gain a better understanding of the phenomena involved, a test was made with the rear part of the model rigidly attached to the test carriage. In this case the model inertia forces were eliminated and only hydrodynamic forces resulting from wave motions were present. A much larger moment was recorded in this condition. This test clearly demonstrated the load relief which had resulted from the model freedom to

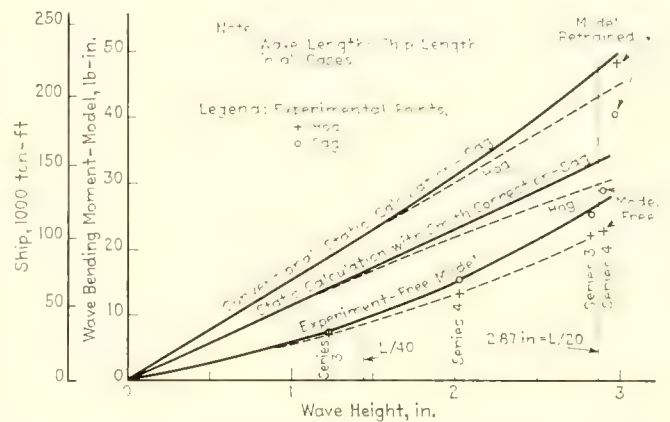


Fig. 12 Comparison of calculated bending moments with experimental values, model of T2-SE-A1 tanker at zero speed, both restrained and free (from E. V. Lewis, 1954)

yield to the waves. E. V. Lewis' explanation of this phenomenon is shown in Fig. 13. At zero model velocity the phase relationships are favorable in that the maxima of moments occur when the model and water move in the same direction. This reduces the forces exerted by waves.

Fig. 14 shows the effect of the forward speed of the model on bending moments and motions in regular waves. It appears that ship speed has no significant effect on bending moments as long as it remains sufficiently below the synchronous one. This speed range probably corresponds to a ship's operation in rough weather. In following seas the sagging and hogging moments are approximately equal, but in head seas the sagging moment is increased by about 30 per cent.

Model tests also included higher than practical speeds in order to investigate conditions which may occur in faster ships. The hogging moment was found to increase gently and the sagging moment more rapidly, with the further increase of speed beyond the synchronous one. Fig. 14 (also Fig. 16) shows a pronounced reduction of the bending moment at synchronous speed, but this

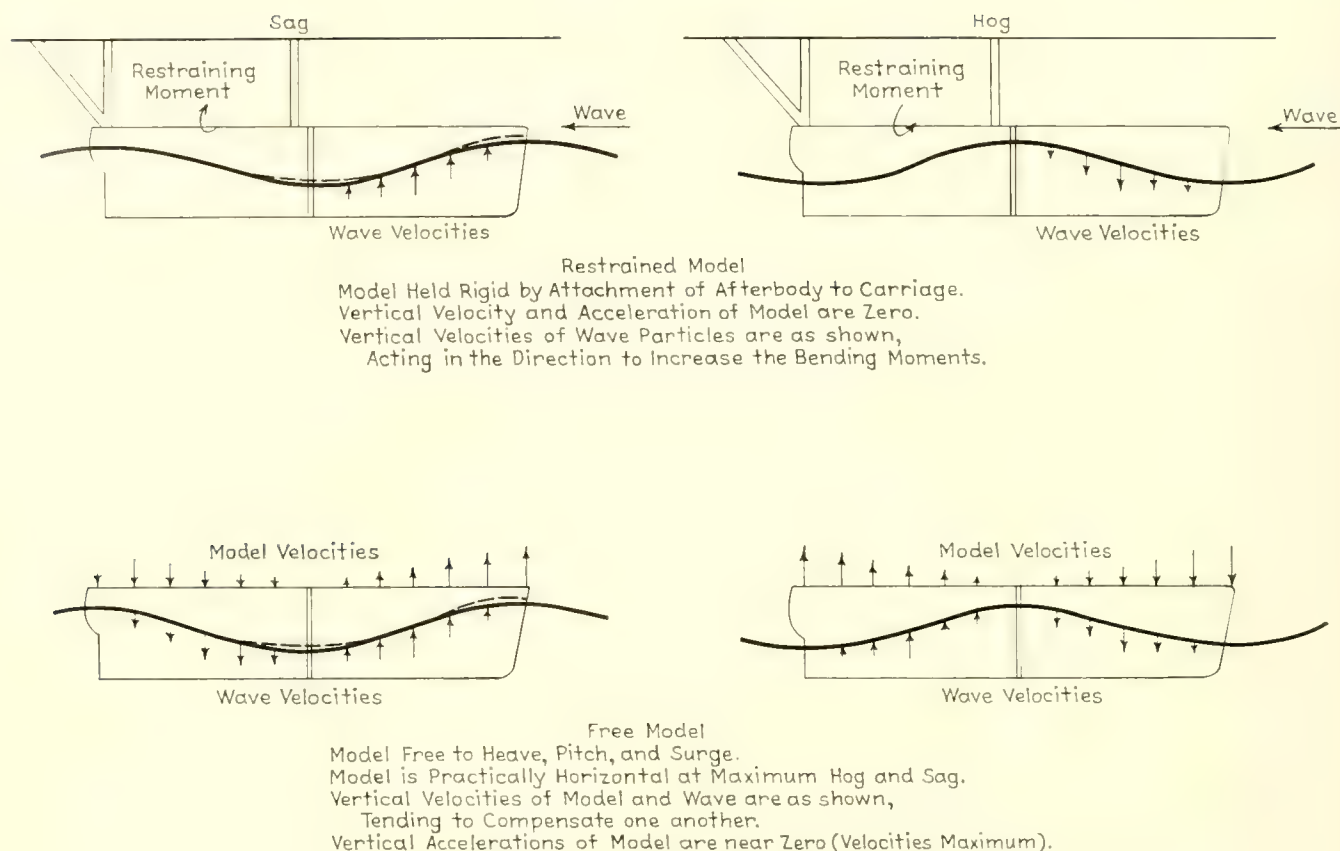


Fig. 13 Diagram comparing dynamic effects on restrained and free models at zero speed (from E. V. Lewis, 1954)

effect was later traced to a defect in instrumentation and does not represent true ship conditions.

Fig. 15 shows the relative ship-wave positions at several consecutive instances and at three ship speeds in regular head waves. Readings of the corresponding bending-moment diagrams are to be taken at vertical lines passing through the instantaneous bow positions. Only the first diagram, at zero ship's speed, represents the practical ship conditions. The lower two diagrams correspond to speeds of 14.5 and 26.6 knots, which are not attainable in the indicated wave size, and represent artificial laboratory conditions.

Fig. 16 shows the bending moments which occurred in irregular sea tests. A paper by St. Denis and Pierson, on ship's behavior in irregular seas, appeared in 1953. Lewis' (1954) paper was the first in which the irregular-wave theory was applied to ship motions and to bending moments in connection with towing-tank tests. An irregular operation of the wavemaker paddle had been devised and it has produced the complex long-crested waves. These were found by analysis to possess the statistical characteristics of the irregular ocean waves. The average height of irregular waves was about 85 per cent of the "standard," $L/20$, wave and the highest waves, therefore, considerably exceeded the height of the standard wave. This led, of course, to higher peak

bending moments as this is shown by dots in Fig. 16. The bending moments in irregular seas have now reached those obtained from a static calculation with Smith correction (in regular waves), but still remained well below the conventional static method. There was no significant variation of bending moments with speed.

Ship positions at peak sagging and hogging conditions and at maximum slamming are shown in Fig. 17. This figure brings out vividly the severity of the model test conditions. It represents, of course, an exaggeration as far as an actual ship is concerned. No captain will or can operate a T-2 tanker at 14.5 knots in head seas in which the average wave height is 12 ft and the mean of 10 per cent highest waves reaches 25 ft. As a result of this exaggeration the slamming was encountered under full-load conditions, while on cargo ships at sea the slamming of fully loaded ships is rare and becomes frequent only in light conditions.

4.12. Analysis of E. V. Lewis' bending-moment data.

At the time E. V. Lewis' tests were started, the rational approach outlined in Section 2 of this chapter was not yet available.¹⁴ It had become available later and was applied to the T-2 tanker data by W. R. Jacobs (1958). In his 1954 paper, however, Lewis gave an extensive

¹⁴ The early theoretical work of Kriloff and of Horn and measurements at sea by Schnadel appear to have been forgotten.

Table 4 Factors Involved in Bending Moments of Ships or Ship Models in Waves (from E. V. Lewis, 1954)

	Calculations			Experiments			Ship pressure integration (9)
	Conv. static	Modif. conv.	<i>Ocean Vulcan</i> analysis (9)	Restrained model, 0 Speed	Free Model 0 Speed	Moving (and ship)	
Wave forces in undisturbed waves; ^a							
1 Buoyancy—Hydrostatic							
(a) Hull assumed level.....	X			X			
(b) Hull at correct pitched and heaved position; Kriloff (17).....		X	X		X	X	X
2 Pressure distribution in wave; Smith effect (13).....		X	X	X	X	X	X
Interaction between hull and wave:							
3 Modification of pressures by presence of hull or by vertical motion in pitch and heave relative to undisturbed wave; damping (9, 24, 26)....			X (partial)	X (partial)	X	X	X
4 Modification of pressures in wave by vertical accelerations of hull and of wave particles; virtual mass (9, 29, 31).....			X (partial)	X (partial)	X	X	X
5 Modification of wave profile by presence of hull—hence pressures and velocities. Reflection, Kreitner (30). (Mainly due directly to forward speed).....				X (minor)	X (minor)	X	X
Effects of motion on virtual weights:							
6 Vertical accelerations; Kriloff (17).....			X		X ^b (minor)	X	
Miscellaneous effects:							
7 Effect of natural period of hull vibration (28).....				X (minor)	X (minor)	X	
8 Effect of eccentricity of compressive forces (9).....				X	X	X	X
9 Direct effect of speed in modifying pressures (33) and causing dynamic lift.....						X	X

NOTE Items 3, 4, and 6 are affected indirectly by changes in phase relationships and in the tuning factor (ratio of natural period to period of encounter) in pitch and heave.

^a Proude-Kriloff hypothesis

^b At 0 speed, phase relationships in this case are such that model accelerations do not affect maximum bending moments.

discussion of the origin and causes of ship bending moment. This discussion represents in fact a qualitative analysis. Of particular interest is Table 4 which lists various causes contributing to bending moments and shows to what extent they are incorporated in various analyses and tests.

An attempt at a quantitative analysis also was made and the results are shown in Fig. 18. The calculations were based on the observed wave profile at the model's side and on measured accelerations. The statically computed buoyancy was not in balance with the hull weights which were reduced by model accelerations. Since the method of accounting for various water-flow effects was not available, the buoyancy curve was adjusted by judgement so as to provide the necessary balance. Quoting from E. V. Lewis: "In order to indi-

cate the order of magnitude of the dynamic effects, which have not been accounted for, a hypothetical curve of effective buoyancy has been drawn in Fig. 18.¹⁵ This curve would bring about a balance between virtual weight and buoyancy and at the same time would give equal forebody and afterbody bending moments of 13.0 lb-in., as measured with the model in the position shown. It is clear that dynamic effects are of appreciable magnitude and therefore that their investigation is of considerable importance."

A complete quantitative analysis of bending moments in regular waves was made later by Jacobs (1958). The analysis followed the pattern outlined in Section 2 of this chapter. Comparison of the experimental and calculated bending moments is shown in Fig. 19 for

¹⁵ The figure number of the present monograph is substituted.

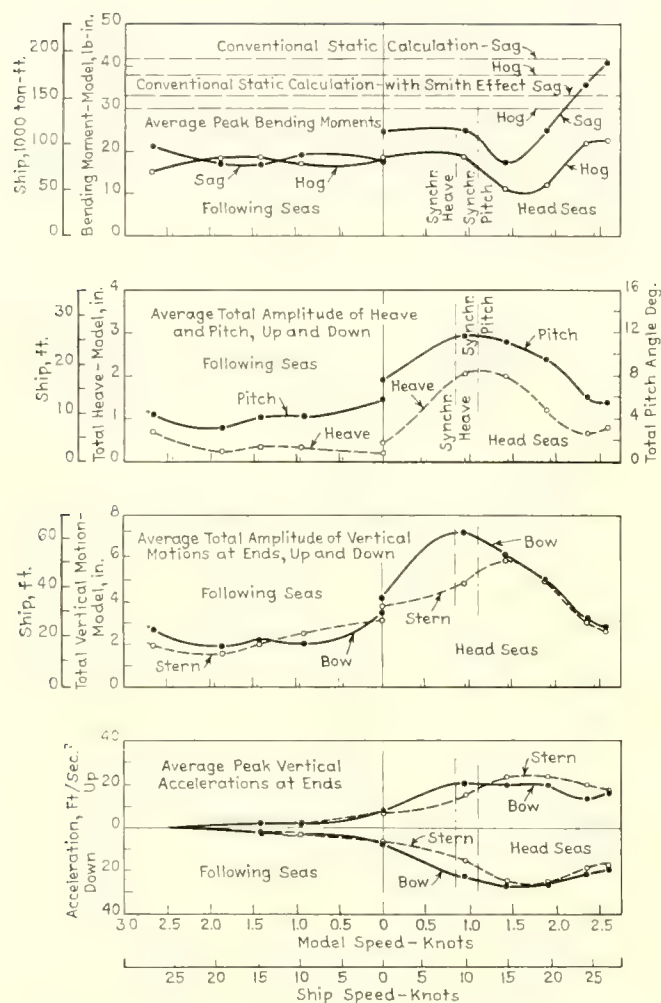


Fig. 14 Effect of speed on bending moments and motions, model of T2-SE-A1 tanker in regular $L/20$ waves (from E. V. Lewis, 1954)

two model speeds, 0 and 2 fps. The computational points are indicated by crosses.

Bending moments for a restrained model in waves of $h \cong \lambda/20$ are shown in Fig. 19. In this case there is no model motion and no acceleration force. The diagram shows, therefore, directly the degree of agreement in the calculation of hydrodynamic forces. These data can be summarized as follows:

Model speed, fps	Total range of bending moment, in-lb	
	Calculated	Experimental
0	101	89
2	100	88

The original test data for a free model in waves of $h \cong \lambda/20$ were shown in Fig. 16 for comparison with tests in irregular waves. As this was already stated, the dip of the bending-moment curve at synchronous speed was later traced to an instrumental error. The results of repeated tests in less steep waves ($h = \lambda/48$), using

different dynamometers, were published by E. V. Lewis (1958). These data are shown in Fig. 20. The comparison of Jacob's (1958) computed bending moments with new experimental data is summarized as follows:

Model speed, fps	Calculated	Experimental
0	23	20
2.4	21	22

The most important feature of Jacobs' (1958) analysis is the subdivision of the total bending moments into its several components. This is shown in Fig. 21 for the free model at 2.4 fps. The diagram corresponds to the instant, $t = 0$, at which the wave crest is at the front perpendicular. At this instant, the calculated and experimental sagging bending moments are near their maxima. In this analyzed case the waves were low, and the Smith effect, shown by the dotted line, is relatively small. The inertial effect of the added mass of water (dash-double dot curve), however, is very strong and is deducted from the displacement force. The effect of the forces caused by water velocities (dash-dot curve) is also significant over the forebody and tends to increase the bending moment. The final loading curve is the result of summation of the hull inertia forces and of four components of water pressure forces. It is therefore very sensitive to the errors in computation of individual components. *The bending moment can be considered in fact as a "second-order effect" in that it is the result of a relatively small difference of large components.*

4.2 K. Ochi—A Cargo Ship. K. Ochi (1956a and b, 1957, 1958a) published the results of extensive towing-tank tests of two cargo-ship models with U and V-bow sections. The body plans of models were shown in Fig. 2-25, and the particulars are given in Table 5. One of the major objectives of tests was to obtain the information on slamming. The discussions of this subject, however, will be deferred to later sections and only normal bending moments now will be considered.

Six-meter-long (19.7 ft) models were made of brass and were tested in a 655-ft-long towing tank.¹⁶ All runs were made in head seas under the self-propulsion method and the models were free to pitch, heave and surge. Model motions are shown in Figs. 22 and 23 and phase relationships are shown in Fig. 24. These figures refer to the wave length, λ , equal to the ship's length, L , and to the wave height of $L/30$. Bending stresses at the deck amidships are shown in Fig. 25 for different values of the ship's draft, d . The draft ratio $d/L = 0.059$ corresponds to a fully loaded ship. Sagging stresses are plotted as negative and hogging stresses as positive. Ochi's data, shown in Fig. 25 are in agreement with Lewis' Fig. 14 in that below synchronous speed the bending moment or stress is essentially independent of speed. Above synchronous speed there is a gradual increase of stress.

¹⁶ See Ochi (1958a) for detailed description of the models, test schedules, and results obtained.

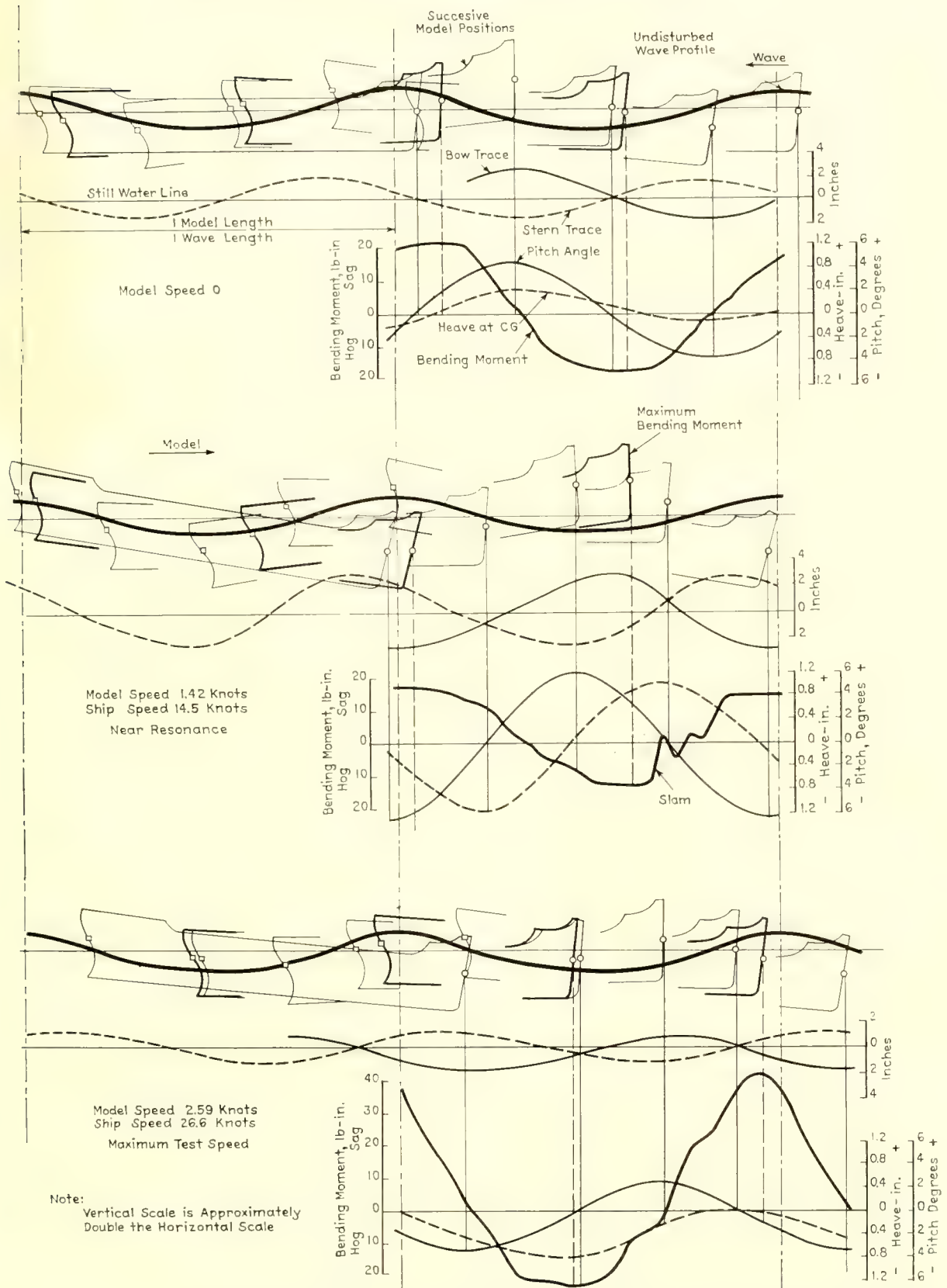


Fig. 15 Typical cycles of motions in regular $L/20$ head seas, model of T2-SE-A1 tanker (from E. V. Lewis, 1954)

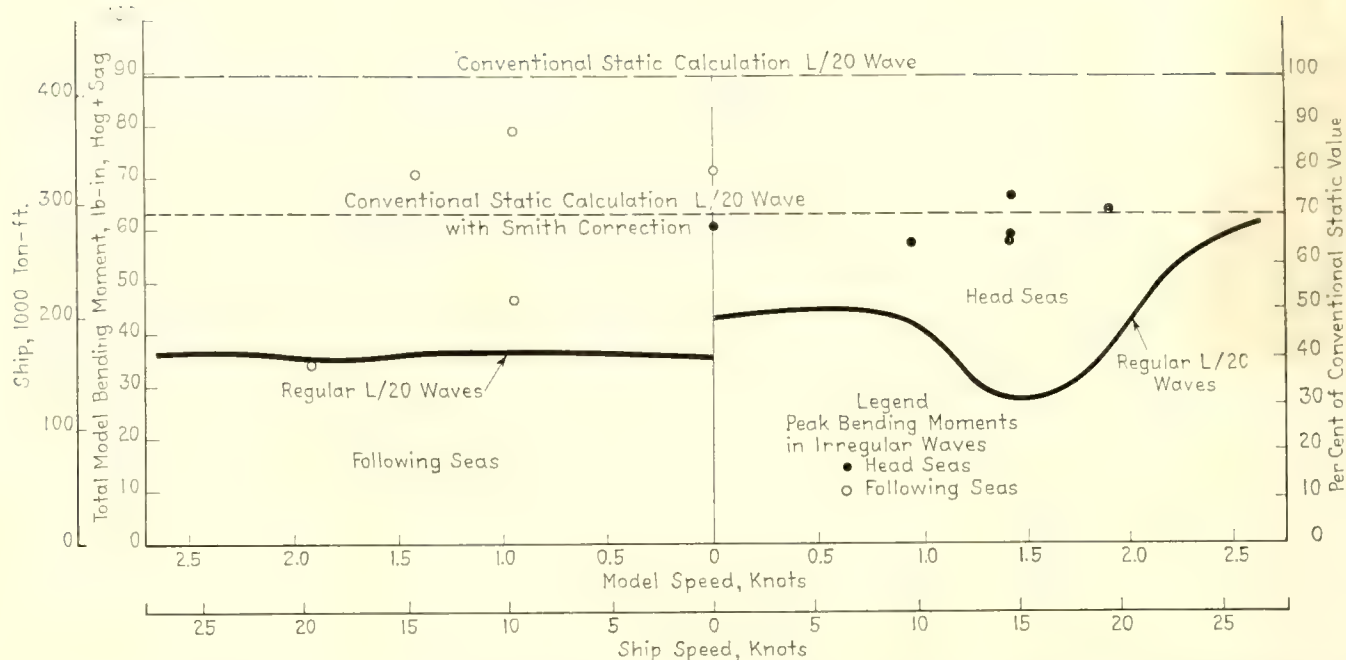


Fig. 16 Comparison of peak experimental bending moments in irregular head and following seas with values obtained in regular waves, model T2-SE-A1 tanker (from E. V. Lewis, 1954)

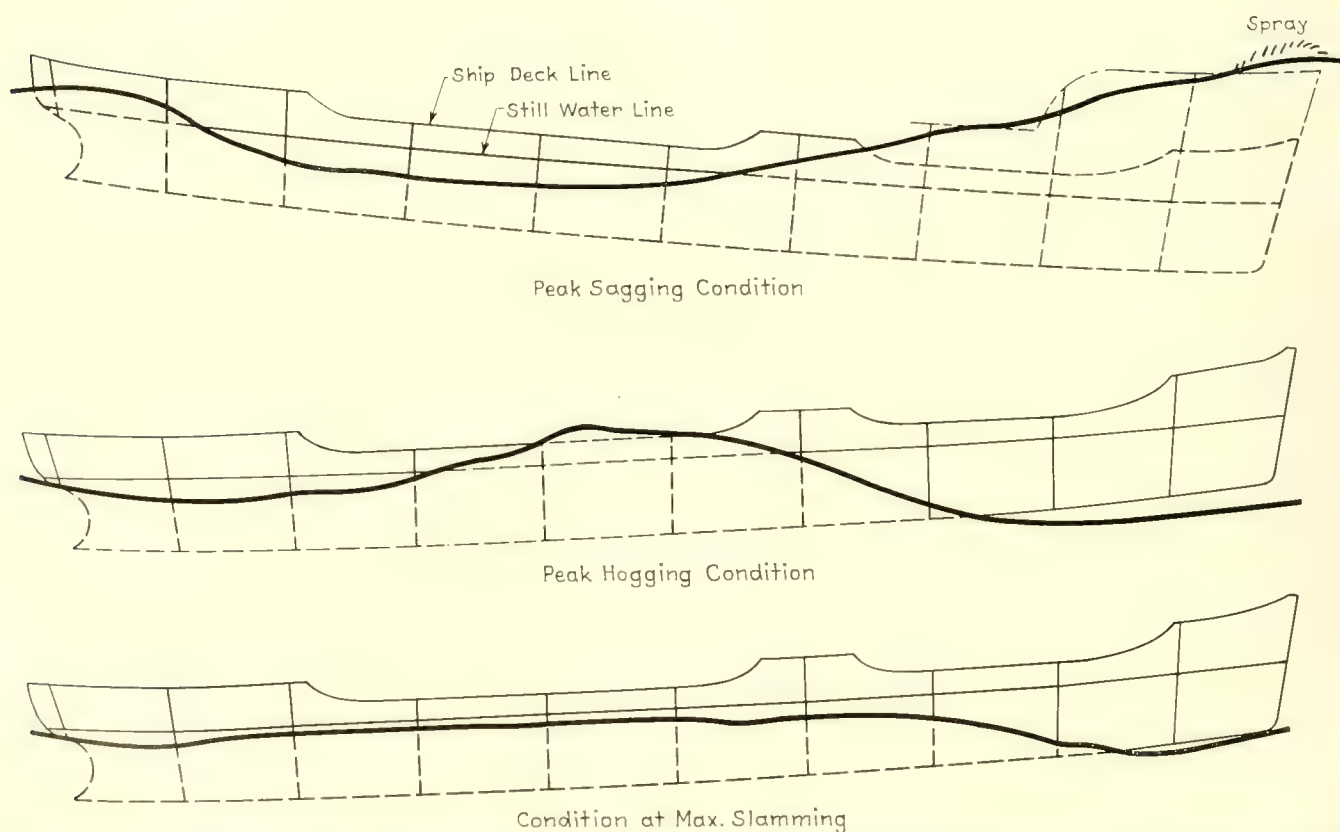


Fig. 17 Typical observed wave profiles at speed corresponding to 14 1/2 knots in irregular head seas, showing maximum bending moments and slamming. From moving picture records of T2-SE-A1 tanker model (from E. V. Lewis, 1954)

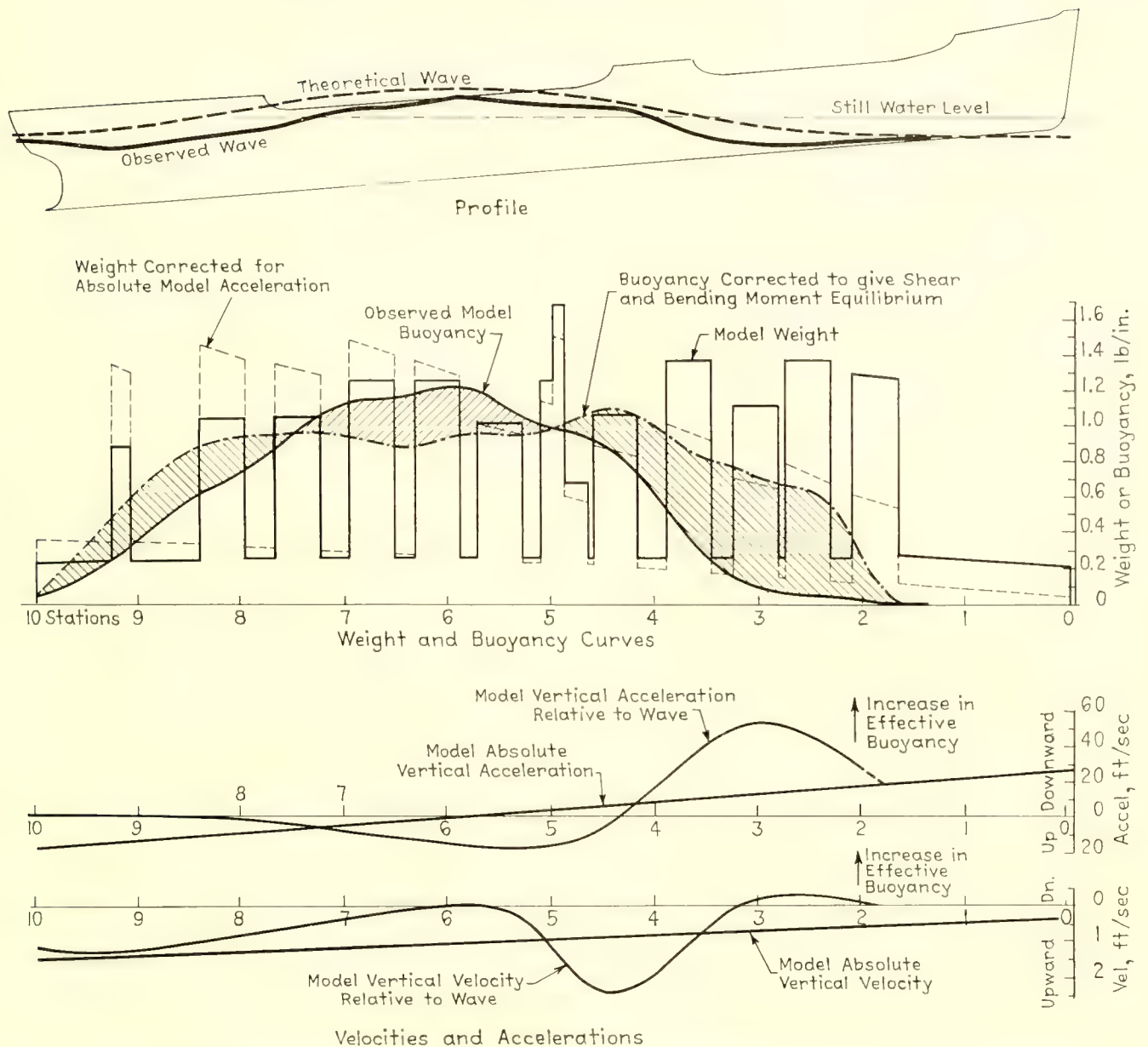


Fig. 18 Bending-moment calculation including dynamic effects, at speed corresponding to $14\frac{1}{2}$ knots. T2-SE-A1 tanker model in hogging condition, regular $L/20$ head seas (from E. V. Lewis, 1954)

Ochi's results are in qualitative agreement with Schnadel's (MS *San Francisco*) and Lewis' data in that the sagging moment is larger than the hogging one. Quantitatively, however, there are large differences. Schnadel found the sagging moment to be some 20 per cent larger than the hogging one. Lewis' Figure 14 shows the sagging bending moment to exceed the hogging one by about 30 per cent. Ochi's Fig. 25, on the other hand, shows the hogging stress at zero speed of about 0.115 kg/mm^2 while the sagging stress is almost 0.2 kg/mm^2 , or 74 per cent larger than the hogging one.

Ochi adopted Schnadel's definition of the "effective wave height" as the height of a wave which, by conventional static calculations, produced the same stress as the actual wave. He called the "effective wave height ratio," η ,¹⁷ the ratio of the height of this static wave to the actual wave height. He made static calculations alternately on the basis of the displacement and the displacement with correction for the Smith effect.

¹⁷ With subscripts H for hogging, s for sagging, and primes denoting computations made with Smith effect taken into account.

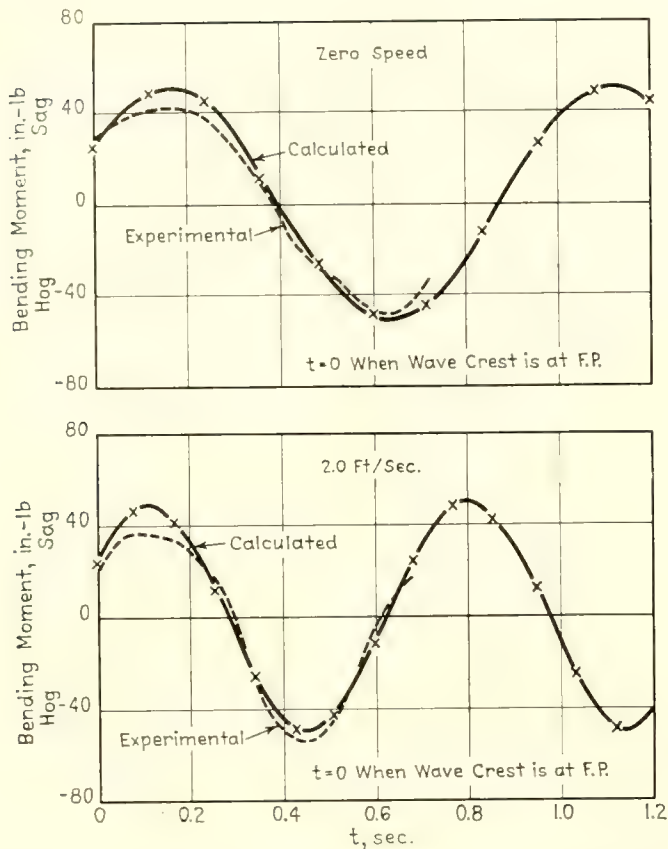


Fig. 19 Calculated and measured bending moments at mid-section restrained 48-ft model of T2-SF-AI tanker in waves 4.8 ft \times 3.0 in. (from Jacobs, 1957)

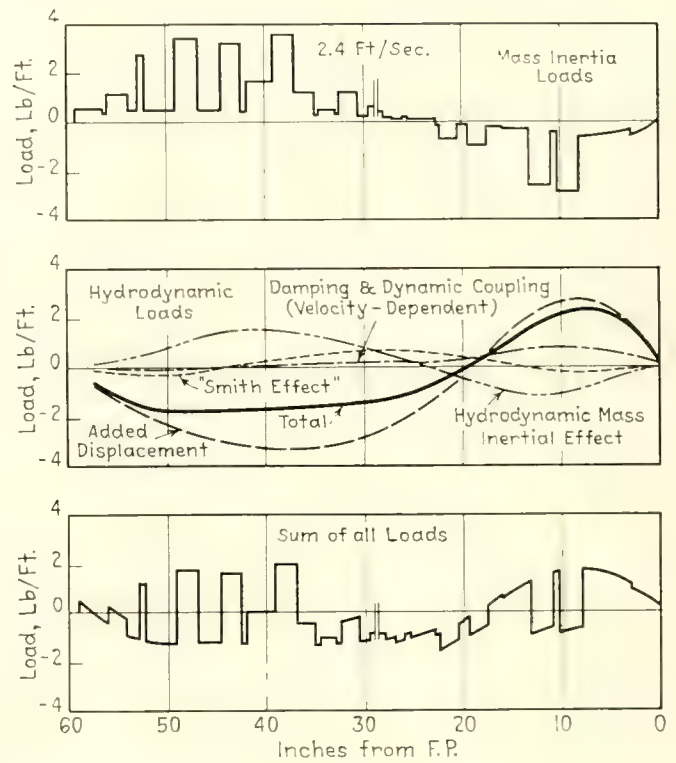


Fig. 21 Loads on free model of T2-SF-AI tanker in waves 4.8 ft \times 1.2 in. at 2.4 (fps for condition when wave crest is at FP) (from Jacobs, 1958)

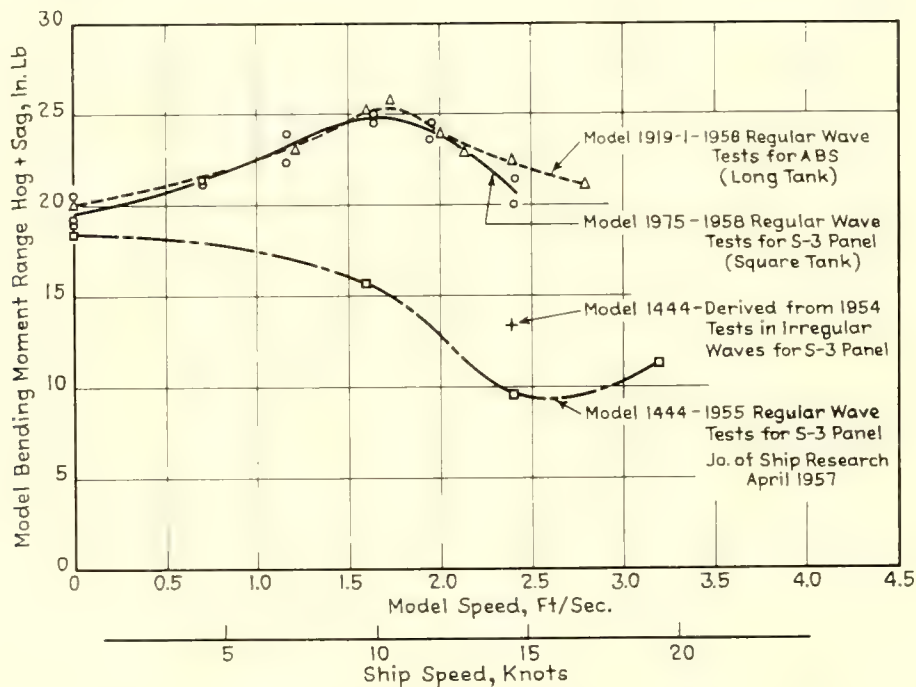


Fig. 20 Comparison of vertical bending moments from four independent tests. $1/105$ -scale models of T2-SF-A tanker; wave length = model length; wave height = $1/45$ \times model length; head seas (from Lewis, 1958)

Table 5 Model Ship Characteristics (from Ochi, 1956b)

Model Ship Characteristics		
Type	U-form ship	V-form ship
Length L_{pp}	600.0cm	600.0
Breadth B_m	82.6cm	82.6
Depth D_m	53.0cm	53.0
Draft d_{max}	35.5cm	35.5
Displacement ∇_{max}	1,334kg	1,334
Block coeff. C_b	0.741	0.741
Prismatic coeff. C_p	0.751	0.751
Midship area coeff. C_M	0.986	0.986
Water plane coeff. C_W	0.829	0.829
Ship form coeff. k	0.812	0.656
C. B. from \mathcal{Q}	2.7cm fore	2.7
C. B. above B. L.	18.5cm	18.5
Radius of gyration in air	—	0.298L
Natural pitching period, afloat ($d=20$ cm)	1.30sec.	1.20
Natural heaving period, afloat ($d=20$ cm)	1.37sec	1.25
Hull weight	262.8kg	258.0
I/y at \mathcal{Q} to deck	352cm ³	352
I/y at \mathcal{Q} to bottom	459cm ³	459

The resulting effective wave-height ratios are shown in Figs. 26(a) and (b). Ochi's towing-tank data are in agreement with Schnadel's sea observations in that the effective wave-height ratio is appreciably less than unity even when the Smith effect is taken into account. The speed parameter F in the caption of Fig. 26 is the speed-length ratio, V/\sqrt{L} , with V in meters per second and L in meters.

4.3 M. Sato—Destroyer Model. M. Sato (1951) presented an outline of the towing-tank experiments conducted prior to the end of World War II in 1945. The experimental study was initiated in 1935, following the breaking up of two destroyers in a severe typhoon. The major part of the data was lost because of the war damage and subsequent confusion but Sato's paper nevertheless contains much valuable information.

The body plan and principal dimensions of the model are shown in Fig. 27. The model was made of half-hard brass plate. The longitudinal members (keelson and two deck stringers) were made continuous as far as possible and the transverse members were intercostal and riveted. The frame spacing was 50 mm in the forward quarter of the ship's length and 100 mm in the remaining length. The appendages, i.e., the bilge keels, rudder, propellers, propeller shafts and brackets as well as superstructure and deck equipment, were omitted. The model was free to heave and pitch and these motions were measured by potentiometers and recorded by an oscillograph.

Wave heights were measured electronically by a pair of brass rods 5 mm (0.195 in.) in diameter and were recorded on an oscillograph.

Strain meters were of mechanical type with gage length of 80 and 100 mm (3.15 and 4 in.). The relative displacements of two legs of a gage affected inductances

of two coils and were recorded on an oscillograph. Several strain meters were installed at the deck and bottom at seven locations along the model's length.

The model's elastic properties were verified by a preliminary bending test. The model ship was supported at two points and was loaded by dead weights on the deck. The displacement of the keel centerline was measured by dial gages at both supporting points and at ten suitably spaced locations. Fig. 28 shows the calculated and experimental values of deflections under a dead load of 100 kg applied at the points indicated. "Calculated" values represent the sum of the bending and shear deflections and are based on the hundred per cent effectiveness of all longitudinal members.

The amplitudes of heaving and pitching as well as the bending moments are shown in Figs. 29 and 30. The first of these shows the variations of the indicated quantities with wave height at zero ship's speed and the second—with model speed. As this can be seen in Fig. 29, the amplitude of heaving motion is proportional to wave height but the up-heave is some 20 per cent greater than down-heave. The bow-up pitching amplitude is also proportional to wave height but the bow-down one is increasing with the wave height. With the steepest waves, $h = L/19$, the down-pitching is 30 per cent greater than the bow-up one.

For very low waves, up to $h = 100 \text{ mm} = L/75$, the hogging and sagging moments are equal. With the increase of wave height, an added sagging moment develops so that the hogging moment is diminished by it and the sagging moment is increased. At the wave height of 400 mm or $L/19$, the sagging moment is 40 per cent greater than the hogging one. The photographs, attached to Sato's paper, show that, at wave height $h = L/20$, the water reaches the height of the

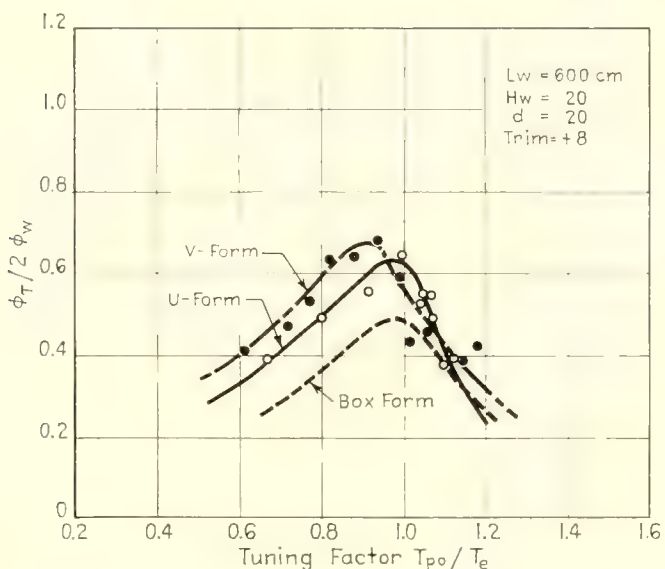


Fig. 22 Pitch-angle ratio $\phi_T/2\phi_w$ versus tuning factor T_{p0}/T_e (from Ochi, 1956b)

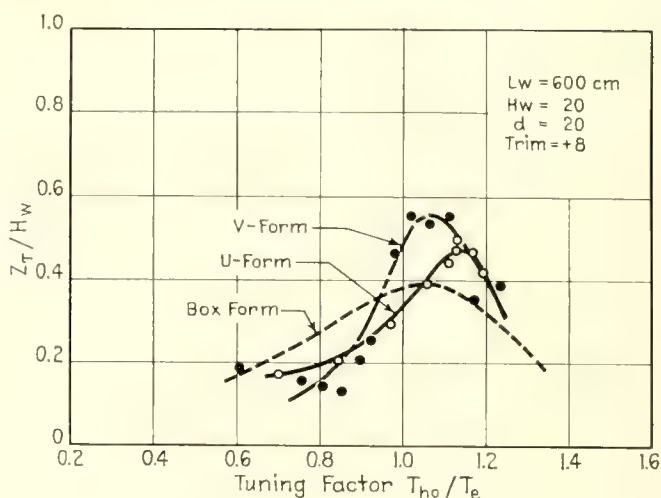


Fig. 23 Heave-amplitude ratio Z/H_w versus tuning factor T_{h0}/T_e (from Ochi, 1956b)

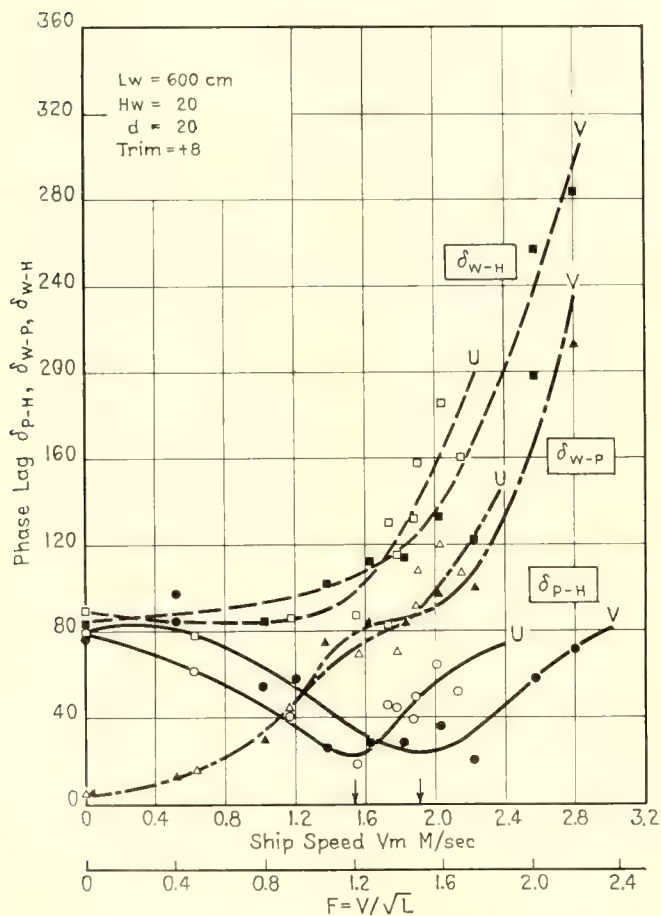


Fig. 24 Phase lag between pitch, heave and wave against ship speed (from Ochi, 1956b)

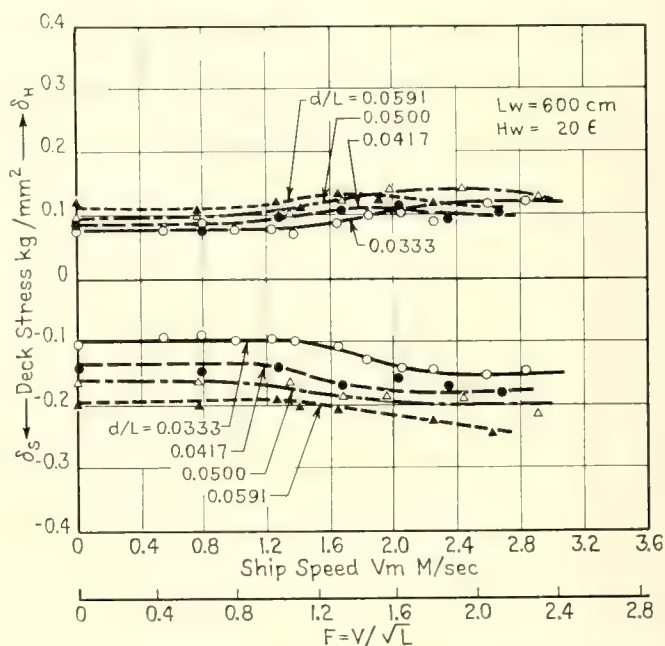


Fig. 25 Hogging, sagging deck stress of the V-form ship (from Ochi, 1957)

forecastle deck. It appears, therefore, that the increase of the wetted beam at the bow flare contributes to the increase of the sagging moment.

Fig. 30 shows two interesting features of bending-moment variation with speed at a constant wave height, $h = L/38$. The first of these is the continuous decrease of the hogging moment and the increase of the sagging moment with increase of speed up to 3 m/sec for the model or 23 knots for a full-size ship. Lewis and Dalzell (1958) have found that this is caused by the addition of the time-independent sagging moment which results

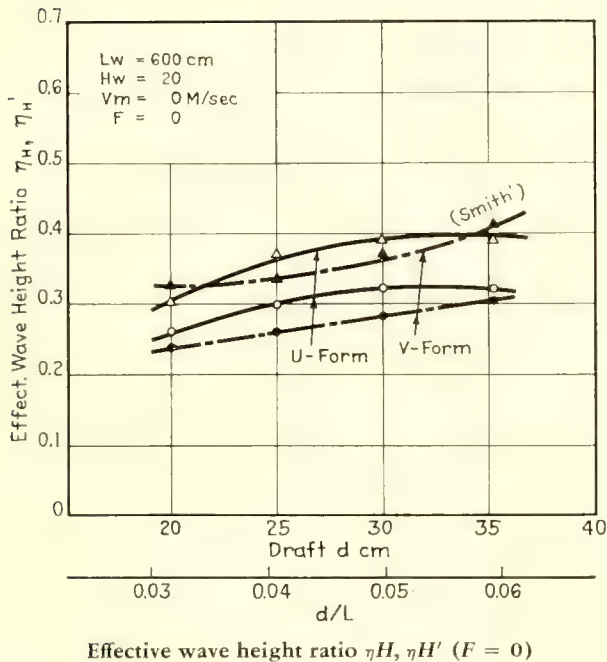
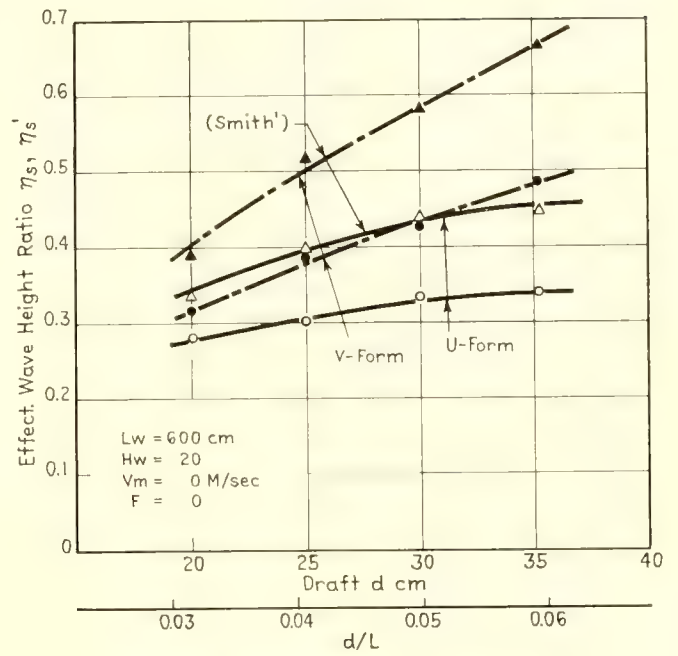
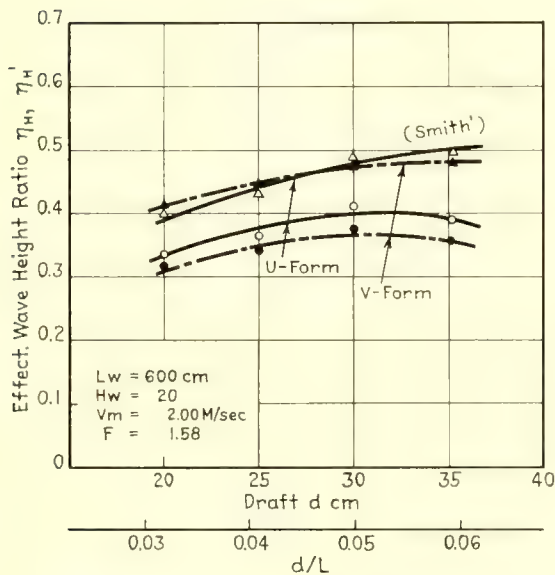
Effective wave height ratio η_H, η_H' ($F = 0$)Effective wave height ratio η_S, η_S' ($F = 0$)Effective wave height ratio η_H, η_H' ($F = 1.58$)

Fig. 26(a) From Ochi, 1957

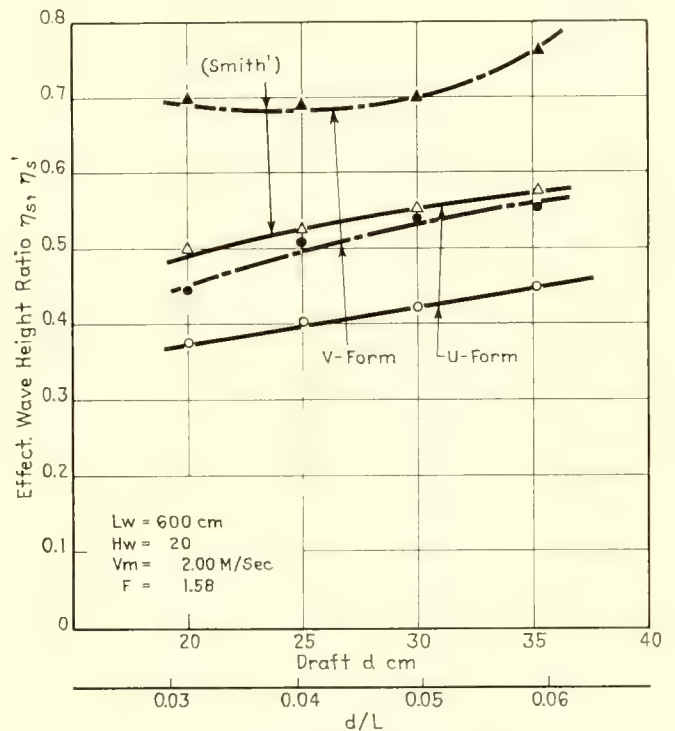
Effective wave height ratio η_S, η_S' ($F = 1.58$)

Fig. 26(b) From Ochi, 1957

from the ship-wave formation. The second conspicuous feature is the sudden increase of the sagging bending moment at 3 m/sec. At this speed the synchronism of pitching motion is approached and motions become large. It is probable that this increase of the bending moment is caused by the water impact on the unusually sharp bow-flare curvature at the fore-castle deck.

Quoting a part of Sato's concluding remarks:

"1 In the drifting state,¹⁸ the curves of strain, pitch-

¹⁸ I.e., at zero forward speed.

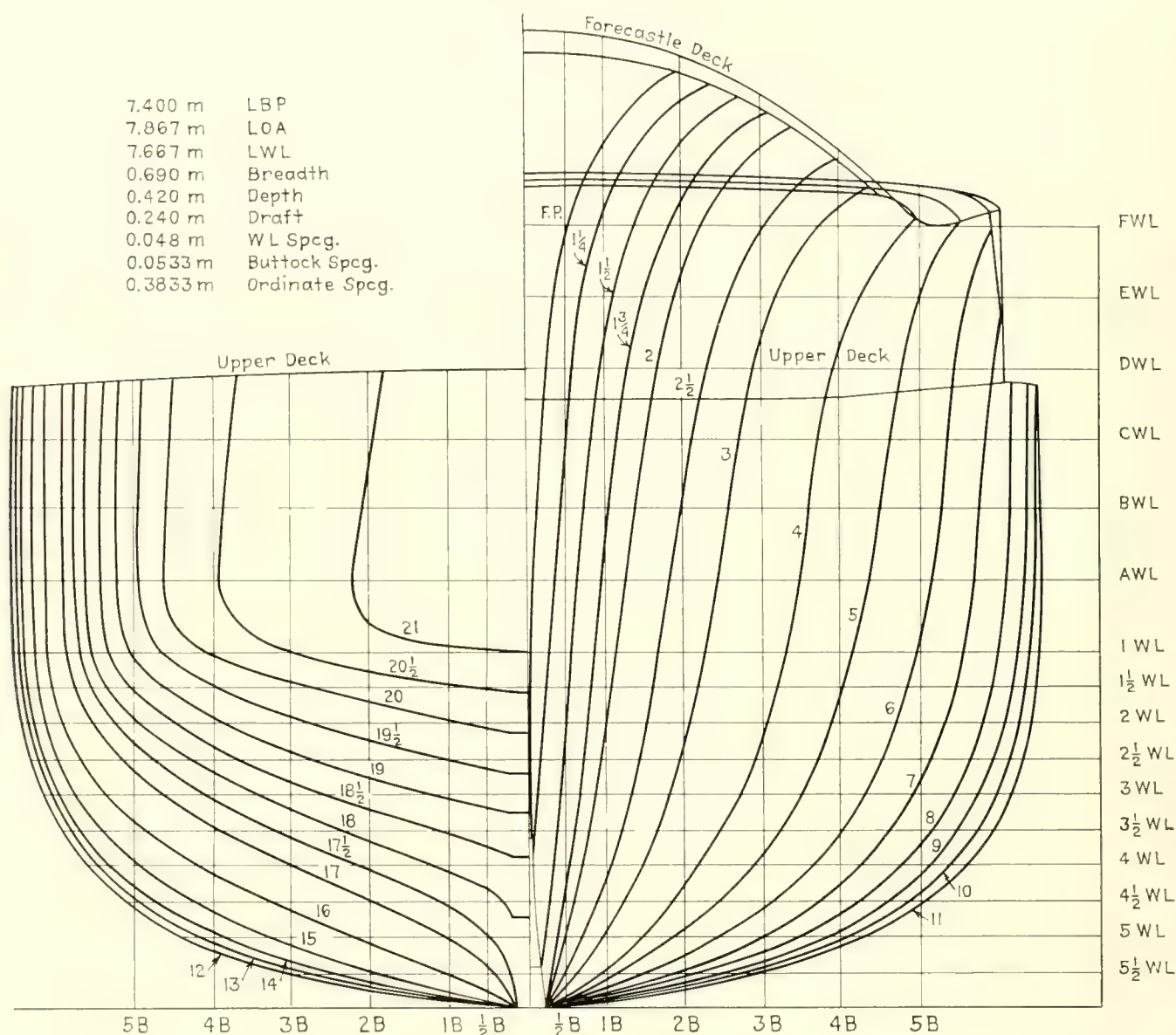


Fig. 27 Body plan and particulars of a destroyer model (from Sato, 1951)

ing and heaving are similar to sinusoidal curves. The amplitudes of the curves increase in proportion to wave height.

"2 The strain curves show a disturbance with increase in ship speed. Vibrational strain is observed at both ship deck and bottom in the section near amidships after a heavy blow is delivered to the bottom.¹⁹ It was ascertained that the frequency of the vibrational strain coincided with the natural frequency of two-noded flexural vibration."

¹⁹ Considering the sharpness of the V-bottom and the suddenness of the flare development at the deck, the present author believes that the "blows" were delivered to the flare.

4.4 Lewis and Dalzell—Destroyer Model. Lewis and Dalzell (1958) reported on experiments with a destroyer model. The body plan is shown in Fig. 31 and the particulars are given in Table 6. The model arrangement was similar to the one E. V. Lewis had used on the T-2 tanker and which was described in Section 4.1 except that the shear force as well as the bending moment was measured. The following data were recorded simultaneously on an oscillograph tape: pitch, heave, surge (or alternately vertical bow acceleration), wave, bending moment, and shear force.

The measured amplitudes of hogging and sagging bending moments are plotted against speed in Fig. 32. As in Sato's experiments, a pronounced increase of the

Table 6 Main Particulars of Model 1723 (Destroyer)
(in feet)

Length, overall.....	5.821
Length between perpendiculars.....	5.710
Load waterline length.....	5.717
Beam maximum.....	0.608
Drafts:	
Forward.....	0.197
Aft.....	0.220
Displacement, lb (FW).....	24.50
Center of gravity, abaft amidships.....	0.175
Longitudinal radius of gyration.....	1.349
Natural period of vertical two-node vibration, cps.....	10.9
Natural periods of oscillation:	
Heave ^a	0.62
Pitch ^a	0.60

^a In seconds by manual oscillation in calm water.

sagging bending moment is observed at a ship speed exceeding 20 knots. In the present case, however, a corresponding decrease of the hogging bending moment is shown. The measurement of the bending moment in smooth water demonstrated that this change is caused by the formation of the ship-made wave. Fig. 33 shows the bending moments with still water moment subtracted. The curves are now seen to be smooth and show a gradual increase of the sagging bending moment with speed. It appears to be established that the total bending moment can be considered as the sum of the bending moment developed in smooth water and the moment developed by waves and ship motions.

Fig. 35 shows a sample of an oscillograph record. A conspicuous feature of it is the persistence of oscillations of the bending moment about a mean (approximately sinusoidal) value. These oscillations were disregarded and the mean values were shown in Figs. 32 and 33. The oscillations, however, appreciably increase the maxima of the bending moment. They have the same period as the two-node period of the ship's vibration. It will be recalled that Sato also commented on the occurrence of such oscillations.

Three conspicuous features of the bending-moment behavior are present in a very fast ship. These are the gradual increase of the sagging bending moment with speed, a sharp hump in the bending-moment curve apparently caused by the flare impact, and the essentially continuous state of vibration. These features are absent in a typical cargo ship. The vibration caused by slamming will be considered later in Section 5.5.

Lewis and Dalzell's (1958) work on a destroyer model included also a comprehensive series of tests in irregular waves and the statistical analyses connected with it. The data on the model's behavior in irregular seas are summarized in Table 7 and in Fig. 36. Comparing this latter with Fig. 34, it is observed that the variation of bending moments with speed is reduced. For the speed which is practically attainable under given wave conditions, probably not over 15 knots, the bending moments can be considered approximately as independent of speed.

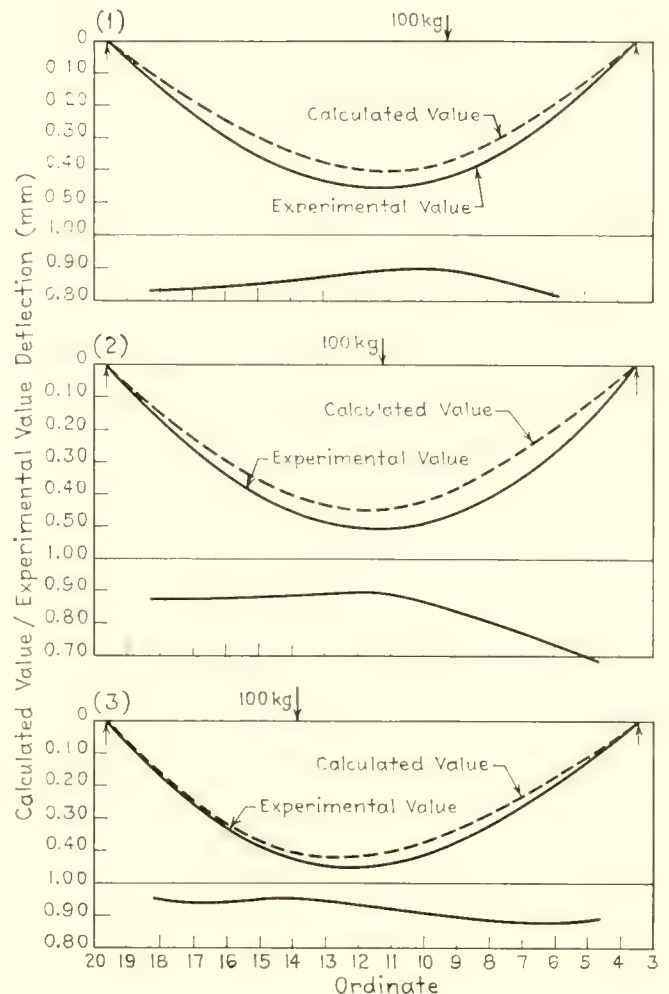
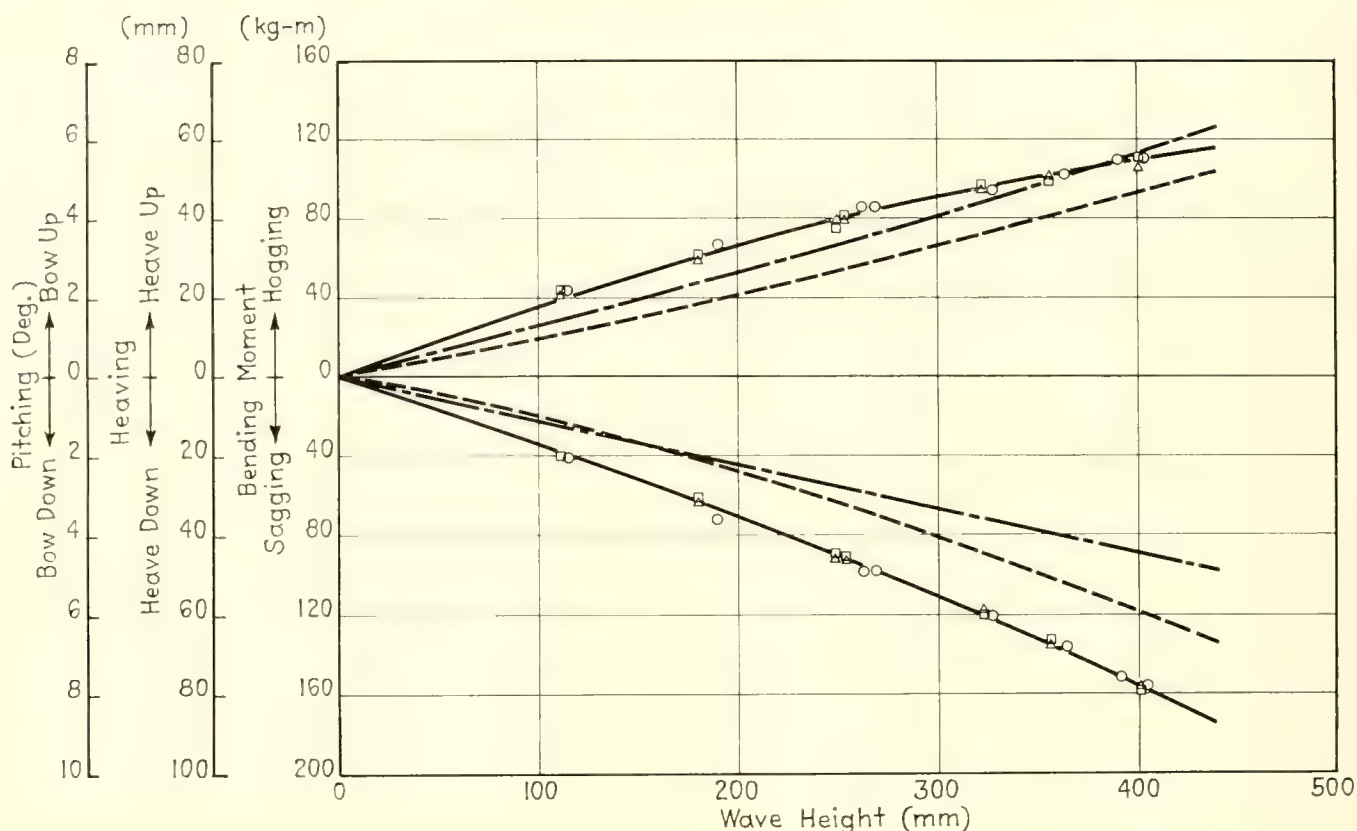


Fig. 28 Calculated and experimentally measured model deflections for three positions of a 100-kg loading weight (from Sato, 1951)

A comparison was made between the average bending moments which have been directly measured in irregular seas and the moments which had been computed on the basis of wave spectra and ship response operators. The degree of agreement shown is given in Table 8.

At the ship's speed of 12.2 knots agreement within 12 per cent can be considered to be good. Increase of the error to 22 per cent at zero speed corresponds to the difference of the ship's-speed effect in regular and irregular seas which already has been commented upon. This error casts some doubt on the accuracy of the zero-speed data. It is generally known that towing-tank data are often affected at zero and low speeds by the waves reflected from tank sides. Examination of Fig. 32 shows in fact a pronounced local dip of several curves at zero speed. It appears that the authors placed too much reliance on the individual zero-speed measurements and could have achieved better results by using instead a smooth extrapolation of several low-speed measurements to zero speed.



Wave Length 7.667m

Location of Strain Meter S5

(150 mm Forward from Midship)

— Bending Moment

--- Heaving

--- Pitching

○ On Upper Deck (Center Line)

□ On Upper Deck (Starboard)

△ On Keel (Right Side of Center Girder)

Fig. 29 Example of experimental results of ships drifting in waves showing effect of wave height; wave length constant (from Sato, 1951)

Table 7 Summary of Results Obtained on a Model 1723 (Destroyer) in High Irregular Seas (From Lewis and Dalzell, 1958)
Average Wave Height 2.19 in. All bending moments in inch-pounds

Model speed, fps	0	1.87	2.53	2.92	3.72	4.12	5.05
Ship speed, knots	0	9.1	12.2	14.1	18.0	20.0	24.5
Average sagging moment.....	22.6	23.7	23.7	25.7	26.4	28.7	38.7
Average hogging moment.....	22.3	23.5	22.6	23.7	21.9	21.8	18.8
Average range.....	44.9	47.2	46.3	49.4	48.3	50.5	57.5
Average of 10 pct highest sagging moments....	46.4	46.4	44.0	50.0	52.8	58.7	69.4
Average of 10 pct highest hogging moments....	42.9	44.6	42.0	42.2	39.0	40.4	39.1
Average 10 pct highest ranges.....	87.7	87.3	83.3	83.1	83.5	89.6	98.9
Largest single sagging moment measured.....	59.6	55.4	52.3	59.3	64.3	78.2	79.9
Largest single hogging moment measured.....	54.6	53.8	48.4	50.4	44.3	49.2	49.4
Largest single range measured.....	108.5	103.9	99.5	95.6	99.7	104.0	115.5
Number of waves encountered.....	102	156	150	154	153	159	179
Number of times model shipped water.....	13	46	52	61	66	77	99
Number of times model slammed ^a	0	3	8	11	18	21	37
Number of times model pounded ^a	1	20	18	30	33	36	37

^a See text for definition.

4.5 Severity of Experimental Waves. It is one of the recognized advantages of the laboratory technique that ambient conditions can be pushed to a much greater

severity than is possible on an actual ship. By doing so, certain important features of a ship's behavior may be discovered. On the other hand, it is important to

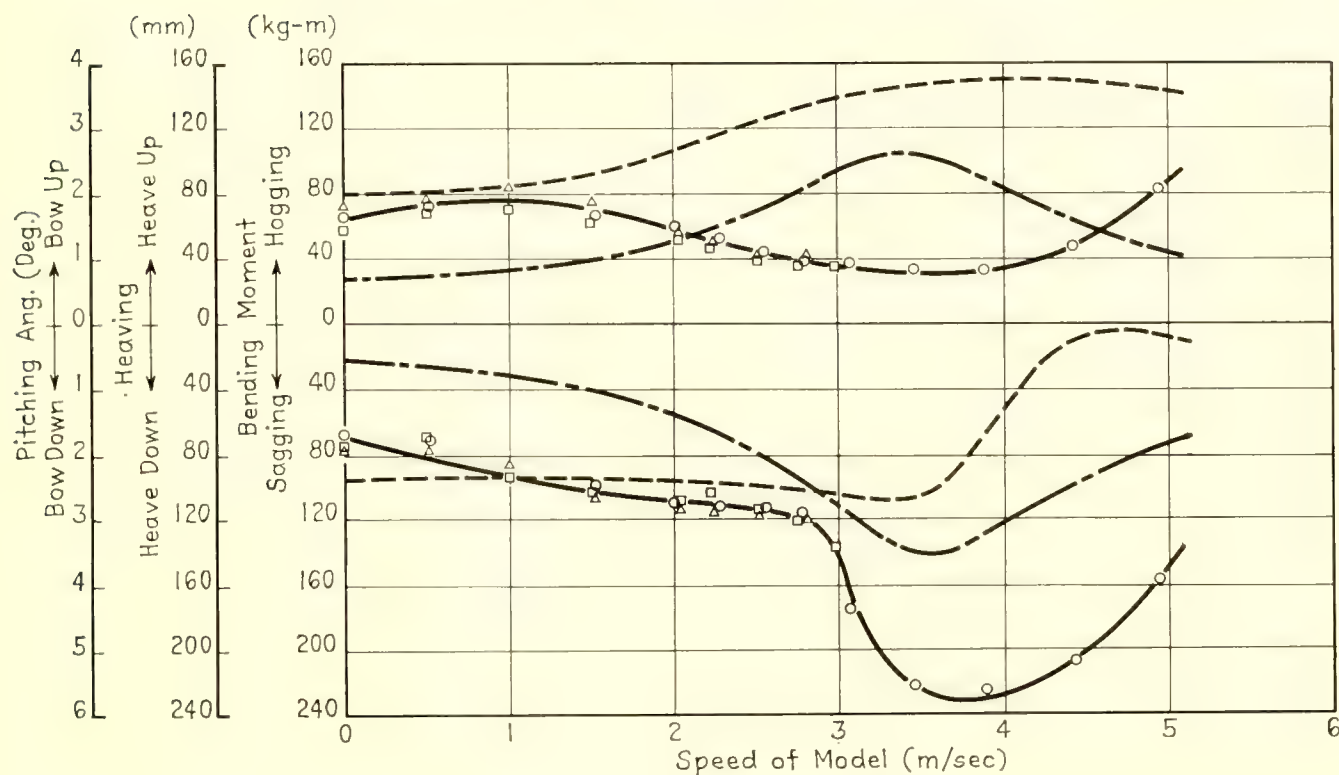


Fig. 30 An example of results of ships moving in waves showing effect of speed; wave height and length constant (from Sato, 1951)

Table 8

	Measured BM, ft-lb	Computed BM, ft-lb	Ratio: measured computed
At zero speed.....	4.15	3.40	1.22
At 2.53 fps (ship 12.2 knots).....	4.27	3.80	1.12

realize that this practice may give an unrealistic picture of a ship's behavior within practically attainable operating conditions. It can be recommended that in all ship-model tests in waves a clear distinction be made between the conditions practically attainable and the exaggerated conditions.

The difficulty in following the foregoing recommendation lies in the scarcity of information on the limiting speed of ships at sea in adverse conditions. For the particular case of destroyers, considered in the two preceding sections, some information is available from the trials of three destroyers of the Royal Netherlands Navy described by Szebehely (1956) and Warnsinek and St. Denis (3-1957). These are preliminary and very incomplete descriptions. Sea conditions unfortunately were recorded only by visual observations. However,

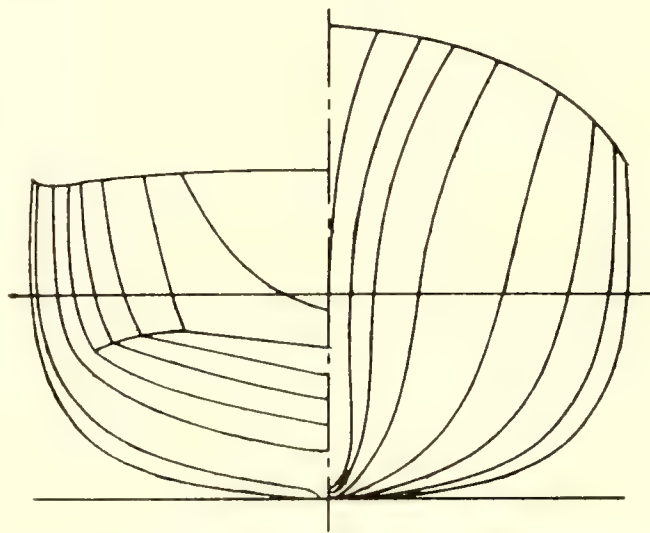


Fig. 31 Destroyer body plan, ETT Model 1723 (from Lewis and Dalzell, 1957)

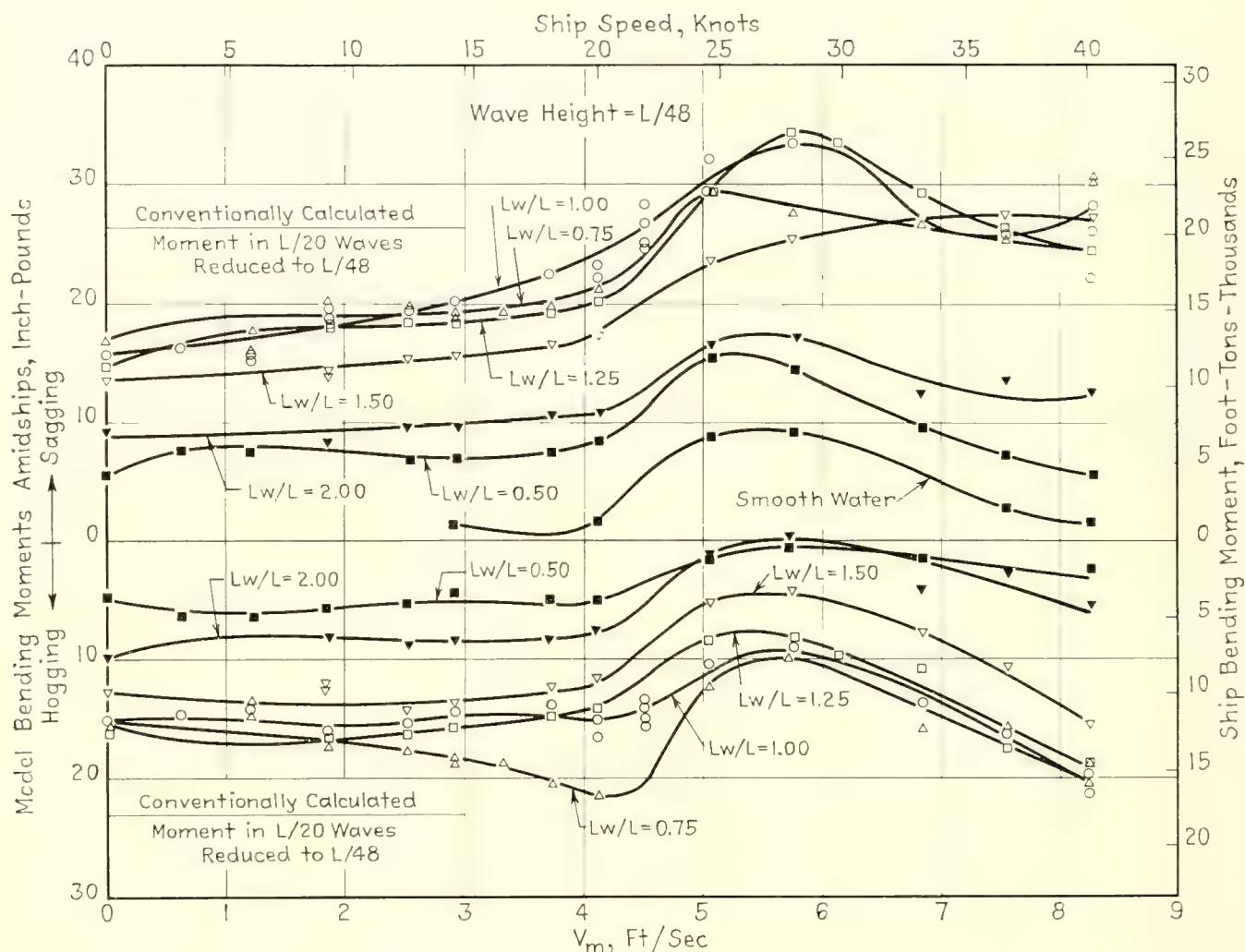


Fig. 32 Bending-moment amplitudes in regular waves (from Lewis and Dalzell, 1957)

additional judgment as to the severity of conditions can be formed on the basis of instrumentally recorded ship motions.

Two sea conditions were met. The first consisted of a long swell of an apparent dominant length ranging between 250 and 350 ft and of significant height between 5 and 6 ft. On this a moderate local sea was superposed of shorter wave length. The combination resulted in occasional wave heights of some 10 ft. Assuming this 10 ft height to correspond to 10 per cent highest waves and using Longuet-Higgins relationships, the average wave height can be estimated as 4.9 ft or $\frac{1}{12}$ of the length of the intermediate of three ships.

The second weather condition is described as fully developed state 5 or 6 sea superposed over a swell. Maximum wave lengths and wave heights were of the order of 600 ft and 12 ft, respectively. If the 12-ft height again was assumed to correspond to the 10 per cent highest waves, the average height would be 5.9 ft or $\frac{1}{60}$ of the ship's length. This latter description would,

however, exaggerate the wave steepness, since the lengths of the largest waves were indicated to be 1.7 times the ship's length.

A series of maneuvers was carried out under sea condition I at a series of speeds up to 28 knots. Under sea condition II the maneuvers were carried out at the 17-knot speed. Only two runs at higher speed—between 17 and 25 knots were attempted in condition II. At 25 knots the trials were suspended because of excessive motion.

The data on pitching of these destroyers are found in Fig. 3-51 on page 205. The average of $\frac{1}{10}$ highest pitching amplitudes of $3\frac{1}{2}$ to 4 degrees was observed on tests of these destroyers at 17 knots in head and bow seas. A table given by Warnsinck and St. Denis (1957) shows that, in a condition II head sea at 17 knots, slamming occurred at an average rate of 6 per 10 min of run, or roughly per 100 dominant waves. There was no slamming in sea condition I.

On the basis of the foregoing information, limiting

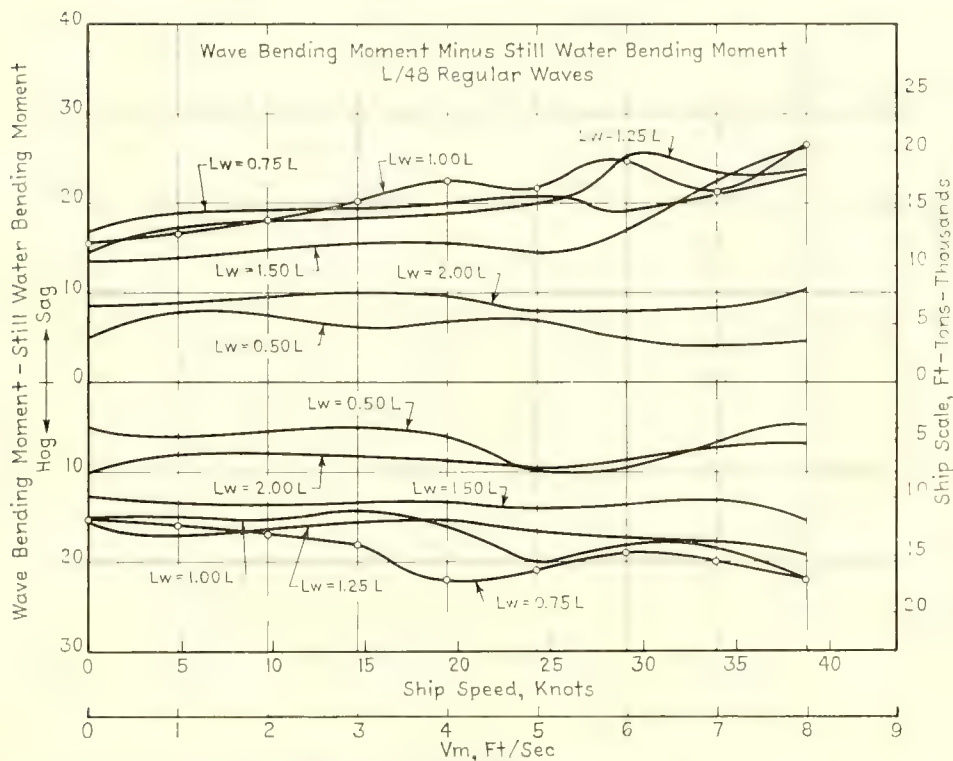


Fig. 33 Wave bending moment minus still-water bending moment; $L/48$ regular waves (from Lewis and Dalzell, 1957)

conditions for a destroyer operation at the speed of 20 knots can be judged to consist of an average wave height of $1/65$ of ship's length, average double amplitude of pitching of 4 to $4\frac{1}{2}$ deg²⁰ and the slamming at the rate of 6 to 8 per 100 waves.

In tests of Lewis and Dalzell, described in the preceding section, more severe wave conditions were used. In regular wave tests, shown in Fig. 32 the wave height was $1/48$ of ship's length. In irregular waves, used in Table 7, the average wave height was $1/32$ of model's length; i.e., the waves were twice as steep as limiting waves in the Szebehely-Warnsinck-St. Denis destroyer tests. Therefore, the maximum practical speed under these conditions must be well below 20 knots.

The average pitching double-amplitude of 4 deg, corresponding to the ship limiting condition, occurred in model tests in regular $h = L/48$ waves at about a 10-knot speed. The practically applicable range of Fig. 32 appears, therefore, to be limited to 10 knots (2 fps for the model). The irregular sea used in model tests was decidedly more severe. At the mean $L/32$ wave height, the average pitching double amplitude of 5.22 deg was already observed at zero speed. The practical applicability of Fig. 36 and Table 7 appears, therefore, to be limited to hove-to, or, say, the speed well below 10 knots.

²⁰ Using Longuet-Higgins relationships for conversion from the mean of 10 per cent highest waves on Figure 3-51 to the average height.

Several spectacular features, indicated in Sato's and in Lewis' and Dalzell's tests at higher speeds, appear to lie entirely outside the range of practical application. None of the data of Figs. 34 and 36 in $L/20$ waves at significant forward speeds is applicable to the practical ship operations. Only the hove-to condition is practical.

In order to represent the practical conditions of the destroyer operation, much lighter sea conditions should have been used at high speeds. On the other hand, the most severe storm conditions to be found at sea should have been used in hove-to conditions and at very low speed.

5 Rapidly Applied Loads (Slamming, Pounding)

The hydrodynamic loads to be considered in this section are applied with such a rapidity that static methods of stress calculation are not applicable. It is necessary to consider the elastic characteristics of a ship.

In the introduction to slamming loads in Section 2-7 Kent's definitions of slamming and pounding were quoted. These definitions are, however, not universally accepted and more often both words are used interchangeably to describe slamming. This was ordinarily understood to mean an impact of a ship's bottom on water surface after previous emersion. Oc-

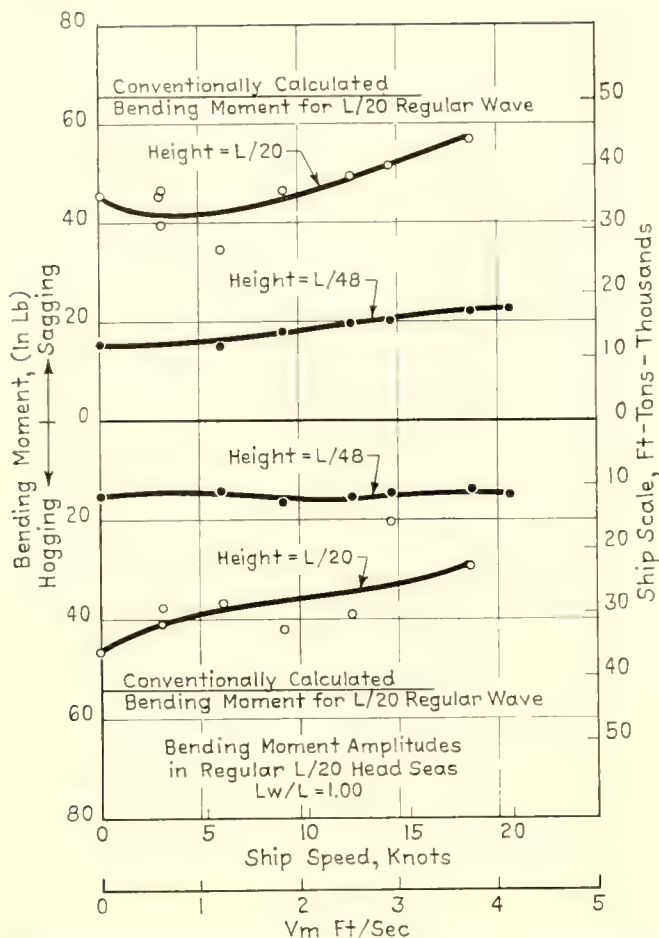


Fig. 34 Bending moment amplitudes in regular $L/20$ head seas; $L_w/L = 1.00$ (from Lewis and Dalzell, 1957)

casionally it was claimed that the prior emersion of the bow is not necessary in order to produce a slam. It appears that any wave shock applied at the bow and severe enough to excite hull vibration has been described as a slam. The subject is apparently confused because too many events are included in one concept of excessive generality. In order to clarify the issue it is necessary to decompose this broad concept into several more narrowly defined ones. In Section 2-7 attention was called to the distinction between high local pressure and the large total force at a lower mean pressure. The first causes damage to the bottom plating in the early stages of an impact and the second causes vibration of a ship and affects bending moments occurring in the later stages of impact. In the present section the distinction between the nature of slamming of slow ships and of high-speed ships also will be introduced. In addition, the relationship between hydrodynamic loads in slamming and the bending stress in a ship will be considered.

5.1 Slamming of Slow Ships. The term "slow ship" means here the usual type of a cargo ship which is characterized by rather full lines, a relatively broad bottom area at the bow, and a very small deadrise. These ships

are known to slam frequently in head seas when in light-draft condition. In the Admiralty Ship Welding Committee Report No. 8 (on SS *Ocean Vulcan*), a statement is made that the ship slammed during one out of three days in open ocean under light load conditions. There exists a large literature on the bottom-plating damage by slamming but too little emphasis has been placed on the speed loss resulting from the necessity to guard against slamming.

In cargo ships, the impact of the water on the ship's bottom is the most conspicuous part of the slamming. Because of the small deadrise, the water-impact pressures are very high and frequently cause damage to the bottom plating. Also, because of a small deadrise, a large area of the bottom is wetted in an extremely short interval of time. A large total force is generated almost instantly and is felt as a sharp shock. When the edge of the wetted bottom area reaches the turn of the bilge, local pressures and the total pressure force diminish rapidly. This follows from Wagner's theory of impact which was outlined in Section 2-7.1. It was shown that pressures vary inversely as the square of the deadrise-angle tangent. They are very high when the angle is small and they diminish rapidly as the angle increases at the bilges.

The experimental data for a complete investigation of slamming events are very meager. The towing-tank experiments of Szebehely and Lum (3-1955), E. V. Lewis (1954), and Akita and Ochi (3-1955) established that slamming occurs when a ship is heaved up and is in a nearly level attitude. At this time the bow has nearly maximum downward velocity. In a cargo ship, the prior emergence of the bow is a necessary prerequisite to slamming. However, these experimenters provided no data for making a detailed analysis of the slamming process.

Attention should be called to the fact that the most conspicuous result of the slamming, the hull vibration, has not been reproduced correctly on models. On a ship, the slamming vibrations at a two-node frequency are often felt for 60 cycles or over 30 sec. An example of such a vibration following a slam is shown in Fig. 37. In model experiments on the other hand the vibrations are extinguished very quickly. Typical examples are shown in Fig. 2-36 and in Fig. 41. These were taken from the work of Akita and Ochi (1955) and Ochi (1956a) which was done with large-size models built of brass sheets. The model construction was generally similar to the one normally used in ships. A plausible explanation may lie in the higher frequency of a model vibration as compared to a full-size ship. Lockwood Taylor (1930) showed that the damping of ship vibrations is caused almost entirely by the hysteresis of the structure, that it is very small at a usual two-node frequency of ships' vibration, and that it increases rapidly with the increase of the frequency.²¹

²¹ Also see Section 5.53.

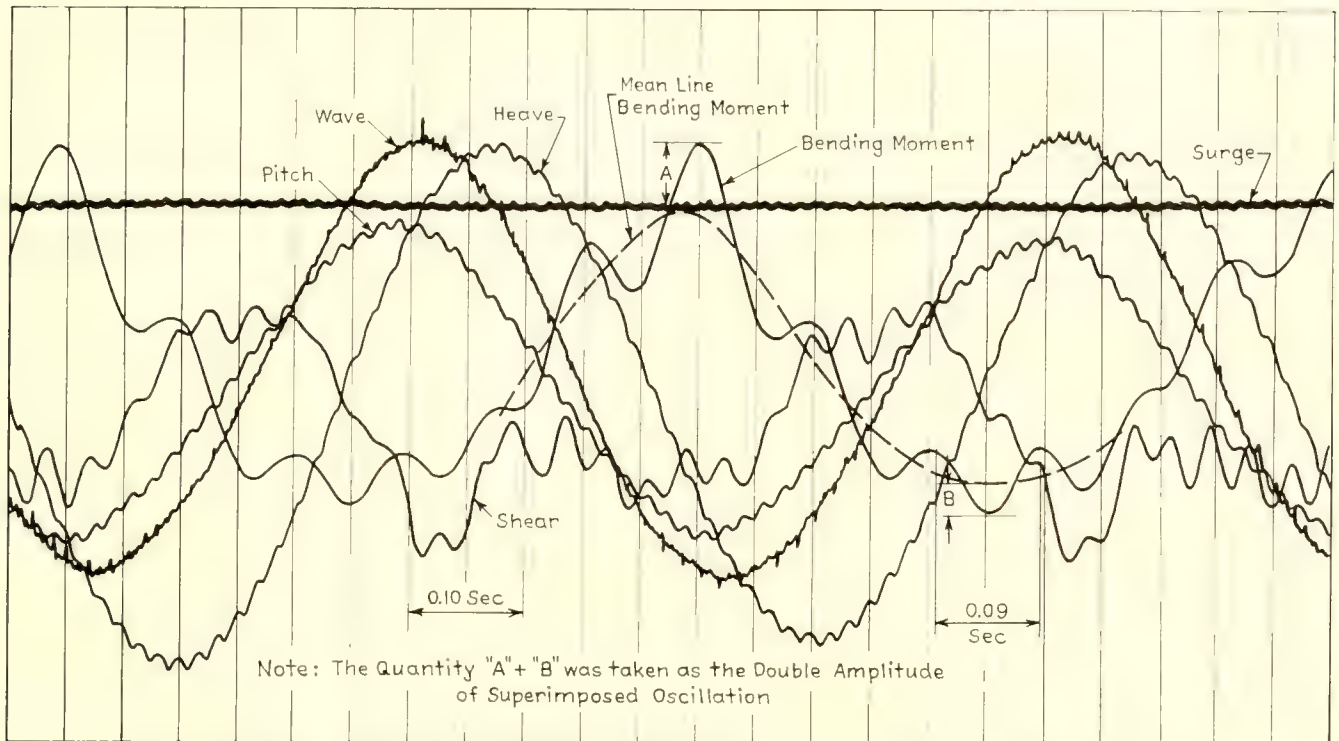


Fig. 35 Sample oscillograph record Model 1723, test 1K, run 10, $V_m = 6.84$ fps, $L_w/L = 1.25$, $b = L/48$ (from Lewis and Dalzell, 1958)

Ochi (1956a, b, 1957) made a very extensive series of slamming experiments. However, in reporting on these, he has neglected to show the dependence of various events on time. This makes the data not suitable for the detailed investigation of slamming phenomena. Only over-all results, such as the maxima of forces and the envelopes of pressures can be used for empirical or semi-empirical studies to be discussed later in Section 5.4.

The full-scale data are equally meager. On the MS *San Francisco* and SS *Ocean Vulcan* the instrumentation was not suitable for recording slamming and slamming accelerations. The bottom pressures occurring in slamming will be discussed in Section 5.4.

5.2 Observations of USCGC Unimak. Greenspon (3-1956) and Greenspon, Jasper, and Birmingham (3-1956) have reported on the observations of slamming on the USCGC *Unimak* which was operated at two weather stations in the North Atlantic. Quoting from the original papers: "The ship operated in very heavy seas, and the Captain allowed the ship to undergo severe slamming so that records could be taken." Neither the sea description nor the speed of the ship is given in these papers.²² The subject is narrowly limited to the conditions at the bottom plating and the general effects of slams on the entire ship are not discussed. However,

the data on the bottom pressures are instructive. They are particularly valuable for the purpose of the present exposition because they permit demonstration of the contrast with the behavior of a destroyer which will be discussed in the next section.

A coast guard cutter has finer lines than the cargo ship discussed in the preceding section. Nevertheless a small part of the bottom near the keel is relatively flat and demonstrates the nature of the slamming impact on a small deadrise surface. The locations of the pressure gages and strain gages used on USCGC *Unimak* are shown in Fig. 38 and a sample of a slamming record is given in Fig. 39. The magnitudes of pressures will be discussed in Section 5.4. At present, attention will be called only to the conspicuous characteristics of the slamming impact. These are:

- 1 Shortness of the pressure pulse—about 0.015 sec.
- 2 Simultaneous pressure pulse at several gages spaced along a longitudinal line 8.25 in. from the centerline; i.e., gages 2, 3, and 4.
- 3 Very low pressure at gage 6 which is located farther from the keel. At this location the deadrise angle was probably much larger than at gages 2, 3, and 4.
- 4 A very short interval of time, about 0.01 sec, needed for the pressure pulse to travel in an outboard direction from gage 4 to gage 6.

The shortness of the pressure impulse on a surface of a small deadrise is an agreement with Wagner's theory outlined in Section 2-7.1.

²² The present author was advised by one of the authors of these papers that the wave data are available at the David Taylor Model Basin.

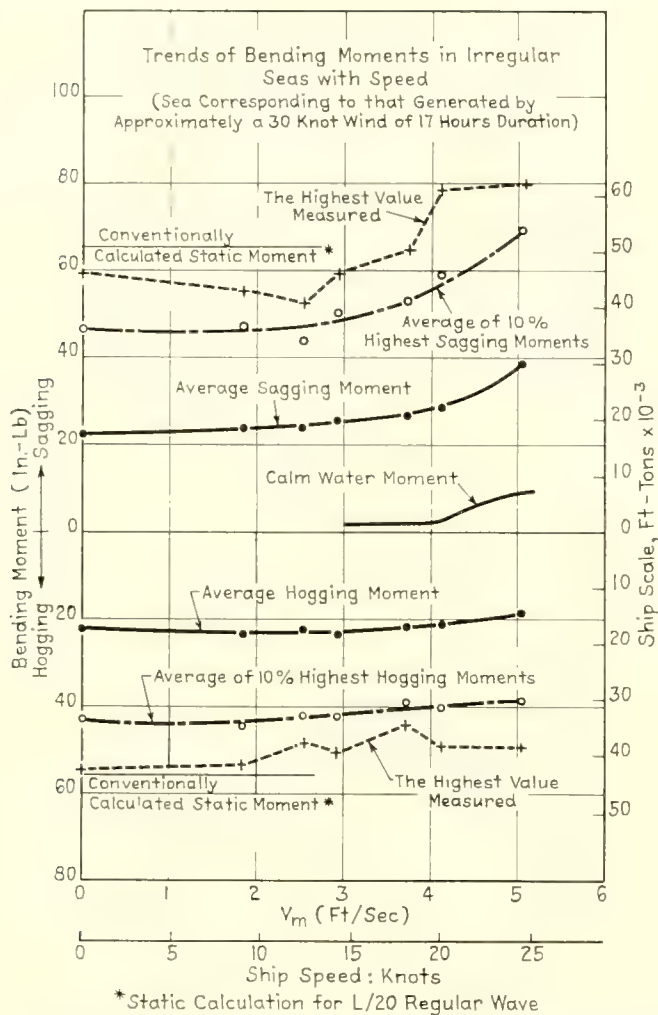


Fig. 36 Trends of bending moments in irregular seas with speed (from Lewis and Dalzell, 1958)

5.3 Observations on a Destroyer. Fig. 40, taken from Warnsinck and St. Denis (1957), shows a sample of a record taken on a destroyer at sea. A brief description of instrumentation was given in Section 3-5.16, and in Figs. 3-49. Wave conditions were outlined in Section 4.5 of the present Chapter. The recorded data are given by diagrams in the middle column of Fig. 40, and the photographs in the right and left-hand columns, respectively, correspond to various instants during the recorded period.

The time scale in seconds is given along the lower edge of Fig. 40. It is measured from an arbitrary instant. The numbers just above the time scale designate the film frame numbers. The broken vertical lines, corresponding to the instants the photographs were taken, are drawn through all sections of diagrams.

The short horizontal stretches found between 7 and 8.5 sec on four upper pressure curves of Fig. 40, correspond to the atmospheric pressure and show that these gages emerged from the water. The instant of slam is

shown by a short vertical rise of pressure curves at the scale time of 8.5 sec. This instantaneous rise on the time scale of this diagram corresponds to the very rapid occurrence of pressure pulse on the USCGC *Unimak*. In the present case this pressure pulse at 8.5 sec reaches only a small pressure of 8 to 10 psi.

The lines of the particular destroyer, to which these data apply, are not available. However, typical destroyer lines such as are shown in Figs. 27 and 31 can be assumed. These body plans show a sharp V-form with a large deadrise angle for the first 60 to 80 ft of the ship's length from the bow. With such a large deadrise angle, only small impact pressures can be expected on the basis of Wagner's theory. Qualitatively this is in agreement with 8 to 10 psi shown by the rise of the vertical line at 8.5 sec in Fig. 40.

Reference to the curve of pitching angles on the upper part of Fig. 40 shows that at the instant of slam (at 8.5 sec) the ship was on an even keel. This feature is in agreement with towing-tank tests of E. V. Lewis (1954), Szebehely and Lum (3-1955), and Akita and Ochi (3-1955). The data on these were given by Figs. 2-34, 35 and 36. In these figures the instants of slams were shown by the readings of accelerometers at the bows.

In the present case, however, the shock at the first instant of slamming, at 8.5 sec, is mild as compared to the final development of the impact force later, and the accelerometer records are more difficult to interpret.

On the records of the USCGC *Unimak*, the pressure pulse was of very short duration. In the case of the destroyer, shown in Fig. 40, the drop of the pressure following the pressure pulse is absent and, instead, the pressure rises relatively slowly with time and reaches the maximum value in about 1 sec after initial impact; i.e., at the time scale of 9.5 sec. Reference to the pitching curve shows that at this instant the ship's bow is in its lowest position. This is confirmed by the photograph, No. 24, which corresponds to an instant at 10 sec.

It appears that with the sharp V-bottom sections of a destroyer, the bottom impact plays a relatively small part in a slamming process and that maximum water pressure is associated with the full immersion of the bow. The pressure, however, is only partly hydrostatic and contains a large dynamic component. The peak pressure at Frame 195 is shown in Fig. 40 to be 27 psi. Assuming a height of the stem of 33 ft, the hydrostatic pressure at full submersion would be expected to reach only 16.5 psi. There is therefore an added dynamic increment of 10.5 psi. Wagner's theory, outlined in Section 2-7.1, indicates that there must necessarily occur an increase of local water pressure as the water level rises to the flare at the deck. The references cited in connection with Sections 2-7.1 and 7.2 show, moreover, that a significant pressure increase over the entire periphery of a submerged body occurs simultaneously with the sharp rise of the local pressure at the water level and the formation of the spray. *The submersion of the bow and the dynamic effect of the bow flare appear, therefore, to be the major causes of "slams" in ships of a destroyer type.*

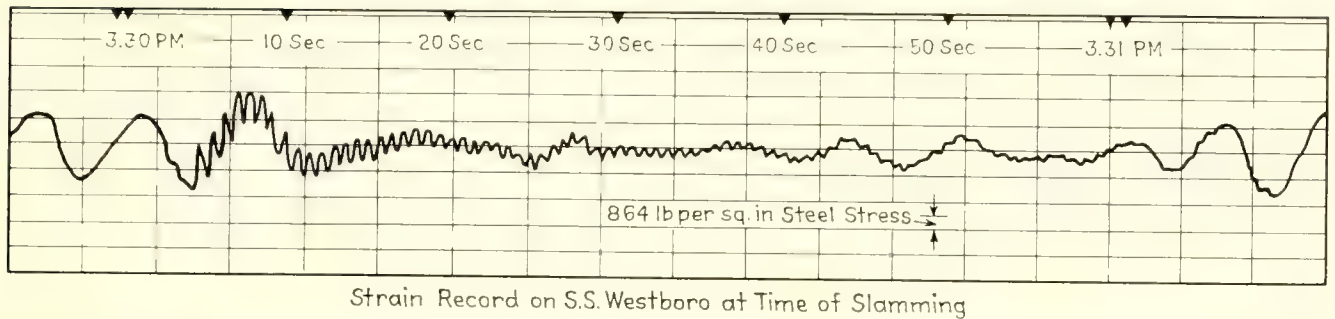


Fig. 37 Strain record on SS Westboro at time of slamming (from Adm. Ship Weld. Comm. Rep. No. 8, 1953)

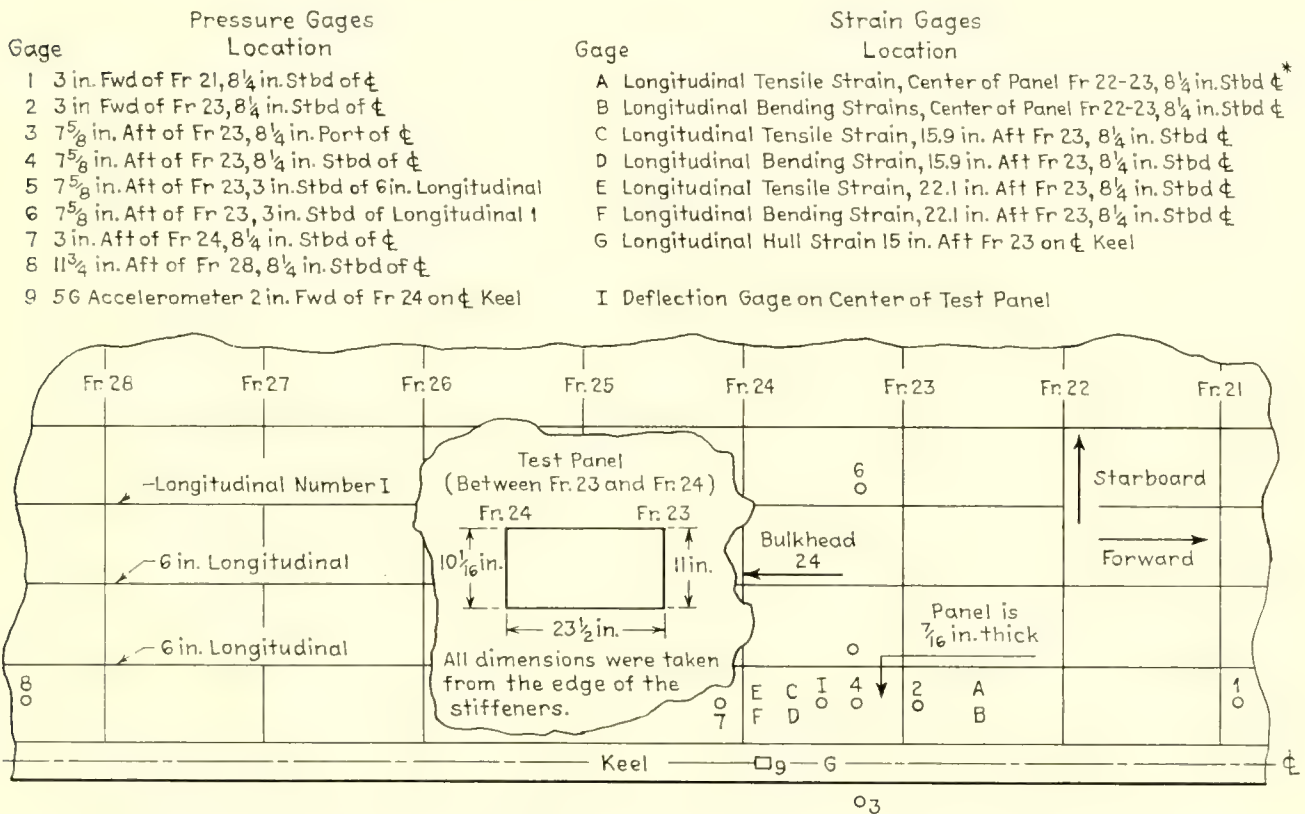


Fig. 38 Location of gages and shell expansion on USCGC Unimak (from Greenspon, Jasper, and Birmingham, 1957)

This may also be the case, with fast cruisers, aircraft carriers and even fast passenger liners.²³

In the process outlined in the foregoing the bottom impact plays only a secondary part and the bow emersion is therefore not a necessary prerequisite to the occurrence of a slam. This is illustrated by Table 9 taken from Warnsinck and St. Denis (3-1957, NSMB Symp.). In two most severe slams, in which the bending moment amidships was approximately doubled, there was no bow emersion.

²³ See Jasper and Birmingham (3-1958).

In addition to the severe shocks recorded as slams, the frequent bow immersion in the head seas causes shocks of sufficient magnitude to maintain the hull in a continuous state of vibration. This was noted in model tests by Sato and by Lewis and Dalzell.

The effect of slamming shocks on the bending moments will be discussed in Section 5.5.

5.4 Water Pressures in Slamming. Theories of Wagner and their adaptation to ships by Szebehely and M. A. Todd, model tests, and sea observations outlined in previous sections, taken together, form qualitatively

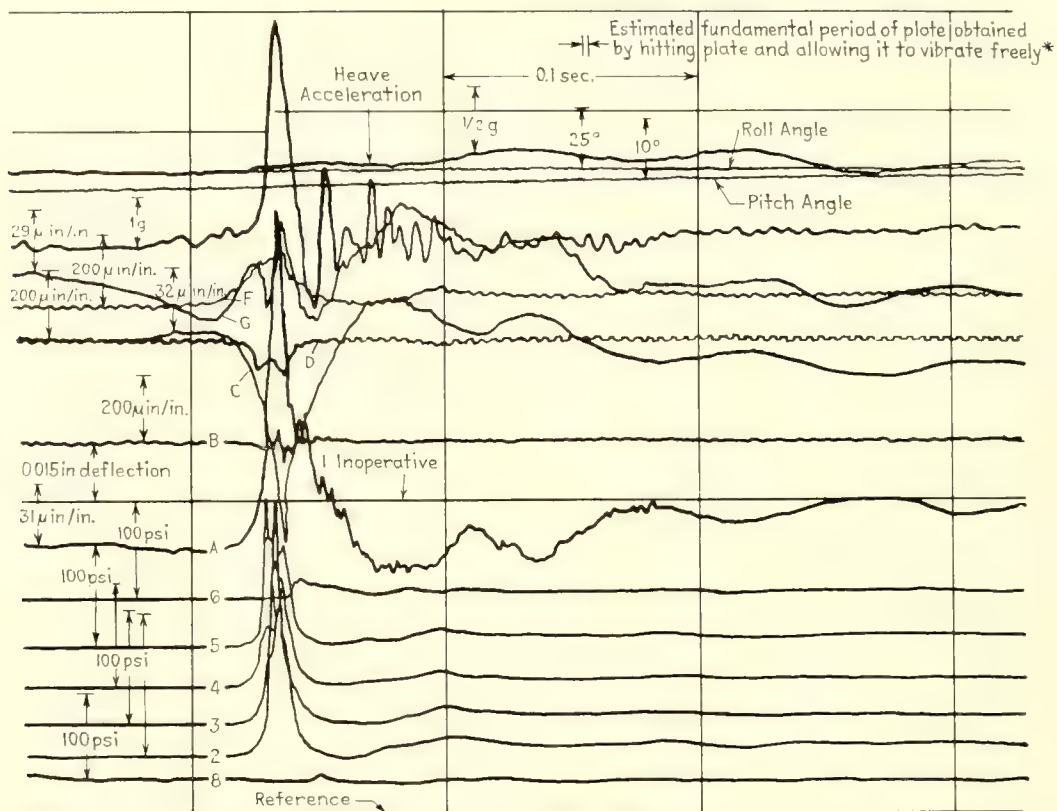


Fig. 39 Representative slamming record on USCGC Unimak (from Greenspon, Jasper, and Birmingham, 1957)

a clear picture of the slamming process. The quantitative evaluation of pressures is, however, uncertain. Theoretically, the pressures over 1300 psi were computed by Bledsoe (3-1956). Such pressures were not experimentally observed. In Chapter 2, the present author expressed an opinion that these high pressures extending over a very narrow peak have little practical significance, and only a mean pressure over an unsupported bottom-plate area is significant. When pressures are measured on ship models, the relatively large size of a gage with respect to a model and the time-response characteristics of the gage provide an averaging action of an uncertain degree. The slamming pressures were measured on two models of cargo ships by Ochi (1956a, b, 1957). The body plan of the model used is shown in Fig. 2-37. For the U-form ship Ochi found the slamming pressures to be six times the hydrostatic pressure in smooth water. For the V-form ship the slamming pressure was four times hydrostatic. Assuming a draft of 28 ft, the pressures can be estimated, therefore, at 75 and 50 psi, respectively. Partly by theoretical means and partly from consideration of damage to ships' plating, Watanabe (1957) arrived at the pressure 7 times hydrostatic, or 87 psi in the foregoing example. These rather mild pressures may be explained by the fact that Ochi used gages of large diameter and experi-

Table 9 Slamming Stresses. From Warnsinck and St. Denis, 1957 NSMB Symposium

Ship	Slamming stress, kips/sq in.	Longitudinal bending stress, kips/sq in.	Ratio	Bow emergence
P	2.83	9.63	0.294	yes
	2.71	10.63	0.255	no
	3.38	9.47	0.357	yes
	4.05	9.76	0.415	no
	11.20	10.61	1.055	no
	4.05	12.62	0.321	no
Q	4.30	14.73	0.292	no
	3.06	11.13	0.275	no
	4.72	13.04	0.362	yes
	8.80	9.29	0.947	no
	5.13	9.00	0.570	yes

NOTE: It is pertinent to note that the highest values of slamming stresses were not obtained with bow emergence. Both of the slams giving rise to the highest slamming stresses occurred in bow, not head, seas.

mented only with regular waves. It is well known that slamming is more readily brought about by irregular waves. Watanabe's work has reflected the conditions actually met by cargo ships in service. It represents therefore an appraisal of a "residual slamming" after the more severe cases were eliminated by good seaman-ship.

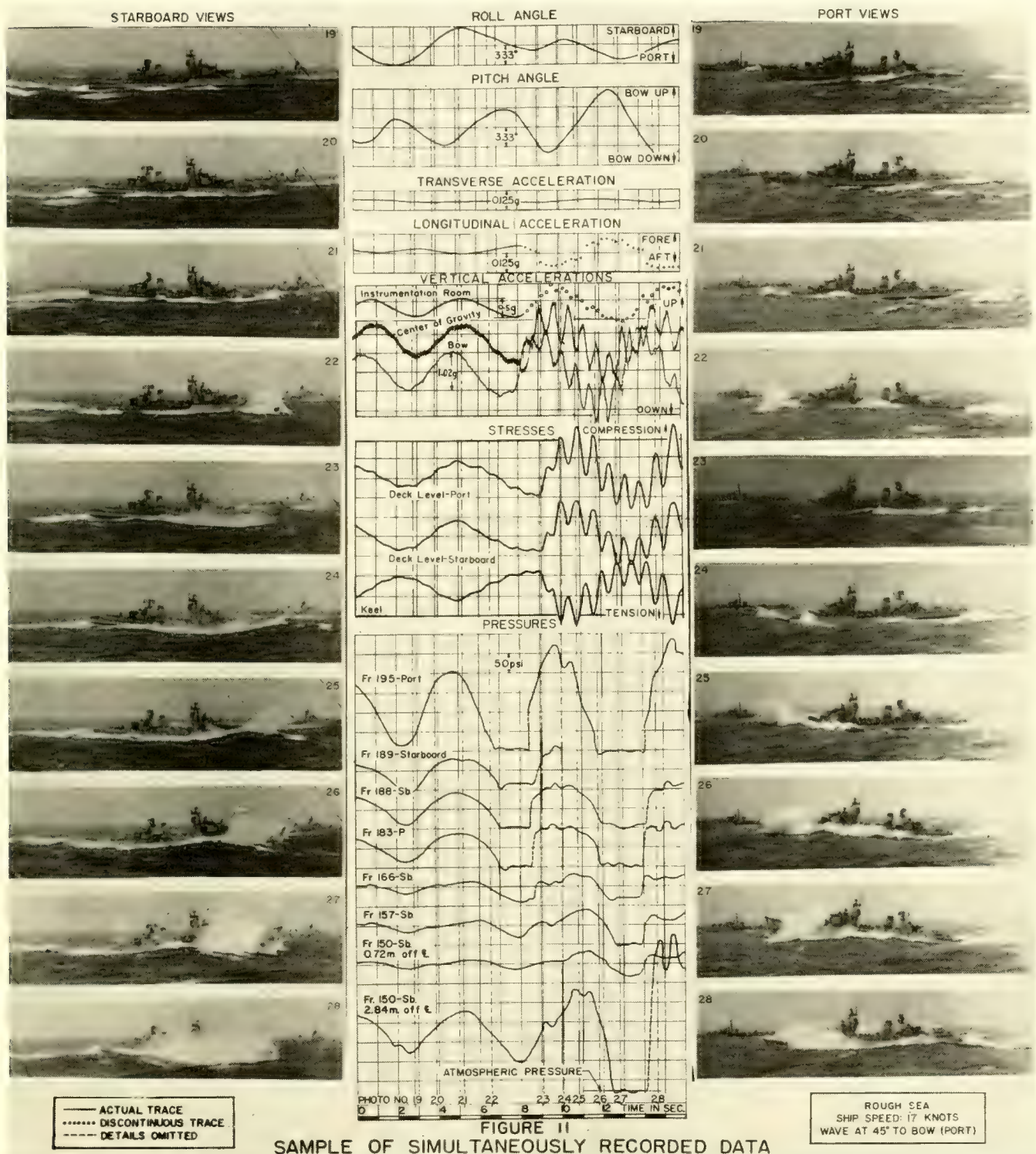


Fig. 40 Pitching, bending stress and bottom-pressure records taken at sea on a destroyer, and photographs corresponding to the record (from Bledsoe, Bussemaker, and Cummins, 1960)

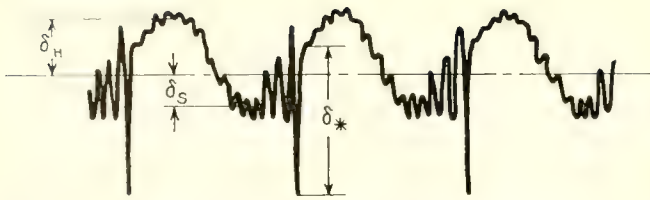


Fig. 41 Typical form of measured stress on ship deck at slamming speeds (from Ochi, 1956a)

On the USCGC *Unimak*, Greenspon (3-1956) and Greenspon, Jasper and Birmingham (3-1956) found the peak pressure readings of individual gages up to 295 psi, but quote the pressure of 86 psi as the highest mean over a plate at an instant of maximum strain. Although the figure of 86 psi is of the same order of magnitude as was given by Ochi and Watanabe, it is recorded here on a ship of finer lines. The explanation lies in the fact that this is a sturdily built small ship and apparently it was operated more daringly than would be practical in a cargo ship.

Warnsinck and St. Denis stated that in most slams during destroyer tests the peak hydrodynamic pressure did not exceed 25 psi. Only about 15 per cent of slams exceeded a pressure of 50 psi. During one slam a peak pressure in excess of 100 psi was recorded.

Ochi's papers (1956b and 1957) contain a number of diagrams indicating the slamming forces and ship areas affected at different speeds. Most slamming is shown to occur at a speed slightly above the synchronous one. Watanabe (1957) made an extensive theoretical analysis of a ship's slamming conditions. This analysis is rather difficult to follow because of the large number of approximations introduced in the process. Watanabe demonstrated, however, a good correlation of his approximate theoretical results with the data on ship-bottom damages. Various criteria for evaluating the effect of a ship's form on slamming were introduced by King (3-1934/35), Hansen (1935), Lehman (1936), Watanabe (1957), and Ochi. Ochi used as a criterion the section coefficient at 10 per cent of a ship's length at 50 per cent of the design draft.

5.5 Relationship Between Slamming Load and Bending Moment. In the process of heaving and pitching in waves, a ship is subjected to bending moments which vary approximately harmonically with time. In such a case the true bending moment is related to the moment calculated statically by the factor

$$\frac{T_1^2}{T_1^2 - T^2} \quad (5)$$

where T_1 is the period of the wave encounter and T is the natural period of vibration. With the average period of the wave encounter of 6 sec and the natural period of the two-node ship vibration of 0.6 sec, this factor is so near unity that it can be neglected. The bending stress is related to the imposed loading essen-

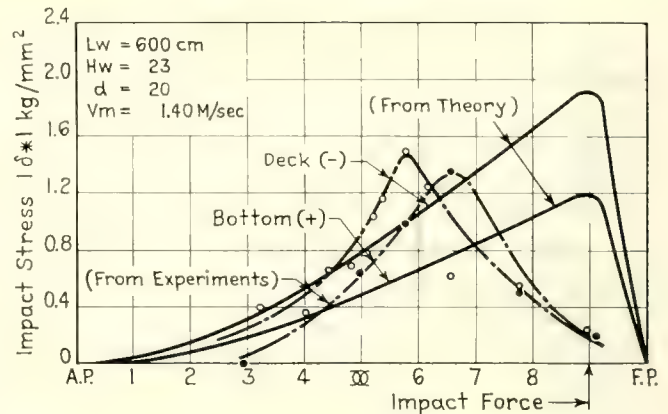


Fig. 42 Comparison between computed and measured stress (from Ochi, 1956a)

tially by the rules of statics. This condition is often referred to as the "quasi-static."

Different conditions exist when a ship is subjected to a local impact as in a slam. The deflection of a loaded ship's part is permitted by the elasticity of the adjacent structure and the impulse of the impact force is first absorbed by the momentum of the masses in its immediate vicinity. After the impact impulse is expended, the structure is left in a strained state and the interplay of elastic and inertial forces produces a state of vibration. Immediately at the end of an impulse, the maximum bending moment (hogging) apparently is found at the point of the load application. The bending-moment diagram has essentially the form labelled "from theory" in Fig. 42. This is a transient state of short duration and there appears to be little probability that it can be spotted in model experiments. Experience shows that a two-node vibration is established so quickly that it is the only one commented upon in sea observations. There is no doubt, however, that a transient state exists and could be recorded by suitable instrumentation. Investigation of this transient state is important in order to establish the redistribution of stresses from the maximum hogging at the point of slamming-load application to the maximum hogging-sagging at approximately amidship in the final two-node vibration. Ochi (1956a) shows in Fig. 42 ("from Experiment") that the effect of the slamming is to cause the maximum bending moment to occur somewhat forward of the midship section. In his report Ochi neglected, however, to represent the impact stresses as functions of time and the stress distribution labelled "from Experiment" apparently refers to an undetermined instant of a transient process.

A theoretical analysis of the response of a typical ship structure to an impact load apparently has not been attempted. A simplified analysis, treating a ship as a beam of uniform section, was made by Lockwood Taylor (1946), Ormondroyd, et al (1948, 1951), and Ochi (1956a, 1958d and e). Papers on the response of a beam to an impact also were published by Arnold (1937), Frankland (1942, 1948) and Salvadori (1947). The paper by

Frankland (1942) can be cited as giving a simple and logical introduction to a structure's response to an impact load. A slender beam may vibrate in a number of modes which can be conveniently described by the number, n , of nodal points of the elastic curve. All vibration modes are excited by an impact and the deflection y is expressed by a summation of contributions from all modes.

Let L designate the length of a beam and L_1 the distance from one end to the point of application of an impulse $P\tau$. The deflection y of the beam at a distance x from the end apparently can be represented as

$$y = (\text{const})P\tau \sum_{n=2}^{\infty} f\left(\frac{x}{L}, \frac{L_1}{L}, n\right) \varphi(t, n) \quad (6)$$

where n is the number of nodal points. The function of x/L , L_1/L , and n is rather complicated. The function (t, n) is essentially sinusoidal with the period depending on n . The deflection, y , is represented as the summation of all possible modes of vibration for which the point of impact application does not coincide with a nodal point. The deflection, y , represented by such a sum of harmonic functions, is certain to display a transient behavior. Lockwood Taylor (1946) demonstrated mathematically that one of the manifestations of transients is a gradual shortening of the apparent period of vibration with time.

Further insight into the behavior of a slender beam, indicated by equation (6), can be obtained by examining a simple case reported by Frankland (1948, equation 39). Frankland considered the undamped vibration of a simply supported beam of length L and mass m per unit length subjected to a load p per unit length, uniformly distributed over the centrally located length a . The load p was assumed to be suddenly applied at the time $t = 0$ and suddenly released at the time $t = \tau$. The time-dependent deflection y of the beam at any point x and at a time $t > \tau$ has been found to be

$$y(x, t) = \frac{4pL^4}{EI\pi^5} \sum_{N=1}^{\infty} \frac{2}{N^5} \left[\sin \frac{N\pi}{2} \sin \frac{N\pi a}{2L} \sin \frac{N\pi x}{L} \right. \\ \left. \sin \frac{N^2\omega_n\tau}{2} \sin N^2\omega_n(t - \tau/2) \right] \quad (7)$$

where N is the vibration mode, and ω_n the circular frequency corresponding to it. N is related to the number n of nodal points in equation (6) by $N = n - 1$.

The bending moment M acting at a section of the beam located at x is

$$M = -EI \frac{\partial^2 y}{\partial x^2} = \frac{4pL^2}{\pi^3} \sum_{N=1}^{\infty} \frac{2}{N^3} [\dots] \quad (8)$$

where the dashes in square brackets designate five trigonometric factors identical with those of equation (7).

Equation (8) can be simplified by considering only the fundamental mode, $N = 1$, and by evaluating the ampli-

tude of the bending moment at mid-point, in which case the first, third, and last trigonometric factors are equal to unity. Equation (8) can represent a central concentrated impact if the distance a is made sufficiently small, $a \ll L$. The duration τ also can be assumed to be small, $\tau \ll T$, corresponding to a slamming impact on a cargo ship. With these assumptions the sine of an angle in second and fourth factors in square brackets can be replaced by the angle, and equation (8), after letting $\omega = 2\pi/T$, is reduced to

$$M_{\max} = (4/\pi)(pa\tau)L/T \quad (9)$$

The amplitude of the dynamic bending moment for a small duration of an impact τ is shown to be proportional to the momentum of the impulse, $pa\tau$, and inversely proportional to the natural period of vibration T . For a given small impact duration τ , the ratio τ/T increases for higher harmonics which have smaller T . These harmonics are, therefore, easily excited by an impact and significantly contribute to the bending moment, as this was shown by Frankland (1948) and Ochi (see Section 5.52).

With the foregoing background in mind, the stress history shown in Fig. 40 will be examined. First it is noted that at the instant of the bottom slam, 8.5 sec, there is no significant effect of the slam on the stresses amidships. The first and very weak sagging stress maximum is found 1 sec later, at 9.5 sec. The amplitude of the vibratory stress is then observed to increase during the next two oscillations and reaches the maximum at the third peak at 11 sec. This is $2\frac{1}{2}$ sec after the initial impact. This peak coincides with the maximum of sagging stress in pitching oscillation and more than doubles this stress. The vibratory amplitude thereafter decays very slowly but is boosted by the second slam at 14 sec on the time scale. This example demonstrates the fact that evaluation of the slamming force does not lead directly to the knowledge of ship stresses. In order to evaluate these stresses, it is necessary to solve the problem of the elastic response of a ship to an impact, to evaluate the transients, and to represent stresses as functions of time.

5.51 Free vibrations. An investigation of the free vibration of ships can be considered as a prerequisite to the subsequent consideration of forced vibrations. Also it has been shown in the preceding section that the maximum bending stress in a ship usually occurs at some time after a slam when the bending stress of free vibrations is superposed on the wave-caused sagging bending stress. The following quotation from McGoldrick, et al (1953) can serve as an introduction to this subject:

"The beam-like nature of a ship's hull is self-evident and has formed the basis for the ordinary strength calculations universally used in design wherein the ship is assumed supported on trochoidal waves which exert a buoyant force per unit length which varies with distance from the end but is considered constant in time, that is, the analysis is carried out as a problem in statics.

"The simple bending theory of beams has been used

widely in deriving a differential equation for the free transverse or flexural vibrations of uniform slender bars, the differential equation being

$$EI \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} = 0 \quad (10)$$

where

E = Young's modulus

I = moment of inertia of area of section with respect to its neutral axis.

y = displacement in vertical plane

x = co-ordinate along axis of bar

m = mass of bar per unit length

t = time

"The solution of this equation for a uniform bar with free ends yields the formula for the natural frequencies:

$$\omega_n = \alpha_n^2 \left(\frac{EI}{mL^4} \right)^{1/2} \quad (11)$$

where

ω_n = circular frequency

L = length

α_n 'characteristic numbers' arising in the solution of this differential equation with these specific boundary conditions.

"As given by Rayleigh in his 'Theory of Sound' the characteristic numbers fall closely in the ratios of the odd numbers starting with 3; that is, 3, 5, 7, 9, etc. The first three characteristic numbers are 4.73; 7.853; and 10.996."

The foregoing expressions were based on the consideration of pure flexural deflections neglecting the inertial effects involved in the inclination (or rotation) of beam sections and neglecting damping. Further development consists of introducing shear deflections, rotary inertia, and damping. The damping is composed of the internal damping of the structure and external damping caused by surrounding water. This latter appears to be small, as will be shown later. The following expression is abstracted from Kumai (1958), neglecting the external damping considered by him:

$$EI \left(1 + \frac{\xi}{E} \frac{\partial}{\partial t} \right) \left(1 + \frac{\eta}{G} \frac{\partial}{\partial t} \right) \frac{\partial^4 y}{\partial x^4} - m \left\{ \frac{EI}{k'AG} \left(1 + \frac{\xi}{E} \frac{\partial}{\partial t} \right) + r^2 \left(1 + \frac{\eta}{G} \frac{\partial}{\partial t} \right) \right\} \frac{\partial^4 y}{\partial x^2 \partial t^2} + m \left(1 + \frac{\xi}{G} \frac{\partial}{\partial t} \right) \frac{\partial^2 y}{\partial t^2} + m^2 \frac{r}{k'AG} \frac{\partial^4 y}{\partial t^4} = 0 \quad (12)$$

where

G = shear modulus

$k'AG$ = effective shear rigidity of hull section

r = radius of gyration of mass moment of inertia

ξ = coefficient of normal viscosity

η = coefficient of tangential viscosity

The solution of equation (12) with suitable boundary conditions gives the frequencies of various vibration modes.

McGoldrick et al (1953) cite three DTMB reports²⁴ pertaining to theoretical and experimental investigation of vertical vibration characteristics of USS *Niagara*. The results of digital computations, based on the vibration theory, are given in the following quotation: "... The principal facts disclosed were the following: The calculation based on bending only is in fair agreement for the first vertical mode but becomes progressively too high beyond the first mode; the calculation based on shear deflection only is quite high for the first mode but becomes progressively nearer the true value as the order of the mode increases; in the case of USS *Niagara* the inclusion of rotary inertia had a negligible effect on the results.

"As will be seen from the tabulation in [Mathewson, 1949], the calculations based on shear and bending with rotary inertia neglected check the experimental values up to the sixth mode within 5 percent with the exception of the fundamental mode.

"From the profile of this vessel it can be seen that its island or superstructure comprising three decks extends for about 30 percent of the length of the hull. When the moment of inertia of this island was added to the moment of inertia previously computed up to the weather deck and the calculation repeated, it was found that the first mode checked within 1 percent but that the remaining frequencies were all too high. It thus appeared that the stiffening effect of a superstructure of such proportions cannot be neglected in the first mode but that it has little effect beyond the first mode. This does not seem at all unreasonable as the first mode is the only one in which bending predominates and the superstructure probably adds very little to the shear stiffness."

There exists a vast amount of literature on the subject of free vibration of ships. Some of the references are listed in the bibliography at the end of this chapter and some will be found in McGoldrick et al (1953), Csopor (1957), and Lewis and Gerard (1958). Detailed discussion of the subject is outside the scope of the present monograph and the foregoing brief outline was presented merely in order to bring out the salient features of a ship's response to slamming to be discussed further in the following sections.

5.52 Forced and transient vibrations. The term "forced vibrations" is used in the present exposition for the continuously acting excitation such as is caused in ships by propellers and machinery. Equations (10) and (12) apply in this case, provided the time-dependent force $P(t)$ is inserted on the right-hand side in place of zero. This is usually a sinusoidal function. While all vibration modes are excited in principle, the response of a particular mode, with the natural frequency nearest to that of the exciting cause, strongly predominates. A practical engineering problem is concentrated, therefore, on avoidance of synchronism between any one of the

²⁴ Jasper (1948) and Mathewson (1949, 1950).

natural frequencies and the frequency of the exciting force.

The term "transient vibrations" will be used to define the vibratory response of a structure to a force $P(t, x)$ variable in time and in the point of application and usually of a short duration. In the present exposition the attention is concentrated on vibrations caused by a slamming impact. In nearly flat-bottomed cargo ships it may have a duration of a fraction of a second, in a destroyer it may last 1.5 sec. Two separate phases are involved here: (a) The development of the momentum and energy content in a ship's hull during force action. (b) The decay of free oscillations after the exciting force ceases to act. The second phase is more conspicuous to an observer aboard a ship and is important in defining the vibration-caused bending stress, which is superposed on several cycles of the wave-caused quasi-static stress. An investigation of the first phase is necessary, however, in order to define the dynamic condition of the hull at the end on the impact, after which a slow decay of the vibration begins.

Vibrations in many modes are excited by a slam. Unlike the response to a specified frequency of a forced oscillation, the distribution of the momenta and energy among many vibration modes cannot be determined *a priori*. The determination of this distribution is the fundamental part of the problem discussed here. Another important part of the problem is the evaluation of the damping connected with each vibration mode. The damping affects the ability of a structure to absorb the impact momentum and it controls directly the rate of the vibration decay in each mode. Superposition of vibrations of all modes, with different momentum contents, different initial time lags and different rates of decay, determines the entire behavior of the structure in time. The question of damping will be further considered in the next section.

Three methods of attacking the problem, outlined in the foregoing, appear to be available; namely, digital calculations, electric analog, and theoretical analysis.

The first method was outlined by McGoldrick et al (1953), with reference to Jasper (1948) and Mathewson (1949, 1950), and by Polacheck (1957). The differential equations of the vibration theory had been converted into finite-difference form and calculations were performed by means of high-speed electronic computers. In this approach it is neither necessary nor possible to distinguish between the behavior of different vibration modes. The calculations give directly the total behavior of a structure, as it could have been observed in an experiment without furnishing explanation as to reasons for this behavior. This appears to be the only method which currently can be applied to actual ships in which section properties vary along the length. Two important approximations have to be made, however, with respect to the damping. A certain mean value of the damping coefficient has to be assumed, disregarding its dependence on the vibration frequency, and this coefficient has to be assumed as proportional to the masses of ship sections.

The applicability of the electrical analog method to ship-vibration problem was discussed by McGoldrick, et al, with reference to Kron (1944) and Kapiloff (DTMB Rep. 742). It appears that attention was concentrated on determination of normal modes and natural frequencies rather than on transient responses.

Theoretical analyses of forced vibrations and transient phenomena caused by slamming have been found to be difficult, and so far were only applied to bars of uniform section. The following is quoted from McGoldrick et al (1953):

"The group working at the University of Michigan solved by means of operational calculus the partial differential equation for the uniform bar subject to bending deflection only, having a uniformly distributed viscous damping and acted upon by a transverse load which was an arbitrary function of time and position along the bar. This required the solution of the differential equation

$$EI \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} + b \frac{\partial y}{\partial t} = P(x, t) \quad (13)$$

"where b is the damping force per unit length of bar per unit velocity, and $P(x, t)$ is the external force per unit length varying both with x and t .²⁵

"A solution of this equation was found by the operational method employing the Laplace transformation. The derivation is given in the second progress report of the University of Michigan on its contract with the Office of Naval Research.²⁶

"The solution shows that, whatever form the function $P(x, t)$ takes, the response of the bar is expressible in a series of normal modes; in other words, the system behaves in general like the systems whose small oscillations were studied by Rayleigh... Moreover, such a system does not partake of wave motion in the ordinary sense in that there is no fixed rate of propagation of a flexural wave. If the bar is struck at one end, a finite time will be required before a finite motion takes place at the other end, but the process is the result of compounding motions in normal modes in each of which the system deflects simultaneously at all points rather than the result of a flexural wave traveling back and forth.

"It also follows from the solution of the uniform bar problem that in each normal mode the system behaves as a system of one degree of freedom would behave and as though this mode only were present. The amplitude produced in each mode by a given simple harmonic driving force depends on the magnitude of the force, the influence function, the effective mass, stiffness, and damping constant of the system in that mode and on the ratio of the frequency of the force to the natural frequency of the mode. The ordinary resonance curve for a system of one degree of freedom is applicable to each normal mode individually..."

Ochi (1956a, 1958d, 1958e) also obtained the solution of

²⁵ Previously defined symbols are omitted from the quotation.

²⁶ Ormondroyd et al (1948).

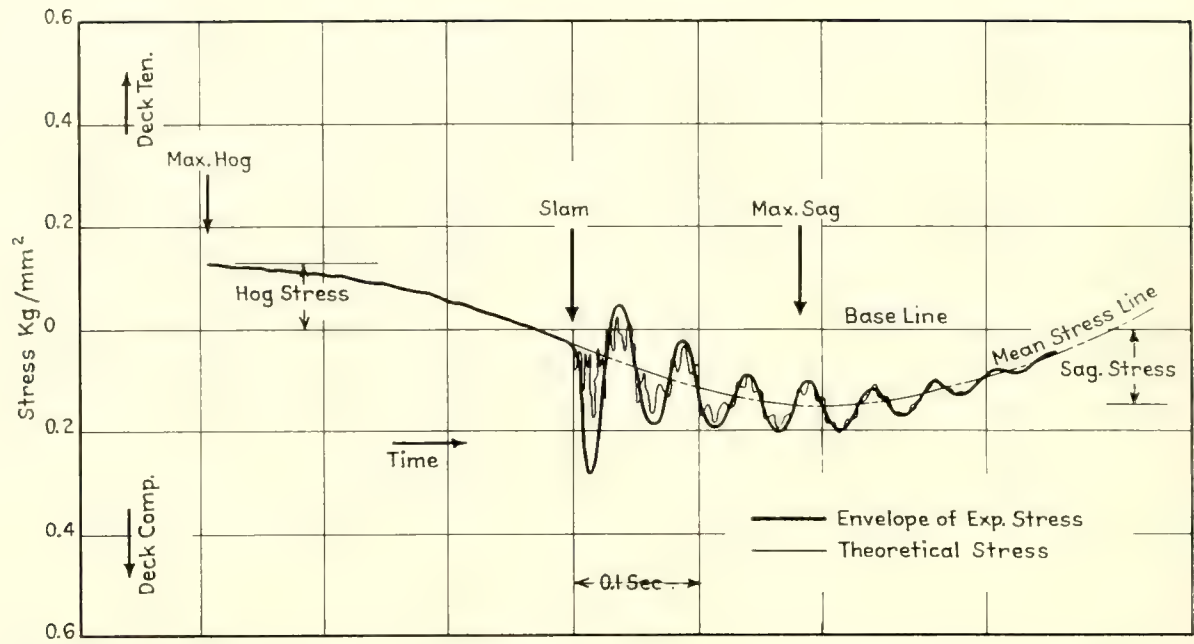


Fig. 43 Comparison of experimental and theoretical values of superimposed stress on deck amidship; V-form model (from Ochi, 1958)

equation (13) for a flexural deflection of a bar of uniform section assuming a constant value of the coefficient of accession to inertia. In this connection he represented the damping coefficient b as

$$b = \frac{\partial}{\partial t} \left(b_f y + \xi I \frac{\partial^4 y}{\partial x^4} \right) \quad (14)$$

The resultant partial differential equation is identical with equation (12) if the terms connected with shear deflection and damping are omitted. Ochi (1958*e*) obtained the solutions for two phases of the vibration process, $0 \leq t \leq \tau$ and $t > \tau$, where τ is the duration of the impact force. The solution for the deflection and bending stress was given as the sum of deflections and stresses in various vibration modes and the investigation was carried up to the seventh mode. A novel and particularly important part of this work is the evaluation of the impact-momentum distribution among various modes by the least-work analysis. The damping was taken as a function of the vibration frequency; i.e., was different for each mode.

Ochi made computations for a bar with structural properties corresponding to the brass ship model which he had tested in a towing tank. The comparison of the computed and experimental vibration response to a slam is shown in Fig. 43. The agreement is shown to be good except in the first vibration cycle. Conceivably, at the first instant the impact momentum is primarily absorbed in shear deflection and the neglect of this in computations led to the incorrect evaluation of the first vibration cycle. A satisfactory agreement in the subsequent cycles can be interpreted as a confirmation

Table 10

Mode	Contributin to the bending moment ^a
1	-191.8
2	—
3	-61.2
4	—
5	+35.8
6	—
7	-78.4

^a In foot-tons for a full-size ship.

of Ochi's impact-momentum distribution among vibration modes and of his damping estimates. On the other hand, the results can be taken as fortuitous since neither shear deflections nor the variation of the mass and structural properties along the length of the ship were taken into account.

A contribution of the impact momentum to a particular vibration mode depends on the location of the impacting force with respect to nodal points of the elastic curve. Therefore, the distribution of a concentrated impact among various modes varies rapidly with the location of the impact. In reality the impact force is always distributed over a certain bottom area near a ship's bow. Ochi (1957, 1958*a*) measured the pressure distribution during model slams in a towing tank and approximated this distribution in the vibration analysis by five localized forces. The resultant contributions of various modes to the bending moment amidships is shown in Table 10.

It should be noted that contributions of even modes

(odd number of nodal points) vanish amidships. However, they will be significant at a certain point forward of amidships where the total vibration-caused bending moment should be at its maximum. Ochi's experiments, Fig. 42, show indeed the maximum to occur about 7 per cent forward of amidships.

5.53 Added mass and damping. The vibration of ships is governed by the same laws of dynamics as oscillations in waves and the reader is referred to Chapter 2 for much of the discussion. The vibration analysis is based on the strip method of evaluating a ship's mass and structural-property distributions. The effective mass is a mass of a ship's section of unit length plus a certain imaginary added mass, the acceleration of which gives the same force as that caused by water pressures. The evaluation of the added mass is usually based on theoretical work of F. M. Lewis (3-1929), J. Lockwood Taylor (3-1930b), and Prohaska (3-1947). No complete and reliable experimental verification of this material appears to be available. Model experiments may indeed be questionable because of certain water-separation effects during a part of an oscillation cycle (Keuligan and Carpenter, 3-1956; T. B. Abell, 3-1916). Theory indicates that free-water surface effects (wave making) should not be significant at vibration frequencies and the data, given by the investigators mentioned, should, therefore, be directly applicable.

The largest uncertainty is in the effect of three-dimensionality on the added masses obtained by two-dimensional strip theory. F. M. Lewis (3-1929) and J. Lockwood Taylor (3-1930b) made analyses of this effect at various vibration modes, and Lewis evaluated the results for the first and second modes. The importance of higher modes in slam-caused vibrations makes it desirable (if not mandatory) to extend these calculations to higher modes. In this connection it is necessary to mention the paper by Macagno and Landweber (3-1958) in which it was demonstrated that results of the analysis strongly depend on the assumed nature of a body's deflection; i.e., on movements of a body surface element in shear and bending deflections, including rotation of sections.

The evaluation of the damping in vibration appears to be more uncertain than evaluation of added masses. The following quotation from Ochi (1958e) may serve as an introduction to this subject: "We have little data which is sufficient to estimate the damping coefficient in ship vibration, especially to estimate the external (water) and the internal (structural) damping coefficient. Moreover, there are some differences between numerical values given in the following papers:

"[J. Lockwood Taylor (3-1930b)] gives the following simple values for small amplitude of two node vibration in full size ship:

$$\begin{aligned} b/\rho A &= 0.025 \text{ (for 80 rpm)} \\ &= 0.032 \text{ (for 99 rpm)} \\ &= 0.066 \text{ (for 148 rpm)} \end{aligned}$$

"where

$$\begin{aligned} b &= \text{damping coefficient} \\ \rho A &= \text{mass of ship per unit length} \end{aligned}$$

"McGoldrick (1954) mentions from the analysis of many data on full-scale experimental works that the damping in ship vibration appears to increase with frequency and the value of $b/\rho A \omega$ (where ω is a frequency in radians) is constant. He gives 0.034 as the mean value of $b/\rho A \omega$ for all modes of ship vibration. Kumai (1958) recently discussed damping factors in the higher modes of ship vibration taking into account the effects of shear deflection, rotary inertia, internal damping and also made some experiments. On the other hand, Sezewa (1936) made a theoretical consideration on four damping factors in ship vibration, namely, (1) water friction, (2) generation of pressure wave, (3) generation of surface wave, (4) structural damping force. He concluded that a generation of surface waves as well as a structural damping force are the main sources of damping, especially the former is pronounced in light draft condition."

As the result of his investigation, Ochi found that the internal damping of his brass towing-tank model formed 80 per cent of the total in the first mode and essentially 100 per cent in higher modes. He also found theoretically that the damping is proportional to the fourth power of the ratio α_n/L , where α_n is the characteristic number of the n th mode and L the length of the bar.

The reader's attention is called to three forms of expressing the damping of a vibrating system:

(a) The term "damping coefficient" has been used for the coefficient b of the velocity-dependent term (\dot{y} or \dot{z}) of differential equations of oscillatory motion [for instance equation (13)].

(b) The factor e^{-qt} occurs in solutions of free vibrating systems and is a measure of the rate of amplitude decay. Kumai (1958) refers to the quantity q as the "damping factor."

(c) The "logarithmic decrement," δ , which is connected with the damping factor by the relationship

$$\delta = 2\pi q/\omega \quad (15)$$

where ω is the natural frequency.

Ochi's solution of equation (13) (for pure flexural deflection) resulted in the evaluation of the damping factor

$$q = - \frac{b_f + \xi I(\alpha_n/L)^4}{2m} \quad (16)$$

where he used $\xi = 4.07 \times 10^4$ for a steel structure.

Kumai (1958) computed Table 11 which shows contributions of various factors to the logarithmic decrement based on calculations for a 32,000-ton tanker. The last column gives the empirically obtained logarithmic decrement for vertical vibration of ships from 260 to 660 ft long. This relationship for the two-noded vibration is

$$\delta_2 = C/L \quad (16)$$

where C is a coefficient with the value between 3 and 4, and L is a ship's length. For higher modes the logarithmic

Table 11 Numerical Results of Components of Logarithmic Decrement in Higher Symmetric Modes of Vertical Vibration of a Ship. From Kumai (1958)

No. of nodes	N_v cpm	δ_ξ	δ_η	δ_ζ	δ	δ_{emp}
2	62.04	0.01813	0.00313	0.00032	0.0216	0.0216
4	173.80	0.02410	0.02110	0.00010	0.0453	0.0467
6	289.30	0.02260	0.04070	0.00006	0.0634	0.0687
8	401.00	0.02180	0.06000	0.00004	0.0818	0.0890

Contributed by: ξ —normal viscosity, η —tangential viscosity, ζ —external damping.

mic decrement δ_n is related to the one at two-noded frequency,

$$\delta_n = \delta_2(\omega_n/\omega_2)^{3/4} \quad (17)$$

where ω_n is the natural frequency of the n -noded vibration and ω_2 that of the fundamental two-noded mode.

5.54 Similarity conditions in ship vibrations. The rapid growth of the damping coefficient with vibration frequency indicates that towing-tank models cannot directly represent the conditions of a full-size ship. Slam-caused vibrations in models are attenuated rapidly because of the heavy damping associated with high vibration frequency. In ships, on the other hand, the damping associated with a low frequency is small and vibrations persist a long time. This observation does not make model tests less valuable but it shows that interpretation of model tests can only be made with proper regard to the vibration-response theory.

In metal models used by Ochi and Sato, (Sections 4.2 and 4.3), the structural characteristics affecting bending deflections were similar to those of full-size ships. However, the reduced number of longitudinals and reduced support of the skin may have resulted in different shear characteristics. Careful consideration of these appears to be needed in view of the large effect of tangential viscosity on damping which is shown by Table 11.

Jasper (1958 appendix) investigated the scaling law for geometrically and structurally similar ships operating in similar seas. The results are summarized in the following quotation: "Thus it has been shown that the wave-induced stresses in similar ships, operating at the same speed-length ratio in similar seas, vary as the length of the ship for a suddenly applied step load, and as the square root of the length for an instantaneously applied impulsive load. The ordinary, slow varying, wave-induced stresses may be expected to vary as the length of the ship." This summary indicates that quasi-static wave-caused bending stress and vibration-caused bending stress should be considered as two distinct components to be added to obtain the total stress. A percentage increase of the wave-caused bending stress by slamming appears to be an untenable concept, except when comparing ships of similar size and type.

5.55 Dalzell's analysis of destroyer model vibration. Only the dynamics of the vibratory ship response to a slam was discussed in the foregoing sections. The principles on the basis of which the impact force in a slam can be estimated were outlined in Chapter 2 but it appears

that no actual evaluation of the force in a typical slam has been made. Ochi used for his vibration analysis the forces measured on a model in a towing tank. Furthermore, the available methods, based on Wagner's work, apply only to a sharp slam (i.e., bottom impact), similar in nature to the impact experienced in a seaplane landing. No methods of analysis were heretofore published for a relatively slow developing bow immersion, such as was observed by Warnsinck and St. Denis (3-1957). Dalzell's (1959) work appears to be the first attempt at an analysis of hydrodynamic forces involved in such a case, as well as of the resultant vibratory response of a towing-tank model.

The scope and objectives of this work best can be stated by quoting from Dalzell's introduction: "A number of investigations of the midship bending moments experienced by jointed wooden models in regular head waves at the Davidson Laboratory (Lewis, 1954; Lewis and Dalzell, 3-1958; Dalzell, 1959) have indicated the presence of nonsinusoidal forces at some time during each cycle, usually when wave length and model speed produce large motion amplitudes. The evidence takes the form of records showing vibration of the jointed model. Tests in irregular head waves show that the vibration occurs under conditions thought to be similar to those under which full-size ship slamming takes place. The purpose of this investigation was to determine whether the impulsive forces on the model could be calculated, utilizing existing theoretical methods and data."

"It was felt worthwhile to work with a regular wave case for simplicity, and the case chosen for study was that of a 5.71-ft destroyer model²⁷ at 6.0 ft/sec in regular waves 7.14 ft long. Experimental records for this case had been obtained and indicated both large motions and large model vibrations. (A complete description of the model and experiments may be found in Lewis and Dalzell, 1958). Further, the ship motion theory developed by Korvin-Kroukovsky had been used to predict motions for this case with reasonably good agreement with experiment (see Korvin-Kroukovsky and Jacobs, 3-1957). The theory can also be used to predict bending moment response in regular waves, and this had been done for the case under study by Jacobs (1958). Since these methods utilize coupled equations of motion with constant coefficients for a rigid body, the calculated motion and force responses to an assumed sinusoidal wave are also

²⁷ The model represents a destroyer 383 ft in length.

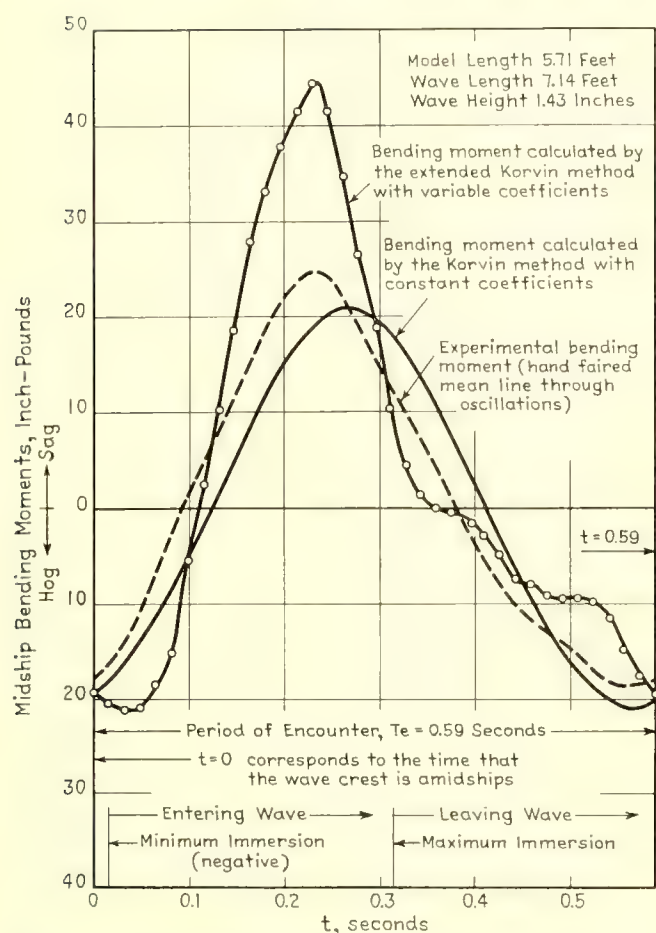


Fig. 44 Calculated and experimental rigid-body midship bending moments for a destroyer model in regular waves at 6 fps. Model length 5.71 ft, wave length 7.14 ft, wave height 1.43 in. (from Dalzell, 1959)

sinusoidal and thus do not reveal any impulsive forces. Experimental data indicate that the motions (heave and pitch) of the model seem to be little affected by whatever impulsive forces cause the vibration. Hence, it was decided to utilize the Korvin-Kroukovsky method to predict motions, then to evaluate the hydrodynamic loads at successive instants of time during the cycle assuming the calculated motions and sinusoidal waves. This evaluation was to be done taking into account the variation during the cycle of buoyancy and added hydrodynamic mass brought about by the geometry of the model."

An inertial force exerted by a fluid on a body results from the rate of change of the fluid momentum:

$$\text{Force} = \frac{d}{dt} (m''w) \quad (18)$$

where m'' is the hydrodynamic mass and w the relative velocity of the body with respect to the fluid.²⁸ In a seaplane landing (used by Szebehely to formulate ship

slamming analysis) the duration of the impact is short, the velocity is assumed to remain constant, and the force is defined as

$$\text{Force} = w \, dm''/dt \quad (19)$$

In the linearized analysis of ship motions, as developed by Korvin-Kroukovsky and Jacobs (3-1957), a wall-sided ship is assumed, the hydrodynamic mass is held constant, and the force is expressed as

$$\text{Force} = m'' \, dw/dt \quad (20)$$

Considering a relatively long duration of a destroyer impact and variable draft and wetted beam, Dalzell modified Korvin Kroukovsky and Jacobs' expressions to include variations of both the hydrodynamic mass and velocity; i.e., expressed the force as

$$\text{Force} = \dot{m}''w + m''\dot{w} \quad (21)$$

Quasi-static wave-caused bending moment resulting from this analysis is shown in Fig. 44 (extended Korvin method) for comparison with the linearized analysis of Jacobs (1958) (constant coefficients), and the mean line drawn through the oscillatory experimental data.

In discussing results of a pioneering attempt at a rational analysis, it is often advisable to consider separately the magnitudes and curve shapes (or trends). In the present case an apparently improved method of analysis resulted in large exaggeration of the bending moment. On the other hand, the analysis demonstrated a nonsinusoidal behavior of the bending moment and thus confirmed earlier findings of Horn (1910) and Hazen and Nims (3-1940). Demonstrated departures from a sinusoidal curve include a sharper peak at the maximum amplitude and steeper flanks of the curve, which are capable of exciting vibrations of a slender hull.

Calculations of the vibratory responses were made using calculated quasi-static bending-moment curve as an exciting function. Basic expressions for the vibratory response were taken from Frankland (1942) and Timoshenko.²⁹ Natural frequency and damping were determined experimentally. Only a single vibratory mode was present since the model consisted of two rigid halves jointed and kept aligned by the dynamometer flexure bar. The integration of equations of motion was replaced by the summation of finite differences, and calculations were carried through seven wave-encounter cycles in order to eliminate the transient response caused by uncertain initial conditions. The cyclic bending-moment variations were found to repeat themselves beginning with the fourth cycle.

Model speed of 6.0 fps was chosen for the analysis because at this speed the period of wave encounter was an exact multiple of the model's natural frequency and the calculations were thereby simplified. Unfortunately, at the nearest test speed of 6.1 fps test results exhibited a rather exaggerated amount of vibration, which did

²⁸ Symbol w is used here for the vertical velocity of an impacting body.

²⁹ "Vibration Problems in Engineering," D. Van Nostrand Co., New York, N. Y., third edition, 1955.

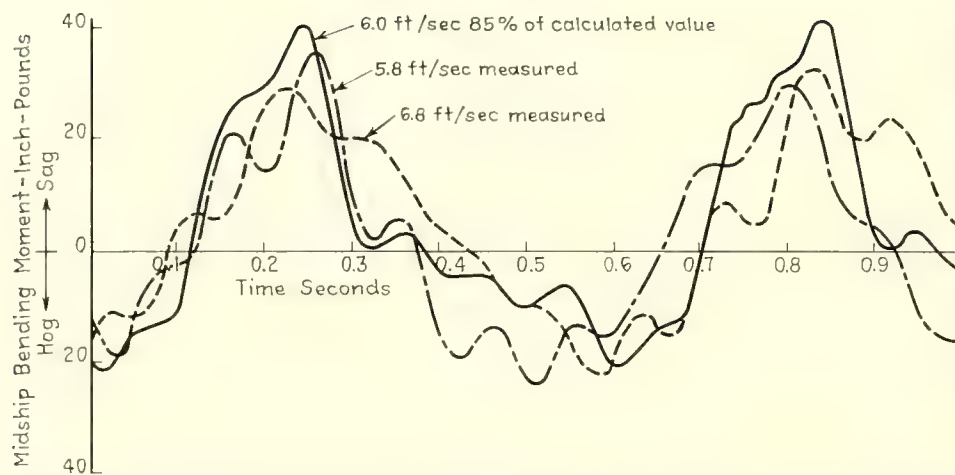


Fig. 45 Calculated and measured elastic responses of a destroyer model (from Dalzell, 1959)

not exist at slightly lower and higher speeds of 5.8 and 6.8 fps. The comparison of calculated and experimental data in Fig. 45 is based, therefore, on these latter speeds. The exaggeration of the exciting bending moment, shown in Fig. 44 would lead to the exaggeration of computed responses. The "calculated 6.0 fps" curve in Fig. 44 was drawn, therefore, taking 83 per cent of the original Dalzell's data in order to bring out more clearly the comparison of the time behavior of bending moments. The calculations brought out the, experimentally observed large magnitude of the sagging moment as compared to the hogging one. Calculations also brought out the existence of experimentally observed vibrations, although details of the calculated and observed vibrations did not agree. Indeed, it appears to be almost hopeless to reproduce a detailed pattern of vibrations by calculation considering the rapidity with which this pattern changes with small changes of model speed in experiments. All one can expect is to evaluate the amplitude of the vibratory response which must be added to the quasi-static wave-caused bending moment.

5.6 Statistical Theory of Slamming. It is apparently impossible to predict the exact shape of a wave in an irregular sea at a given instant and to predict a ship's attitude with respect to it. The detailed description of the conditions leading to a slam in irregular sea, therefore, cannot be formed. Valuable information, however, can be obtained by the modern methods of mathematical statistics. Any characteristic of a ship's response to regular waves can be established either by calculation or by towing-tank tests. Such characteristics can include, for instance, the vertical motions of a ship's forefoot and the vertical velocity of the bow. From experience, a set of characteristics significant to slamming can be specified and the joint probability of their occurrence can be statistically evaluated. Thus, L. J. Tick (3-1954) has established the joint probability of the bow emergence and a vertical velocity of the bow exceed-

ing a specified value. However, a slam will occur only if a ship's keel is nearly parallel to the water surface over an appreciable part of the ship's length near the bow. In computing the joint probability, L. J. Tick (3-1958) extended his work to include a specification of a certain small keel-to-water angle. Typical slamming conditions are well defined by the joint probability of the bow emergence, a certain small angle of the keel to water and a certain vertical velocity of the bow. The significant values of two necessary parameters, the keel-to-water angle and the bow velocity, can next be defined by the analysis of slamming records at sea. They evidently will depend on the ship-form parameter, such, for instance, as Ochi's section coefficient at 10 per cent of the ship's length at 50 per cent of design draft.

The statistical methods developed by Tick do not directly indicate the severity of a slam. This, however, can be evaluated indirectly. The probability of occurrence of various vertical bow velocities can be computed. For any specified vertical velocity the hydrodynamic impact force can be computed by the Wagner-Szebehely-Todd method.

6 Statistical Data on Ship Bending Stresses—Related to Sea Conditions

In this section, attention will be called to statistical data on ship bending stresses which are related to sea conditions. These latter are not measured but are given on the basis of visual observations, ship's log data, or sometimes even wave forecasts. Only the general conditions prevailing at the time of stress measurement are given. Detailed relationships between wave profile and stresses, which were discussed in earlier sections, are not available in the records which are discussed here.

The data of the type just described are available from the following sources:

Table 12 Record of Stress Ranges Exceeding 8000 psi—SS Mormacmail. Abstracted from E. V. Lewis (1957c)

Date	Avg. speed, knots	Sea		Wind		Number of counts ^a in stress range, psi		
		Description	Relative direction	Beaufort scale	Relative direction	8-12000	12-16000	16-20000
1955								
1/26	17.08	Moderate	Beam	3	Var	1	0	0
1/27	16.70	Moderate	Quarter	4	Var	1	0	0
1/30	15.87	Rough	Head	5	2S	3	0	0
1/31	13.00	Rough	Head	8	3S	7	0	0
2/3	15.0	Rough	Head	6	Var	9 ^b	0	0
3/3	14.2	Rough	Head	6	Var	7	0	0
3/5	16.0	Moderate	Beam	5	Var	2	0	0
3/25	17.14	Moderate	Following	6	160P	1 ^c	0	0
3/26	17.13	Rough	Following	6	Var	1	0	0
3/27	16.00	Rough	Beam	6	Var	5	0	0
3/29	14.4	Heavy	Confused	8	Var	1	0	0
3/30	12.00	Rough	Head	9	Var	36	0	0
3/31	5.08	Heavy	Head	8	Var	53	0	0
4/1	6.83	Rough-heavy	Head	9	25S	32	0	0
4/2	15.39	Moderate	Head	5	Var	19	0	0
4/3	13.0	Rough to mod.	Quarter	9	Hove to— 9 hr	1	0	0
1956								
2/23	16.17	Moderate	Beam	5	129P	1	0	0
2/28	16.7	Rough	Following	8	177P	69	0	0
2/29	16.66	Rough	Quartering	8	154P	69	0	0
4/3	16.72	Moderate	Beam	5	Var	1	0	0
4/11	13.2	Moderate	Beam	4	104P	6	0	0
4/12	15.16	Rough	Beam	9	Var	1	0	0
4/16	19.32	Rough	Following	9	Var	4	0	0
4/24	15.72	Moderate	Beam	4	285S	1	1	0
4/26	16.00	Light	Beam	2	266S	1	0	0
4/29	16.43	Light	Beam	2	58S	1	0	0
4/30	16.61	Light	Beam	2	115S	1	1	0
5/1	17.13	Light	Quarter	4	Var	0	1	0
5/26	15.41	Moderate	Head	5	Var	1	0	0
5/27	11.33	Heavy	Head	5	Var	66	2	0
5/28	5.76	Very rough	Head	10	40S	1	0	0
5/29	14.36	Rough	Head	6	12S	44	0	0
5/20	10.8	Rough	Head	8	15S	2	0	0
6/3	16.9	Light	Beam	2	77P	1	0	0
6/26	—	1	Quarter	4	112P	1	0	0
6/27	—	4	Following	5	153P	1	1	0
7/20	—	6	Beam	6	86S	1	1	0
Total.....						452	7	0
Per cent of total counts over 500 psi ^d						0.10	0.0015	0

^a Per day if not otherwise noted.^b In 10.6-hr period.^c In 14-hr period.^d Total number of counts over 500 psi was approximately 450,500.

1 Kent, Kempf, and Hoppe data described in Section 3-5.11 and Table 3-3.

2 Schnadel's data described in Section 3-5.12 and Tables 3-4 and 3-5.

3 Data of the Admiralty Ship Welding Committee on SS *Ocean Vulcan*. General description of the observations was given in Section 3-5.13. Tables of stresses measured under different weather conditions are given at the end of Report No. 8. This report also contains a summary of stresses measured by previous investigators on several ships.

4 Data obtained on SS *Nissei Maru* as described in Section 3-5.15 and in Tables 3-8 and 3-9.

5 Data obtained on SS *Mormacmail* and SS *Mormacpenn* as reported by E. V. Lewis (1957c).

For the purpose of the present monograph it will be sufficient to discuss the last set of data. Strain gages

and stress cycle counters were installed by the David Taylor Model Basin under weather decks of two C-3 type cargo ships. On each ship there was a gage on one side of the ship only. The counters were arranged so that they counted all of the ranges of stress (hog + sag) exceeding each of the following levels: 500, 4000, 8000, 12,000, 16,000, and 20,000 psi. The counters were read once each watch by the ship's officers. Once a day weather, sea conditions, ship speed, and course also were recorded. These data were tabulated and presented to S-10 panel of The Society of Naval Architects and Marine Engineers by E. V. Lewis (1957c). The data were obtained during seven voyages of SS *Mormacmail* and five voyages of SS *Mormacpenn*. The voyages (double crossings) were made over the north part of the North Atlantic between northeast coast ports of the United States and Scandinavian and Baltic sea ports, occasion-

Table 13 Record of Stress Ranges Exceeding 8000 psi—SS *Mormacpenn*. Abstracted from E. V. Lewis (1957c)

	Avg. speed, knots	Sea		Wind		Number of counts in stress range, psi		
		Description	Relative direction	Beaufort scale	Relative direction	8-12000	12-16000	16-20000
1955								
9/12	15.5	Moderate	Bow	5	33P	2	0	0
9/13	14.2	Rough	Head	7	15P	106	0	0
9/14	11.2	Rough	Var	7	Var	8	0	0
9/15	13.4	Very rough	Bow	8	42S	188	4	0
9/16	11.2	Very rough	Var	11	—	40	0	0
9/17	12.0	Very rough	Var	11	—	302	13	—
9/18	6.6	Very rough	Bow	10	85S	13	0	0
9/19	16.2	Moderate	Beam	3	85S	2	0	0
9/20	16.0	Moderate	Head	3	9S	8	0	0
9/21	9.1	Very rough	Beam	7	72S	198	17	1
10/8	16.59	Moderate	Quartering	4	122S	1	0	0
11/20	15.9	Rough	Beam	6-7	69P	55	0	0
11/21	12.2	Very rough	Bow	7-8	40P	1	0	0
11/22	14.04	Very rough	Bow	7	54P	34	0	0
12/14	15.62	—	Bow	5	82P	44	1	0
12/15	10.82	—	Bow	9	79P	124	1	0
12/16	12.17	—	Bow	6	82P	69	0	0
12/17	14.83	—	Bow	5	29S	30	0	0
12/18	14.04	—	Bow	6	39S	13	0	0
12/25	15.56	—	Quartering	7-8	24S	1	0	0
1956								
1/12	16.96	—	Quartering	4	64P	3	0	0
2/7	15.30	—	Beam	7	—	34	0	0
2/8	7.61	X9	Bow	10	Hove to	244	13	0
2/15	15.00	—	Following	4	—	1	0	0
3/17	16.46	—	Following	3	100P	1	0	0
3/20	16.08	—	Beam	4	68S	4	2	0
3/21	14.64	—	Bow	6	84S	1	0	0
3/22	16.33	—	Bow	4	84S	1	0	0
3/24	16.64	—	Quartering	2	18P	1	0	0
4/8	14.66	Rough	—	3	Var	154	6	0
4/9	16.08	Moderate	—	2	Var	5	0	0
4/10	15.41	Moderate	—	1	112S	1	0	0
4/11	15.56	Moderate	—	1	22P	4	0	0
5/29	15.92	Moderate	Following	2	158P	2	0	0
5/30	—	—	—	—	—	1	0	0
5/31	14.11	Rough	—	3	Var	61	3	0
6/1	15.88	Moderate	Beam	2	188P	1	0	0
6/2	16.32	Light	Bow	2	58S	1	0	0
6/5	17.32	Calm	—	0	25P	1	0	0
Total.....						1760	60	1
Per cent of total counts over 500 psi ^a ...						0.91	0.03	—

^a Total number of counts was approximately 193,300.

ally touching Iceland. The parts of the ocean and the times of the year covered by these voyages are known for the great frequency of adverse weather.

Lewis' (1957c) tabulations included all days of the voyages and all stress ranges just cited. The abstract of these data, is given in Tables 12 and 13 and includes only the data on stress ranges exceeding 8000 psi. Figs. 46 and 47 show the number of counts per day of stress ranges between 8000 and 12,000 plotted against average wind force for head seas and for beam, quartering and following seas, respectively.

A perusal of the described material shows that:

1 The stress range of 12,000 psi is exceeded in a relatively small number of cases. There is but a single reading over 16,000 psi.

2 The number of stress counts in the range 8-12,000 psi in head and beam seas increases with wind force.

3 At the wind strength 7, the stress range 8-12,000

psi is encountered more often in beam seas than in head seas.

4 The stress range 8-12,000 psi is encountered in all sea directions and in all winds from Beaufort 1 to 11.

5 Sea directions and wind strength appear to affect only the frequency of occurrence of stress in this range and not the stress level.

6 The conventional method of describing wind and sea does not provide indication of stress magnitude, although it provides indication of the frequency of the stress occurrence.

The difference in stress response of two apparently identical ships is of interest. In the SS *Mormacpenn* the stress range 8-12,000 psi was encountered 9 times more often (in relation to the total number of counts) than on the SS *Mormacmail*. The stress range 12-16,000 psi was encountered 20 times more often.

The author had the privilege of being aboard the SS

Mormacpenn on the eastward crossing of the fourth voyage. This included February 8, 1956, on which the second largest numbers of counts in the ranges 8–12,000 and 12–16,000 psi were recorded. Numerous observations during this crossing indicated that the ship had an abnormally short rolling period. As the result of this, it rolled violently with the amplitude of ± 25 deg in head seas at the time of maximum pitching. Since the strain gage was installed on the port side only, it is possible that high frequency of stress in the foregoing ranges resulted from the addition of the vertical and lateral bending moments. Attention should be called to the fact that terms vertical and lateral refer to ship coordinates. A large lateral component may occur simultaneously with the vertical one when the ship's bow plunges into a wave while the ship is at a large roll angle.

In the literature on ship stresses (for instance, SS *Ocean Vulcan*) the impression is given that the addition of the lateral moment is seldom important. With ships of reasonably long rolling period, significant rolling does not occur in head or bow seas. With an abnormally short rolling period, however, a severe rolling in head seas may occur. In such a case there is a great probability of the significant effect of the lateral moment. This indicates that the information on rolling should be included with other data describing ship conditions during stress measurements.

7 Statistical Data on Ship Bending Moments— Unrelated to Sea Conditions

Jasper (3–1956) presented a large collection of statistical data on bending stresses measured on several ships. These data were obtained from unmanned stress gages and stress-cycle counters, and the ambient weather conditions were not recorded. The nature of the data permits, therefore, studies to be made of various statistical laws to which stress distributions are subjected. It also indicates the stress levels which can be expected on ships at sea. However, the relationship of these stress levels to the conditions which caused them is not determinable.

Since the statistical stress-distribution laws were investigated separately on several ships, it would appear to be possible to determine the effect of a ship form and loading on stresses. However, the analysis of the relationship between the ship form and stress level was not included in Jasper's paper.

Additional data on Jasper's paper will be found in Section 3–5.2. This section includes two quotations defining the utility of the statistical information which was collected and developed by Jasper.

7.1 Maximum Bending Moment Expected at Sea. In his paper, Jasper (3–1956) included the discussion of the maximum bending moment expected at sea. This is based on the extrapolation of the frequency distribution of observed data using strictly statistical methods. The author does not believe that this approach can be fruitful. This opinion is based on the following:

1 In a random distribution of measured stresses, any arbitrarily large stress can be indicated provided sufficiently low probability of occurrence is specified.

2 The extrapolation is based on the observational data in which a very small number of occurrences of a high stress is found.³⁰ The starting point for the extrapolation is therefore uncertain.

3 A statistical extrapolation necessarily assumes an unchanged environment. However, the physical conditions of a ship among waves change with excessive increase of sea severity. The nonlinearities, which normally may not be important, grow in importance in waves of extreme steepness.

It is the author's opinion that the evaluation of extreme stresses should be based on the consideration of the physical conditions of ship operation. It has been shown by calculations and by model tests that maximum bending stresses are caused by waves of the length in the range of 0.75 to 1.25 of a ship's length. Within this length range, the stresses increase with wave steepness; i.e., with the ratio of the wave height to wave length. The wave steepness, however, cannot increase without limits. In regular waves the limiting steepness is $1/7$. Statistical methods, used in describing an irregular sea, do not permit at present prediction of the steepness to be expected in waves of a specified length. The research in formulating the necessary mathematical methods for this purpose is recommended.

Sea observations on wave steepness as a function of wave length had been made and were described in Chapter 3.³¹ Relatively mild steepnesses were observed. The extrapolation of these empirical observations to the extreme conditions is, however, extremely uncertain. Therefore, a theoretical research, mentioned previously, is recommended.

Ship stresses do not increase in proportion to wave steepness. As the wave steepness increases, the action of the relieving dynamic effects rapidly increases. An effective limit to the maximum stress can therefore be expected.

It may be of interest to imagine a case of a small ship carried on a crest of a regular wave of the extreme $1/7$ steepness. In appearance, the ship is in the hogging condition as it is supported by the sharp crest of the wave. In reality, the dynamic acceleration effects are such that the ship and the water are essentially weightless, being in the condition of a free falling body. The bending stresses are nil in this extreme condition.

Theoretical and experimental model research are recommended in order to evaluate the extreme bending moment conditions on physical grounds.

8 Concluding Remarks and General Research Suggestions

Chapter 5 of the present monograph is concerned with the methods of evaluating the loads on the basis of which

³⁰ This for instance is illustrated by Tables 12 and 13.

³¹ See Figs. 3–37, 38 and 44.

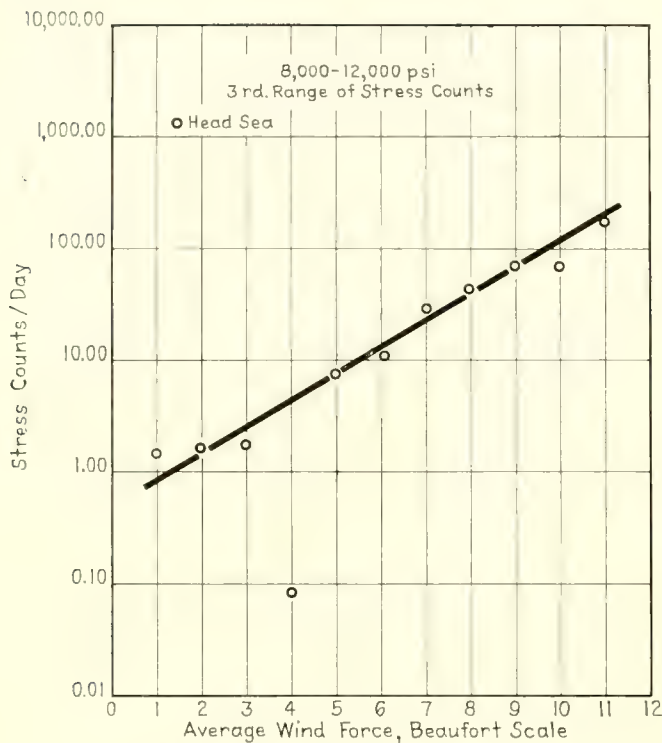


Fig. 46 From E. V. Lewis, 1957

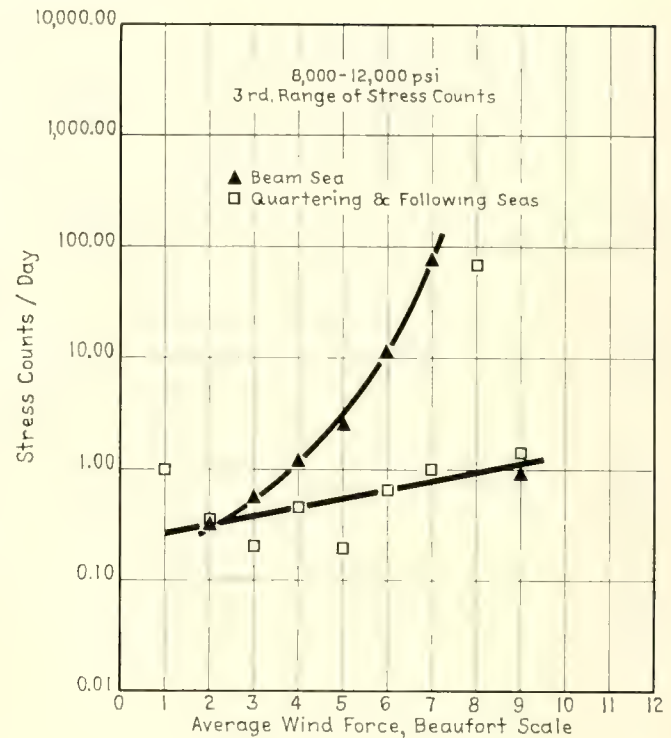


Fig. 47 From E. V. Lewis, 1957

ship bending moments and stresses can be computed. The conventional static method has served well as a yardstick for the evaluation of bending moments. However, it does not represent the physical conditions of a ship moving among waves. During several decades in the past, isolated attempts were made to include in the analysis various aspects based on the wave and ship dynamics. In the past few years it became possible to combine these separate attempts into a single logical system of bending-moment calculation. This system was outlined in Section 2 and it can now serve as the basis for further research.

8.1 Ship Motions and Damping Forces. It has been demonstrated that the load distribution is composed of a number of components defined by displacements, velocities, and accelerations of ship masses as well as of water in waves. The evaluation of ship motions is therefore a necessary prerequisite to the evaluation of the structural loading curve and of bending moments. Therefore all suggestions for research, made in connection with Chapters 2 and 3 are directly relevant to calculations of bending moments. In particular, the poor state of knowledge in regard to damping forces must be emphasized. Neither of the two available methods of damping-force calculations³² is found to agree with experimental data.³³ There is an acute need for both theoretical and experimental data on damping forces of prismatic bodies as

well as on the distribution of these forces along a ship. The inadequacy of the available data adversely affects but does not invalidate ship-motion data. However, the bending-moment calculations require greater accuracy and are more strongly affected.

8.2 Need for Ocean-Wave Data. Bending moments as well as ship motions at sea can be estimated in two steps: (a) The response of a ship to regular waves must be evaluated; and (2) a ship's behavior in irregular sea waves can be determined by the methods of mathematical statistics. The second step is based on the knowledge of ship responses to regular waves and of the irregular sea spectrum. Review of the ocean-wave knowledge was made in Chapter 1 and it was shown that the available knowledge is inadequate. Even if only the wave spectrum at a point is considered, there is no unanimity of opinion among different research groups as to the spectrum form and as to the wave height corresponding to given atmospheric conditions. There appears to be practically no data on the directional spectrum in an open ocean. Yet, the directional spectrum, which causes the wave short-crestedness, is very important in defining ship motions and ship stresses. In Section 6 and in Figs. 46 and 47, it is shown that high stresses occur frequently in a beam sea and this fact is evidently caused by the wave short-crestedness. As far as the rational evaluation of a ship's bending moment at sea is concerned, the lack of reliable data on the sea wave spectrum appears to be by far the most important factor.

In view of the foregoing, the evaluation of the sea

³² Holstein-Havelock's and Grim's methods.

³³ Golovato (3-1956, 1957); Gerritsma (1957).

spectrum in various weather conditions must be considered as the most important direction of research leading to the rational evaluation of ship stresses. Since apparently insurmountable differences of opinion exist between certain currently dominant research groups, it is suggested that efforts be directed to interest and support additional research groups, not yet associated with one or another of the mutually opposed opinions. The foregoing remarks are made here only as a reminder on the importance of research on conditions of the sea. For a more detailed exposition the reader is referred to Chapter 1.

8.3 Linear Theories and Model Tests. The linearized ship-motion and ship vertical bending-moment theories in head or following seas appear to be in a reasonably good state. They were formulated in a general form which includes all aspects affecting a ship's behavior. Further improvements are necessary, of course. However, they will be directed primarily to the evaluation of various coefficients entering into differential equations of motions. The most important of these are the coefficients of damping forces, and the acute need of further research on these was pointed out earlier.

Towing tank data on ship-model motions in head regular seas appear to be satisfactory and are in reasonably good agreement with calculated motions at model speeds which are higher than synchronous speed. Model test data appear to be unreliable for speeds below synchronism because of the interference of waves reflected from the towing tank walls, Abkowitz (3-1956a). The only measurements available to date in a wide maneuvering tank, Numata (1957), indicate considerable differences in model motions from the results obtained in long tanks at and below synchronous speed. Since ships are always operated below synchronous speed in heavy weather, the additional experimental research in wide tanks in the low speed range is necessary.

Model tests in towing tanks have given clear qualitative information on the magnitude of bending moments as functions of encountered waves. This information, in agreement with theory and with meager data obtained at sea, shows that a ship's bending moment is much smaller than is indicated by static calculations. Static calculations exaggerate bending moments even if the Smith effect is taken into account. The inertial forces of ship masses and hydrodynamic forces resulting from interaction of a ship and waves further reduce bending moments. Rationally computed as well as experimentally measured bending moments are much smaller than statically computed ones in any given regular wave. On the other hand, individual waves in a typical irregular sea are often much steeper than the 1:20 wave used in conventional static calculations. As a result, the actual bending moments in irregular sea are of the same order of magnitude as conventionally computed ones in the case of normal cargo ships.

Nevertheless it is recommended that the research in further development of rational methods of bending-moment calculations be pursued. The research in this

domain already has indicated that the bending moment experienced by a ship is a relatively small difference of several large static and dynamic components of different signs. It can be imagined readily that, in an irregular sea, an unfavorable combination of several components may occur from time to time. Unusually high stresses may then result. Such an occasional unfavorable combination may explain the recorded occurrence of high stresses under apparently mild-weather conditions, Section 6.

Quantitatively, significant differences are observed in bending-moment behavior as measured on models by different investigators. These differences are particularly conspicuous in the variation of bending moments with model speed and in the amount by which the sagging stresses exceed the hogging ones. Further model research in bending-moment and shear distribution is, therefore, recommended. The probability of important wall interference in long tanks indicates that research programs in wide tanks are needed.

8.4 Nonlinear Theories. Previous remarks on the rational theory of ship motions and bending moments were based on a linear theory. The significance of nonlinearity in defining ship motions at large amplitudes is uncertain, and carrying out of nonlinear calculations of motions was suggested in Chapter 3. However, effect of nonlinearity on motions is not believed to be important for ships of normal form, and this direction of research can be assigned a lower priority than the calculations and measurements of mean added masses and damping forces. This opinion is based on the fact that ship motions are obtained by double integration of accelerations caused by hydrodynamic forces. Nonlinearities cause certain short-duration deviations of instantaneous forces from the mean harmonic variation. The effect of these short-duration variations is smoothed out in the process of double integration³⁴ and the motions are mainly defined by the mean energy transfer between a ship and waves.

A different situation exists in the case of bending moments. The variations of local displacement and hydrodynamic forces are felt instantly by a ship's structure while inertial forces depend on the ship's motions. The bending moments are caused by the unbalance among hydrodynamic and inertial forces and, therefore, can be expected to be sensitive to nonlinearities. The increase of sagging bending moments of destroyers with the wave height and a ship's speed (Sato, 1951; Lewis and Dalzel, 1958) is suspected to be caused by nonlinearities resulting from the submersion of a flared bow in waves. The foregoing discussion and the model experiments indicate that the motions remain nearly harmonic in regular waves and are not changed appreciably even by slamming. Hence, the instantaneous hydrodynamic forces can be calculated considering the actual

³⁴ The expression "double integration" should not be considered as applicable to calculations but rather as the description of a physical process in which time is needed for accelerations to develop velocities, and for velocities to develop body displacements.

conditions of submergence, velocity, and acceleration at each ship section while computing these latter on the basis of the linear theory of motions. Only a pioneering attempt in this direction was made by Dalzell (1959) and further work of this type is suggested.

8.5 Ship Motions in Seven Degrees of Freedom.³⁵ Measurements of stresses on ships at sea (E. V. Lewis, 1957c), shown in Figs. 46 and 47, indicate frequent occurrences of high bending stresses in other than head waves. The necessary material for rational analysis is completely lacking in this case. First of all, a reliable knowledge of the directional sea spectrum is necessary and is not yet available. Next, the calculations of ship motions in seven degrees of freedom³⁵ must be formulated. The formulation of the equations of motions has been accomplished by Kriloff (3-1898) and is also found in aeronautical literature. A large amount of theoretical and experimental research is needed, however, for evaluation of the necessary coefficients of differential equations of motion. This subject was discussed in greater detail in Chapter 3. However, the subject is again emphasized here because the data of E. V. Lewis' (1957c) report indicate the significance of oblique sea conditions in causing large ship bending moments.

8.6—Slamming: 8.61—Bottom impact. The problem of ship slamming has been treated quite inadequately in the past. No clear distinction has been made between (a) the high pressure significant for the bottom plating, (b) the total bottom impact force causing ship vibrations and affecting ship bending moments, and (c) the force resulting from the submersion of a flared bow of ships designed for high speed. The loads, listed under (b) and (c), are applied to a ship's structure with such a rapidity that ship vibrations are excited. The relationship between the load and the stress cannot be determined in this case by the rules of statics and it is necessary to investigate the elastic response of a ship's structure. It appears that little attention has been given so far to the study of these elastic effects.

Theoretical evaluation of bottom pressures in slamming has been developed only in the past few years by Szebehely and his associates at the David Taylor Model Basin. This activity was based on theories of H. Wagner which had been found valid in seaplane engineering for surfaces of large deadrise (say over 10 deg). In the case of ships, the theory apparently gave good results in the estimate of the total impact force when the edges of the wetted area reached the turn of the bilge. It indicated, however, peak pressures sometimes exceeding 1300 psi, which apparently are many times higher than those measured at sea or estimated from ship-damage observations.

Attention should be called to the fact that these high pressures are indicated both by Wagner's expanding-plate and Wagner's spray-root theories. The first of these, however, is intended to represent the total force

acting on a plate, but does not represent correctly the flow condition at the edge of the wetted area. The spray-root theory represents these conditions correctly, provided the deadrise angle is not too small. It is a potential-flow theory and it remains valid as long as the fluid velocities change not too rapidly with distance; i.e., the velocity gradient is small. If the velocity gradient is large, the viscosity becomes significant and the potential flow breaks down. These conditions can be expected to occur at a spray root of a V-section with a very small deadrise angle. Flat bottoms or bottoms with excessively small deadrise should be avoided in the fore parts of ships. As long as they are used, however, research on the impact of such surfaces is recommended. It should represent an extension of Wagner's work but should not be an indiscriminate application of it. The physical conditions existing at a low deadrise should be taken into account.

The application of Wagner's expanding-plate theory to the calculation of the total impact force involves the concept of an added mass. Very high forces at low deadrises result from the consideration of the added masses of water as though they were real masses of a fixed magnitude. In this approach a force of infinite magnitude is predicted in the case of a flat-bottom impact. After reaching this point, the writers on the subject usually invoke the elasticity of water or of a body's structure in order to explain this physically impossible result. However, a quantitative analysis of the effect of these elasticities is lacking. Two alternate research programs can be suggested here: (a) The development of an impact theory in which the hydrodynamic mass is derived on the basis of true physical conditions³⁶ and is not assumed in advance. (b) The quantitative evaluation of the effects of water's and ship structure's elasticity.

The research directed to obtaining reliable experimental data on impact pressures is recommended. As this was suggested in Chapter 2, the mean pressure over a certain bottom area should be measured rather than a peak pressure at an isolated gage. The size of the area should correspond to a typical area of a ship's bottom plating between supports.

8.62 Slamming—submersion of a flared bow. In the foregoing paragraphs attention was concentrated on the aspects of slamming which affect the bottom plating and which define the total forces resulting from bottom impacts of cargo ships. It is suggested that such calculations as were made by Bledsoe (3-1956) for cargo-type ships be extended to cover the full submersion of a flared destroyer bow. In this case the impact lasts an appreciable amount of time³⁷ and the impact force must be presented as a function of time. Typical combinations of the ship and wave motions should be assumed. These can be obtained either from model tests or from

³⁵ Not forgetting the extremely rapid changes of the flow pattern with time, and the effects of viscosity when the velocity gradient becomes excessive.

³⁷ Of the order of 1 sec.

³⁶ Surge, sideways, heave, rolling, yawing, pitching and rudder motion.

calculations of motions. Basing the entire investigation on calculations can be expected to yield more consistent results from which it will be easier to derive various trends. Unlike the bottom impact of cargo ships, the force acting on a submerged destroyer bow consists of the static buoyancy and of dynamic pressure forces of the same order of magnitude. The measurements of the time history of the hydrodynamic force during flared bow submergence is also recommended. This can be done on models in towing tanks. Care must be taken to consider the hydrodynamic force and not the model's elastic response. The latter appears to differ from the response of a full-size ship.

8.7 A Ship's Elastic Response. By far the most neglected and most needed field of research in ship stresses lies in the determination of a ship's elastic response to a slamming impact. This subject was outlined in Section 5.5. The importance of it was demonstrated by the preliminary data on destroyer tests published by Warnsinck and St. Denis (3-1957, NSMB Symp.). In these tests the bending stress amidships was more than doubled by the slamming impact.³⁸ The important feature of the impact action is the fact that the critical stress does not occur at the time of impact but some time later. The first effect of the impact is to excite ship vibrations of all modes. The superposition of several modes of vibration defines the transient state. During the transient state, the initial peak hogging bending moment at the point of an impact gradually but rapidly changes to the peak alternating hogging-sagging moment slightly forward of amidships. This is associated with a two-node mode of vibration. In the transient process, dangerous peaks of stress may occur at structural discontinuities and also at discontinuities of mass distribution. The bending stress of the two-node vibration is superposed over the maxima of several bending-moment cycles which occur at the frequency of wave encounter. Not only the maximum stress is often more than doubled but also a large number of stress fluctuations is thereby produced.

8.71 Suggested procedure in research on elastic response. As has been stated already, the research in the elastic response of ships appears to be in its infancy. Very few references were found in the literature and these were mentioned in Section 5.5. These references had given a rather incomplete treatment of an impact on a slender beam and have not yet considered a complete ship structure. In its entirety, the problem is evidently extremely complicated. In order to formulate a fruitful research and to make rapid progress, it is necessary to subdivide it in its composite parts.

It appears to the author that it is possible in the first approximation to separate the impact investigations from the investigations of the vibratory characteristics of ships. A large literature already exists in regard to the

latter.³⁹ It deals with free vibrations and with the steady-state forced harmonic vibrations. The most important results of investigations are the evaluation of the locations of nodal points and of frequency of different vibration modes. Next in importance, and apparently less developed, is the experimental evaluation of damping characteristics for different modes. The determination of these characteristics for higher modes is important in evaluating the transient vibration after impact.

An analysis of the impact response can be simplified by assuming the location of the nodal points and of frequencies of vibration as they are found from the aforementioned analysis of harmonic-vibration characteristics. The analysis of an impact response can then be made assuming the structure to be an artificial simple slender beam limited to the modes of vibration described. The problem is reduced to finding the amplitude of each mode of vibration as a function of an impulse $P\tau$ and its location with respect to the locations of nodal points.

It is believed that the simplest form of solution will be obtained with a rectangular impulse of a localized uniform force P acting over an extremely short time τ . A certain distribution of impulses along a ship's length will be necessary in order to represent a pressure area of a given magnitude. For the more slowly applied impacts of flared bows, such as are found in destroyers, an additional distribution of impacts with time will be necessary.

In the form of research suggested above the problem is subdivided in four stages:

1 Determination of frequencies and locations of nodal points for a number of vibrational modes. An evaluation of the corresponding damping characteristics (by analysis of experiments).

2 Distribution of amplitudes excited in all significant modes by a single impulse. A simple beam theory is used, but the beam is artificial in that it is limited to the vibration characteristics found under (1):

3 After completion of the foregoing solution, the transient time history of deflection and stresses can be computed readily by simple summation of deflections caused by all vibratory modes along the entire length of a ship.

4 Superposition of deflections caused by impulses distributed along a ship's length and with time in order to represent an actual force distribution.

Some readers may not agree with the simplifying order of procedure suggested and may formulate another approach. The author only wishes to emphasize the importance of the research in this field and the need to simplify the complicated problem. It appears to him that a principle "to divide and to conquer" is a promising approach, which moreover, permits spreading the work among many investigators. Apart from the funda-

³⁸ Previous data were obtained on slow cargo ships and indicated bending moment increases of the order of 20 per cent.

³⁹ The investigation of a ship's structural characteristics is outside the scope of the present monograph. The following two references can be given, however, as illustrating the latest achievements in this field: McGoldrick and Russo (1955) and Csopor (1956).

mental difficulties in formulating a satisfactory approach, the difficulties are expected in the attainable accuracy of evaluating characteristics of higher modes of vibration. These may be important in the early stages of the transient process. They also may be important in defining stress upsurge at structural and mass discontinuities.

8.72 Experimental data on elastic response. The experimental data on structural responses to impulses are evidently needed. The organization of experiments and the reporting of test data should correspond to theoretical outline given. The deflections and stresses must be reported as functions of time as well as of the location along the beam. Experiments with simple rods in a structural laboratory will serve to verify the simple beam theory. Built-up ship-like structures may be used in a laboratory in order to verify the "artificial-beam" approach suggested earlier. Observations on ships at sea must include the installation of a series of strain gages along a ship's length. Simultaneous records must be taken so as to bring out the time history of all gage readings. Care must be taken in locating the gages to make them represent the general structure and to be free from areas of structural or mass discontinuities. The effects of these latter must be investigated separately by additional groups of gages.

It has been mentioned frequently that slamming has a great effect on bending stresses in destroyers and only a mild effect on cargo ships. It is suggested therefore that sea research on elastic response to slamming be made on destroyers or other ships designed for high speed. This research will require a more elaborate strain-gage installation than is common in the bending-moment research. As much investigation as possible should be done in harbor since the difficulties of maintaining the extensive equipment at sea were emphasized in the work on the *SS Ocean Vulcan*.

The strain-gage installation should be supplemented by the optical deflection measurement similar to those used by Kempf on the *SS Hamburg* and Schnadel on the *MS San Francisco*. However, a continuous optical recording must be devised to investigate the time dependence of deflections and to permit the harmonic analyses of these.

It is conceivable that the entire research project on a ship at sea can be based on the optical method of deflection measurement. The extreme difficulty and tediousness of an extensive strain-gage installation thereby can be avoided and only a few strain gages in accessible locations need be used. The optical equipment can be of semi-portable type requiring a minimum modification of a ship's structure.

8.8 Statistical Studies. The statistical studies of ship stresses in relation to the ambient weather were illustrated in Section 6 by an abstract of data obtained on two C-3 cargo ships. The work of this kind should be continued but more attention should be given to reporting weather conditions. It is suggested that a complete copy of a ship's weather log be included in the

reports on stresses. The author does not consider the wave forecasts or hindcasts on the basis of meteorological data to be sufficiently accurate for judging the relationships to ship stresses. A weak part of the weather descriptions, as it is used at present, is the lack of clear distinction between swell and wind sea. It is very difficult to make this distinction visually even in moderate weather and impossible to do so in a wind of Beaufort 6 and above. The value of stress measurements, such as were made on C-3 ships, would be enhanced if ships were equipped with shipborne wave recorders. From the analysis of wave records the presence of a swell can be detected. On large ships, however, the available wave recorders (Tucker's) do not give satisfactory records of moderate wind sea. This is caused by the depth of the gage installation and the resultant attenuation of the effects of small waves.

E. V. Lewis' (1957c) report on stresses on two C-3 type ships indicated a frequent occurrence of high stresses in moderate wind conditions. It is almost certain that in such conditions the stresses are caused primarily by swells. A simple method of photographing the radar screen (Roll, 3-1952) gives a valuable quantitative information regarding the direction and wave lengths of the swells. It is recommended that the simple equipment needed for this purpose and the routine operating procedures were developed for making these recordings in connection with the measurement of ship stresses. The swell heights can be determined if a shipborne wave recorder also is available.

The research in evaluating the maximum bending moments, expected at sea, must continue. It should not be limited, however, to pure statistical extrapolation of existing stress measurements. Instead a rational method must be developed in which the physical sea conditions and ship dynamics are taken into account. This was discussed in greater detail in Section 7.1.

8.9 Condensed Summary. In the research aimed at defining the bending stresses acting on ships at sea, two broad domains particularly must be emphasized. These are:

- 1 The evaluation of wave conditions at sea in terms of directional spectra.
- 2 Evaluation of the elastic response of a ship to a slamming impact. This appears to be particularly important for ship forms designed for high speed.
- In these two domains the available knowledge is insufficient and the uncertainties of stress estimates are excessively large. Immediately next in importance is:
- 3 Evaluation of ship motions with seven degrees of freedom (including rudder motions) and resultant ship stresses. These motions and stresses occur in oblique irregular waves.

9 Condensed List of Suggested Research Topics

1 **Bending Moment Analysis** of several ships is suggested using equations of motion with variable coefficients, Section 2.2. The linearized theory (using con-

stant coefficients) can be assumed to hold for ship motions, but instantaneous values of coefficients, taken as functions of the draft, are to be taken for bending moment analysis.

2 Bending Moment Analysis of ships to which the linear theory does not apply is suggested. This includes such forms, for instance, as the yacht discussed by Korvin-Kroukovsky and Jacobs (3-1957) and a shallow-draft vessel of Akita and Ochi (3-1955). In this case the variability of coefficients must be considered in calculations of motions, as well as bending stresses, and the project involves step-by-step numerical integration or the use of an analog computer.

3 Effect of Attenuation of the Wave Profile as the wave progresses along the ship should be investigated. In the strip method of calculation, based on Korvin-Kroukovsky and Jacobs (3-1957) and Jacobs (1958), this effect is neglected. Therefore, usually it is not possible to achieve the closing of the bending-moment diagram, and calculations are based on the front half of a ship's length. The method of correcting for the wave attenuation can be based on Grim (3-1957*b*, second part). In addition to closing of the bending-moment diagram, the improvement can be expected in the evaluation of phase relationships. These relationships are sensitive to cross-coupling coefficients which are directly affected by the wave attenuation. Stresses are in turn sensitive to phase relationships.

4 Development of the Control Over Ship's Loading is suggested in order to avoid excessive ship stresses. It was shown in Sections 3.1 and 3.2 (MS *San Francisco* and SS *Ocean Vulcan*) that bending moment caused by a ship's loading often doubled the wave-caused moment.

5 Correlation of the Computed and Measured Bending Moment on ships at sea can be made broadly following the technique of Schnadel and Admiralty Ship Welding Committee, Sections 3.1 and 3.2. However, considerable refinement in the instrumentation and analysis is needed. Continuous and simultaneous recording of all data must be provided. The method of recording must be co-ordinated with the available methods of analysis so that a larger proportion of the obtained data could be utilized than heretofore has been possible. The instrumentation should include a shipborne wave-height indicator (at present Tucker's), preferably installed somewhat ahead of the front end of bilge keels in order to reduce the ship's hull interference. The properties of irregular waves must be considered in reconstruction of wave profiles. The wave height at positions different from the place at which the height is measured conceivably can be evaluated by methods of Fuchs (3-1952) and Cote (1-1954).

The author does not believe that it is practical to use enough pressure gages to evaluate the total hydrodynamic force with sufficient precision. He suggests, therefore, that pressure measurements and their correlation with theory be made only on one or two ship sections. Other correlations would be limited to ship motions, bending moments and deflections.

Past experience indicates the difficulty of maintaining at sea the instruments installed at the ship's bottom. Therefore, it is suggested that strain gages at the deck only be used at sea and that their relationship to bending moments be established by calibration in port. As in the case of pressure gages, one or two ship sections in accessible location (engine room) can be equipped completely with strain gages.

It is recommended that a suitable towing-tank test be conducted on the model of a ship used in sea observations. The data from such an auxiliary project will help in interpretation of the data obtained at sea, and also may be used to fill in the missing data, for instance, because of malfunctioning of isolated transducers.

6 Relationship Between Hogging and Sagging Bending Moments should be investigated further by theory and model tests. The data of Schnadel, Lewis, and Ochi agree in showing that the sagging moment is larger, but disagree on the relative magnitudes (see the end of Section 4.2). The ratio of the sagging to hogging bending moments depends, of course, on the type of ship.

7 Limiting Value of Bending Moments in head seas should be investigated by using nonlinear theory and computational procedure. The technique is similar to that indicated in project 2; ship motions and bending moments are to be evaluated employing step-by-step integration of the coupled pitch-heave equations of motion with time-dependent coefficients evaluated on the basis of instantaneous conditions. A suitable section of a typical long-crested irregular wave pattern is to be chosen and calculations should cover several wave cycles, starting with a quiescent period and running through the group of highest and steepest waves. It is suggested that the investigation be made in irregular waves, since the phase relationships in a continuous train of regular waves may be different from those in a group of a few high irregular waves, and since bending moments are strongly affected by phase relationships. Calculations should be repeated for a few sea conditions of increasing severity in order to establish the relationship among bending moments and statistically defined sea conditions. Once the computational programming is developed, the additional cases can be evaluated on high-speed computers with but little additional labor.

8 Bending-Moment Measurements on Models should be made under conditions analogous to the ones outlined in project 7. It should be emphasized that project 7 has for its aim the detailed explanation of the limiting ship stress conditions; it is not a statistical project. Project 8 has for its first objective the comparison of theoretical (from project 7) and experimental values. However, this project can be continued to the second objective; namely, to establish statistically the frequency distribution of high bending moments in a sufficiently long typical irregular wave pattern. In a towing tank of limited length, the long irregular sea pattern can be composed of a number of short test runs in successive sections of a continuous pattern.

9 Bending-Moment Measurements on Models in

Short-Crested Oblique Seas should be made in wide tanks, a number of which is now becoming available. The ship log studies of E. V. Lewis, Section 6, indicated that high stresses occurred in light seas and in beam seas although at a lower frequency than in high and head seas. It is theoretically conceivable that critical bending-moment conditions may occur in steep oblique or beam short-crested waves of too small a length to be critical in head waves.

The vertical and lateral bending moments should be measured in these tests and/or strains should be recorded on both sides of the deck. In particular, the models representing ships with large metacentric height may develop severe rolling and in so doing generate large unsymmetrical stresses (see end of Section 6). These programs, therefore, must include variation of the metacentric height. Realistic short-crested irregular wave system should be used in these tests, and exaggeration of natural sea conditions should be avoided.

10 Model Experiments on Slamming of cargo-type ships are needed. Previous experiments of Lum, E. V. Lewis, Akita and Ochi, Section 5.1, clarified the general pattern of events, but more complete and detailed information is needed for the correlation with theory, which is necessary for the proper understanding of the slamming process. In particular, it is important to record all events simultaneously as functions of time, and to distinguish among approach conditions, the impact process and subsequent vibration. It is suggested that experiments be made in reasonably realistic irregular sea conditions at practical values of a light ship draft. Most of the experiments in the past were made in regular waves and the models were made to slam by artificial means, such as unusually steep regular waves or unusually shallow draft and unusual ship forms.

In the approach conditions, it is desired to evaluate the relative ship-wave vertical velocity and the local wave profile in the impact area. In the impact process, the development of the wetted area and of the total hydrodynamic force with time should be recorded. This process in cargo-type ships is extremely rapid and high-speed recording is needed. The low-speed records, such as are shown in Figs. 39 and 41, do not permit sufficiently accurate evaluation of the force development with time. The time at which a slam occurs can be established by preliminary tests in reproducible irregular wave pattern in a towing tank. The short interval of time of particular interest in a continuous wave pattern can be covered in subsequent tests with high-speed recording; i.e., open time scale.

Open-scale recording also is needed for the history of slam-caused free vibrations in order to evaluate the effect of transients and to correlate with a theory. It is emphasized that it is impossible to obtain the combined hydrodynamic and structural response similarity for the model to represent directly a full-size ship. Therefore the history of model vibrations should be used for verification of the elastic-response theory and the latter subsequently applied to full-size ships.

In pressure measurements, a thorough investigation must be made of the transducer and recording-system frequency response. Also it should be remembered that, according to Wagner's theory, the recorded pressure will be a function of the pressure-gage area.

11 Records of Slams of Ships at Sea should be obtained. The observations reported by Warnsinck and St. Denis, Section 5.3 and Fig. 40, can be taken as a guide, but should be improved by instrumental recording of wave profiles. The use of the more open time scale is suggested in order to permit a more complete analysis of the initial stages of the impact and of the subsequent free vibrations. Strain-gage instrumentation should permit tracing of the time history of shear and bending-moment variation along the length of a ship. Tests in waves at sea should be supplemented by investigation of ship structural properties in calm water, including forced-vibration tests and derivation of damping coefficients in several vibration modes. These calibration tests should be made in a sufficiently deep water, since hydrodynamic masses and damping are strongly affected by water shallowness.

12 Effort to Evaluate Water Pressure Experienced in Slamming should be continued using theoretical methods, model tests in towing tanks, and observations at sea. The first two of these approaches may err in stipulating unrealistically severe conditions. Therefore, sea observations are particularly valuable in establishing pressures encountered in practical operating conditions.

It should be remembered that there is no uniquely defined figure for the magnitude of the water impact pressure. It depends on ship-motion characteristics, details of the bottom shape, and operating conditions of a ship, Section 5.4. In model tests the size and frequency response of a gage also affect recorded pressure.

13 Efforts to Develop a Suitable Theory for Flat-Bottom Impact are needed. Presently available Wagner's theories fail in case of a flat bottom.

14 Methods of Theoretical Analysis Should Be Sought for the Impact of Surfaces of Very Small (Vanishing) Deadrise. An approach different from Wagner's expanding-plate theory should be sought in that the evaluation of the hydrodynamic mass may differ from the one derived by analogy with a fully submerged plate in a steady flow. The approach also may differ from Wagner's spray-root theory in that the potential theory is expected to fail locally as the result of large velocity gradient in the spray-root region at a small deadrise surface. This is expected to cause a reduction of peak pressures given by potential theory.

15 Impact of a Flat or Small Deadrise Surface on the Rippled Water Surface Should be Investigated. The impact of a body on smooth water surface heretofore has been investigated in theoretical and model-test approaches to the impact problem. It corresponds to a ship slamming in a smooth-surfaced swell in the absence of any wind. While such a condition may occur

at sea, it is by no means common. Usually swells, or waves of sufficient length to cause ship motions, are covered by an endless variety of small waves, chop, and ripple. These have the effect of cushioning the impact of the nearly flat bottom found in most cargo ships. Theoretical efforts and model tests are suggested for the investigation of the impact pressure and total impact force in these conditions. Theoretical work is visualized as progressing in two stages: (a) The impact on a single sharp wave crest can be investigated and the history of the force development can be expressed as a function of the wave steepness; i.e., the degree of the crest sharpness. (b) The results can be generalized by methods of mathematical statistics to represent the impact on a set of small waves which are characterized by the high-frequency end of a wave-energy spectrum.

16 Evaluation of Hydrodynamic Masses for High Vibration Modes is needed. The solution of the problem was given initially by F. M. Lewis (3-1929) and J. Lockwood Taylor (3-1930b) and the masses for the two lowest modes were computed. The differences between these two investigators were resolved, and a more complete solution was presented by Macagno and Landweber (3-1958) together with numerical data for the two-node vibration. The extension of this work to higher modes (at least to the seventh) is visualized under the project here suggested, Section 5.53.

17 Evaluation of the Internal Damping Characteristics of ship hulls is needed for use in the analysis of slam-caused transient vibrations. The discussion given in Section 5.53 indicates the inadequacy of the currently available data.

18 Methods of Introducing Variable (Frequency-Dependent) Damping Into the Finite-Difference Computational Solution of slam-excited vibration should be developed. This is needed in order to represent faithfully the transient state of the slam-excited vibration and the experimentally observed variability of the apparent vibration period with time. While it is not practical to distinguish among vibration modes in this method, the variable damping can be set as a function of instantaneous values of $\partial^4 y / \partial x^4$, $\partial^4 y / \partial x^2 \partial t^2$, and $\partial^4 y / \partial t^4$ as shown by equation (12).

19 Active Theoretical Work in Analytical Solution of Slam-Excited Vibration is Recommended, and is given a high priority in the present list of research topics. This can be considered as a continuation of the work of Ormondroyd and of Ochi, Section 5.52. Mathematical difficulties so far have limited this work to the vibration of beams of uniform section, considering only flexural deflections. While a complete solution of the problem for a true ship structure is desirable in principle, the possibility of obtaining it in the near future appears to be remote. However, it appears to the author that a simple short cut in this procedure exists and may bring about a quick and comprehensive solution. It may be assumed that positions of nodal points and the normal mode frequencies computed for a free vibration will

remain sensibly unchanged in a slam-caused vibration.⁴⁰ A number of methods now exist for the evaluation of these for ship hulls (McGoldrick et al, 1953; McGoldrick, 1954; Csupor, 1956). These methods take into consideration the variability of structural properties along ship's length, flexural and shear deflections, and rotary inertia. The activity under this proposed project can be directed to evaluating the distribution of the momenta and energies among various vibration modes. The work of Ochi (1958c) in applying the method of least work for this evaluation can be used as a starting point. Once the energy contained in each mode (and therefore the corresponding amplitude) is evaluated, the time history of the entire process is obtained by a simple summation of time and position dependent deflections, using appropriate added masses and damping.

20 Experimental Measurements of Elastic Response of Structures⁴¹ to slam-like impact are needed for the dual purpose of 1) providing the knowledge of the physical behavior of structures and thereby setting targets for the future analytical work; and 2) providing experimental data for comparison with completed sections of analytical work. In connection with 2), structural tests must be planned for a series of structures of increasing complexity. The following steps are suggested:

(a) Slender steel and brass bars. A uniform section bar flexural theory is expected to be applicable.

(b) A thin-walled tube (steel and brass) in the air and also floating half submerged on water. A uniform-bar theory based on flexural and shear deflections is expected to be applicable.

(c) A built-up steel girder of uniform section and of structure generally similar to those used in ships. Longitudinal and transverse-framing types may be compared. A uniform bar theory based on flexural and shear deflections and considering rotary inertia is expected to be applicable with greater emphasis on shear than in sub-project (b).

(d) A complete ship model, reproducing a ship in essential structural properties, such as for instance used by Ochi and Sato, Sections 4.2 and 4.3. Models with transverse and longitudinal framing are suggested.

In all cases a slam-like impact of a known force-time pattern should be applied and simultaneous time history of deflections at a number of points along the beam length should be recorded. Sufficiently close spacing of measuring points should be used to permit the comparison of experimental and analytical data at higher vibration modes. In steps (c) and (d) the design of instrumentation should aim at distinguishing between flexural and shear deflections. In all cases the impact tests should be supplemented by forced-vibration tests to establish elastic characteristics of structures at all vibration modes.

⁴⁰ This assumption appears to be implicitly included in Ochi's work.

⁴¹ This project corresponds to Lewis and Gerard (1958) project III-3-2.

Direct objectives of the tests are to provide data for comparison with analytical work, to evaluate internal damping,⁴² and, in structures alternately tested while floating on water, to evaluate added masses.

In tests made in the air a free-free condition may be achieved by suspending a structure on a number of light-weight long springs of low spring constant.

21 Investigation and Well-Defined Statement of Similarity Relationships should be developed for ships and models involving hydrodynamic loading and the elastic response of the structure, Section 5.54. It appears at the moment that the strong dependence of the structural damping (hysteresis) on vibration frequency prevents direct comparison of models and full-size ships. It is necessary to establish what information can be obtained in model tests and how it should be interpreted to indicate the ship behavior. The investigation suggested here also has bearing on the comparison of sea-observation data obtained on ships of different sizes and types.

22 Parallel Investigation of Transient Elastic Vibrations on a Ship and a Model is suggested. For this purpose the model listed in project 20 item (d) should correspond to an available ship. Also the model test conditions should include those practical for the ship, such for instance, as the dropping of an anchor. The supplementary steady-state vibration tests on the model should be produced in a manner similar to those produced by the vibrator available for the ship.

23 Detailed Step-by-Step Calculation of Forces Acting on a High-Speed Ship Model and the Resultant Model Vibrations is recommended as a further development of the pioneering attempt by Dalzell, 1959, Section 5.55. It is suggested that such investigation be made on a destroyer model in an irregular sea imitating the condition reported by Warnsinck and St. Denis (1957). A short sample of a wave record can be chosen starting with a quiescent period and running through a group of high waves. Corresponding ship motions can be estimated by the linear theory of Fuchs and McCamy (3-1953). Such a project will help to develop understanding of the wave and slam-excited vibration of destroyers.

24 Statistical Collection of Bending-Moment Data Related to Sea Conditions is recommended. This is to be a continuation and further development of E. V. Lewis' activity described in Section 6. It is suggested that the weather log as well as the deck log be used in reporting sea conditions. It is also suggested that ships be equipped by two strain gages, one on the port and one on the starboard sides, since unusually high stresses conceivably may be caused by the rolling of the ships (with excessive metacentric height) in head seas. Therefore, the amount of rolling should be recorded, or at least commented upon.

25 Development of a Statistical Method for Prediction of Wave Steepness in a Specified Wave Length and in Specified Wave-Energy Spectrum is recommended. This is needed in connection with establishing the maximum bending moment expected in ships at sea. A rather high priority is suggested for this project.

26 Effect of Discontinuities on the Elastic Response should be investigated by experimental and analytic means. The experimental data should be obtained to establish the pattern of physical events. A suitable mathematical model can be formulated after examination of this pattern. On the other hand, analytical work can be pursued concurrently with the particular aim of checking whether the existing vibration theories will yield satisfactory results by mere insertion of discontinuous mass and structural property distributions. It is emphasized that discontinuities in mass as well as sectional property distributions are important.

27 Experimental Investigation of the Impact of Prismatic and Cylindrical Bodies⁴³ of the section typical of bow sections of ships is suggested in order to help the analysis of pressure and forces experienced by ships in slamming impact. Two distinct cases are visualized here. In sections typical for cargo ships the deadrise angle is small, the duration of the force is short, and the vertical velocity remains constant during impact. In destroyers the duration of the impact is from the first contact to the full submergence of the bow; i.e., about a quarter of the wave-encounter cycle. The vertical velocity is variable from maximum at the beginning to zero at the end. Attention should be paid to choosing realistic velocities. The results of this investigation subsequently can be used to evaluate the total impact force by the strip method.

Nomenclature

Symbols listed at the end of Chapter 2 and in Appendix C apply unless otherwise stated. Additional symbols are as follows:

- b = damping coefficient
- b_f = damping coefficient caused by water
- E = modulus of elasticity
- G = shear modulus
- I = moment of inertia of a hull section
- $k'AG$ = effective shear rigidity of a hull section
- m = a mass; mass per unit of a beam's length
- \bar{m} = mass per unit length of ship structure, equipment and load (the overbar may be omitted when there is no ambiguity)
- m'' = hydrodynamic mass
- n = number of nodal points; also used as a subscript for a vibration mode designation
- P = impacting force
- $P(t)$ = impacting force as a function of time
- $P(x, t)$ = exciting force as a function of position and time
- $p(t)$ = exciting force per unit of mass, also per unit of a beam's length
- q = damping factor (defined in Section 5.53)
- r = radius of gyration of a hull section
- α_a = characteristic numbers defined under equation (11)
- δ = logarithmic decrement

⁴² The evaluation of internal damping is emphasized by Lewis and Gerard (1958) in their suggested project III-3-4.

⁴³ Lewis and Gerard (1958) project 4-3-1 is here relevant. An extension of this project to investigate the effect of rigidity of a built-up structure was suggested by Lewis and Gerard in the project III-3-1.

η_n, η_s = wave-height ratios (Ochi, Section 4.2)
 η = coefficient of tangential viscosity } equations (12)
 ξ = coefficient of normal viscosity } and (14)
 τ = duration (usually short) of an impacting force
 ω = circular frequency
 ω_n = circular frequency of n -node vibration

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(For code of abbreviations, see page 218.)

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1 The Trochoidal Wave

In the literature on naval architecture, the “trochoidal” wave has been most often used as a representation of a simple wave. This is a purely geometric description, shown in Fig. 1. The shape of the water surface is generated by a point on the smaller of two concentric circles (radius r) rotating together with the larger one of radius R . The latter rolls through an angle θ on an imaginary horizontal track located at a certain height above the still-water surface. By virtue of this geometry, the instantaneous co-ordinates of the point P on the small circle, i.e., the co-ordinates x and y of the described trochoidal curve, are given by the parametric equations

$$\begin{aligned}x &= R\theta + r \sin \theta \\y &= R + r \cos \theta\end{aligned}\quad (1)$$

The origin of the horizontal co-ordinates x is at the wave hollow and the angle θ is measured from the lower vertical, passing through the center of the generating circle. The ordinates y are measured downwards from the track $x - x$. A conspicuous feature of the trochoidal curve is sharper curvature at the wave crest and a lesser curvature at the wave troughs. This difference is aggravated as r increases and approaches R . At $r = R$, the wave crest takes the form of a sharp cusp. In this case the wave height would be $2R$ and, since wave length $\lambda = 2\pi R$, the minimum ratio of wave length to height would be 3.14. Such a wave height never occurs in nature, and is not compatible with the theory of potential waves discussed in the next section. For waves having a height-to-length ratio often found in nature, such as $\frac{1}{20}$ for instance, the trochoidal curve agrees well with experimental tank observations.

Equations (1) merely define the trochoidal form, but do not give any indication of its relation to the still-water level. This relation is defined by the condition that the total water volume above this level must be equal to the one below. In order to satisfy this condition, the line of centers of the rolling circles must be located $r^2/2R$ above the still-water level.

The dynamic characteristics of the wave, such as its velocity of propagation and pressure gradients, are derived from the condition that a uniform atmospheric pressure acts on each part of its surface. Since there is no pressure gradient along the surface, there is no fluid acceleration in this direction, and all fluid accelerations at the free surface are normal to it. This fact will be seen

later to play an important part in the theory of ship rolling. In this connection, this was demonstrated experimentally by W. Froude (1861), who fitted a pendulum over a small float and found that the pendulum remained in its still-water position normal to the float, regardless of the inclinations taken by the float in waves in a towing tank. He found this to hold even when the float was partially inverted on the forward face of a breaking wave.

Certain values of the circular frequency ω and of the wave celerity c are needed to fulfill the foregoing conditions. The acceleration of a water particle at the surface must be such that, combined with the acceleration of gravity g , it would give the total acceleration normal to the trochoidal surface. On this basis, the following relationships have been established:

$$\text{Wave celerity } c = \sqrt{g\lambda/2\pi}$$

$$= 2.26 \sqrt{\lambda} \text{ ft (fps)} = 1.34 \sqrt{\lambda} \text{ ft (knots)}$$

$$\text{Wave period (sec) } T = \lambda/c = \sqrt{2\pi\lambda/g} = 0.442 \sqrt{\lambda} \text{ ft}$$

$$\begin{aligned}\text{Wave length (ft) } \lambda &= 0.0196c^2 \text{ (fps)} \\&= 0.577c^2 \text{ (knots)} = 5.118 T^2\end{aligned}$$

A water particle, initially at rest in still water, is lifted at the approach of the wave crest, moved forward at the crest, lowered at the approach of the hollow and moved aft at the hollow. It thus describes a complete circle of radius r at the circular frequency ω , the complete circular path being accomplished in the passage of one wave; i.e., in the period T . This is known as the orbital motion, and the particle velocity as the orbital velocity. The whole mass of water in the immediate vicinity of a particle goes through approximately the same motion, so that there is no significant rotation of the particle or of the small mass of water in relation to the ambient water. However, any initial rectangular mass $a b c d$, as shown cross-hatched in Fig. 2, is distorted into $a' b' c' d'$ in such a way that the motion is not “irrotational” or “potential” from the point of view of hydrodynamics, and cannot be represented in a simple form needed for the solution of most hydrodynamic problems. This severely limits the practical usefulness of this theory.

A very simple and well-developed exposition of the trochoidal theory—from the naval architect’s point of view—is given by Manning (1-1942, pages 1-7).¹ A

¹ Reference to sections, equations, figures and bibliography will be designated by chapter number and section, equation, figure, or bibliography; reference noted is to Manning, chapter 1 (1942).

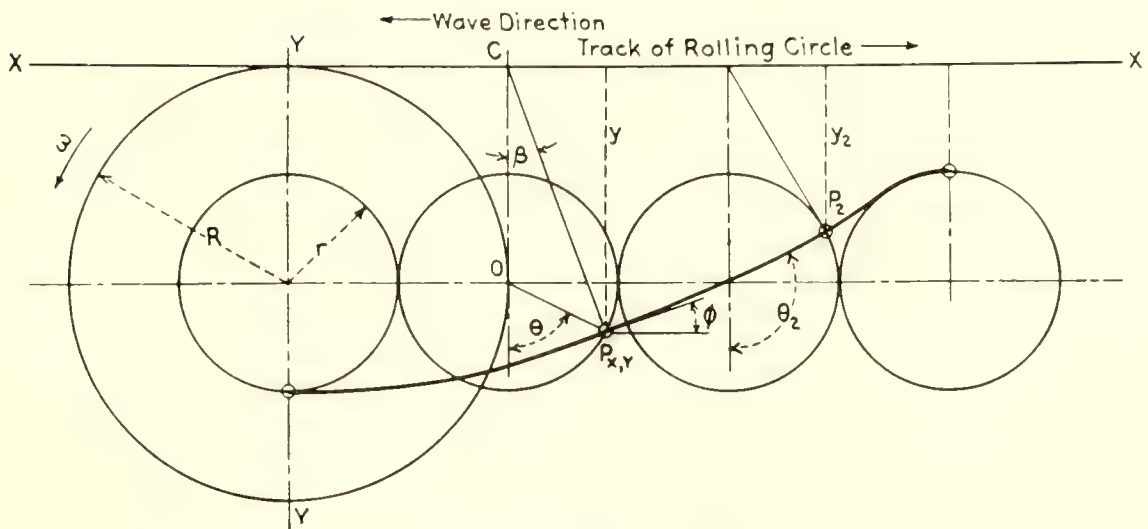


Fig. 1 Geometry of trochoid

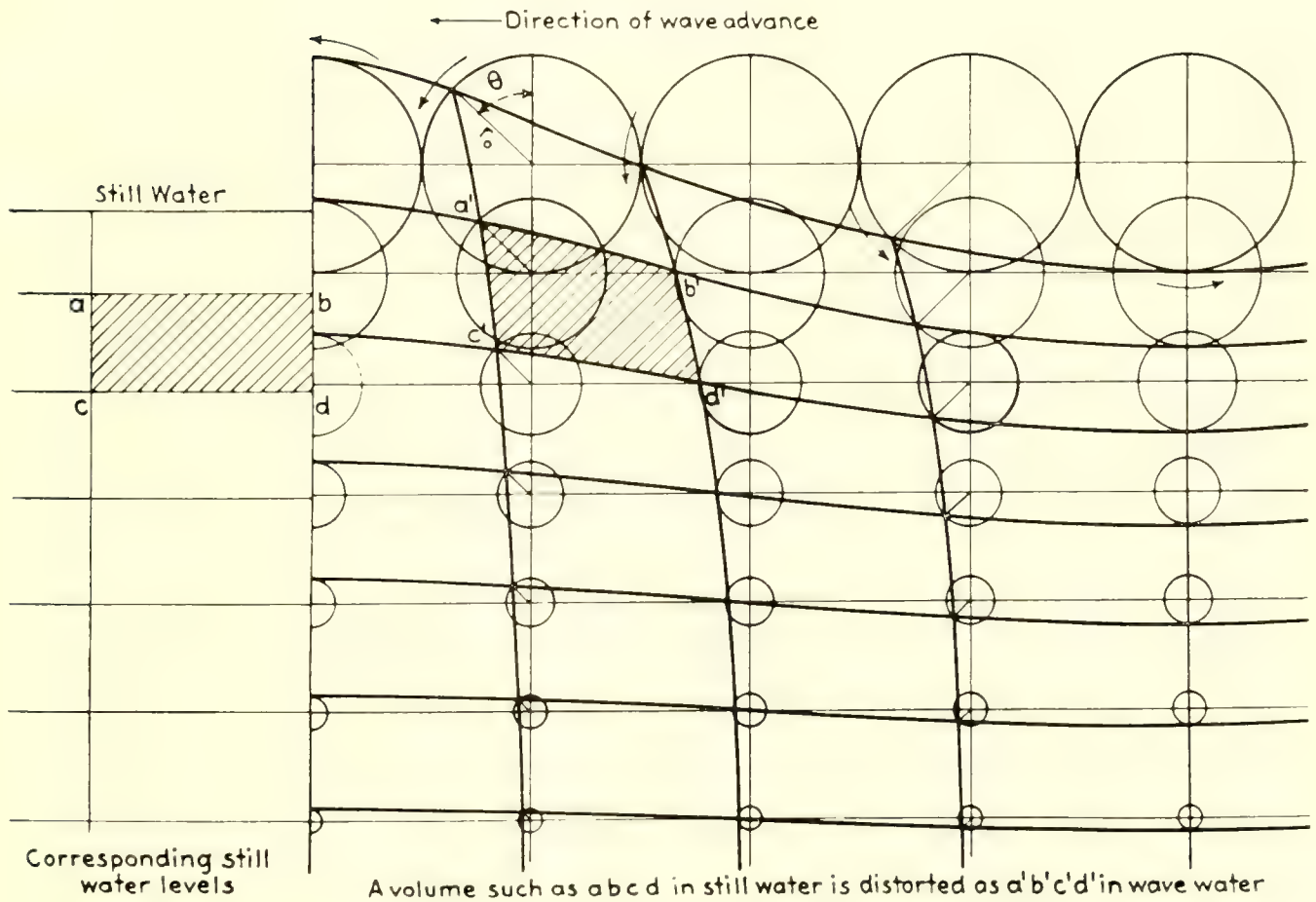


Fig. 2 Trochoidal wave motion

more mathematical presentation will be found in Lamb (1-D pages 421–423), Milne-Thomson (1-F pages 381–385), and, in a more complete form, in Kochin (1-C pages 441–448). The extension of the theory to shallow water is given by O'Brien and Mason (1-G pages 28, 29).

Historically, introduction of the trochoidal theory into Naval Architecture was rather unfortunate. As mentioned before, it cannot be used in most hydrodynamic problems. It fails in defining the maximum possible wave height. The vertical position of the trochoidal curve on the water surface is not included in the basic definition, and has to be evaluated independently. Finally, it does not indicate the “mass transport,” i.e., the greater amount of water surge in the direction of wave propagation at wave crests than in the return backward movement at wave troughs, which is clearly observable in nature. All of these characteristics form an integral part of the potential theory of gravity waves of finite amplitude. Moreover, the potential theory of waves of finite amplitude gives a wave profile which, like the trochoidal one, is in agreement with observations for waves of moderate height. In other words, the trochoidal form and the form of waves of finite height are identical within the obtainable accuracy of experimental observations although mathematically they always differ in the terms of higher order.

2 Potential Theory of Surface Waves

2.1 Outline of Theory, Velocity Potential, Wave Elevation and Celerity. The theory of two-dimensional gravity waves on the surface of water is usually known as “potential theory of surface waves.” The wave crests are of infinite length, uniformly spaced and parallel to each other, and are advancing in the direction normal to them at a certain celerity c . The word “celerity” is used for this velocity of propagation of *wave form*, to distinguish it from the water-particle velocity U , which is usually very much smaller than c . With infinitely long and uniform wave crests, the various functional relationships involved in the wave description remain unchanged for any change of position along the direction parallel to the wave crests. Only the distances x measured in the direction of wave propagation, and the vertical distances y have any effect on these functions. Such waves are therefore often referred to as two-dimensional. Since the wave form advances with celerity c , the wave elevation (as well as all other functions involved, such as pressure for instance) depends on time t , as well as on the distances x and y ; that is, all relationships involved have the general form $F(x, y, t)$.

Water is assumed to be inviscid and incompressible; its motion can therefore start from rest only by pressures or impulses acting normally to any water surface or boundary. Such a motion is called “irrotational” or “potential” and is characterized by the existence of the velocity potential ϕ and of the stream function ψ such that

Horizontal component of water velocity

$$u = -\partial\phi/\partial x = -\partial\psi/\partial y \quad (2)$$

Vertical component of water velocity

$$v = -\partial\phi/\partial y = \partial\psi/\partial x,$$

where the origin of co-ordinates is taken at the water surface, positive x is in the direction of the wave propagation, and y is positive in the upward direction; i.e., it is negative throughout the region occupied by the water.

In any form of potential flow, the functions ϕ and ψ satisfy Laplace's equations:

$$\nabla^2\phi = (\partial^2\phi/\partial x^2) + (\partial^2\phi/\partial y^2) = 0 \quad (3)$$

and

$$\nabla^2\psi = (\partial^2\psi/\partial x^2) + (\partial^2\psi/\partial y^2) = 0 \quad (4)$$

Solution of a hydrodynamic problem (in this case the wave motion) consists of finding relationships between the function ϕ or ψ and the co-ordinates x and y such that the Laplace equations are satisfied. In addition, such a solution must satisfy the geometric and dynamic conditions existing at various boundaries of the fluid. In the case of two-dimensional waves in water of uniform depth h , three boundary conditions are to be satisfied:

(i) At the bottom ($-y = h$), the vertical velocity is nil; i.e.,

$$\frac{\partial\phi}{\partial y} = 0 \quad (5)$$

(ii) At the free surface, the vertical velocity of the water particles, $v = -\partial\phi/\partial y$, must be the same as the vertical velocity of the surface itself, $\partial\eta/\partial t$, where η denotes the wave elevation; i.e.,

$$\frac{\partial\eta}{\partial t} = \left(-\frac{\partial\phi}{\partial y} \right)_{y=\eta} \quad (6)$$

(iii) The atmospheric pressure acting on the free surface is uniform.

In connection with condition (iii), it should be noted that the theory of surface waves treats the propagation of free waves solely under the action of gravitational and inertial forces. Neither the method of generating waves nor the exciting or damping effect of air motions is included here. These are treated separately in Chapter 1. Condition (iii) is put into mathematical form by use of Bernoulli's equation for time-dependent flows with gravity force $g\eta$:

$$g\eta + \frac{1}{2}(u^2 + v^2) - \partial\phi/\partial t = F(t)$$

where $F(t)$ is an arbitrary function of time resulting from integration of Euler's equations of motion.

Including $F(t)$ in $\partial\phi/\partial t$:

$$\eta = \frac{1}{g} \frac{\partial\phi}{\partial t} - \frac{1}{2g} (u^2 + v^2) \quad (7)$$

In the first-order theory, usually referred to as the theory of “waves of small amplitudes,” simplifying as

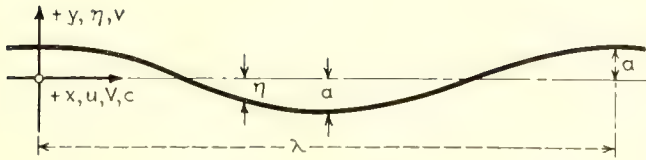


Table 1 Properties of Harmonic Deep-Water Waves

Surface profile.....	$\eta = a \cos k(x - ct)$
Velocity potential.....	$\phi = -ace^{ky} \sin k(x - ct)$
Stream function.....	$\psi = -ace^{ky} \cos k(x - ct)$
Horizontal water velocity.....	$\left\{ \begin{aligned} u &= -\partial\phi/\partial x = -\partial\psi/\partial y \\ &= kace^{ky} \cos k(x - ct) \end{aligned} \right.$
Vertical water velocity.....	$\left\{ \begin{aligned} v &= -\partial\phi/\partial y = \partial\psi/\partial x \\ &= kace^{ky} \sin k(x - ct) \end{aligned} \right.$
Wave celerity.....	$c = \frac{\lambda}{T} = \frac{g}{\omega} = \left(\frac{g\lambda}{2\pi}\right)^{1/2}$ $= \frac{gT'}{2\pi} = 2.26\sqrt{\lambda^*}$
Wave length.....	$\lambda = \frac{2\pi c^2}{g} = \frac{2\pi g}{\omega^2} = \frac{gT^2}{2\pi} = 5.12T^2^*$
Wave number.....	$k = \frac{2\pi}{\lambda} = \frac{\omega^2}{g} = \frac{g}{c^2} = \frac{4\pi^2}{gT^2}$
Pressure**.....	$p = \rho g e^{ky} \cos k(x - ct)$
Horiz. pressure gradient**	$\partial p/\partial x = -\rho g k e^{ky} \sin k(x - ct)$
Vert. pressure gradient**	$\partial p/\partial y = \rho g k e^{ky} \cos k(x - ct)$
Maximum wave slope....	$\bar{\theta} = ka = \frac{2\pi a}{\lambda}$

* λ in feet, c in feet per sec. ** Exclusive of hydrostatic.

sumptions are made in evaluating boundary conditions (ii) and (iii). In the first case it is assumed that condition (ii) is fulfilled at the undisturbed water level $y = 0$ instead of at $y = \eta$, i.e., condition (ii) is written as

$$\text{(ii-simplified)} \quad \partial\eta/\partial t = (-\partial\phi/\partial y)_{y=0} \quad (8)$$

In the second case the squares of the small perturbation velocities u and v are neglected in comparison with $\partial\phi/\partial t$, and (iii) reduces to

$$\text{(iii-simplified)} \quad \eta = \frac{1}{g} \frac{\partial\phi}{\partial t} \quad (9)$$

The simplified conditions (ii) and (iii) can be combined to form

$$\text{(iv)} \quad \frac{\partial^2\phi}{\partial t^2} + g \frac{\partial\phi}{\partial y} = 0_{\text{at } y=0}, \quad (10)$$

which is usually referred to as the free-surface condition.

In the present work the notation conforms to the sketch at the top of Table 1. The origin of co-ordinates ($x = 0, y = 0$ at $t = 0$) is taken at the still-water level under the wave crest. The x -co-ordinates, the wave celerity c , and the horizontal component u of the water velocity are positive to the right. The ordinates y and the vertical components v of the water velocity are positive in the upward direction; i.e. below the still-water

level, y has negative values. The water depth h is taken as a positive quantity.

The Laplace equation (3) and the boundary conditions (i) and (iv) are satisfied by the expression for the complex potential (Milne-Thomson, 1-F, p. 357, modified for change in the origin of the co-ordinates from nodal point to crest and from bottom to still-water level):

$$w = \phi + i\psi = -ac \frac{\sin(kz + ikh - kct)}{\sinh kh} \quad (11)$$

where $z = x + iy$, k is written for $2\pi/\lambda$ and a is the amplitude, or more exactly half the wave height measured from trough to crest. Separation of real and imaginary parts gives the velocity potential:

$$\phi = -ac \frac{\cosh k(y + h)}{\sinh kh} \sin k(x - ct) \quad (12)$$

and the stream function

$$\psi = -ac \frac{\sinh k(y + h)}{\sinh kh} \cos k(x - ct) \quad (13)$$

By the use of expression (19) for c^2 , and noting that $kc = \omega$ (where ω is the circular frequency in radians per sec), equation (12) can be rewritten in an alternate form:

$$\phi = -\frac{ga}{\omega} \frac{\cosh k(y + h)}{\cosh kh} \sin k(x - ct) \quad (14)$$

For very deep water (i.e., large h ; in practice, $h > \lambda/2$), these expressions become

$$\phi = -ac e^{ky} \sin k(x - ct) \quad (15)$$

$$\psi = -ac e^{ky} \cos k(x - ct) \quad (16)$$

The components of the water velocity at any point $p(x, y)$ in the water are obtained by partial differentiation, as shown by equations (2). The resulting expressions are listed in the summary tables of wave properties, (1) and (2).

The elevations of lines of equal pressure are obtained by using expressions (14) and (15) for ϕ in equations (9), as follows:

For water depth h ,

$$\eta = a \frac{\cosh k(y + h)}{\cosh kh} \cos k(x - ct) \quad (17)$$

and for deep water,

$$\eta = ac e^{ky} \cos k(x - ct) \quad (18)$$

In particular, the surface wave profile is obtained by letting $y = 0$. In this case the factors containing hyperbolic functions and the exponential become unity.

The velocity of wave propagation c is found by inserting the expressions for ϕ in the surface condition (iv); i.e., equation (10), so that:

For water of depth h ,

$$c^2 = \frac{g\lambda}{2\pi} \tanh \frac{2\pi h}{\lambda} \quad (19)$$

Table 2 Properties of Harmonic Waves in Water of any Depth

Surface profile...	$\eta = a \cos k(x - ct)$
Complex potential	$w = \phi + i\psi$ $= -ac \frac{\sin(kz + ikh - ckt)}{\sinh kh}$
Velocity potential.....	$\phi = -ac \frac{\cosh k(y + h)}{\sinh kh} \sin k(x - ct)$ $= -\frac{ga}{\omega} \frac{\cosh k(y + h)}{\cosh kh} \sin k(x - ct)$
Stream function.....	$\psi = -ac \frac{\sinh k(y + h)}{\sinh kh} \cos k(x - ct)$
Horiz. water velocity.....	$u = ack \frac{\cosh k(y + h)}{\sinh kh} \cos k(x - ct)$
Vert. water velocity.....	$v = ack \frac{\sinh k(y + h)}{\sinh kh} \sin k(x - ct)$
Wave celerity.....	$c^2 = \frac{g\lambda}{2\pi} \tanh kh$
Pressure.....	$p = apg \frac{\cosh k(y + h)}{\cosh kh} \cos k(x - ct)$
Horiz. pressure gradient..	$\partial p / \partial x$ $= -apgk \frac{\cosh k(y + h)}{\cosh kh} \sin k(x - ct)$
Vert. pressure gradient...	$\partial p / \partial y$ $= apgk \frac{\sinh k(y + h)}{\cosh kh} \cos k(x - ct)$

For very shallow water ($h < \lambda/20$), this becomes

$$c^2 = gh \quad (20)$$

and, for deep water ($h > \lambda/2$),

$$c^2 = \frac{g\lambda}{2\pi} \quad (21)$$

All characteristics of the deep-water gravity waves are summarized for ease of reference in Table 1. The characteristics of waves in water of limited depth are summarized in Table 2. Table 3 shows the relationships among wave length, period, and celerity in deep water.

2.2 Velocity and Path of Fluid Particles. The total velocity of a fluid particle can be obtained by adding vectorially the u and v components:

$$U = u + iv \quad (22)$$

By inserting the values of u and v from Table 1 for deep-water waves and letting $x = 0$ and $2\pi ct/\lambda = \omega t$,

$$U = \frac{2\pi ac}{\lambda} e^{2\pi y/\lambda} (\cos \omega t - i \sin \omega t)$$

or

$$U = \frac{2\pi ac}{\lambda} e^{2\pi y/\lambda} e^{-i\omega t} \quad (23)$$

The particle velocity, therefore, is represented by a vec-

tor whose uniform absolute value is $2\pi ac/\lambda$ at the water surface and which rotates at the angular velocity ω in radians per sec, making a complete revolution in the period of the passing wave. The path of the particle at the surface is therefore a circle of radius a . The absolute value of the velocity vector and the radius of the circular path of the particle diminish with depth as $e^{2\pi y/\lambda}$. This result is identical with the result of the trochoidal-wave theory.

When the wave is in water of limited depth h , the path of a particle is elliptical and is defined by the equation (Milne-Thomson, 1-F, page 356; Coulson, 1-A, page 78)

$$\frac{\xi^2}{\alpha^2} + \frac{\eta^2}{\beta^2} = 1, \quad (24)$$

where ξ and η are the horizontal and vertical displacements of a particle from its initial still-water position, and α and β are semi-axes of the ellipses:

$$\alpha = \frac{a \cosh k(h + y)}{\sinh kh}$$

$$\beta = \frac{a \sinh k(h + y)}{\sinh kh} \quad (25)$$

Here, k is again written for brevity in place of $2\pi/\lambda$. All the ellipses have the same distance $2a/\sinh kh$ between their foci, but the semi-axes α and β diminish with depth and β vanishes at the bottom, $y = -h$, where water particles have a rectilinear motion between foci.

The theory as outlined in the foregoing, i.e., based on the simplifying assumptions (ii) and (iii) shown by equations (8) and (9), is known as the linear theory, or theory of waves of small amplitude. The wave described by equation (18) is known as a "simple harmonic wave." The derivation becomes exact for vanishingly small wave amplitude a . With waves of growing height the true surface profile, which is practically trochoidal, deviates increasingly from equation (18). However, the profile of such a wave can be represented by a Fourier series of simple harmonic waves, and in that case equation (18) represents the first term (the first harmonic) of such a series. Equations (15), (16) and (18) represent the simplest form of gravity wave. As such, they are indispensable as the basic material from which any type wave can be constructed by a suitable superposition of a number of simple forms.

2.3 Pressure and Pressure Gradient. A wave characteristic which is often needed in analyses of engineering structures (including ships) is the pressure acting at any point in water. Within a first approximation, equation (9) for example, the pressure is given by

$$p = \rho \frac{\partial \phi}{\partial t} \quad (26)$$

where ρ is the mass density of water 2 lb per cu ft for sea water. For water of limited depth h , on the basis of equation (12)

Table 3 Properties of Simple Waves. From Robert L. Wiegel (1-K)

T , sec	c , fps	c , knots	λ , ft	T , sec	c , fps	c , knots	λ , ft	T , sec	c , fps	c , knots	λ , ft	T , sec	c , fps	c , knots	λ , ft
0.5	2.56	1.52	1.28	7.0	35.8	21.2	251	13.5	69.1	40.9	933	20.0	102.4	60.6	2047
0.6	3.07	1.82	1.84	7.1	36.3	21.5	258	13.6	69.6	41.2	947	20.1	102.9	60.9	2068
0.7	3.58	2.12	2.51	7.2	36.8	21.8	265	13.7	70.1	41.5	961	20.2	103.4	61.2	2088
0.8	4.09	2.42	3.28	7.3	37.4	22.1	273	13.8	70.6	41.8	975	20.3	103.9	61.5	2109
0.9	4.61	2.73	4.15	7.4	37.9	22.4	280	13.9	71.1	42.1	989	20.4	104.4	61.8	2130
1.0	5.12	3.03	5.12	7.5	38.4	22.7	288	14.0	71.6	42.4	1004	20.5	104.9	62.1	2151
1.1	5.63	3.33	6.19	7.6	38.9	23.0	296	14.1	72.2	42.7	1018	20.6	105.4	62.4	2172
1.2	6.14	3.64	7.37	7.7	39.4	23.3	304	14.2	72.7	43.0	1032	20.7	105.9	62.7	2193
1.3	6.65	3.94	8.65	7.8	39.9	23.6	312	14.3	73.2	43.3	1047	20.8	106.4	63.0	2214
1.4	7.17	4.24	10.0	7.9	40.4	23.9	320	14.4	73.7	43.6	1062	20.9	107.0	63.3	2235
1.5	7.68	4.55	11.5	8.0	40.9	24.2	328	14.5	74.2	43.9	1076	21.0	107.5	63.6	2257
1.6	8.19	4.85	13.1	8.1	41.4	24.5	336	14.6	74.7	44.2	1091	21.1	108.0	63.9	2278
1.7	8.70	5.15	14.8	8.2	42.0	24.8	344	14.7	75.2	44.5	1106	21.2	108.5	64.2	2300
1.8	9.21	5.45	16.6	8.3	42.5	25.1	353	14.8	75.7	44.8	1121	21.3	109.0	64.5	2322
1.9	9.72	5.76	18.5	8.4	43.0	25.4	361	14.9	76.2	45.1	1137	21.4	109.5	64.8	2344
2.0	10.2	6.06	20.5	8.5	43.5	25.7	370	15.0	76.8	45.4	1152	21.5	110	65.1	2366
2.1	10.7	6.36	22.6	8.6	44.0	26.1	379	15.1	77.3	45.8	1167	21.6	110.5	65.4	2388
2.2	11.2	6.67	24.8	8.7	44.5	26.4	388	15.2	77.8	46.1	1183	21.7	111.1	65.8	2410
2.3	11.8	6.97	27.1	8.8	45.0	26.7	397	15.3	78.3	46.4	1199	21.8	111.6	66.0	2432
2.4	12.3	7.27	29.5	8.9	45.6	27.0	406	15.4	78.8	46.7	1214	21.9	112.1	66.4	2455
2.5	12.8	7.58	32.0	9.0	46.1	27.3	415	15.5	79.3	47.0	1230	22.0	112.6	66.7	2477
2.6	13.3	7.88	34.6	9.1	46.6	27.6	424	15.6	79.8	47.3	1246	22.1	113.1	67.0	2500
2.7	13.8	8.18	37.3	9.2	47.1	27.9	433	15.7	80.4	47.6	1262	22.2	113.6	67.3	2522
2.8	14.3	8.48	40.1	9.3	47.6	28.2	442	15.8	80.9	47.9	1277	22.3	114.1	67.6	2545
2.9	14.8	8.79	43.0	9.4	48.1	28.5	452	15.9	81.4	48.2	1293	22.4	114.6	67.9	2568
3.0	15.4	9.1	46.1	9.5	48.6	28.8	461	16.0	81.9	48.5	1310	22.5	115.2	68.2	2591
3.1	15.9	9.4	49.2	9.6	49.1	29.1	471	16.1	82.4	48.8	1326	22.6	115.7	68.5	2614
3.2	16.4	9.7	52.4	9.7	49.6	29.4	481	16.2	82.9	49.1	1343	22.7	116.2	68.8	2637
3.3	16.9	10.0	55.8	9.8	50.2	29.7	491	16.3	83.4	49.4	1359	22.8	116.7	69.1	2661
3.4	17.4	10.3	59.2	9.9	50.7	30.0	502	16.4	83.9	49.7	1376	22.9	117.2	69.4	2684
3.5	17.9	10.6	62.7	10.0	51.2	30.3	512	16.5	84.4	50.0	1393	23.0	117.7	69.7	2707
3.6	18.4	10.9	66.4	10.1	51.7	30.6	522	16.6	85.0	50.3	1410	23.1	118.2	70.0	2731
3.7	18.9	11.2	70.1	10.2	52.2	30.9	533	16.7	85.5	50.6	1427	23.2	118.7	70.3	2755
3.8	19.4	11.5	73.9	10.3	52.7	31.2	543	16.8	86.0	50.9	1444	23.3	119.2	70.6	2779
3.9	20.0	11.8	77.9	10.4	53.2	31.5	554	16.9	86.5	51.2	1461	23.4	119.8	70.9	2803
4.0	20.5	12.1	81.9	10.5	53.7	31.8	564	17.0	87.0	51.5	1479	23.5	120.3	71.2	2827
4.1	21.0	12.4	86.1	10.6	54.2	32.1	575	17.1	87.5	51.8	1496	23.6	120.8	71.5	2851
4.2	21.5	12.7	90.3	10.7	54.8	32.4	586	17.2	88.0	52.1	1514	23.7	121.3	71.8	2875
4.3	22.0	13.0	94.7	10.8	55.3	32.7	597	17.3	88.5	52.4	1531	23.8	121.8	72.1	2899
4.4	22.5	13.3	99.1	10.9	55.8	33.0	608	17.4	89.0	52.7	1549	23.9	122.3	72.4	2924
4.5	23.0	13.6	104	11.0	56.3	33.3	620	17.5	89.6	53.0	1567	24.0	122.8	72.7	2948
4.6	23.5	13.9	108	11.1	56.8	33.6	631	17.6	90.1	53.3	1585	24.1	123.3	73.0	2973
4.7	24.0	14.2	113	11.2	57.3	33.9	642	17.7	90.6	53.6	1603	24.2	123.8	73.3	2997
4.8	24.6	14.5	118	11.3	57.8	34.2	654	17.8	91.1	53.9	1621	24.3	124.4	73.6	3022
4.9	25.1	14.8	123	11.4	58.3	34.5	665	17.9	91.6	54.2	1639	24.4	124.9	73.9	3047
5.0	25.6	15.2	128	11.5	58.9	34.8	677	18.0	92.1	54.5	1658	24.5	125.4	74.2	3072
5.1	26.1	15.5	133	11.6	59.4	35.1	689	18.1	92.6	54.8	1677	24.6	125.9	74.5	3097
5.2	26.6	15.8	138	11.7	59.9	35.4	701	18.2	93.1	55.1	1695	24.7	126.4	74.8	3123
5.3	27.1	16.1	144	11.8	60.4	35.8	713	18.3	93.6	55.4	1714	24.8	126.9	75.1	3148
5.4	27.6	16.4	149	11.9	60.9	36.1	725	18.4	94.2	55.8	1732	24.9	127.4	75.4	3173
5.5	28.1	16.7	155	12.0	61.4	36.4	737	18.5	94.7	56.1	1751	25.0	128.0	75.7	3199
5.6	28.7	17.0	161	12.1	61.9	36.7	750	18.6	95.2	56.4	1770	25.1	128.5	76.0	3225
5.7	29.2	17.3	166	12.2	62.4	37.0	762	18.7	95.7	56.7	1789	25.2	129.0	76.4	3250
5.8	29.7	17.6	172	12.3	63.0	37.3	775	18.8	96.2	57.0	1809	25.3	129.5	76.7	3276
5.9	30.2	17.9	178	12.4	63.5	37.6	787	18.9	96.7	57.3	1828	25.4	130.0	77.0	3302
6.0	30.7	18.2	184	12.5	64.0	37.9	800	19.0	97.2	57.6	1847	25.5	130.5	77.3	3328
6.1	31.2	18.5	191	12.6	64.5	38.2	813	19.1	97.8	57.9	1867	25.6	131.0	77.6	3354
6.2	31.7	18.8	197	12.7	65.0	38.5	826	19.2	98.3	58.2	1886	25.7	131.5	77.9	3380
6.3	32.2	19.1	203	12.8	65.5	38.8	839	19.3	98.8	58.5	1906	25.8	132.0	78.2	3407
6.4	32.8	19.4	210	12.9	66.0	39.1	852	19.4	99.3	58.8	1926	25.9	132.6	78.5	3433
6.5	33.3	19.7	216	13.0	66.5	39.4	865	19.5	99.8	59.1	1946	26.0	133.1	78.8	3460
6.6	33.8	20.0	223	13.1	67.0	39.7	879	19.6	100.3	59.4	1966	26.1	133.6	79.1	3486
6.7	34.3	20.3	230	13.2	67.6	40.0	892	19.7	100.8	59.7	1986	26.2	134.1	79.4	3513
6.8	34.8	20.6	237	13.3	68.1	40.3	906	19.8	101.3	60.0	2006	26.3	134.6	79.7	3540
6.9	35.3	20.9	244	13.4	68.6	40.6	919	19.9	101.8	60.3	2027	26.4	135.1	80.0	3567

$$p = \rho g \frac{\cosh k(y + h)}{\cosh kh} \cos k(x - ct) \quad (27)$$

and for deep water, on the basis of equation (15)

$$p = \rho g e^{ky} \cos k(x - ct) \quad (28)$$

The pressures indicated by equations (27) and (28) are those due to wave formation only; i.e., the pressures which act over and above the hydrostatic pressure $p_0 = -\rho g y$ in still water.

When the dimensions of a body are small as compared to the wave length, the total force exerted on it by waves can be represented in terms of the body volume and pressure gradient existing instantaneously at a particular point under the surface wave. This will often prove to be much less laborious than integration of pressures over the body surface, particularly for a complicated body form. The pressure gradients are obtained by partial differentiation of equations (27) and (28) with respect to

x or y , giving: In the horizontal direction, for shallow water,

$$(\partial p / \partial x) = -\rho g k \frac{\cosh k(y + h)}{\cosh kh} \sin k(x - ct), \quad (29)$$

and for deep water

$$(\partial p / \partial x) = -\rho g k e^{ky} \sin k(x - ct), \quad (30)$$

in the vertical direction, for shallow water

$$(\partial p / \partial y) = \rho g k \frac{\sinh k(y + h)}{\cosh kh} \cos k(x - ct), \quad (31)$$

and for deep water

$$(\partial p / \partial y) = \rho g k e^{ky} \cos k(x - ct) \quad (32)$$

In applying the expressions for the pressure gradients, the distinction between the effects of the hydrostatic and dynamic gradients should be remembered. In a hydrostatic case, the force acting on a body is given by $-(\partial p / \partial x)$ times the volume of the body. It has been shown by G. I. Taylor (1-1928) that when the pressure gradient is caused by velocities and accelerations in the fluid, this relationship is modified and becomes:

$$\text{Force} = -(\partial p / \partial x)(\text{body volume})(1 + k_x) \quad (33)$$

where k_x is the coefficient of accession to inertia in the x -direction in the case of the horizontal force. A similar expression is obtained for the vertical force by using the derivative $\partial p / \partial y$, and the k_y -value for the vertical direction. For a cylindrical body $k_y = 1$, and the force due to a dynamic pressure gradient is double that due to a hydrostatic one.

Now the complete forms $\sin k(x - ct)$ and $\cos k(x - ct)$, which had to be kept throughout the development, since various derivatives had to be taken, can be simplified for two typical cases.

If a ship, relatively long with respect to the wave length, is analyzed at a specified instantaneous location on the wave, the time t at this particular instant can be considered zero, so that the expressions become $\sin(2\pi x / \lambda)$ and $\cos(2\pi x / \lambda)$. In another case an analysis may be required of time-dependent pressures and forces acting on a small body at a fixed location, which can be taken as $x = 0$. The expressions become then functions of $(2\pi ct / \lambda)$, and are expressed more conveniently in terms of the circular frequency ω as $\sin \omega t$ and $\cos \omega t$.

3 Waves of Finite Height

3.1 Stokes' Waves. In the previous section the approximations made in writing the boundary condition equations (8), (9) and (10) made the analysis exact only for vanishingly small wave height. It has been mentioned already that in the case of a finite wave height this analysis gives correct expressions for the first harmonic of the wave form expressed by a Fourier series. A brief discussion of the theory of waves of finite height will now be given in order to support this statement, to demon-

strate the significance of the approximation made, and to compare the results of the potential theory with the trochoidal one. The theory of waves of finite amplitude was formulated first by Stokes (1-1847), and was further developed and confirmed by Levi-Civita (1-1925), and Struick (1-1926). A brief exposition of it is given by Lamb (1-C par. 250, pages 417-420), and the results are stated in a simple and convenient form by O'Brien and Mason (1-G, pages 14-24). The subject is completely missing in other basic texts on hydrodynamics.

Stokes (1-1847) gives the following expressions for the wave in deep water:

To the second order,

$$(\eta / \lambda) = (a / \lambda) \cos kx + \pi (a / \lambda)^2 \cos 2kx \quad (34)$$

and

$$c = (g\lambda / 2\pi)^{1/2} \text{ (as in the linear case)}$$

to the third order,

$$(\eta / \lambda) = (a / \lambda) \cos kx + \pi (a / \lambda)^2 \cos 2kx + 3\pi^2 (a / \lambda)^3 \cos 3kx \quad (35)$$

and

$$c = (g\lambda / 2\pi)^{1/2} \left[1 + 2\pi^2 \left(\frac{a}{\lambda} \right)^2 \right]$$

For a wave of length-to-height ratio $\lambda / 2a = 20$, the correction to c in the latter case is only 1.25 per cent.

According to Stokes, equation (35) for a potential wave of finite amplitude to the third order of approximation coincides with the sum of the first three harmonics of the trochoidal form. This is above the accuracy attainable in practice in wave generation and measurement. For the equation of η to the fourth order, and for corresponding expressions for water of depth h the reader is referred to O'Brien and Mason (1-G pp. 15-22).

From the work of Rayleigh (1-1876) (see Lamb, 1-D, p. 417, equation 1) it is seen that expression (15) for the velocity potential ϕ in deep water is valid within a second approximation; correction is needed only for third-order and higher approximations. Correction of the first-order theory to a second approximation, therefore, involves only the relationship between wave height and potential, which was mutilated in making the approximations shown by equations (8) and (9). Part of the correction is to reinstate the terms in velocities u and v of equation (7). Denoting this correction to η by $\Delta_1 \eta$ using expressions for u and v listed in Table 1 (for $y = 0$) and substituting $c^2 = g\lambda / 2\pi$,

$$\Delta_1 \eta = -\frac{\pi a^2}{\lambda} [\sin^2 k(x - ct) + \cos^2 k(x - ct)] \quad (36)$$

The second necessary correction $\Delta_2 \eta$ to the η given by equation (8) can be accomplished by the method of R. Guillon (see Korvin-Kroukovsky and Jacobs, 1-1954, pages 26, 27). To the first order of approximation equation (8) represents a function $F(x, 0)$ and it is desired to find $F(x, \eta)$. Taking the first two terms of the expansion in a Taylor's Series

$$F(x, \eta) = F(x, 0) + (\eta \partial F(x, 0) / \partial y) \quad (37)$$

$\Delta_2 \eta$ then amounts to the second term of expression (37). Substituting expression (18) for η , taking its derivative with respect to y , and letting $y = 0$

$$\Delta_2 \eta = 2(\pi a^2 / \lambda) \cos^2 k(x - ct) \quad (38)$$

The total correction is therefore

$$\begin{aligned} \Delta(\eta/\lambda) &= (\Delta_1 \eta + \Delta_2 \eta) / \lambda \\ &= \pi \left(\frac{a}{\lambda} \right)^2 [\cos^2 k(x - ct) - \sin^2 k(x - ct)] \\ &= \pi \left(\frac{a}{\lambda} \right)^2 \cos 2k(x - ct) \end{aligned} \quad (39)$$

After letting $t = 0$ this is seen to be identical with the second term of Stoke's equation (34).

In a great majority of potential-flow problems connected with waves, the work is based not on the wave height itself, but on the velocity potential of the wave motion. The primary purpose of the foregoing discussion has been to demonstrate that *work based on the wave-velocity potential is correct within a second-order approximation in the potential gravity wave theory*. While a simple harmonic wave is usually referred to in the introductory text of any such work, the results apply in reality to very nearly trochoidal wave form.

3.2 Maximum Wave Height. As the wave height increases, and the terms of higher order become significant, the crest of the wave becomes sharper and sharper, until finally it takes the form of a sharp cusp. The value of the minimum included angle at the cusp was derived by Stokes on the following basis: A uniform velocity $-c$ is imposed on the water, so that the waves become stationary in space. At the cusp the velocity must be zero, and in close vicinity to it the water flow corresponds to the one occurring in a re-entrant angle. Using polar coordinates and the notation indicated in Fig. 3, the velocity

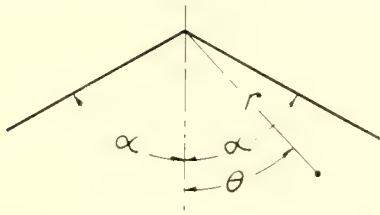


Fig. 3. Sketch used in connection with Section 3.2

potential of such a flow is (Lamb, 1-D, par, 63, p. 68 and p. 418)

$$\phi = Ar^n \sin n\theta \quad (40)$$

The velocity normal to the bounding surface (i.e., at $\theta = \pm \alpha$)

$$-(\partial \phi / r \partial \theta) = 0 = nAr^{n-1} \cos n\theta \quad (41)$$

and therefore

$$n\theta = \pi/2. \quad (42)$$

The velocity along the surface is

$$U = -(\partial \phi / \partial r) = -nAr^{n-1} \sin n\theta \quad \text{at } \theta = \pm \alpha \quad (43)$$

Since the water motion in the wave is solely due to gravity and inertial forces, and since the velocity is zero at the cusp, the velocity at any vertical distance $r \cos \theta$ below the cusp is

$$U = (2gr \cos \theta)^{1/2} \quad (44)$$

By equating the exponents of r , $(n - 1)$ in equation (43) and $1/2$ in equation (44), n is evaluated as $3/2$. From equation (42) it then follows that $\max \theta = \alpha = \pi/3$; therefore the total included angle 2α is 120 deg.

Michell (1-1893) and Havelock (1-1918) found that the corresponding *limiting wave height from trough to crest is 0.14λ* (expressed to two decimal points). Such a limiting height is approached in experiments only with a perfect wave form. Any irregularity of the wave causes a premature breaking of crests.

3.3 Mass Transport. The volume of water flowing between the free surface and a depth y below it is expressed by the stream function²

$$\psi = -ace^{ky} \cos k(x - ct) \quad (45)$$

where to the second order of approximation

$$y = y_0 + ae^{ky_0} \cos k(x - ct) \quad (46)$$

Substituting equation (46) into (45) and retaining only the first two terms of a serial expansion of the exponential,

$$\psi = -ace^{ky_0} [1 + ka^{ky_0} \cos k(x - ct)] \cos k(x - ct) \quad (47)$$

The mean rate of flow for the water layer contained between the surface and the depth y_0 can be obtained by integration of ψ over the cycle period T . The mean rate of flow is independent of the position x , and without loss of generality can be evaluated at $x = 0$. Noting further that only terms containing cosine squared in the integral contribute to the value of the integral, the mean value of ψ , designated here by $\bar{\psi}$, is

$$\bar{\psi} = -\frac{1}{T} ka^2 ce^{2ky_0} \int_0^T \cos^2 2\pi t/T dt \quad (48)$$

$$= -1/2 ka^2 ce^{2ky_0}$$

The mean velocity \bar{u} of the water flow at the depth y_0 is obtained by partial differentiation

$$\bar{u} = -\partial \bar{\psi} / \partial y_0 = k^2 a^2 ce^{2ky_0} \quad (49)$$

which is identical with the expression derived by Lamb (1-D, p. 419, equation 16).

As an example, a wave 600 ft long and 30 ft high ($a = 15$) can be considered. Its celerity is 55.4 fps. The surface layer of water is shown by equation (49) to drift at

² Table 1 in this Appendix.

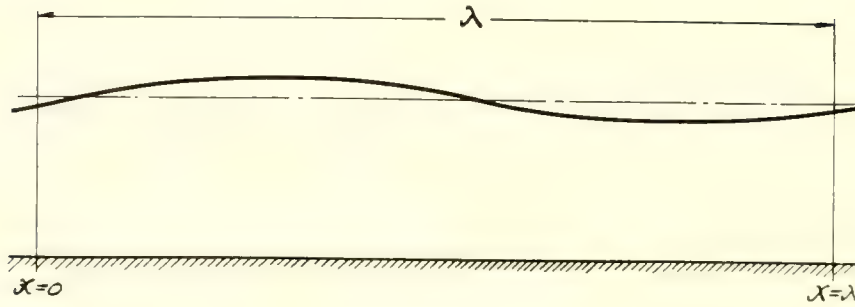


Fig. 4 Sketch used in connection with Section 4.1

1.36 fps; i.e., a shallow floating object will drift 19.3 nautical miles in a day. Because of the factor of 2 in the exponential, the drift velocity diminishes very rapidly with depth.

4 Wave Energy and Group Velocity

4.1 Energy per Unit of Sea Surface. The energy connected with water motion can be considered from two points of view: (a) Energy contained per unit area of sea surface, and (b) energy transported by waves through a vertical plane normal to the direction of wave propagation.

In considering the first of these aspects, distinction is made between the potential and kinetic energy. The potential energy is due to the weight of water ρg , and its elevation or depression with respect to the still-water level. For a unit width of the sea surface area, and for one wave length, it is defined as

$$E_p = \frac{1}{2} \rho g \int_0^\lambda \eta^2 dx \quad (50)$$

Using the artifice of making the wave motion steady by superposing the water velocity $-c$, which is equivalent to taking η at $t = 0$

$$\eta = a \cos kx \quad (51)$$

and therefore

$$E_p = \frac{1}{4} a^2 \rho g \lambda \quad (52)$$

The kinetic energy contained in a body of fluid is defined in terms of the conditions existing at its boundaries

$$E_k = \frac{1}{2} \int_0^\lambda \phi (\partial \phi / \partial n) ds \quad (53)$$

where ds is an increment of length taken along the boundary. Consider a mass of water contained between the free surface, the bottom and two vertical control planes spaced one wave length λ apart, as shown in Fig. 4. As the integral is taken following these boundaries in the clockwise direction, the contribution due to the control planes vanishes, as it is equal and opposite in sign on the two planes. At the bottom $(\partial \phi / \partial n) = 0$, so that the only non-vanishing contribution comes from the free water surface. With the low wave height assumed in the linear theory, the normal surface velocity $-\partial \phi / \partial n$ can be replaced by the vertical one $-\partial \phi / \partial y$. Substituting

the values of

$$\phi = cx - ac \sin kx \quad \text{for } y = 0 \quad (54)$$

and

$$\partial \phi / \partial y = -ack \sin kx \quad (55)$$

and omitting the constant term resulting from cx , the kinetic energy due to wave motion is found to be

$$E_k = \frac{1}{4} a^2 \rho g \lambda \quad (56)$$

i.e., the potential and kinetic energies in wave motion are equal and the total energy is

$$E = E_p + E_k = \frac{1}{2} a^2 \rho g \lambda \text{ per wave length} \quad (57)$$

or

$$E = \frac{1}{2} a^2 \rho g \text{ per unit area of sea surface} \quad (58)$$

4.2 Energy Transfer. The rate at which the wave energy is transferred in the direction of wave propagation can be evaluated by considering the rate at which the work is being done on the water to the right of a vertical control plane (Lamb, 1-D, page 383). The control plane can be assumed to be located at $x = 0$. This rate of work is evidently a product of the pressure p and the horizontal water-particle velocity u . Using the previously derived values of these, the instantaneous rate of work is

$$\begin{aligned} \partial E / \partial t &= \int_{-\infty}^0 p u dy = \rho g a^2 k c \cos^2 k(x - ct) \int_{-\infty}^0 e^{2ky} dy \\ &= \frac{1}{2} \rho g a^2 c \cos^2 k(x - ct) \end{aligned} \quad (59)$$

The mean value of $\cos^2 k(x - ct)$ over a cycle is $1/2$, so that finally

$$\text{Mean } \partial E / \partial t = \frac{1}{2} \rho g a^2 (c/2) \quad (60)$$

i.e., the total energy of wave motion in deep water is transferred at half the wave celerity.

4.3 Group Energy. The following is essentially quoted from Lamb (1-D Art. 236, pp. 380, 381), with minor changes and some abbreviation:

It has often been noticed that when an isolated group of waves, of sensibly the same length, is advancing over relatively deep water, the velocity of the group as a whole is less than that of the individual waves composing it. If attention be fixed on a particular wave, it is seen to advance through the group, gradually dying out as it

approaches the front, while its former place in the group is occupied in succession by other waves which have come forward from the rear.

The simplest analytical representation of such a group is obtained by the superposition of two systems of waves of the same amplitude, and of nearly, but not quite, the same wave length. The corresponding equation of the free surface will be of the form

$$\begin{aligned}\eta &= a \sin(kx - \omega t) + a \sin(k'x - \omega' t) \\ &= 2a \cos\left\{\frac{1}{2}(k - k')x - \frac{1}{2}(\omega - \omega')t\right\} \\ &\quad \sin\left\{\frac{1}{2}(k + k')x - \frac{1}{2}(\omega + \omega')t\right\} \quad (61)\end{aligned}$$

where ω is written for the circular frequency $kc = 2\pi c/\lambda$.

If k, k' be very nearly equal, the cosine in this expression varies very slowly with x , so that the wave profile at any instant has the form of a curve of sines in which the amplitude alternates gradually between the values 0 and $2a$. The surface, therefore, presents the appearance of a series of groups of waves separated at equal intervals by bands of nearly smooth water. Since the distance between the centers of two successive groups is $2\pi/(k - k')$, and the time occupied by the system in shifting through this space is $2\pi(\omega - \omega')/(k - k')$, the group-velocity (U , say) is $=(\omega - \omega')/(k - k')$

$$U = d\omega/dk \quad (62)$$

This result holds for any case of waves traveling through a uniform medium. In application to waves on water of depth h ,

$$c = \left(\frac{g \tanh kh}{k}\right)^{1/2}$$

and therefore, for the group velocity,

$$U = d(kc)/dk = \frac{1}{2}c \left(1 + \frac{2kh}{\sinh 2kh}\right) \quad (63)$$

The ratio which this bears to the wave celerity c increases as kh diminishes, being $1/2$ when the depth is very great, and unity when it is very small, compared with the wave length.

It is observed that the group velocity is identical with the rate of transmission of the energy in waves. Indeed, an isolated group of waves cannot advance into still water unless its energy is transmitted at the velocity of the group.

4.4 Damping of Waves. The theory of waves discussed up to now neglects the viscosity and assumes a perfect or inviscid fluid. The waves described by this theory represent therefore a "conservative system"; i.e., there is no loss or gain of energy, and the total energy contained in the system remains constant. Under "energy" here is meant the sum of the potential (gravity) and kinetic energies. Loss of energy means conversion of it into the energy of molecular motion; i.e., heat. In an actual fluid possessing a small viscosity, as is the case with water, the equations resulting from the potential theory are still found to describe the flow correctly

(Lamb, 1-D, Art. 346, pages 623-625), but there is a certain small rate of dissipation of energy due to internal friction resulting from the distortion of fluid elements. The waves, therefore, cannot propagate without change of form, but propagate with gradually and very slowly decreasing amplitude. Equation (58) for the total energy per unit area of sea surface can be rewritten as

$$E = \frac{1}{2}\rho k c^2 a^2 \quad (64)$$

and the rate of energy loss expressed as

$$d/dt(\frac{1}{2}\rho k c^2 a^2) \quad (65)$$

From a consideration of the work done by viscous forces (Lamb, 1-D, p. 624) the rate of energy dissipation is

$$-2\mu k^3 c^2 a^2 \quad (66)$$

where μ is the viscosity of water. By equating (65) and (66) the rate of decrease of wave amplitude is

$$\frac{da}{dt} = -2\nu k^2 a \quad (67)$$

or

$$a = a_0 e^{-2\nu k^2 t} \quad (68)$$

where a_0 is the initial wave amplitude at $t = 0$, a the amplitude at the time t , and ν is the kinematic viscosity $= \mu/\rho$. The time necessary for the wave amplitude to be reduced to $1/e$ of its original amplitude, called the modulus of decay, is

$$t = \lambda^2/8\nu\pi^2 \quad (69)$$

Since k is squared in the exponential, the short waves with large k are damped rapidly, while the long waves with a small k are attenuated extremely slowly. Table 4 (taken from O'Brien and Mason, 1-G, page 42) gives the length of the wave travel in which the amplitude is reduced to $1/e$ of its initial value.

Table 4

Wave period, sec	Wave length, ft	Wave celerity, fps	Wave travel, miles
1	5.1	5.1	32.5
5	128	25.6	102000
10	512	51.2	3250000

The foregoing theoretical computations are confirmed qualitatively by observing waves in nature. The small ripple is found to disappear in a few minutes, while the long swells are found to travel hundreds or thousands of miles without appreciable attenuation. This slow rate of damping of long swells makes it possible to forecast the surf conditions, which often have their origin in a far distant storm. The rate of amplitude decay just given is entirely due to internal friction. In the actual sea there is additional and apparently larger dissipation of energy due to turbulence and to the breaking of steep waves.

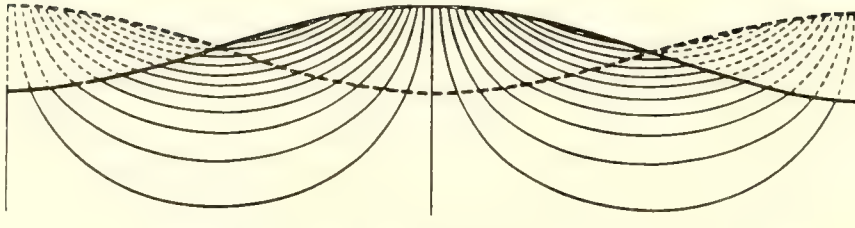


Fig. 5 Streamlines of water flow in standing waves (from Lamb, I-D)

5 Standing Waves

The waves discussed in the foregoing propagate in one direction with celerity c and are known as "progressive waves," as distinguished from "standing waves"; in these latter, the water particles at crests and troughs move up and down in a vertical straight line, while the particles at nodal points move to and fro along horizontal straight lines. This system can be represented mathematically by superposing two progressive wave trains moving in opposite directions, each with the amplitude $a/2$. It will be convenient first to modify equations (13) and (14) by replacing $\cos k(x - ct)$ by $\sin k(x - ct)$ and vice versa. This brings the notation to the form used by Lamb (1-D, Art. 228, page 364). Substituting $(a/2)$ for (a) in these equations, forming two pairs of equations with $+c$ and $-c$, and using trigonometric relationships for the sum and difference of two angles, the following expressions for the velocity potential and stream function are obtained:

$$\phi = -\frac{ag}{\omega} \frac{\cosh k(y+h)}{\cosh kh} \cos kx \cos \omega t \quad (70)$$

$$\psi = +\frac{ag}{\omega} \frac{\sinh k(y+h)}{\cosh kh} \sin kx \cos \omega t \quad (71)$$

The corresponding streamlines are shown in Fig. 5. The relationship between the circular frequency ω , wave length λ (through $k = 2\pi/\lambda$) and water depth h is given by (Lamb 1-D, page 364):

$$\omega^2 = gk \tanh kh \quad (72)$$

The water velocities are found by partial differentiation of equation (70) with respect to x and y ,

$$u = -\frac{\partial \phi}{\partial x} = -\frac{agk}{\omega} \frac{\cosh k(y+h)}{\cosh kh} \sin kx \cos \omega t \quad (73)$$

$$v = -\frac{\partial \phi}{\partial y} = +\frac{agk}{\omega} \frac{\sinh k(y+h)}{\cosh kh} \cos kx \cos \omega t \quad (74)$$

and displacements of water particles ξ and η from the still-water position (x, y) are obtained by integration of the foregoing equations with respect to t , and combining with equation (72) for ω^2 ,

$$\xi = -a \frac{\cosh k(y+h)}{\sinh kh} \sin kx \sin \omega t \quad (75)$$

$$\eta = a \frac{\sinh k(y+h)}{\sinh kh} \cos kx \sin \omega t \quad (76)$$

These equations show that the motion of each particle is rectilinear and simple harmonic, the direction of each motion varying from vertical beneath the crests and hollows ($kx = 0, \pi, 2\pi$, etc.) to horizontal beneath the nodes ($kx = \pi/2, 3\pi/2$, etc.).

The shape of the free surface is obtained from (76) by letting $y = 0$,

$$\eta = a \cos kx \sin \omega t \quad (77)$$

Two cases of standing waves are of greatest interest to naval architects; namely, that formed by interaction of the wave reflected by a beach or a vertical cliff with the oncoming sea wave, and that formed by water oscillating in a rectangular tank. The first case needs only to be pointed out, since all relationships given in the foregoing apply for the case of a complete reflection. In the case of partial reflection, the result is a weaker standing-wave system superposed on the progressive wave system. This is often found in towing tanks when the wave-absorbing beach at the end of a tank is not sufficiently good. In the actual sea this phenomenon is often manifested in a sea condition which is much more unfavorable for ships near the shore than in the open sea; the Bay of Biscay is particularly known in this respect. It has been derived theoretically and verified experimentally (Penny and Prince, 1-1952; Taylor, 1-1953) that the minimum included angle at the wave crest of a standing wave is 90 deg, as against 120 deg for the progressive wave system; i.e., much steeper waves can be encountered in a standing-wave system than in a progressive one.

5.1 Waves in Rectangular Tanks. The sloshing of water in rectangular tanks is an instance of a standing-wave system which is of interest to naval architects. The basic principles of the problem are discussed by Coulson (1-A, Art. 49, page 75) and Lamb (1-D, Art. 190, pages 283 and Art. 257, pages 440, 441), and in application to swimming pools on board ships by Comstock (1-1947). All the boundary conditions discussed earlier in connection with waves in water of limited depth h are applicable in this case, and in addition the normal velocity at the tank walls must vanish. Equation (72), connecting the wave frequency, wave length, and tank depth applies in this case, but the possible choice of k -values is now limited to certain ratios of tank length or width to wave length, $1/2, 1, 2, \dots$, or specifically

$$k^2 = (2\pi/\lambda)^2 = \pi^2[(m^2/L^2) + (n^2/W^2)] \quad (78)$$

where m and n are whole numbers 1, 2, 3, . . ., L is the tank length and W the width. If $L > W$, the component oscillation of the longest period is obtained by making $m = 1$, $n = 0$; the wave motion is then parallel to the long side L of the rectangle and the wave length $\lambda = 2L$. The crest is at one end of the tank simultaneously with the trough at the other end. The wave component or mode corresponding to $m = 2$, i.e., $\lambda = L$, is often also of interest. In this case crests occur simultaneously at both ends with the trough in the middle, or vice versa. Generally all modes can occur simultaneously, and the

wave elevation is represented by the double Fourier's series.

$$\eta = \sum \sum a_{m,n} \cos (m\pi x/L) \cos (n\pi z/W) \quad (79)$$

and the velocity potential of each individual mode is

$$\phi = \frac{ga}{\omega} \frac{\cosh k(y+h)}{\cosh kh} \cos (m\pi x/L) \cos (n\pi z/W) \quad (80)$$

In writing these equations the origin of co-ordinates is taken at one corner of the tank, so that the tank is bounded by the vertical planes at $x = 0$ and L , $z = 0$ and W , with y negative downwards, and h the depth of the tank.

Nomenclature

a = wave amplitude (more exactly, half the height)	p = pressure	ξ, η = horizontal and vertical displacements of water particles in orbital motion
A = a coefficient	r = a polar co-ordinate	η = wave surface elevation
c = wave celerity	t = time	θ = a polar co-ordinate
E = wave energy	T = wave period	$\bar{\theta}$ = maximum wave slope
E_k = kinetic wave energy	u, v = horizontal and vertical components of orbital water velocity	λ = wave length
E_p = potential wave energy	U = water velocity	μ = coefficient of viscosity
h = water depth	w = complex potential	ν = kinematic viscosity, μ/ρ
$k = 2\pi/\lambda$. Wave number	α = included half-angle at wave cusp	ρ = mass density
g = acceleration of gravity	α, β = horizontal and vertical semi-axes of water-particle path in water of limited depth	ϕ = velocity potential
k_x, k_y = coefficients of accession to inertia in directions of x and y -axes		ψ = stream function
n = an index		ω = circular frequency = $2\pi/T$

Nomenclature

The nomenclature on page 90 of Chapter 1 applies in general to Appendix B. Additional symbols are as follows:

A = a coefficient; energy	
A_n = energy transmitted by normal component of drag force, equation (19)	
A_t = energy transmitted by tangential component of drag force, equation (20)	
$B(\beta)$ = "effective dissipative factor," equation (56)	
$C(\beta)$ = "effective friction factor" or "resistance factor" in work on drag force, equations (37 and 38)	
H_1, H_2, H_3 = equivalent heights of three major waves, $\beta_m, \beta(1)$ and β_m^*	
T_1, T_2, T_3 = characteristic mean periods of above waves	
α = a coefficient	
β_m = "sea" with $c < U$ $\beta(1)$ = intermediate wave $c = U$ β_m^* = long wave $c = 1.37 U$	Designations of waves, defined as characteristic mean values for certain period intervals when a continuous spectrum is visualized as subdivided into three major intervals (Neumann, 1952b)
$\bar{\mu}$ = value of turbulent viscosity $\mu^*(U)$ in a fully developed seaway with $\beta = \text{const}$	

1 Introduction

In this Appendix the work of Neumann on wave generation (1-1948, 1949a and b, 1950, 1952a and b)¹ will be outlined. This work is based either on consideration of the significant wave, as by Sverdrup and Munk (1-1946, 1947), or on taking the sea as composed of three predominant wave systems. Neumann's formulation of the continuous sea spectrum (1-1953, 1954, and Pierson and Neumann, 1-H) was presented in Section 1-6.2.

Neumann's work on wave generation is outstanding because:

(a) The experimental material, either taken from previous observations of other oceanographers or obtained by Neumann himself, covers a very wide range of wind and sea conditions and of geographic locations.

(b) A determined attempt is made in the analysis to adhere as closely to the rational procedure as available means permit.

(c) It appears to be the only work on wave development in which the balance of energies transmitted from the wind and dissipated in waves is fully discussed.

(d) The mathematical and statistical formulations are preceded or accompanied by a large amount of physical description of sea conditions.

The author considers the foregoing features (particularly item c) to be of sufficient importance to warrant a comprehensive summary. While the detailed work based on the concept of a significant wave or that of three wave components is outdated, the work as a whole can still be used as a valuable guide for future research. At least it has to serve as such until a more modern work of comparable scope appears. Contributions to the theory of wave generation found in the literature appear to be concerned only with individual facets of the problem. Neumann's 1948-1952 work appears to be the only one treating the problem in its entirety.

In the following exposition an attempt will be made to present, in as logical and as short a form as possible, the content of six of Neumann's papers published in the years 1948 to 1952. While the mathematics and basic concepts involved in these papers are simple, it is not always easy to follow Neumann's logic. The reader is warned therefore not to accept the following as an adequate presentation of Neumann's work, but to consult the original papers for a serious study. The difficulties mentioned result primarily from two causes: The work represents a transition period from the concept of a simple harmonic sea to the spectral presentation developed later; also it represents a gradual development of ideas from paper to paper, but not quite a continuous one. New ideas appear without conspicuous announcement, older ones are abandoned without a word, although a good deal of the older material is used and is referred to. Under no condition, however, can this set of six papers be considered as being superseded by the subsequent work on the sea-energy spectrum.

The work on the spectrum is concerned entirely with the statistical study of observational material, without any regard to its physical or hydro-aerodynamic properties. It is a good presentation of observed facts without regard to the hydro-mechanical causes of their existence. Contrary to this, Neumann's work of 1948-1952 represents the most complete investigation of the mechanics of wave formation available to date, while his description of the observable characteristics of a seaway must be kept in mind by all theoretical investigators of lesser sea experience.

¹ References to sections, equations, figures, and bibliography will be designated by chapter number and section, and so on; reference noted is to Neumann, Chapter 1 (1948).

Table 1 Sin ψ Versus U : Observations Made in 1925; Average Depth 70 M

U , meters ps.....	1	2	3	4	5	6	7	8	9.5	12.5	18
$10^3 \sin \psi$	0.2	0.8	1.2	1.4	2.0	2.6	3.0	3.6	4.6	7.7	14.4
Number of days											
observed.....	34	50	51	66	44	32	23	14	13	15	5

Table 2 Sin ψ Versus U : Single Observations; Average Depth 50 M

	In a storm from October 4 to 6, 1936										Jan. 2 and 4 1935
U , meters ps.	9.3	11.8	13.0	16.1	17.6	19.4	23.4	24.6	26.0	10.5	18.0
$10^3 \sin \psi$	6.8	9.7	12.4	19.1	23.3	25.7	31.5	36.3	43.6	6.5	16.0

In the Section 6 of Chapter 1 it was demonstrated that large differences exist among various spectra obtained on the basis of empirical data. In the introduction to Chapter 1 the author stated his belief that the purely empirical approach is inadequate and that concurrent theoretical development is needed. The Neumann's work, here outlined, can be taken as a general guide for such an approach. However, it has to be supplemented by, and blended with, the recently developed mathematical statistics of irregular sea.

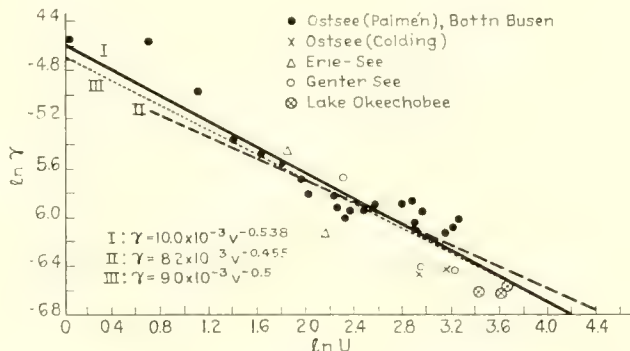
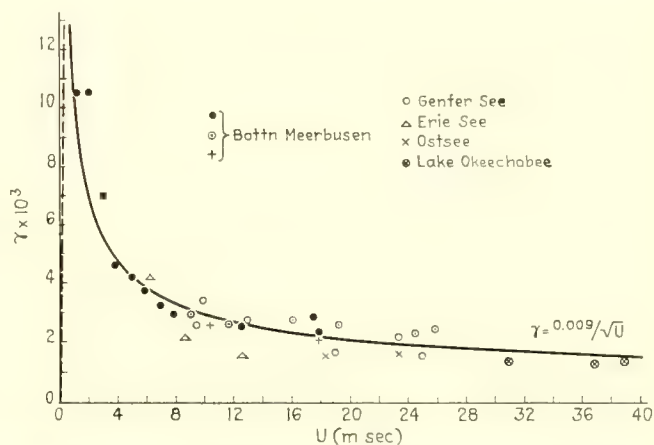
2 Horizontal Drag Force

In accordance with items (b) and (c) of the introduction, the question of the energy balance in wave growth is given a large amount of attention. The primary information (experimentally obtained) needed in this connection is the horizontal drag exerted by wind on a water surface. This forms the specific subject of Neumann (1-1948). In Sections 1-2.3 and 2.7, an attempt was made to correlate the drag force with the wave form which, in turn, depends on wind speed, fetch, and so forth. Since quantitative data on the wave form at sea are not available, Neumann attempts a direct correlation of drag force with wind speed. For this purpose he examines the large number of observations of wind-caused inclination of sea surface made by Palmén (1-1932 *a* and *b*, 1-1936, Palmén and Laurila, 1-1938). Observations were made in the Bay of Bothnia in the northern part of the Baltic Sea, where the relatively uniform, small water depth and steady wind conditions over a large area provided the opportunity for the most accurate observations. The mean observed data on wind and sea-surface slope are given in Table 1, together with the number of observations involved. Table 2 gives a set of isolated observations made during a storm in the Baltic Sea where satisfactory observation conditions also existed.

For observations made on land, i.e., with a constant degree of roughness, G. I. Taylor expressed the tangential force τ as

$$\tau = \rho \gamma U^2 \quad (1)$$

with the coefficient γ between 0.0022 and 0.0032 based on the wind velocity U measured at the height of 30 m. With particular emphasis on strong winds, Palmén and

Fig. 1. Plot of $\log \gamma$ versus $\log U$ (from Neumann, 1948)Fig. 2. Variation of drag-force coefficient γ with wind velocity U (from Neumann, 1948)

Laurila (1-1938) also derive the same expression with $\gamma = 0.0024$ (but presumably with U measured at the more usual 6 or 10-m height). Since the wave form at sea is variable and depends on the wind, Neumann assumes a relationship of the form

$$\gamma = f(U) = KU^a \quad (2)$$

His plot of $\log \gamma$ (observed) versus $\log U$ is shown in Fig. 1, and his line III is found to give the simple relationship

$$\gamma = 0.09U^{-0.5} \quad (\text{with } U \text{ in cm/sec}) \quad (3)$$

and

$$\tau = 0.09\rho'U^{3/2} \quad (4)$$

The final plot of the coefficient γ versus U is shown in Fig. 2.

The drag coefficient γ diminishes with stronger winds; i.e., from the point of view of the energy transmission from wind, the sea becomes less rough in a stronger wind. While startling at first, this result is consistent with the fact shown earlier by Sverdrup and Munk (1-1946, 1-1947) that wave steepness decreases with increase of wind strength (see Fig. 1-18). Neumann (1-1948) independently obtains a rapid reduction of wave steepness from the limiting value of $\lambda/H = 7.33$ at $U = 1$ m per sec to the maximum $\lambda/H = 25$ at 12 m per sec. Darbyshire (1-1952), attempting to use Jeffreys' (1-1925) theory to explain some of his findings, came to the conclusion that the sheltering coefficient s is not constant, but can be assumed to be proportional to the wave slope $\partial\eta/\partial x$. This slope decreases with H/λ and with increase of wave length at increasing c/U ratio.

Additional confirmation of the decrease of drag-force coefficient with increase of wind is found in the measurements of wind-velocity gradient made by Wüst (1-1937) and Jeffreys (1-1920). The theory of such measurements is outlined in Section 1-2.7, but will now be repeated following Neumann (1-1948).² It is assumed that the flow of air along the sea surface is of the same nature as along a rough plate. L. Prandtl expressed the tangential force per unit of plate area as

$$\tau = Adu/dz = \rho'l^2(du/dz)^2 \quad (5)$$

where u is the air velocity at the distance z from the plate, and l is the "mixing length," expressed in the vicinity of the plate as

$$l = k_0(z + z_0) \quad (6)$$

Here k_0 is a constant evaluated theoretically by von Kármán as 0.38, and experimentally measured as 0.40; z_0 is a length dimension which defines the "roughness" of the plate. From (5 and 6), it follows that

$$\tau = \rho'k_0^2(z + z_0)^2(du/dz)^2 \quad (7)$$

or

$$\frac{du}{dz} = \frac{1}{k_0(z + z_0)} \left(\frac{\tau}{\rho'} \right)^{1/2} \quad (8)$$

Assuming that shear stress in the vicinity of the plate can be taken as independent of z , it follows after integration, that

$$u = \frac{1}{k_0} \left(\frac{\tau}{\rho'} \right)^{1/2} \ln \frac{z + z_0}{z_0} \quad (9)$$

and

$$\tau = \frac{\rho'k_0^2}{\left(\ln \frac{z + z_0}{z_0} \right)^2} u^2 \quad (10)$$

Comparing (10) with Taylor's expression (1), it follows that

² For a recent review of this subject see Neumann (1-1955).

$$\gamma = \frac{k_0^2}{\left(\ln \frac{z + z_0}{z_0} \right)^2} \quad (11)$$

From measurements of velocity u at several heights z the tangential force τ and the roughness parameter z_0 can be computed. The results of such measurements and computations made by Wüst (1-1937) and Jeffreys (1-1920) are given in Table 3. The roughness parameter z_0 is seen to diminish very rapidly with increase in wind speed at low wind speeds, and more slowly at higher wind speeds, when it generally has a very low value.

Table 3 Roughness Parameter z_0 Versus Wind Speed

Source	Method	Roughness z_0 , cm	Wind speed, m/s
Wüst (1-1937)	Wind velocity gradient in lowest air layers	5	3.7
		17	4.3
		1.6	4.3
		4.3	5.5
		0.6	5.8
		1.4	6.2
		0.8	6.2
		0.8	6.3
		0.9	7.0
		0.7	10.25
Jeffreys (1-1920)	Angle between surface wind and gradient wind	0.49	15.25
		0.32	22(?)
		0.85	10.25
Jeffreys (1-1920)	Relationship between wind velocity on the surface and strength of gradient wind	0.36	15.25
		0.64	22(?)

AUTHOR'S REMARK: The reduction of the tangential-force coefficient γ with wind speed can logically be explained on the basis of Section 1-2, taking into consideration the complex nature of sea waves. In moderate to strong winds, it can be expected that the surface of larger waves is entirely covered by small waves and ripples, for which $\beta = c/U < 0.4$ and which attain the limiting steepness of $1/7$ and develop sharp crests. The drag coefficient C_d , on the basis of Motzfeld's (1-1937) wind tunnel tests as well as the wind flume tests of Francis (1-1951) and of Johnson and Rice (1-1952), can be expected to have attained its maximum value and to remain approximately constant. The coefficient C_d is defined, however, with respect to the relative velocity $U-C$, where C is the facet velocity of Munk (1-1955) and probably can be identified with the formula velocity of Keuligan (1-1951) and Van Dorn (1-1953). The coefficient γ on the other hand is referred to U . It follows then that

$$\gamma \approx C_d U^2 / (U - C)^2 \quad (12)$$

and decrease of γ with increase of U occurs if C increases less rapidly than U , even in the case of constant maximum value of C_d .

Moreover, the description of a complex sea given by Neumann in the references here considered indicates that the small ripples are not uniformly distributed over the surface of the sea. They are more developed on the wind-

ward side of large wave crests and appear much smoother on the lee side. The turbulence caused by breaking wave crests also damps out small waves temporarily, but they are again regenerated by wind. Therefore, with the development of larger waves the mean value of C_d can be expected to assume values smaller than the maximum and a further reduction of γ can be expected.

3 Drag as Function of Wave Slope

For later use, Neumann develops an expression for the tangential stress as a function of the maximum wave surface slope $(\partial\eta/\partial x)_{\max} = 2\pi a/\lambda = ak$, where k is the wave number $2\pi/\lambda$. In terms of the drag coefficients,

$$\tau = C_d \frac{\rho'}{2} (U - c)^2 \quad (13)$$

and in terms of the wave slope and a coefficient s (similar to Jeffreys' except for the factor of $1/2$),

$$\tau = s \frac{\rho'}{2} a^2 k^2 (U - c)^2 \quad (14)$$

It follows that

$$C_d = sa^2 k^2 \quad (15)$$

Noting that Motzfeld (1-1937) obtained different powers for a and k from his wind-tunnel experiments, Neumann (1-1949) writes C_d generally as

$$C_d = sa^n k^n \quad (16)$$

Using equation (4) for the total stress τ , C_d is computed by equation (13) on the assumption of $c = U/3$. By using the empirical relationship for a/λ derived in Neumann (1-1948), simultaneous solutions for s and n are obtained for a number of wind speeds. These show that mean constant values of $n = 1$ and $s = 0.095$ can be assumed. The final relationship

$$\tau = s \frac{\rho'}{2} ak(U - c)^2 \quad (17)$$

is then adopted. The restriction to $c = U/3$ in the derivation of C_d appears to be neglected in the subsequent work, and a constant value of $s = 0.095$ is used independently of $\beta = c/U$.

4 Description of the Sea

As mentioned under (d) in the introduction, Neumann's work is valuable not only for its formal derivations, but also for the descriptions of the observed sea conditions. These descriptions are inserted throughout the cited references, and it would be impractical to reproduce them with any degree of completeness. However, a few quotations (in an informal translation) will be useful here. Thus from Neumann (1-1950, page 42), speaking about initial waves: "... at a wind of about 1 m/sec they are about 7 cm long, and the height of these 'ripples'

amounts to about 0.5 cm. As the wind becomes stronger, the sea surface takes on a pronounced 'rough' appearance. The early regularity of initial wave is destroyed; with relatively rapidly growing waves, all possible wave lengths appear beginning with small ripples up to the maximal wave. In this wave confusion certain waves show signs of breaking, or the beginning of breaking, of sharp crests. This condition establishes itself relatively fast with freshening wind. The steepness of waves ($H/\lambda = \delta$) is relatively large. According to the results of various observers, the maximum ratio δ appears to lie at $1/8$, which comes close to the theoretical maximum $\delta_{\max} = 1/7$, which Michell calculated for the steepest waves according to Stokes' theory."

"When the wind strength further increases, the wave lengths and wave heights grow rapidly. A fact known to all sea travellers is that with a suddenly developing wind, and quickly whipped-up sea, the wave crests are steeper than with an older sea at corresponding wind strength. With a steady wind velocity and with waves reaching full development, a certain maximal wave corresponding to the wind strength always develops. It is what a seaman designates as 'sea.' These seas can be individually recognized, sometimes more, sometimes less clearly, in the wave confusion in a seaway. But the more fully developed the sea is, the more conspicuously these waves appear; further indication of a fully developed seaway in a storm wind is the increasing length of wave crests. The 'sea' is entirely overlaid by shorter waves down to ripples. The stereophotographs in the work on "Meteor" (Schumacher, 1-1925) show how the wave-agitated sea surface consists of overlaying of the large forms by the smaller and the smallest. Particularly in 'storm seas' this is a well known occurrence. The long 'rolling' sea is almost always clearly seen in a seaway, and its surface appears to be strongly roughened."

"The wave confusion of overlaying small waves stands close to a 'continuous spectrum,' which can be observed in a young seaway thrown up by a sudden onset of wind. Again and again these small waves are rebuilt by absorption of the energy from wind, grow to maximum steepness and break; they represent an important factor in the question of energy transfer from wind to water. In the development of the complex sea they act in a certain sense as 'roughness protuberances' for the wind sweeping over the main wave profile of the sea. The resulting shearing forces form a substantial part of the total thrust exerted by wind on the sea surface. They also can be effective in the sense of energy transfer when those waves occur whose celerity exceeds the wind velocity."

On the basis of the plot of $\delta = H/\lambda$ versus c/U , shown in Fig. 1-19, Neumann (1-1950, page 45) writes: "Among the waves in the appearance of the complex seaway three types will play particular roles:—

"1 The short but steep breaking $c = U/3$ wave.³ It is of small practical significance, but theoretically it

³ Symbols are given here in the notation of the present monograph.

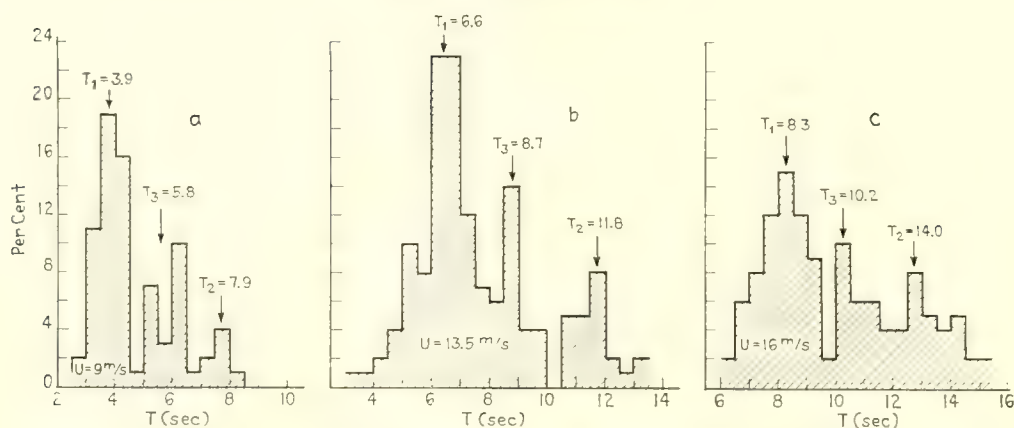


Fig. 3 Observed frequency distribution of apparent periods in a seaway at different wind speeds (from Neumann, 1952a)

plays a part in the question of energy transfer and 'roughness'.

"2 The relatively steep, ready-to-break 'sea.' This is of greatest interest to a practical man at sea. Its dimensions depend on the wind strength: $c = f(U)$, $\delta = F(U)$.

"3 The gently crested, swell-like advancing 'long wave' for which we will take $c \sim 1.37 U$. It does not attract the immediate attention of an observer, since in the wind region it can be covered by the steeper waves."

From Neumann (1-1952a pages 96, 97): "In a seaway waves of different height and length occur side by side and are superimposed. From this the sea surface obtains its pronounced rough appearance, particularly in the development stages of wave motion. But the longer the wind acts and the more the sea attains full development, i.e., its greatest height, the clearer the characteristic waves, corresponding to wind strength, work out of the wave confusion. At the same time the individual hill-like wave crests unite to an increasing extent into long crests: the 'sea' builds up.

"A further sign of developing or fully developed seaway is the occurrence of irregularities in the sequence of 'seas' following one after another as well as the group-like occurrence of wave systems. As one tries to follow for a time an isolated wave crest, one recognizes that it does not exist an arbitrarily long time, but gradually loses height and disappears in the wave confusion, while another one grows and later similarly fades away. One can observe the swelling and diminishing of the waves from a not-too-fast ship in a following sea, when the wave celerity is approximately the same as the ship speed. Two wave crests alongside the ship often change their heights so that the rear one grows in size, while the front one fades away. Eventually the rear one also seems to disappear, and at another place alongside the ship a new one begins to grow. Such wave-group appearances and irregularities are typical of the nature of sea surface.

"The seaway in a fully or almost fully developed state is not described by a single characteristic 'sea,' although

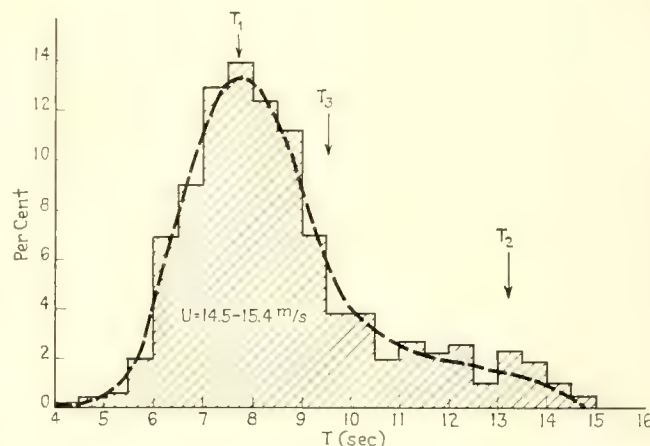


Fig. 4 Frequency distribution of apparent periods in a seaway in wind speed interval 14.5-15.4 m/sec (from Neumann, 1952a)

one appears most conspicuous at almost all wind speeds. With progressive development, a category of longer and substantially flatter waves apparently builds up in the seaway. The mutual action of these main waves, which are covered by waves of lower order, gives the visible irregular picture of the sea surface, and makes understandable the occurrence of characteristic wave groups as well as irregularities observed at sea."

Observational data used by Neumann originate primarily from four sources: The material of many observers collected by Sverdrup and Munk (1-1946), the observations on the mean water slope in wind by Palmén (1-1932a, b, 1-1936), observations of Roll (1-1951) for small c/U , and very extensive observations made by Neumann himself. These were made during the voyage on MS *Heidelberg* starting from Hamburg on October 14, 1950. The outgoing course to the Caribbean Sea was via the Azores, and the return passing close to Bermuda. Neumann (1-1952a, page 79) writes: "... the number of individual measurements of periods is estimated at about

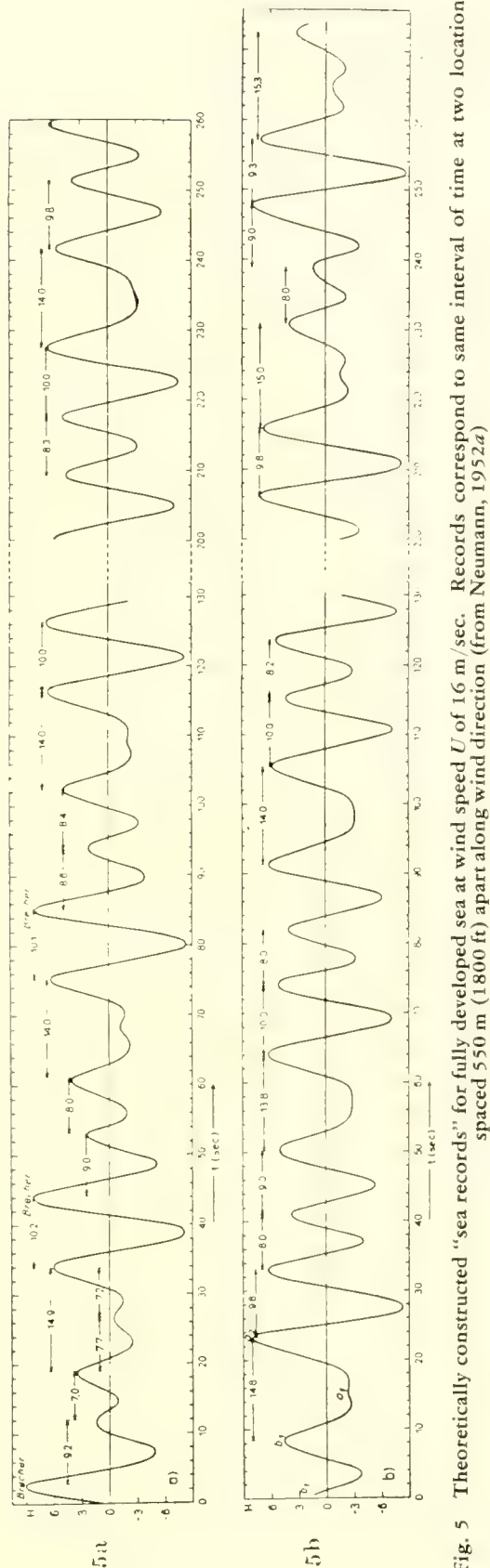


Fig. 5 Theoretically constructed "sea records" for fully developed sea at wind speed U of 16 m/sec. Records correspond to same interval of time at two locations spaced 550 m (1800 ft) apart along wind direction (from Neumann, 1952a)

27,000 The working out of the material followed the system in which the individual series measurements were divided into class intervals of 0.5 second, were counted, and were presented in the form of distribution of frequency of occurrence (period spectra). Altogether about 250 such series were investigated.

"In Fig. 3a to c, three such diagrams are presented as examples of three series of observations under quasi-stationary wind conditions. . . Individual 'bands' come to attention as characteristic in the 'period spectra.' The regular appearance of these concentrations of periods in certain class intervals indicates possibly a certain measure of law in a seaway at different wind velocities.

"The series in Fig. 3(a) shows, besides the frequency maximum in the class interval 3.0–4.5 sec, three additional concentrations. This series gives observations at a wind of 9 m/sec from a stopped ship (observations in the sea region southwest of the Azores in a steady wind from an easterly direction). The arrows designated T_1 and T_2 give the computed periods in accordance with earlier theoretical results (Neumann, 1-1950) for the characteristic predominant waves corresponding to the wind speed. T_1 designates the period for the 'sea' and T_2 the period for the 'long wave' The periods in the interval 3.0–4.5 sec are evidently connected with measurements of steeper 'sea' for which the theoretical period was given as 3.9 sec. In the band of periods between 7.0 and 8.5 sec we find again the 'long wave' which ought to have a period of 7.9 sec (G. Neumann, 1-1950).

"The concentration of periods in an interval between the above named main waves should be noted. In the case of Fig. 3(a) the intermediate maximum occurs in periods from 5.0 to 6.5 sec. . . .

"Fig. 3(b) gives a series of measurements at a wind speed of 13.5 m/sec. The most conspicuous is the sea with periods between 5 and 8 sec. The calculated period is $T_1 = 6.6$ sec. The 'long wave' should have the period $T_2 = 11.8$ sec, which could also agree with the concentration of periods in the interval 10.5–12.5 sec. As the third concentration we find again the 'intermediate wave,' clearly separated from the other main waves in the seaway, with periods in the interval 8.5–9.0 sec. In Fig. 3(c) quite similar relationships also occur. The period of the 'sea' rises most clearly, always with a certain scatter of individual measurements. The diagram in Fig. 4 unites all measurements in the wind strength interval of 14.5–15.4 m/sec and shows a smoother 'spectrum' in which the periods around 8 seconds strongly predominate."

As an example, the following data are given for three predominating waves in the fully developed sea in wind strength of 16 m/sec (52.5 fps or 31 knots). The subscript 1 denotes "sea" 2—"long wave," and 3—"intermediate wave."

$\lambda_1 = 107$ m	$\lambda_2 = 307$ m	$\lambda_3 = 163$ m
$= 350$ ft	$= 1000$ ft	$= 535$ ft
$T_1 = 8.3$ sec	$T_2 = 14.0$ sec	$T_3 = 10.2$ sec

$$c_1 = 12.95 \text{ m/sec} \quad c_2 = 21.9 \text{ m/sec} \quad c_3 = 16 \text{ m/sec}$$

$$= 25 \text{ knots} \quad = 42.7 \text{ knots} \quad = 31 \text{ knots}$$

amplitudes:

$$a_1 = 3.0 \text{ m} \quad a_2 = 3.4 \text{ m} \quad a_3 = 3.2 \text{ m}$$

Assuming each of the three "partial waves" to be sinusoidal, the resultant profile of the total waves is represented as

$$y = a_1 \sin 2\pi \left(\frac{c_1 t - x}{\lambda_1} \right) + a_2 \sin 2\pi \left(\frac{c_2 t - x}{\lambda_2} \right) + a_3 \sin 2\pi \left(\frac{c_3 t - x}{\lambda_3} \right) \quad (18)$$

Quoting from Neumann (1-1952a, pages 106, 107): "The time-dependent fluctuations of the seaway, resulting from the superposition of the three designated waves, are shown in Fig. 5. The upper wave train *a* represents the sequence of waves at the place $x = 0$ for the duration from $t = 0$ to $t = 260$ sec. The lower wave train *b* gives the corresponding fluctuations for the same time interval at a place $x = 550$ m; i.e., at a position located almost 1/3 of a nautical mile from $x = 0$ in the direction of wave propagation. The constructed wave records show that outstanding wave groups can be expected only from time to time at a fixed place. At the location $x = 0$, they appear at intervals of about 45 sec at the beginning, at the location $x = 550$ m at the end of the observation period. In these 'groups' certain wave crests grow to particularly great heights. In nature they probably do not reach their full theoretical height, since after exceeding a certain maximum steepness they become unstable and break.

"Between the groups at the beginning of the wave train *a*, and correspondingly at the end of *b*, lie mostly three low waves, the heights of which change with time and place. Also the interference described as 'double waves' appears at certain intervals of time. After passage of a sequence of outstanding groups the seaway picture at a fixed place changes. The conspicuous difference between particularly high waves and the low waviness between groups evens out more and more, until after a certain time the typical group character of the seaway again predominates."

Quoting from Neumann (1-1952b pages 253, 254): "The complex picture of the development of a seaway must be strongly taken into account in the question of energy transfer from wind to water; the shearing forces of wind at the rough wave boundary surface are determined to a very large extent by the short wave overlay of the main wave profiles."

"In the following, an attempt will be made to calculate the growth of a complex seaway for different wind velocities as dependent on the duration of wind and on the length of effective wind path (fetch). In this, the turbulent nature of the event must be taken as much as possible into account, and the fact must be considered that, with increasing development of a seaway, the 'width of the

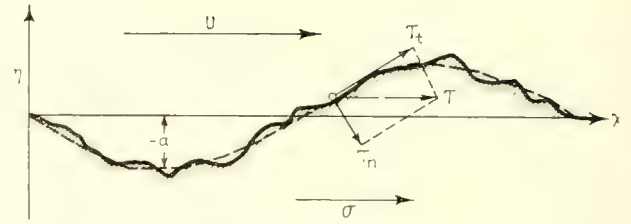


Fig. 6 Wave profile with rough surface and resolution of effective drag τ into normal and tangential components (from Neumann, 1952b)

spectrum of periods' of characteristic waves also grows.

"The whipping-up of the seaway to the fully developed condition of all waves appears to be a not continuous process. Rather one must assume discontinuities in certain stages of wave growth; the relatively short 'seas' even in a fully developed state must be accepted as steep waves breaking from time to time. These steep breaking waves (with various 'apparent' wave lengths) are probably necessary for the further development of the complex seaway, so that longer waves with greater energy content can build up, until the energy transmitted from wind and dissipated in wave confusion reaches a balance. . . . The width of the spectrum of periods as well as the mean and maximum wave heights depend on the wind strength, and in a not-fully developed seaway, on the state of development of individual wave components. . . ."

5 Energy Transmitted from Wind

On the basis of the foregoing descriptions, Neumann formulates a quantitative wave theory, considering the shearing force of wind due to small waves and ripples, and the air pressures on the surfaces of three dominating waves. Fig. 6 shows Neumann's concept of the total horizontal force τ caused by the wind as composed of the normal pressure component and tangential shear force. The former is taken as proportional to the slope $\partial\eta/\partial x$ of the wave under consideration and the second as caused by a "roughness" resulting from the overlay of smaller waves and ripples. The mean energy A transmitted by these two components is:

For the normal component

$$A_n = -\frac{1}{\lambda} \int_0^\lambda \tau_n w dx \quad (19)$$

For the tangential component

$$A_t = \frac{1}{\lambda} \int_0^\lambda \tau_t (u + u') dx$$

where u is the horizontal component of the orbital velocity of a water particle at the surface, u' is the mean transport velocity of Stokes' waves of finite height, and w is the vertical component of the orbital velocity.

For the wave profile expressed as

$$\eta = a \sin k(x - ct), \quad (20)$$

the vertical velocity of a water particle is

$$w = \partial\eta/\partial t = -ake \cos k(x - ct) \quad (21)$$

and the horizontal velocity (including the mean mass transport)

$$u + u' = ake \sin k(x - ct) + a^2 k^2 c \quad (22)$$

The wind velocity U over a sinusoidal wave profile is expressed as

$$U = \bar{U}[1 + \pi\delta \sin k(x - ct)] \quad (23)$$

where δ is the wave steepness $2a/\lambda$, and \bar{U} a constant mean value of the velocity over the wave profile. Neglecting the δ^2 , U^2 is written as

$$U^2 = \bar{U}^2[1 + 2\pi\delta \sin k(x - ct)] \quad (24)$$

and the tangential stress

$$\tau_t = \rho' f_t U^2 = \rho' f_t \bar{U}^2 [1 + 2\pi\delta \sin k(x - ct)] \quad (25)$$

where f_t is the "effective" tangential-force coefficient.

The asymmetry of the normal pressure distribution over the windward and leeward wave faces is assumed to be

$$\tau_n = \bar{\tau}_n + \tau_n' \cos k(x - ct) \quad (26)$$

where $\bar{\tau}_n$ denotes a mean value of the normal force over the wave profile.

Substitution of equations (23), (25) and (26) into equations (19) and integration over the wave length leads to

$$\begin{aligned} A_n &= \frac{1}{2} \pi \delta \tau_n' c \\ A_t &= 2\pi^2 \rho' \delta^2 f_t \bar{U}^2 c \end{aligned} \quad (27)$$

Since data on the coefficients τ_n' and f_t are not available, Neumann writes finally for the total energy ($E_p = A_n + A_t$) transferred from wind to waves

$$E_p = \rho' U^3 \beta C(\beta) \quad (28)$$

where U is now the wind velocity at anemometer height, $\beta = c/U$, and $C(\beta)$ is a coefficient which is a function of β . To the energy equation (28) corresponds the effective tangential stress

$$\tau_{\text{eff}} = C(\beta) \rho' U^2 \quad (29)$$

By comparison with equation (1), $C(\beta)$ is seen to be identical with γ which was previously evaluated, equation (3), empirically as a function of U on the basis of water-surface inclination and wind-velocity gradient at sea. Neumann, however, points out repeatedly that evaluation of the seaway in terms of a single mean wave has relatively little value, since it does not explain many features of the complex wave formation. He proceeds therefore with an analysis of seaway growth due to wind in three stages corresponding to the plot of wave steepness $\delta = H/\lambda$ versus wave age $\beta = c/U$ as shown in Fig. 1-19. The stages are $\beta < 1/3$, $1/3 < \beta < 1.37$ and $\beta > 1.37$. The objective of this work is to derive functional relationships for three dominant wave systems. The im-

portant features of the complex seaway (irregularity, wave-group formations) can be approximated by considering the joint action of these wave systems.

Quoting from Neumann (1-1952b): "Under the assumption that the seaway passes consecutively through three main stages of development until it is fully arisen, the relationships among wave characteristics (wind speed, duration of its action, the fetch) can be derived; these represent a pragmatic approximation for the prediction of wave conditions. The waves defined at a given point, Neumann (1-1952a), β_m , $\beta(1)$, β_m^* with the periods T_1 , T_2 , T_3 , must be visualized as the characteristic mean values for certain intervals of periods, when we think of the continuous spectrum as subdivided into these three major intervals; the heights H_1 , H_2 , H_3 are then visualized as 'equivalent' heights for these major intervals." In the foregoing, $\beta(1)$ is the "intermediate wave" with $c = U$, β_m the "sea" with $c < U$, and β_m^* the long wave with $c = 1.37 U$.

6 Energy in Terms of Wave Age and Slope

The starting point is empirical evaluation of $\delta = f(\beta)$. At the beginning of the development outlined here, very little empirical data were available for the range $\beta < 1/3$, as can be seen from the plot in Fig. 18. At the same time, on theoretical grounds, maximum values of δ could be associated with $\beta = 1/3$ at which the maximum energy transfer from wind occurs according to Jeffreys (1925). Neumann assumed therefore that

$$\delta = 1.111\beta^2 \quad \text{for } \beta < 1/3 \quad (30)$$

Later Fig. 19 was constructed in which many experimental points at $\beta < 1/3$ were added, mostly from Neumann and from Roll (1-1951). On this basis Neumann sets

$$\delta = \text{const} = 0.124 \quad \text{for } \beta < 1/3 \quad (31)$$

For the other stages of wave development,

$$\delta = 0.215e^{-1.667\beta} \quad \text{for } 1/3 < \beta < 1.37 \quad (32)$$

and

$$\delta = \text{const} = 0.022 \quad \text{for } \beta > 1.37 \quad (33)$$

Next it is assumed that the effective shear-force coefficient is a function of the maximum wave slope $2\pi a/\lambda = \pi\delta$, in terms of a coefficient of proportionality s (similar to Jeffreys'), and the relative air-wave velocity $U - c$. Neumann (1-1948) estimated the coefficient s to have approximately a constant value of 0.095. With this value for s and the previously established expressions for δ , the mean value of the coefficient $C(\beta)$ is computed by integrations over the relevant ranges of variation of β . The "effective friction" force is caused mostly by the work of the normal forces on the sharp profiles of small waves by which the mean wave is covered. The true skin friction of a smooth water surface is here neglected in comparison with the much higher effective force due to

Table 4

β	0.37	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.4
$10^3 \gamma$	5.75	5.62	4.55	4.05	3.64	3.27	3.01	2.78	2.68	2.57	2.52

Table 5

U (m/sec)	2	4	6	8	10	12	14	16	18	20	24
μ^*	10.3	58	161	332	577	912	1350	1860	2520	3260	5720
$\mu^*(1)$	0.57	5.25	20	51	99	174	274	409	583	800	1382
$\mu^*(\beta)$	0.08	1.1	5.4	16.4	36.5	73	132	216	315	538	1134

Table 6

β	0.15	0.25	0.45	0.7	1.0	1.15	1.25	1.37
$\mu^*(\beta)$...	5.85	8.15	15.8	36.5	99	202	326	577

pressure distributions; this follows from an examination of the figures for Møtsfeldt's Model No. 4 in Table 1-1.

The stress acting on the simple wave profile is

$$\tau = \text{const } (\rho'/2)(U - c)^2 \quad (34)$$

and the mean stress resulting from contributions by waves of all lengths; i.e., all celerities c' , using relationships (30) and (32), is

$$\tau' = 3\pi \left(\frac{\rho'}{2} \right) \frac{1.111 s'}{U} \int_0^{U/3} c'^2 \left(1 - \frac{c'}{U} \right)^2 dc' + \left(\frac{\rho'}{2} \right) \frac{0.215 \pi s' U^2}{(c - U/3)} \int_{U/3}^c e^{-1.677 c'/U} \left(1 - \frac{c'}{U} \right)^2 dc' \quad (35)$$

where c' is the celerity of the overlaying waves and c the celerity of the main wave. Taking the lower limit of the first integral as 0 instead of the few centimeters per second of an initial wave represents a negligible error. s' is taken as $s/2 = 0.0475$ in order to allow for the nonuniformity of distribution of the steepest small waves along the profile of the main wave. After integration, introducing $\beta = c/U$ and defining the effective tangential force as

$$\tau' = \rho' \gamma(\beta) U^2$$

the coefficient⁴ $\gamma(\beta)$ is evaluated as

$$10^3 \gamma(\beta) = 1.75 + 16.2 \frac{1}{\beta - 1/3} \{ e^{-1.667\beta} [0.48(\beta + 0.6) - 0.6(1 + \beta^2)] + 0.126 \} \quad (36)$$

The resulting values of $\gamma(\beta)$ versus β are given in Table 4.

Adding the work of the normal forces on the mean wave profile to the work of "effective friction" due to smaller wave components (evaluated in the foregoing) the total coefficient of the effective wind force is given as follows:

For the range $1/3 \leq \beta \leq 1$,

$$C_2(\beta) = \gamma(\beta) + 0.1075 s' \pi e^{-1.667\beta} (1 - \beta^2) \quad (37)$$

Designating by β_m the values of β in the range $1/3 < \beta < 1$, and by β_m^* the values of $\beta > 1$, the expression for the range $1 \leq \beta_m^* \leq 1.37$ is

⁴ The notation $\gamma(\beta)$ is used in this monograph for $\gamma^2(\beta)$ written by Neumann.

$$C_3(\beta) = \gamma(\beta_m) + 0.1075 \pi s' e^{-1.667\beta_m} (1 - \beta_m)^2 - 0.1075 \pi s^* e^{-1.667\beta_m^*} (1 - \beta_m^*)^2 \quad (38)$$

where Neumann takes $s^* = 2s$. In a fully developed seaway $\beta_m^* = 1.37$.

7 Energy Dissipation

With the coefficient $C(\beta)$ now evaluated for various stages of wave development (wave age) β , the energy input from wind can be computed by equation (28). The next step in evaluating the energy balance is to establish the rate of energy dissipation by waves. The dissipation by viscous forces was given by equation (66) in Appendix A as

$$E_{ds} = 2\mu(2\pi/\lambda)^3 c^2 a^2 \quad (39)$$

Substituting $c^2 = g\lambda/2\pi$ and $2a/\lambda = \delta$, and replacing μ by μ^* , the energy dissipated per second per unit area is

$$E_{ds} = 2\mu^* \pi^2 g \delta^2 \quad (40)$$

where μ^* , which may be called turbulent viscosity, is no longer a constant but a function of β , or, more generally, depends on the wind and conditions of a seaway.

In a fully developed seaway, i.e., with fully developed "sea" and fully developed "long wave," the energy is in balance; i.e., $E_p = E_{ds}$. μ^* no longer depends on β , but only on wind strength, and is designated $\bar{\mu}$. By using expression (40) for E_{ds} and (28) for the energy E_p , letting $C(\beta) = \gamma(U)$ from equation (1), and using equation (32) for δ , at $\beta = 1.37$ the expression for $\bar{\mu}$ is obtained as

$$\bar{\mu} = \frac{E_p}{2\pi^2 g \delta^2} = \frac{1.37 p' \gamma(U) U^3}{8 \times 0.1075^2 g \pi^2} e^{3.334 \times 1.37} \quad (41)$$

$\gamma(U)$ is taken as evaluated empirically by equation (3), except that in the present case Neumann changes the coefficient 0.09 to 0.10. With $\rho' = 0.00125$ and $g = 980$ cm/sec²,

$$\bar{\mu} = 0.00001825 U^{5/2} \text{ (in cm}^{-1} \text{ g sec}^{-1}) \quad (42)$$

The foregoing value of $\bar{\mu}$ is the maximum value, cor-

responding to the fully developed seaway. At earlier stages of wave development the turbulent viscosity, designated as $\mu^*(\beta)$, is an unknown function of the wave age. Neumann assumes for it an exponential relationship: for $1 < \beta_m^* < 1.37$,

$$\mu^*(\beta_{mm}) = \bar{\mu} \exp \left(-3.334 \frac{1.37 - \beta_m^*}{\beta_m} \right) \quad (43)$$

and for $0.1 < \beta < 1$,

$$\mu^*(\beta) = \mu^*(1) \exp [-3.334(1 - \beta)] \quad (44)$$

where $\mu^*(1)$ is the value of $\mu^*(\beta)$ at $\beta = \beta_m^* = 1$. Table 5 shows the values of μ^* versus wind speed. Table 6 shows the coefficient $\mu^*(\beta)$ ($\text{cm}^{-1}\text{g sec}^{-1}$) at a wind speed of 10 m/sec at various stages of seaway development.

AUTHOR'S REMARK: The rapid growth of the coefficient μ^* with wind speed U and wave age β gives a distorted impression as to the magnitude of the actual energy dissipation E_{ds} . This growth of μ^* results from the presence of δ^2 in the denominator of equation (41), and the fact that $\bar{\mu}$ is referred to the very low δ of the "long wave," as shown by the value for $\beta_m^* = 1.37$. The actual energy dissipation, however, according to equation (40) is mostly caused by "young waves" of low β and large δ , and is proportional to the product $\mu^*(\beta)\delta^2$. It would appear that consistency would require the evaluation of $\mu^*(\beta)$ by integration over all values of β , following the same procedure as was used earlier for the evaluation of $\gamma(\beta)$ in equation (35).

8 Energy Balance

With the necessary expressions developed for the energy E_p received from the wind, and E_{ds} dissipated by turbulence, the energy balance can be set up and from this the rate of wave growth can be computed. The symbols E_p and E_{ds} designate the energy per unit area of sea surface per second. Neumann, following the procedure previously used by Sverdrup and Munk, writes the expression for the rate of change in energy for an area of unit width and wave length λ in length. The wave energy advances at the group velocity $c/2$. The wave length λ and amplitude a change with time t and position x so that

$$\frac{d}{dt} (E\lambda) = (E_p - E_{ds})\lambda \quad (45)$$

and

$$\lambda \left(\frac{\partial E}{\partial t} + \frac{c}{2} \frac{\partial E}{\partial x} \right) + E \left(\frac{\partial \lambda}{\partial t} + \frac{c}{2} \frac{\partial \lambda}{\partial x} \right) = (E_p - E_{ds})\lambda \quad (46)$$

Two cases are now considered: In Case A a constant wind of velocity U blows over an unlimited space. Waves in all positions grow with time at an equal rate. The space derivatives vanish and

$$\frac{\partial E}{\partial t} + \frac{E}{\lambda} \frac{\partial \lambda}{\partial t} = E_p - E_{ds} \quad (47)$$

or, substituting the expressions for E (equation 58, Appendix A) and λ (Table 1, Appendix A)

$$\rho g a \frac{da}{dt} + \frac{\rho g a^2}{c} \frac{dc}{dt} = E_p - E_{ds} \quad (48)$$

In Case B a wind has blown for a sufficiently long time along a fetch of limited length x . Thus, time derivatives vanish and

$$\frac{c}{2} \left(\frac{\partial E}{\partial x} + \frac{E}{\lambda} \frac{\partial \lambda}{\partial x} \right) = E_p - E_{ds} \quad (49)$$

or

$$^{1/2} g \rho a^2 \frac{\partial c}{\partial x} + g \frac{c}{2} \rho a \frac{\partial a}{\partial x} = E_p - E_{ds} \quad (50)$$

Introducing now $\beta = f(\delta)$ according to equations (31) and (32):

Case A for $\beta \leq 1/3$,

$$\frac{12 \rho p^2 \pi^2}{g} U^4 \beta^3 \frac{\partial \beta}{\partial t} = E_p - E_{ds} \quad (51)$$

and for $1/3 \leq \beta \leq \beta_m^*$,

$$\rho \frac{4 \pi^2}{g} n^2 U^4 \beta^3 e^{-2r\beta} (3 - r\beta) \frac{\partial \beta}{\partial t} = E_p - E_{ds} \quad (52)$$

Case B, for $\beta \leq 1/3$,

$$\frac{6 \rho p^2 \pi^2}{g} U^5 \beta^4 \frac{\partial \beta}{\partial x} = E_p - E_{ds} \quad (53)$$

and for $1/3 \leq \beta \leq \beta_m^*$,

$$\rho \frac{2 \pi^2}{g} n^2 U^5 \beta^4 e^{-2r\beta} (3 - r\beta) \frac{\partial \beta}{\partial x} = E_p - E_{ds} \quad (54)$$

In all cases $(E_p - E_{ds})$ is a function of β . In writing the foregoing, the symbol p was used for the number 0.062, r for the number 1.667 and n for the number 0.1075 found in equations (31) and (32), and $\beta_m^* = 1.37$.

Substituting the previously derived expressions for E_p and E_{ds} and transposing leads to differential equations as follows:

Case A, for $\beta \leq 1/3$

$$dt = \frac{\rho}{\rho'} \frac{12 \pi^2 p^2}{g} U \frac{\beta^2}{C_1(\beta) - B_1(\beta)} d\beta \quad (55)$$

where

$$B_1(\beta) = \frac{8 \pi^2 g p^2}{\rho'} \frac{\mu^*(\beta)}{U^3 \beta} \quad (56)$$

Case A, for $1/3 \leq \beta \leq \beta_m^*$,

$$dt = \frac{\rho 4 \pi^2}{\rho' g} n^2 U \frac{\beta^2 e^{-2r\beta} (3 - r\beta)}{C_2(\beta) - B_2(\beta)} d\beta \quad (57)$$

where

$$B_2(\beta) = \frac{8 \pi^2 g n^2}{\rho'} \frac{\mu^*(\beta) e^{-2r\beta}}{U^3 \beta} \quad (58)$$

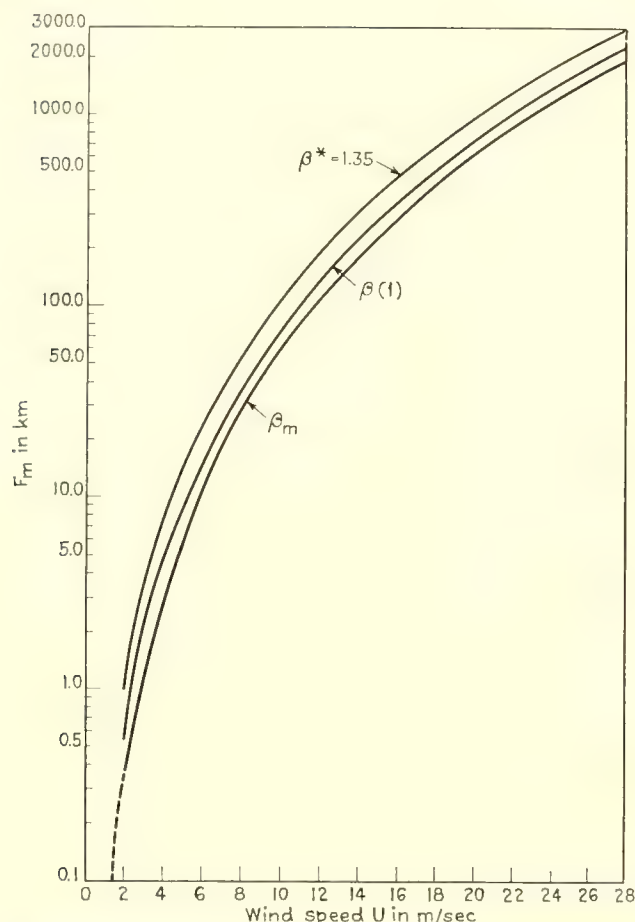


Fig. 7 Minimum fetch needed for development of three main wave stages. At $\beta^* = 1.35$ the seaway is practically fully developed (from Neumann, 1952b)

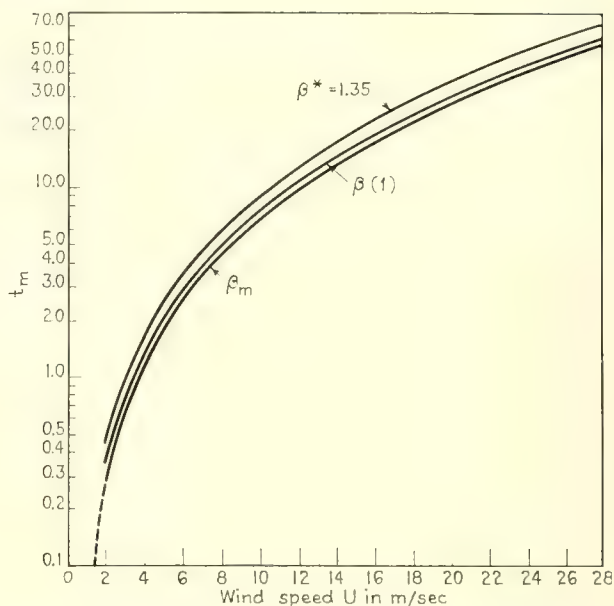


Fig. 8 Minimum wind duration (in hours) needed for development of three main wave stages (from Neumann, 1952b)

fetch and the duration are obtained as functions of $\beta = c/U$. The minimum fetch and duration required for the full development of the three "equivalent" wave components are shown in Figs. 7 and 8. Here β_m corresponds to the celerity of the wave of the most frequently occurring period.

Citing an example from Neumann (1-1952b, page 270): "At a wind speed of 5 m/sec a sea space (fetch) about 14 km long (in the wind direction) and a duration of wind action of 2.25 hr is sufficient for the full development of a complex seaway. At a wind speed of 10 m/sec, it is already 106 km and 8.6 hr. These minimum values rise rapidly with increasing wind strength. In a storm with wind velocity of 20 m/sec the fetch must be at least 1050 km or 600 sea miles long, and the wind must last 35.6 hr in order to build a fully developed seaway. At 24 m/sec it would be 1800 km or 1000 sea miles and 51.8 hr. Observations of storm seas, which could be applied to verification of these results are very scarce and are very difficult to establish. We find the orders of magnitude of the computed minimum values to be completely in agreement with findings of experienced wave observers. So V. Cornish (see H. Thorade, 1942) estimates the minimum fetch needed for the full development of storm waves at 600 to 1000 sea miles."

The conditions of seaway at all stages of development are shown in Figs. 1-22 to 25. Here nondimensional ratios are introduced:

$$\begin{aligned}\beta &= \varphi(gx/U^2) \\ \beta &= F(gt/U) \\ gH/U^2 &= \Phi(gx/U^2) \\ gH/U^2 &= f(ft/U)\end{aligned}$$

Case B, for $\beta \leq 1/3$

$$dx = \frac{\rho}{\rho'} \frac{6\pi^2 p^2}{g} U^2 \frac{\beta^3}{C_1(\beta) - B_1(\beta)} d\beta \quad (59)$$

Case B for $1/3 \leq \beta \leq \beta_m^*$

$$dx = \frac{\rho}{\rho'} \frac{2\pi^2}{g} n^2 U^2 \frac{\beta^3 e^{-2r\beta}(3 - r\beta)}{C_2(\beta) - B_2(\beta)} d\beta \quad (60)$$

Quoting from Neumann (1-1952b, page 265): "Just as the quantity $C(\beta)$ was designated the 'effective friction factor' or 'resistance factor' for the wind friction on the sea surface, so the dimensionless quantity $B(\beta)$ is named the 'effective dissipation factor' for the turbulent wave motion. The difference between the two represents the 'effective energy factor' for the whipping-up of the wave motion."

9 Final Results

For the remaining part of the discussion the reader is referred to the original paper, Neumann (1-1952b). Suffice it to say here that by numerical integration the

Quoting from Neumann (1-1952b, page 275), in connection with the foregoing figures: "The curves . . . give the conditions of growth, and the horizontal branching lines the condition of the fully arisen three main waves in a complex sea. In the region of lower values gx/U^2 , the observed β and gH/U^2 values arrange themselves very well about the curves. At larger values of the fetch parameter gx/U^2 the observations deviate to the right from the computed curves; this can be expected when the first mean wave (the β_m wave) has reached its fully developed state, and remains as a dominant wave independent of the buildup of longer waves. In the diagrams it is indicated by straight lines branching to the right from the curve. As the development of the seaway proceeds further, at sufficiently long fetches and duration of wind action, the $\beta(1)$ waves and β_m^* waves (with $\beta_m^* = 1.37$) also appear in addition to the β_m waves. For these, the corresponding values of x and t for a given wind strength can be taken from the upward branching curves. In a fully developed seaway, β numbers and corresponding periods can thus be expected, which lie substantially between the upper horizontal line and the lower lines for the corresponding wind strength.

"In order to give an example of application of the diagrams, let us consider the development of a seaway in wind strength of 16 m/sec. [31 knots].⁵

"The first 'main wave' with $\beta = 0.81$, $c = 13$ m/sec will be reached at the fetch parameter $gx/U^2 = 11,500$ and time parameter $gt/U = 37,200$; this wave is fully built up after a fetch of 300 km [162 sea miles] or after 16.8 hr with a sufficiently long fetch. It has the following 'dimensions':

$\lambda_1 = 107$ m [350 ft], $H_1 = 5.9$ m [19.3 ft], the corresponding period of 8.3 sec indicates the most frequently occurring 'period,' which is observed between wave crests following one-after-another at a certain particular point on the sea surface.

"The $\beta(1)$ wave appears at $g\lambda/U^2 = 13,210$ and $gt/U = 40,560$ or after 345 km (186 sm) and 18.4 hr. Its dimensions are $\lambda_3 = 163$ m [535 ft], $H_3 = 6.6$ m [21.6 ft], period = 10.2 sec. Development of the waves proceeds further under action of the wind, and at $gx/U^2 = 18,060$ and $gt/U = 48,610$ the longest wave in a complex seaway—age $\beta^* = 1.35$ becomes fully developed; with it the seaway is practically fully developed. This occurs at a fetch $x = 472$ km [254 sm] and with a minimum wind duration of $t = 22.1$ hr. The dimensions of this longest 'characteristic' wave in a seaway is then about

$\lambda_2 = 298$ m [980 ft], $H_2 = 6.75$ m [22.2 ft],
period 13.8 sec

Were we to take, for the longest wave, the β_m^* wave with $c = 1.37U$, then λ_2 would be 307 m [1002 ft] and $H_2 = 6.8$ m [22.3 ft] at $x > 500$ km [271 sm].

"These three 'main waves' characterize the seaway at 16 m/sec [31 knots] and embrace the region of 'significant waves.' The characteristic periods lie between 8

and 15 sec. The observed frequency distribution of the time intervals between wave crests, following one-after-another at a fixed point of the sea surface, gave a scatter of periods between 6 and 16 sec, with the maximum of the frequency distribution at 8.3 sec at the mean wave height of 6 m [19.7 ft]."

10 Summary

The observational evidence on which Neumann's development is based consists of

(a) The plot of $\delta = H/\lambda$ versus $\beta = c/U$ originated by Sverdrup and Munk (1-1946) and supplemented by extensive observations by Neumann himself, and (for low β) by Roll (1-1951) and Francis (1-1951). Empirical relationships were fitted to these plots, as shown by equations (30) to (33).

(b) Evaluation of the horizontal drag of the sea surface, Neumann (1-1948) on the basis of the inclination of the mean water level, supplemented qualitatively by observations of the wind-velocity gradient. The inclination of the water surface was measured on large sea areas. The resultant empirical relationships are given by equations (1), (3) and (4).

With reference to (a), many observational points were added to the original plot of Sverdrup and Munk (1-1946) by Bretschneider (1-1952) and the Neumann (1-1952a). For the range $\beta > 1/3$ this material represents probably the most reliable empirical information available on sea waves. The values of δ corresponding to the range $\beta < 1/3$ are rather uncertain, but this does not represent an important factor in the over-all pattern of Neumann's work. The evaluation of δ for $\beta < 1/3$ will be important in the future for the estimates of the distribution of very short and steep waves which will be needed for a more rational evaluation of the wind-drag force.

With reference to (b), the number of observations made and the care exercised by Neumann in choosing the data can scarcely leave any doubt as to the validity of the evaluation for wave lengths smaller than double the water depth, i.e., $\lambda < 140$ m [460 ft]. On the basis of Neumann's (1-1948) data this corresponds to a wind speed of about 35 knots. For stronger winds and longer waves the results can be considered as extrapolation.

The most important first step, in working out the aforementioned empirical data, is establishment of the drag exerted by wind as a function of wave steepness δ . Jeffreys' expression (14) was not based on observed physical facts, and Neumann finds that, in order to agree with available empirical data, it has to be modified as shown by equation (17); the drag is expressed as directly proportional to the maximum wave slope, rather than to its square. In this equation the coefficient s was found to have a constant value of 0.095. It should be noted, however, that this conclusion resulted from the assumption of $c = U/3$; i.e., a constant value of $\beta = 1/3$. Expression (17) was used thereafter in disregard of its limited applicability. This is mentioned here in order to warn future investigators against indiscriminate use of these conclusions.

⁵ Conversion to knots, nautical miles and feet which are inserted in square brackets not in original text.

Calculation of Hydrodynamic Forces by Strip Theory

Evaluation of Hydrodynamic Forces

This Appendix is arranged to follow as closely as possible Appendix 1 of an earlier paper (Korvin-Kroukovsky, 1955b) so that the changes made can be easily seen. Where sufficient discussion was given in that reference, the details will be omitted here.

Formulation of Problem

Consider a ship moving with a constant forward velocity V (i.e., neglecting surging motion) with a train of regular waves of celerity c . Assume the set of co-ordinate axes fixed in the undisturbed water surface, with the origin instantaneously located at the wave nodal point preceding the wave rise, as shown in Fig. 14.² With increase in time t the axes remain fixed in space, so that the water surface rises and falls in relation to them. This vertical displacement at any instant and at any distance x is designated η . Imagine two control planes spaced dx apart at a distance x from the origin, and assume that the ship and water with orbital velocities of wave motion penetrate these control surfaces. Assume that the perturbation velocities due to the presence of the body are confined to the two-dimensional flow between control planes; i.e., neglect the fore-and-aft components of the perturbation velocities due to the body, as in the "slender body theory" of aerodynamics. This form of analysis, also known as the "strip method" or "cross-flow hypothesis," is thus an approximate one in the sense that a certain degree of interaction between adjacent sections is neglected.

The cross section of the ship at x will now be taken as semi-circular; the correction necessary to represent other ship sections will be introduced later. Following F. M. Lewis (1929) and Weinblum and StDenis (1950), the flow about the semi-submerged body used in the basic derivation will be assumed to be identical with that about the lower half of a fully submerged body. Corrections to account for the presence of the free water surface will be brought in later.

In considering the pitching and heaving motions of the body it is necessary to introduce a second co-ordinate system moving with the ship with its origin at the center of gravity of the ship. The longitudinal distance of any section of the ship from the origin is designated ξ

(positive forward). Vertical displacement of the CG (i.e., the heave) is designated by z (positive upwards) and angular displacement or pitching motion is designated by θ (positive for bow displaced upwards). The vertical displacement of the section at x due to pitching is then $\xi\theta$, with θ in radians, for the relatively small angles encountered. (It is also assumed that $\cos \theta \cong 1$.)

The two-dimensional flow pattern between the control planes at x results from three imposed motions:

- 1 Vertical velocity of the center of the circle

$$v = \dot{z} + \xi\dot{\theta} - V\theta \quad (8)$$

- 2 Vertical component of wave orbital velocity

$$v_w = \dot{\eta} e^{2\pi y/\lambda} = \frac{-2\pi h c}{\lambda} e^{2\pi y/\lambda} \cos \frac{2\pi}{\lambda} (x - ct) \quad (9)$$

where h is wave amplitude, λ is wave length, $y = -R \cos \alpha$, the depth below the still-water level to any point in the fluid, and $\eta = h \sin 2\pi (x - ct)/\lambda$.

- 3 Apparent variation of the radius r of the ship section at the control planes with time; $r = r(t)$.

All motions are assumed to be sufficiently small so that the derivatives of the potential can be taken on the surface of the circle as if its center were at its initial position $y = 0$, and the known expression for the potential in a uniform fluid stream can be applied, despite the slight nonuniformity induced by the waves which are assumed to be small.

The vertical hydrodynamic force acting on the length dx of the submerged semi-cylinder is given by

$$\frac{dF}{dx} = 2r \int_0^{\pi/2} p \cos \alpha d\alpha \quad (10)$$

where F denotes the vertical force, α is the polar angle as defined in Fig. 14, and the time-dependent part of the pressure p is obtained from Bernoulli's equation. Neglecting the squares of small perturbation velocities

$$p = \rho \frac{\partial \phi}{\partial t} \quad (11)$$

where ϕ is the velocity potential and ρ the mass density.

The velocity potential of the flow about a cylinder due to the relative vertical velocity $(v - v_w)$ is given by

$$\phi_b = -(v - v_w) \frac{r^2}{R} \cos \alpha \quad (12)$$

The first term of equation (12) may be considered as the potential due to the body motion in smooth water, desig-

¹ Appendix from Korvin-Kroukovsky and Jacobs (3-1957). References in this Appendix will be found at the end of Chapter 3.

² Figure and equation numbers of the original paper are retained in this reproduction.

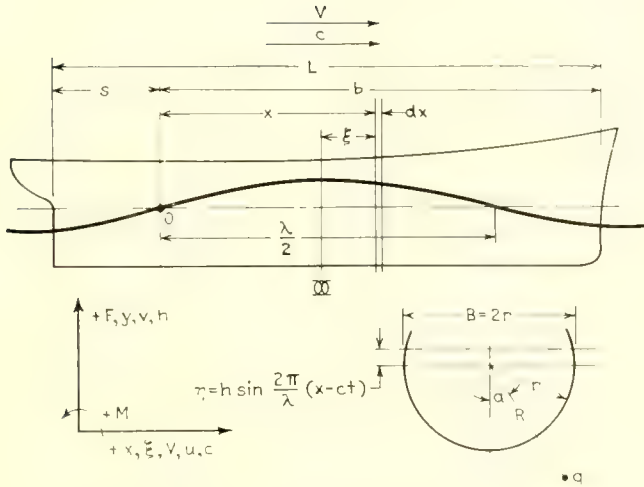


Fig. 14 Sketch illustrating notation used in Appendix

nated ϕ_{bm}

$$\phi_{bm} = -v \frac{r^2}{R} \cos \alpha \quad (13)$$

The second term of equation (12) is the potential due to body-wave interaction, ϕ_{bw}

$$\begin{aligned} \phi_{bw} &= v_w \frac{r^2}{R} \cos \alpha \\ &= -\frac{2\pi h c}{\lambda} \frac{r^2}{R} e^{2\pi y/\lambda} \cos \alpha \cos \frac{2\pi}{\lambda} (x - ct) \end{aligned} \quad (14)$$

The potential due to wave motion alone is ϕ_w

$$\phi_w = h c e^{2\pi y/\lambda} \cos \frac{2\pi}{\lambda} (x - ct) \quad (15)$$

The total velocity potential is the sum of equations (13), (14) and (15)

$$\phi = \phi_{bm} + \phi_{bw} + \phi_w \quad (16)$$

Exciting Forces

Attention will now be concentrated on the second and third terms of equation (16), the two component parts of the velocity potential which give the exciting forces due to waves.

The pressure due to ϕ_{bw} is, from equations (11) and (14)

$$\begin{aligned} p_{bw} &= \rho \frac{\partial \phi_{bw}}{\partial t} = -\frac{4\pi^2 c^2 h}{\lambda^2} \rho \frac{r^2}{R} e^{(-2\pi R \cos \alpha)/\lambda} \\ &\times \cos \alpha \sin \frac{2\pi}{\lambda} (x - ct) - \frac{2r}{R} \dot{r} \rho \\ &\times \frac{2\pi h c}{\lambda} e^{(-2\pi R \cos \alpha)/\lambda} \cos \alpha \cos \frac{2\pi}{\lambda} (x - ct) \end{aligned}$$

Nomenclature

A, B, C, \dots = coefficients of miscellaneous terms of the differential equations of motions, equations (2)
 a, b, c, \dots = coefficients of miscellaneous terms of the differential equations of motions, equations (2)
 \bar{I} = ratio of amplitude of waves made by ship to amplitude of heaving motion
 a_0 = amplitude of vertical acceleration at bow
 B = beam (local)
 b, s = instantaneous distances of ship bow and stern from nodal of wave as defined in Fig. 14
 c = wave celerity
 E = area under spectrum
 F = hydrodynamic heaving force
 \bar{F} = heaving force imposed on a ship by waves ($= F_0 e^{i\sigma}$)
 g = acceleration of gravity
 H_f = force due to water pressures generated by waves and ship's oscillations
 H_m = moment about CG due to water pressures generated by waves and ship's oscillations
 h = wave amplitude

J = longitudinal moment of inertia of a ship in mass units
 K_1, K_2 = coefficients of equation (25)
 k_2 = added mass coefficient in two-dimensional vertical flow about a ship section
 k_4 = correction coefficient for effect of free water surface
 L = ship length
 M = hydrodynamic moment
 \bar{M} = pitching moment imposed on a ship by waves ($= M_0 e^{i\tau}$)
 m = mass of a ship or of a ship section
 $N(\xi)$ = vertical damping force per unit of body length per foot per second
 P, Q, \dots = groupings of coefficients of differential equations of motions defined by equations (7)
 p = pressure
 R = radial distance to a point q in fluid
 r = local radius of semi-cylindrical body
 S = sectional area
 s_0 = amplitude of vertical displacement at bow
 t = time
 u = horizontal component of orbi-

tal velocity of water in waves

V = ship speed
 v = vertical velocity
 v = vertical component of wave orbital velocity
 x = longitudinal co-ordinate with respect to wave nodal point
 y = vertical co-ordinate or local half-breadth of LWL plane
 Z = complex amplitude of heaving motion ($= Z_0 e^{i\delta}$)
 z = vertical co-ordinate or heaving displacement
 α = polar co-ordinate
 β = angle between longitudinal tangent to body surface and x -axis
 δ, ϵ = phase angles of ship motions
 η = local wave-height co-ordinate
 θ = angle of pitch
 $\bar{\theta}$ = complex amplitude of pitching motion ($= \theta_0 e^{i\epsilon}$)
 λ = wave length
 ξ = longitudinal co-ordinate with respect to CG
 ρ = water density
 σ, τ = phase angles of exciting forces
 ϕ = velocity potential
 ω or ω_e = frequency of wave encounter

On the surface of the body where $R = r$ (and since $c^2 = g\lambda/2\pi$)

$$p_{bw} = - \left[\frac{2\pi}{\lambda} \rho gh \sin \frac{2\pi}{\lambda} (x - ct) \right] \times r \cos \alpha e^{(-2\pi r \cos \alpha)/\lambda} - \left[\frac{2\rho gh}{c} \cos \frac{2\pi}{\lambda} (x - ct) \right] \dot{r} \cos \alpha e^{(-2\pi r \cos \alpha)/\lambda} \quad (17)$$

(In the 1955 paper an error was made in substituting $R = r$ before differentiating with respect to time, since r is a function of time.)

The corresponding component of the force due to body-wave interference is obtained by substitution of equation (17) in equation (10)

$$\left(\frac{dF}{dx} \right)_{bw} = \left\{ -4 \frac{\pi}{\lambda} \rho ghr^2 \sin \frac{2\pi}{\lambda} (x - ct) - 4\rho \frac{ghr\dot{r}}{c} \cos \frac{2\pi}{\lambda} (x - ct) \right\} \times \int_0^{\pi/2} \cos^2 \alpha e^{(-2\pi r \cos \alpha)/\lambda} d\alpha$$

The series expansion of the exponential is

$$e^{(-2\pi r \cos \alpha)/\lambda} = 1 - \frac{2\pi r}{\lambda} \cos \alpha + \frac{2\pi^2 r^2 \cos^2 \alpha}{\lambda^2} \dots$$

and the integral is evaluated as

$$\int_0^{\pi/2} \cos^2 \alpha e^{(-2\pi r \cos \alpha)/\lambda} d\alpha = \frac{\pi}{4} - \frac{4\pi r}{3\lambda} + \frac{3\pi^3 r^2}{8\lambda^2} \dots$$

Neglecting cubes and higher powers of the small quantity r/λ in the first term and also r^2/λ^2 in the coefficient of the \dot{r} -term

$$\left(\frac{dF}{dx} \right)_{bw} = -2\rho ghr \left\{ \left(\frac{\pi^2 r}{2\lambda} - \frac{8\pi^2 r^2}{3\lambda^2} \right) \sin \frac{2\pi}{\lambda} (x - ct) + \frac{\dot{r}}{c} \left(\frac{\pi}{2} - \frac{8\pi r}{3\lambda} \right) \cos \frac{2\pi}{\lambda} (x - ct) \right\} \quad (18)$$

The pressure due to the potential of wave motion ϕ_w is from equations (11) and (15)

$$p_w = \rho \frac{\partial \phi_w}{\partial t} = \frac{2\pi \rho h c^2}{\lambda} e^{2\pi y/\lambda} \sin \frac{2\pi}{\lambda} (x - ct)$$

At the surface of the body

$$p_w = \rho g h e^{(-2\pi r \cos \alpha)/\lambda} \sin \frac{2\pi}{\lambda} (x - ct) \quad (19)$$

The corresponding component of the force is obtained by substituting equation (19) in (10)

$$\left(\frac{dF}{dx} \right)_w = 2\rho ghr \sin \frac{2\pi}{\lambda} (x - ct) \times \int_0^{\pi/2} \cos \alpha e^{(-2\pi r \cos \alpha)/\lambda} d\alpha$$

where the integral is equal to

$$1 - \frac{\pi^2 r}{2\lambda} + \frac{4\pi^2 r^2}{3\lambda^2} \dots$$

Again neglecting cubes and higher powers of r/λ

$$\left(\frac{dF}{dx} \right)_w = 2\rho ghr \left(1 - \frac{\pi^2 r}{2\lambda} + \frac{4}{3} \frac{\pi^2 r^2}{\lambda^2} \right) \times \sin \frac{2\pi}{\lambda} (x - ct) \quad (20)$$

The first term of equation (20) is seen to be the displacement force resulting from the wave rise or fall and the accompanying increase or decrease of volume. The second and third terms represent a modification of this force, due to the (approximate) exponential variation of pressure with depth, and this modification is known as the "Smith effect." The entire equation represents the force acting under what is usually referred to as the "Froude-Kriloff hypothesis." This force which is exerted by the waves on the body is reduced by the body-wave interference effect indicated by equation (18); for a semi-cylindrical section at zero speed this amounts to approximately doubling the Smith effect.

Designating by β the angle between the longitudinal tangent to the body surface and the positive x -axis, the derivative r is evaluated as

$$\dot{r} = \frac{dr}{d\xi} \frac{d\xi}{dt} = -V \tan \beta \quad (21)$$

(In the 1955 paper the sign was erroneously taken as positive.) With the substitution of equation (21), the sum of equations (18) and (20) which is the total exciting force, becomes

$$\frac{dF}{dx} = \left(\frac{dF}{dx} \right)_w + \left(\frac{dF}{dx} \right)_{bw} = 2\rho ghr \times \left\{ \left(1 - \frac{\pi^2 r}{\lambda} + \frac{4\pi^2 r^2}{\lambda^2} \right) \sin \frac{2\pi}{\lambda} (x - ct) + \frac{V}{c} \tan \beta \left(\frac{\pi}{2} - \frac{8\pi r}{3\lambda} \right) \cos \frac{2\pi}{\lambda} (x - ct) \right\} \quad (22)$$

This expression replaces equation (26) of the 1955 paper. It differs from it in the sign of the velocity-dependent terms, which are small, and in the value of the coefficient of the $(r \tan \beta)/\lambda$ -term, which is also small.

Two steps remain to be taken: (a) Equation (22) must be generalized for ship sections other than semi-circular, and (b) corrections must be introduced for free-surface effects.

It is clear that the factor

$$2\rho g r h \sin 2\pi(x - ct)/\lambda = 2\rho g r \eta$$

represents the change in displacement force with wave rise and fall; r in this case then is to be taken as the half-beam $B/2$, at the load waterline. Next it is noted that when the body-wave interaction is taken into account the Smith effect on a circular body is doubled. This factor of 2 may be interpreted as $(1 + k_2)$ by analogy with G. I. Taylor's expression (1928) for the force acting on a body placed in a fluid flow with a velocity gradient. Here k_2 is the coefficient of accession to inertia in vertical flow and is equal to 1 for a circular section.³

With a free water surface and the formation of a standing-wave system, the value of $k_2 = 1$ for the circular cylinder is modified by a factor which is designated as k_4 . Ursell (1954, and answer to discussion, Korvin-Kroukovsky, 1955b) has computed the following values of k_4 versus $\omega^2 r/g$ (or $\omega^2 B/2g$) for the circular cylinder:

$\frac{\omega^2 B/2g}{0}$	$\frac{k_4}{\infty}$
0.262	0.818
0.524	0.632
0.785	0.592
1.571	0.673
2.094	0.738
2.356	0.762
3.142	0.818
3.927	0.859
4.712	0.883

In the absence of more complete information it will be assumed that this table of corrections applies to non-circular sections as well.

From experiments with an oscillator, Golovato (1956) derived the coefficients of added (virtual) mass in heaving oscillation for a ship form symmetrical fore and aft with U-sections almost wall-sided at the load waterline. Fig. 8 of that reference shows a curve quite similar in trend to the coefficients of the foregoing table but with values about 20 per cent higher and with the minimum shifted to a somewhat higher frequency. The effect of these differences on the ship response are expected to be small.

The factor $(1 + k_2 k_4)/2$ will then be applied to all terms of equation (22) after the first displacement-force term. In the earlier paper, k_4 was estimated for the ship as a whole and it was applied only to the integrated virtual mass and inertia effects due to body motion in smooth water; it was omitted in the expression for the exciting forces. Subsequently, it was found necessary to apply the k_4 correction to each section for the calculation of bending moments and highly advisable to adopt this more accurate procedure for the motion calculations. Therefore, this omission has been corrected in the present paper.

Since the modified Smith-effect terms are connected with virtual masses and since the effect of section shape

is defined by the coefficient k_2 , the factor r in this case is interpreted as a measure of sectional area; i.e.,

$$r = (2S/\pi)^{1/2} \quad (23)$$

where S is sectional area below the load waterline, and therefore

$$\tan \beta = \frac{dr}{d\xi} = \frac{1}{(2\pi S)^{1/2}} \frac{dS}{d\xi} \quad (24)$$

With the substitutions just indicated, the distribution of vertical heaving forces due to the action of waves on a ship at a particular instantaneous position of the ship on the wave ($t = 0$) is expressed as

$$\frac{dF}{dx} = \rho g h B \left(K_1 \sin \frac{2\pi x}{\lambda} + K_2 \cos \frac{2\pi x}{\lambda} \right) \quad (25)$$

where K_1 and K_2 are nondimensional coefficients

$$K_1 = 1 - \frac{1 + k_2 k_4}{2} \frac{\pi^2}{\lambda} \left(\frac{2S}{\pi} \right)^{1/2} + 4(1 + k_2 k_4) \frac{\pi}{\lambda^2} S$$

$$K_2 = \frac{\pi(1 + k_2 k_4)}{4} \left[1 - \frac{16}{3\lambda} \left(\frac{2S}{\pi} \right)^{1/2} \right] \frac{V}{c} \frac{1}{(2\pi S)^{1/2}} \frac{dS}{d\xi}$$

These coefficients depend on the sectional shape and area, on the wave length and also, because of the presence of the coefficient k_4 , on the frequency of wave encounter. The distribution of forces along the length of the ship given by equation (25) can be used directly in the computation of the bending moments exerted on a ship by waves.

For an analysis of ship motions the force distribution must be integrated to provide the total heaving force F and the total pitching moment M

$$F = \int_s^b \frac{dF}{dx} dx \quad (26)$$

and

$$M = \int_s^b \frac{dF}{dx} \left(\xi + \frac{1}{\pi} \frac{dS}{d\xi} \right) dx \quad (27)$$

where the limits of integration s and b are the values of x at the stern and the bow, respectively. The second term of equation (27) results from the consideration that the water pressure acts normally to the body surface; in the case of a body of varying circular section the moment arm is $(\xi + r \tan \beta)$ and, by the use of the relationships (23) and (24), for a normal ship form the moment arm may be assumed to be $(\xi + dS/\pi d\xi)$. Equations (25), (26), and (27) replace Equations (33) and (34) of the earlier paper.

The integrals of equations (26) and (27) can be evaluated readily by Simpson's rule. By changing the ship's position relative to the wave the maximum values or amplitudes of the exciting force and moment can be found as well as the phase lags σ and τ . Calculations of these amplitudes F_0 and M_0 , were made for a 5-ft-long model of the Series 60, 0.60-block-coefficient hull (ETT Model No. 1445) in waves of ship length ($\lambda/L = 1$) and wave height of 1.5 in., for comparison with the ex-

³ O. Grim (3-1957) proved this interpretation on basis of F. M. Lewis' (1929) transformation.

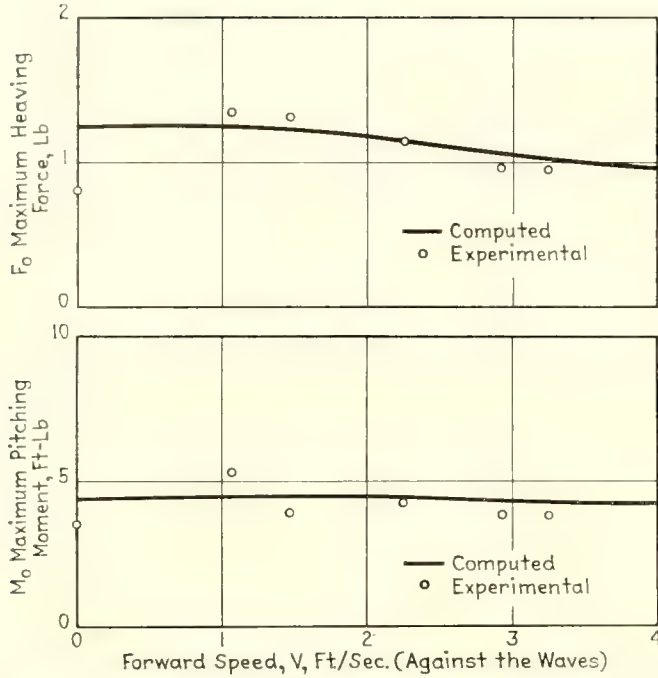


Fig. 15 Comparison of computed and experimentally measured exciting force and moment amplitudes for ETT Model 1445 in waves 5 ft by 1.5 in.

perimental data described in Appendix 2 of the 1955 paper. A very good agreement between calculated and experimental values was obtained except at zero and very low forward speeds, Fig. 15. It has been observed in recent years that at such speeds there is often interference from the waves reflected by the walls of the towing tank, while at greater speeds this interference is no longer encountered and the experimental data become more reliable.

Forces Due to Body Motions

The pressure in the fluid due to the body's own motion is, from equations (11) and (13)

$$p_{bm} = \rho \frac{\partial \phi_{bm}}{\partial t} = -\rho \dot{v} \frac{r^2}{R} \cos \alpha - \frac{2\rho v \dot{r}}{R} \cos \alpha$$

$$= -\rho v r \cos \alpha - 2 \rho v \dot{r} \cos \alpha \quad (28)$$

at the body surface where $R = r$. The distribution of vertical forces is obtained by substituting equation (28) in (10)

$$\left(\frac{dF}{dx} \right)_{bm} = -\rho \frac{\pi}{2} r^2 \dot{v} - \rho \pi r \dot{r} v \quad (29)$$

The apparent vertical velocity v of the center of the circular section, given by equation (8), consists of three parts

$$v = \dot{z} + \xi \dot{\theta} - V\theta$$

where the first is the heaving velocity, the second is the

vertical velocity due to the angular velocity of pitching, and the third is the vertical velocity due to the instantaneous angle of trim θ at the control planes. Since ξ is a function of time and $\dot{\xi} = -V$

$$v = \dot{z} + \xi \dot{\theta} - 2V\theta \quad (30)$$

After substituting equations (8), (21) and (30) in equation (29), the following set of six terms is obtained

$$\left. \begin{aligned} \left(\frac{dF}{dx} \right)_{bm} &= - \left[\rho \frac{\pi}{2} r^2 \right] \dot{z} & (i) \\ &- \left[\rho \frac{\pi}{2} r^2 \xi \right] \dot{\theta} & (ii) \\ &+ 2 \left[\rho \frac{\pi}{2} r^2 V \right] \theta & (iii) \\ &+ [\rho \pi r \tan \beta V] \dot{z} & (iv) \\ &+ [\rho \pi r \tan \beta V \xi] \dot{\theta} & (v) \\ &- [\rho \pi r V^2 \tan \beta] \theta & (vi) \end{aligned} \right\} \quad (31)$$

Equation (31) replaces equation (39) of the 1955 paper. Terms (i) and (ii) are identical with (3) and (6) of equations (39) and term (iii) is the sum of the earlier (1) and (5) with the sign of the latter corrected. Terms (iv), (v), and (vi) are twice terms (4), (7), and (2), respectively, of the earlier equation (39), which were incorrect because substitution of $R = r$ had been made before differentiation with respect to time.

The factor $(\rho \pi r^2/2)$ in the first three terms of equation (31) is evidently the virtual mass of an element of body length, and by introducing k_2 and k_4 on the basis of the reasoning outlined in connection with exciting forces, it can be expressed as $(\rho S k_2 k_4)$. The factor $(\rho \pi r \tan \beta)$ in the last three terms is the derivative with respect to ξ of $(\rho \pi r^2/2)$ and so it can be expressed as $d(\rho S k_2 k_4)/d\xi$. The total force due to the body's own motion is then

$$F_{bm} = -\rho \int S k_2 k_4 (\dot{z} + \xi \dot{\theta} - 2V\theta) d\xi$$

$$+ V \rho \int \frac{dS k_2 k_4}{d\xi} (\dot{z} + \xi \dot{\theta} - \xi \theta - V\theta) d\xi \quad (32)$$

and the moment is

$$M_{bm} = -\rho \int S k_2 k_4 (\dot{z} + \xi \dot{\theta} - 2V\theta) \xi d\xi$$

$$+ V \rho \int \frac{dS k_2 k_4}{d\xi} (\dot{z} + \xi \dot{\theta} - V\theta) \xi d\xi \quad (33)$$

where the integration is carried over the length of the hull.

Since F_{bm} and M_{bm} are functions of z and θ and their derivatives, the terms of equations (32) and (33) may be transposed to the left-hand side of the coupled equations of motion. Their contributions to the coefficients of these equations are designated by the subscript 1. Thus

$$\left. \begin{aligned}
 a_1 &= \rho \mathcal{F}(Sk_2k_4)d\xi \\
 b_1 &= -V\rho \int \frac{d(Sk_2k_4)}{d\xi} d\xi = 0 \\
 d_1 &= \rho \mathcal{F}(Sk_2k_4)\xi d\xi \\
 e_1 &= -2V\rho \mathcal{F}(Sk_2k_4)d\xi - V\rho \int \frac{d(Sk_2k_4)}{d\xi} \xi d\xi \\
 g_1 &= +V^2\rho \int \frac{d(Sk_2k_4)}{d\xi} d\xi = 0 \\
 A_1 &= \rho \mathcal{F}(Sk_2k_4)\xi^2 d\xi \\
 B_1 &= -2V\rho \mathcal{F}(Sk_2k_4)\xi d\xi - V\rho \int \frac{d(Sk_2k_4)}{d\xi} \xi^2 d\xi \\
 C_1 &= V^2\rho \int \frac{d(Sk_2k_4)}{d\xi} \xi d\xi \\
 D_1 &= \rho \mathcal{F}(Sk_2k_4)\xi d\xi \\
 E_1 &= -V\rho \int \frac{dSk_2k_4}{d\xi} \xi d\xi
 \end{aligned} \right\} \quad (34)$$

Dynamic Terms in \dot{z} and $\dot{\theta}$

Attention should be called to the fact that the velocity-dependent terms in \dot{z} and $\dot{\theta}$ of equations (31), (32) and (33) do not involve dissipation of energy, but only the transfer of energy from one mode to another. This was demonstrated by Haskind (1946) and by Havelock (1955), who refers to these terms as "dynamic coupling."

In previous studies of oscillations, velocity-dependent terms appear only in the role of energy dissipation either by viscosity or by wave making in the case of ships. Following these earlier studies it was assumed in the 1955 paper that the velocity-dependent terms in the development of the potential theory merely implied damping and could be replaced by damping terms determined on the basis of energy dissipation by waves as a *quid pro quo*, since the initial statement of the problem did not provide for inclusion of energy-dissipation terms. Later, examination of the work of Fay (1957) suggested that this was an error of judgment which could be responsible for the poor correlation between calculated and experimental phase relationships reported in the earlier paper and also for the poor results obtained when applying the methods of that reference to the calculation of bending moments.

A study of Haskind (1946) and Havelock (1955) confirmed this and therefore the terms in \dot{z} and $\dot{\theta}$, (iii), (iv) and (v) of equation (31) have been reinstated. They yield a heaving force due to pitching velocity $\dot{\theta}$ and a pitching moment due to heave velocity \dot{z} equal to

$$[-2\rho \mathcal{F}(Sk_2k_4)d\xi - \rho \mathcal{F}\xi d(Sk_2k_4)]V\dot{\theta}$$

and

$$[-\rho \mathcal{F}\xi d(Sk_2k_4)]V\dot{z} \quad (35)$$

For the case of a half-immersed spheroid under the condition of a free water surface but neglecting wave

making, Havelock gives the dynamic coupling terms as

$$-pMV\dot{\theta} \text{ for the heaving force}$$

and

$$+qMV\dot{z} \text{ for the pitching moment}$$

where M is the mass of displaced water. In general $p \neq q$, and each is given by a fairly complicated expression in terms of ellipsoidal co-ordinates and associated Legendre functions of the second kind. For a long spheroid Havelock finds that $p = q = (1 + k_1)/2$, or 0.515 for a length-diameter ratio of 8 which is the fineness ratio of the usual ship form. Haskind (1946) had found that $p = q$ for a thin or "Michell" ship symmetrical fore-and-aft.

For a prolate spheroid S is a function of ξ alone and

$$\int_L S d\xi = -\int_L \xi dS = M/\rho$$

Therefore expression (35) of the present development becomes

$$-\overline{k_2k_4}MV\dot{\theta}$$

and

$$+\overline{k_2k_4}MV\dot{z}$$

where the bar indicates the value for the entire body.

Thus $p = q = k_2k_4$. For a circular section $k_2 = 1$ and, at the oscillating frequency in the vicinity of synchronism for most ship forms, k_4 is of the order of 0.75. It appears from the application to the models in this paper that the damping in heave is reduced and in pitch is increased by the addition of the dynamic coupling terms.

Dissipative Damping

It was mentioned previously that the free water surface was not taken into account in the basic derivation of the present paper and that a correction for it must be introduced independently. The effect of the free surface on the virtual mass has been allowed for approximately by the use of the coefficient k_4 derived by Ursell for a semi-cylinder (1954, and discussion, Korvin-Kroukovsky, 1955b). (Grim, 1953, also has calculated this effect for some ship-like forms but his material is not extensive enough for general application.) The other well-known effect of the free surface is the dissipation of energy in the formation of waves which propagate away from the ship in all directions. In the "strip" method of analysis it is assumed that waves from each length segment $d\xi$ propagate laterally. If the ratio of the amplitude of these waves to the amplitude of the heaving motion of a ship section is designed by \bar{A} , the damping force per unit vertical velocity v of the ship segment is expressed as (Holstein 1936, 1937a, 1937b, and Havelock 1942)

$$N(\xi) = \rho g^2 \bar{A}^2 / \omega_e^3 \quad (36)$$

where ω_e is the frequency of the waves radiated by the ship and is equal to the frequency of wave encounter.

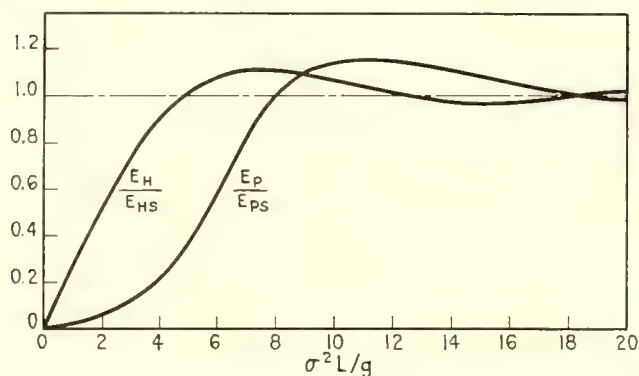


Fig. 16 Ratios of three-dimensional to two-dimensional calculations of damping coefficients for submerged spheroid of $L/B = 8$ (Fig. 1 of Havelock, 1956. Subscript H indicates heaving, P pitching, and S strip method)

Holstein (1936, 1937a and 1937b) and Havelock (1942) represented the heaving body by a distribution of harmonically pulsating sources along the bottom. In that case the amplitude ratio \bar{A} is given by

$$\bar{A} = 2e^{-k_0 y} \sin(k_0 y) \quad (37)$$

where $k_0 = \omega^2/g$, y is the half-beam $B/2$, and $f = S/B$, the mean draft of a ship section. This theoretical result was confirmed by Holstein's own experiments; the results of the many experiments appear consistent, yet some doubt may be felt because of the smallness of the test tank. The theory is approximate and acceptance of it necessarily hinges on agreement with experimental results.

A more nearly exact theory was developed by Ursell (1949, 1953, 1954) for a heaving semicylinder, and by Grim (1953) for a number of analytically defined sections closely approaching practical ship sections. In the case of a semi-circular section Grim's results agree with Ursell's. In their theory the damping force is calculated as a boundary-value problem. Unfortunately no experimental verification has been provided.

Both theories, that of Holstein and Havelock which does not satisfy the boundary condition on the surface of the body and that of Grim which fulfills all the boundary conditions with a good degree of accuracy, give approximately the same results at the frequency for synchronism for a normal ship, but very different results at higher frequencies. The Holstein-Havelock values of \bar{A} , equation (37), were used in the 1955 paper where the results of the calculations generally indicated overdamping. In recent work Grim's \bar{A} -values, which are presented in the form of charts, have been substituted. This has given a better correlation of the calculated and experimental amplitudes of the motions of the ship models.

The experiments of Golovato (1956) have been mentioned already in connection with coefficients of added mass. In those experiments the coefficient b of damping force was also measured. The experimentally measured b was found to be lower than the coefficients computed by the different methods of Havelock and Grim, that

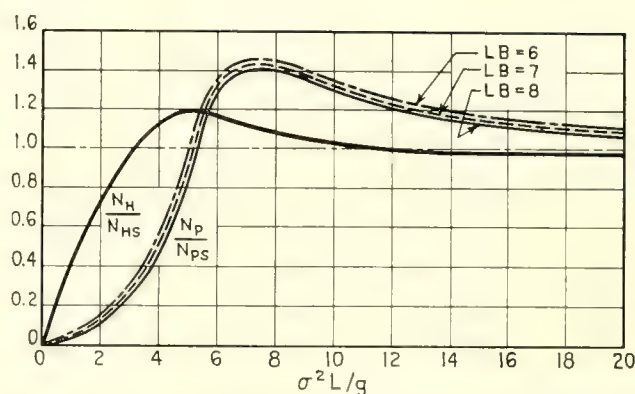


Fig. 17 Ratios of three-dimensional to two-dimensional calculations of damping coefficients for thin "Michell" ship of $B/H = 2$ (Fig. 2 of Vossers, 1956. Subscript H indicates heaving, F pitching, and S strip method)

computed by Grim's method being closer to the measured value. Another comparison by Golovato of computed and measured coefficients using the experimental data of Haskind and Riman (1946) gave similar results.

The computed coefficients are based on the strip method of analysis which assumes two-dimensional fluid flow, while Golovato's and Haskind and Riman's tests were made with ship models, i.e., in three-dimensional flow. The relation of damping in three-dimensional flow to damping in two-dimensional flow can be estimated on the basis of the work of Havelock (1956) and of Vossers (1956). Havelock calculated the damping coefficient by two methods, three-dimensional and strip, for a submerged spheroid with length-beam ratio of 8, a fair value for comparison with ship models. Vossers made similar computations from Haskind's theoretical results (1946) for a "thin ship" in the sense of the approximations introduced by Michell (1898) in his theory of wave resistance of ships. The results of both are reproduced here in Figs. 16, 17 as ratios of the coefficients of damping in heave or pitch by the two methods.⁴ It should be remembered that accurate evaluation of damping is most important in the vicinity of synchronism. The values of the frequency parameter $\omega^2 L/g$ (or $\sigma^2 L/g$ in the figures) at synchronism are given in the last two columns of Table 5 for the eight models to which the computational method outlined here has been applied. (Also included in Table 5 are the parameters L/B , k_2 and k' for comparison with Havelock's submerged spheroid.)

Omitting from consideration the original trawler and yacht, the lowest values of the parameter $\omega^2 L/g$ are 10.4 in heave for the T-2 tanker and 11.7 in pitch for the Series 60 hull, while for the models with displacement-length ratio of 60 $\omega^2 L/g$ is above 17.6. At these values Figs. 16 and 17 indicate negligible corrections for damping in heave and small corrections for damping in pitch (0 to 15 per cent based on Havelock's computations and 10 to 20 per cent from Vossers'). Since the corrections for three-dimensional effect are negligible or small and

⁴ The subscript S in the figures denotes "strip" method.

Table 5 Values of $\omega^2 L/g$ for Synchronous Speed, L/B , k_2 and k' , of the Various Ship Forms for Comparison with Havelock's Submerged Spheroid

Model	L/B	k_2^a	k'^a	ω		$\omega^2 L/g$	
				Pitch	Heave	Pitch	Heave
Havelock's submerged spheroid.....	8	0.945	0.84	—	—	—	—
Series 60 (Model 1445).....	7.5	0.72	0.42	8.7	9.0	11.7	12.5
V-bow (Model 1616).....	7.5	0.74	0.48	9.7	9.0	14.5	12.5
T-2 Tanker (Model 1444).....	7.4	0.68	0.51	9.0	8.4	12.0	10.4
Destroyer (Model 1723).....	9.4	0.90	0.75	10.6	10.3	20.1	18.8
Trawler (Model 1699A).....	5.8	0.66	0.51	8.4	8.6	9.5	9.9
Yacht (Model 1699B).....	3.5	0.70	0.46	7.7	8.1	7.9	8.6
Lengthened trawler (Model 1699C).....	10.3	0.64	0.53	10.6	10.3	20.1	18.8
Lengthened yacht (Model 1699D).....	8.0	0.72	0.48	10.0	10.5	17.6	19.4

^a k_2 for the ship forms (with free surface correction k_4) = $\frac{\text{added mass}}{\text{mass of water displaced by a ship}}$

$k' = \frac{\text{added moment of inertia}}{\text{moment of inertia of water displaced by a ship.}}$

since they have not yet been developed for ships of normal form, they will be ignored in the present work; the damping will be taken as computed for two-dimensional flow by Grim. It is gratifying nevertheless to have measures of possible error such as Figs. 16 and 17 to replace arbitrary assumptions made in the earlier paper as to the applicability of the strip method of analysis.

The damping force for each section is

$$vN(\xi) = [\dot{z} + \xi\theta - V\theta]N(\xi) \quad (38)$$

where $N(\xi)$, the damping force per unit vertical velocity of the ship section, is given by equation (36). The total damping force and moment of the hull are then obtained by integration over the length, thus the damping force is

$$\int N(\xi)[\dot{z} + \xi\theta - V\theta]d\xi \quad (39)$$

and the damping moment is

$$\int N(\xi)[\dot{z} + \xi\theta - V\theta]\xi d\xi \quad (40)$$

The contributions of these expressions to the coefficients of the coupled equations of motion are designated by the subscript 2

$$\begin{aligned} b_2 &= \int N(\xi)d\xi \\ e_2 &= E_2 = \int N(\xi)\xi d\xi \\ g_2 &= -V \int N(\xi)d\xi = -Vb_2 \\ B_2 &= \int N(\xi)\xi^2 d\xi \\ C_2 &= -V \int N(\xi)\xi d\xi \end{aligned} \quad (41)$$

The integrations are carried out numerically as in all cases in this paper, and over the length of the hull.

When all the forces and moments proportional to z and θ and their derivatives with respect to time, which are presented in the foregoing development of the potential theory with corrections for free-surface effects, are combined with the inertial and restoring forces and moments the coefficients of the equations of motion become

$$\begin{aligned} a &= m + \rho \int (Sk_2 k_4) d\xi \\ A &= J + \rho \int (Sk_2 k_4) \xi^2 d\xi \\ d &= D = \rho \int (Sk_2 k_4) \xi d\xi \\ b &= \int N(\xi) d\xi \\ B &= \int N(\xi) \xi^2 d\xi - 2VD - V\rho \int \xi^2 d(Sk_2 k_4) \\ e &= \int N(\xi) \xi d\xi - 2V\rho \int (Sk_2 k_4) d\xi \\ &\quad - V\rho \int \xi d(Sk_2 k_4) \quad (42) \\ E &= \int N(\xi) \xi d\xi - V\rho \int \xi d(Sk_2 k_4) \\ c &= \rho g \int B^* d\xi \\ {}^5C &= \rho g \int B^* \xi^2 d\xi - VE \\ g &= \rho g \int B^* \xi d\xi - Vb \\ G &= \rho g \int B^* \xi d\xi \end{aligned}$$

where the integrations are taken over the length of the hull.

Terms c and G , the coefficients of the displacement in heave z in the force and moment equations respectively, depend only on the changes in the displacement of the ship. With the linearizing assumption these are evaluated on the basis of the beam of a section. Terms g and C , the coefficients of the angular displacement θ , depend on displacement changes and also on the kinematics of the fluid flow resulting from the ship being at an instantaneous angle of trim.

It is seen that the damping-force coefficient b in heave is a function of frequency of encounter ω_e but is independent of forward speed V per se. This appears to be confirmed by the experimental work of Golovato (1956). However, the damping moment coefficient B in pitch and the cross-coupling coefficients e and E are composed of dynamic terms proportional to V and dissipative terms independent of V (except as the frequency of encounter ω_e is a function of V). While the dynamic terms contribute only a little to B , they make very important contributions to the cross-coupling coefficients.

⁵ B^* represents beam.

Analog Equipment for Processing Randomly Fluctuating Data¹

FRANCIS B. SMITH²

Experimental studies of buffeting, flutter, and atmospheric turbulence produce data which are not amenable to analysis by the usual process of measuring or calculating discrete values of static force, pressure, or moment. Instead, where randomly fluctuating quantities are involved, it becomes necessary to use statistical methods to analyze and describe the phenomenon being investigated. The types of information usually required are probability distributions, power spectra, and cross spectra. In order to avoid the time and expense involved in using conventional numerical methods for securing the various statistical analyses, the NACA uses a technique of recording data on magnetic tape and playing it back into electronic analog analyzers which perform the desired analyses and automatically plot the results. The magnetic tape recording system and the analog analyzers are described in this paper and their application to aeronautical problems is illustrated. A discussion of the comparative accuracy and reliability of the numerical and analog methods is also included.

Introduction

IN MUCH of the experimental work undertaken in aeronautical research and development it is possible to set up experiments so that the amplitude of any quantity being measured remains very nearly constant during a given test run. In this case a precise measure of the static level is sought and any fluctuations in the level occurring during the run are considered to be "noise" which must be eliminated by filters or faired through on the time-history record.

However, many phenomena being studied produce data in which random amplitude fluctuations are inherent and measurement of average or faired levels only is of little or no value. Typical examples are gun aiming errors encountered in fire control systems, loads imposed on airplanes by turbulence of buffeting, stresses produced by engine noise or vibration, and aircraft landing gear loads caused by rough runways. In these cases, the information sought must be secured by anal-

ysis and study of the random amplitude fluctuations about the mean value.

The most practical way to describe quantitatively these amplitude variations is to use the statistical techniques which have been devised for analyzing stationary random time series. The simplest and most familiar of these techniques is the probability distribution, which, for example, has been used for years to describe atmospheric turbulence. Recently, the generalized harmonic analysis techniques have been developed and are being successfully applied to many random-type data analysis problems.

The theoretical aspects of these techniques have been discussed by Wiener [1],³ Tukey [2], and Rice [3] and specific applications to aerodynamic problems have been described by Clementson [4], Liepmann [5], Summers [6], Press [7], Chilton [8] and others.

These authors have demonstrated that statistical techniques are extremely useful in aeronautical research. However, if conventional time-history records and numerical data work-up procedures are used to secure the various required analyses, the application of these techniques to actual samples of experimental data is a time-consuming and expensive process. This is especially true in the case of power and cross spectra determinations. In addition to the digital computer expense involved in making the thousands of required numerical calculations, several days may be required to read the necessary number of data points from the time-history record. These factors often seriously restrict the number of statistical analyses that might be made, or force one to limit severely the length of the data samples and to accept the consequent poor statistical reliability.

To overcome these limitations the NACA has combined commercially available electronic equipment with NACA-developed components to provide an analog data processing facility which produces the required statistical analyses rapidly and inexpensively. This facility consists of four basic elements: (1) magnetic-tape data storage and playback system, (2) a probability distribution analyzer, (3) a power spectral density analyzer, and (4) a cross spectral density analyzer. This paper will describe these pieces of equipment,

³ Numbers in brackets designate References at the end of the paper.

¹ Presented at the Electronic Aids to the Aircraft Industry Session, Twenty-Third Annual Meeting, IAS, New York, N. Y., January 24-27, 1955. Reprinted with the kind permission of the *Aeronautical Engineering Review*, May 1955 issue.

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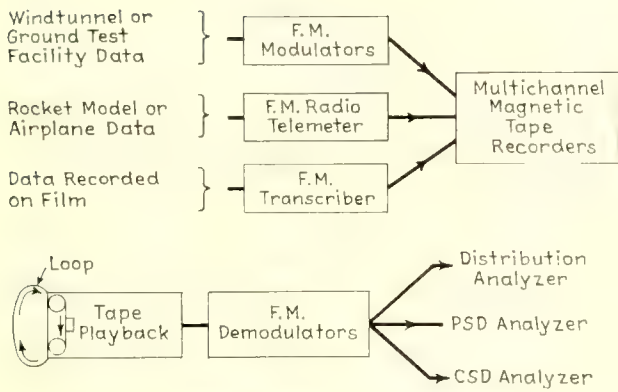


Fig. 1 Magnetic tape-recording system

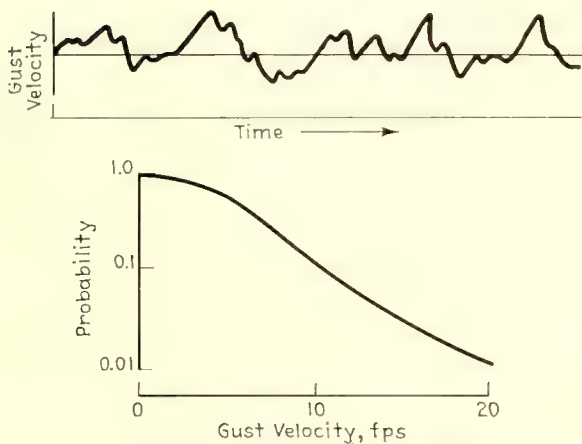


Fig. 2 Probability distribution

in the order listed, and will briefly illustrate the application of the various types of analyses to aeronautical research.

Magnetic Tape Recording and Playback System

An essential part of the data processing facility is the magnetic tape recording equipment illustrated in Fig. 1. This equipment provides a means for storing the data in such a way that it can be accurately reproduced in a form most suitable for analog processing.

Instead of recording data directly on magnetic tape a frequency-modulated carrier system is used. This permits the storage of very low frequency and d-c data and eliminates much of the spurious amplitude variation which would otherwise be caused by tape imperfections and tape wear. It also makes possible the direct recording of FM radio telemeter data.

Where the data to be processed are produced by a stationary facility such as a wind tunnel, the tape recorder and FM modulators are located at the facility. Where the data are produced by an airplane, helicopter, or rocket-propelled missile, the data are transmitted to

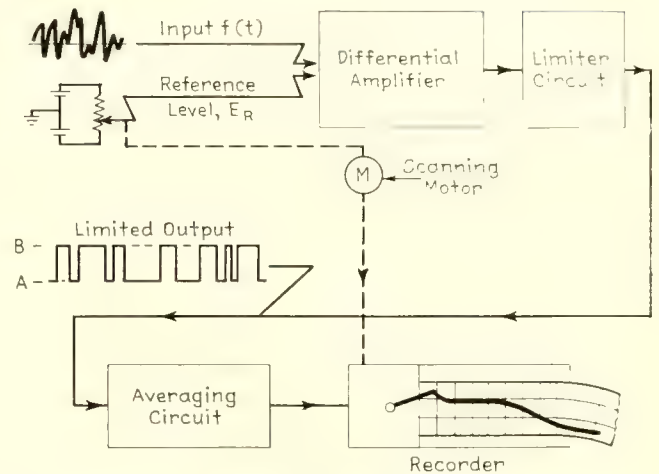


Fig. 3 Probability distribution analyzer

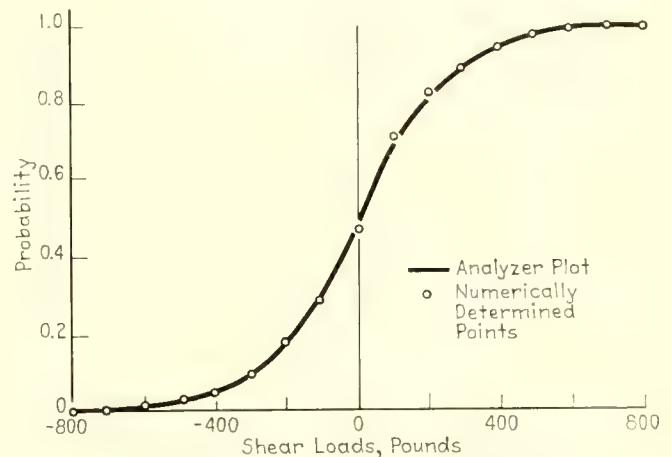


Fig. 4 Distribution of wing shear loads

a tape recorder on the ground by a radio telemeter link. Or, if the data to be processed have been previously recorded as a trace deflection on photographic paper or film, a transcribing machine is used to convert film deflections to a frequency modulated carrier which is recorded on tape.

Use of magnetic tape recorders provides a great deal of flexibility in handling data samples. Sample lengths and frequency ranges can be expanded or compressed by factors as high as 240 to 1 simply by changing the tape speed. Data samples from 1 sec to several minutes in duration and containing frequencies from dc to 6000 cycles per sec or 7000 cycles per sec can be recorded and reproduced by the FM system. If necessary, higher frequencies can be handled by eliminating the frequency modulated carrier and recording directly.

The analog analysis methods require that data samples be scanned a large number of times by the analyz-

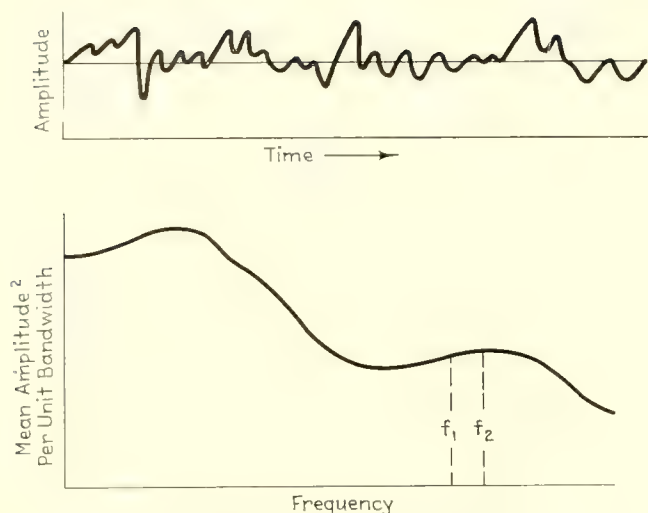


Fig. 5 Power spectral density

ing equipment. Consequently each sample on the tape is spliced into a continuous loop before being played back into the FM demodulators and the analyzers.

The three types of analyzers through which data recorded on tape may be processed are a probability distribution analyzer, a power spectral density analyzer, and a cross spectral density analyzer. These three analyzers and the type of statistical information produced by each will be discussed in the following sections of this paper.

Probability Distribution

The time function shown at the top of Fig. 2 illustrates a randomly varying quantity of the type to be considered throughout this paper. One way to analyze such a function is to determine its probability distribution. This is a measure of the proportion of total time during which the amplitude of the varying quantity exceeds given levels.

For example, suppose the time function shown at the top of the figure were a plot of the gust velocities encountered by an airplane flying through rough air. The intensity of the turbulence could be shown by plotting the probability that the gusts would exceed specific velocities against gust velocity. The gusts in the atmosphere represented by the distribution shown would be expected to exceed 3 or 4 fps about 95 per cent of the time and to exceed 20 fps only about 1 per cent of the time.

The electronic analyzer used to determine probability distributions is illustrated in Fig. 3. The data sample recorded on an endless loop of magnetic tape is continuously played into the analyzer, and its instantaneous amplitude is compared to a reference voltage E_R . The level of the reference voltage is determined by the position of the potentiometer slider. If the input voltage is larger than the reference voltage, the amplifier will

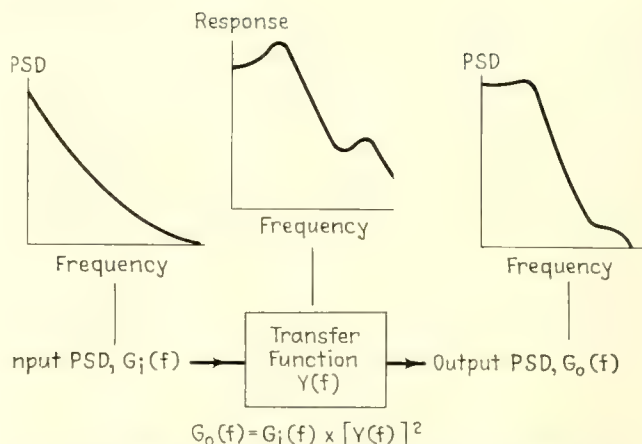


Fig. 6 Input-output power spectral density relationship

be saturated and the output of the limiter will be at level B ; if the input is less than the reference voltage, the amplifier will be cut off and the output of the limiter circuit will be at level A . The percentage of time that the level is at B then represents the probability of the input being above the fixed level E_R . By averaging the output of the limiter this probability may be read directly and recorded. By slowly changing the level of the reference voltage and driving the paper on the recorder in synchronization, a complete plot of the probability distribution may be obtained.

To illustrate the use of the analyzer, Fig. 4 shows a distribution of wing buffeting loads obtained during a gradual pull-up at high speeds. The solid line is the analyzer record and the circled points are values determined numerically. Setup and running time on the electronic analyzer was about 20 to 30 min and about 7 man-hours were required to read the time-history record, calculate the mean, and determine numerically the circled points.

Power Spectral Density

The probability distribution describes the intensity characteristics of the data but does not describe their frequency or spectral characteristics. Nor does it furnish information sufficient for calculating system input-output relationships or transfer functions. If this sort of information is required, a second type of statistical analysis called the power spectral density analysis may be used.

Power spectral density analysis is sometimes referred to as "generalized harmonic analysis" and is similar to the familiar Fourier series harmonic frequency analysis except in the following respects: (1) Fourier series analysis is applicable to repetitive functions while generalized harmonic analysis is applicable to stationary random time functions, (2) as illustrated in Fig. 5, the spectrum is represented as a continuous curve rather than as discrete harmonically-related frequencies, and (3) the spectrum is plotted in terms of mean-squared am-

plitude per unit bandwidth (equivalent to average power per unit bandwidth in the electrical system) instead of simple sine-wave amplitudes.

The power spectrum then represents the distribution of energy over the frequency spectrum. For example, the portion of the total energy included in the frequency band f_1 to f_2 in the figure is represented by the area under the curve between f_1 and f_2 .

One extremely useful feature of the power spectra concept is the simple input-output spectra relationship illustrated in Fig. 6. For linear systems, the power spectral density of the system's output $G_o(f)$ is equal to the power spectral density of the input $G_i(f)$ times the squared absolute value of the transfer function $Y(f)$. Thus, we have a relationship among the three variables such that if two of the three are known, the third may be determined.

This sort of input-output-transfer function relationship is one which is familiar to the electrical engineer who is often fortunate enough to be able to work with strictly sinusoidal signals. It should be noted that the power spectra concept now enables the aeronautical engineer, who often has no choice but to work with randomly fluctuating type of "signals," to perform the same sort of "circuit analysis" as the electrical engineer is accustomed to do.

For example, again consider the problem of an airplane flying through rough air; the airplane can be considered as a mechanism having a transfer function $Y(f)$, the turbulence can be considered to be the input $G_i(f)$, and the airplane's response can be considered as the output $G_o(f)$.

Thus, by measuring an airplane's transfer function and its response to atmospheric turbulence, it is possible to use the plane as an instrument for measuring turbulence, as Clementson [4] and Summers [6] did. Or, if a specific turbulence spectrum is assumed and the airplane's transfer function is known, it is possible to predict the plane's response to the turbulence.

Also, by using the techniques developed by Rice [3], it is possible in many instances to use power spectra to calculate such things as the number of zero crossings and the number of times the amplitude of a randomly varying quantity exceeds certain levels.

Power spectral density may be numerically calculated by a procedure outlined by Tukey [2]. Very briefly summarized, this procedure is as follows: First, read the time-history record point by point; second, calculate the autocorrelation function of the data sample by evaluating the integral

$$\int_0^T f(t) \cdot f(t + \tau) dt$$

for several discrete values of time lag τ ; and third, determine the power spectral density of the data by taking the Fourier cosine transform of the autocorrelation function.

This process is equivalent to passing a tunable, constant-bandwidth filter of known characteristics over the

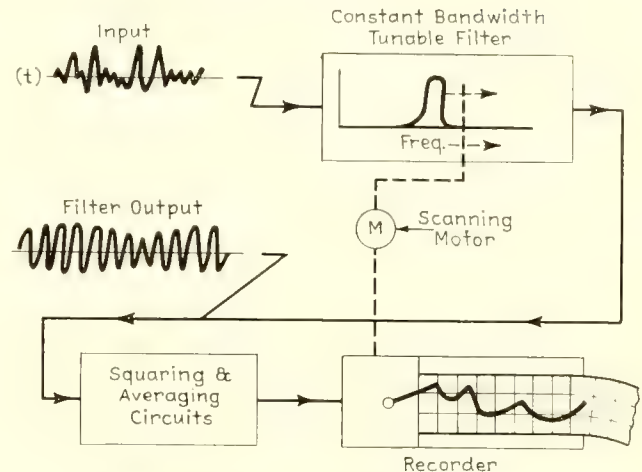


Fig. 7 Power spectral density analyzer

data and measuring the time average of the square of the filter's output. Except for squaring of the filter's output and some difference in the filter's characteristics, the results are almost identical to those obtained from the familiar spectrum analyzers that have been used for years by communications and sound engineers.

This suggests then that there are two types of electronic analog equipment which might be used to determine power spectra: one type parallels the numerical process by first determining the autocorrelation function and then taking its transform to get power spectral density; the other omits the autocorrelation function entirely and measures the spectrum directly by means of a scanning electrical filter.

Since scanning filter types of analyzers are comparatively simple electronic devices and are commercially available, the NACA uses the direct spectrum measurement approach.

This type of analyzer is illustrated in Fig. 7 and operates in the following manner. The data sample stored on a continuous loop of magnetic tape is applied to the bandpass filter. Any frequency components in the data which fall within the filter's pass band will be passed by the filter, squared, averaged, and then applied to a direct-writing recorder. The filter is initially set at the low end of the frequency range and slowly scans upward through the spectrum until the entire frequency range of interest has been covered; at the same time the recorder paper is moving under the stylus so that a continuous plot of power spectral density against frequency is obtained.

The scanning speed of the analyzer is normally conservatively adjusted so that about three passes of the data sample on the loop are made during the time required for the filter to scan one filter bandwidth. Under these conditions, the time required for complete analysis of a typical record is 10 to 15 min. Faster scanning speeds may be used, but some "smearing" of the spectrum might result.

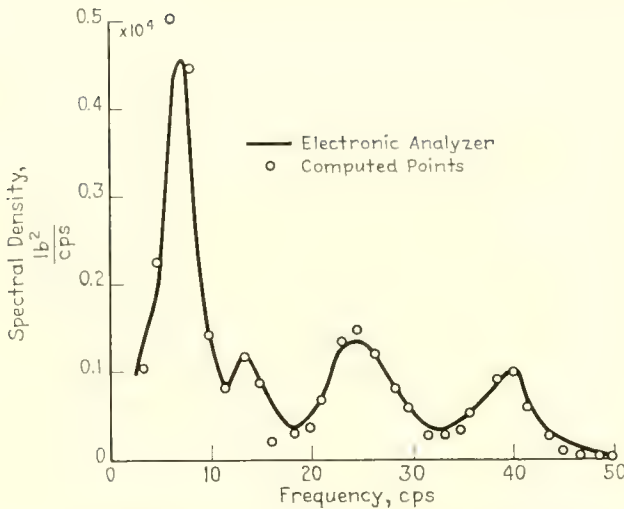


Fig. 8 Power spectral density of wing shear loads

The bandwidth of the scanning filter in the NACA's equipment can be adjusted to values ranging from 1/2 cycles per sec to 200 cycles per sec and the true-time range of frequencies which can be analyzed is from 3 cycles per sec to 15,000 cycles per sec.

By taking advantage of the possible changes in tape speed previously mentioned, it is possible to obtain equivalent filter bandwidths of 0.01 cycles per sec or less and to handle frequencies ranging from a few hundredths of a cycle to 50 or 60 kc. By changing tape speeds where necessary, it is also possible to handle record lengths ranging from a few tenths of a second to several minutes in duration.

To illustrate the use of the power spectrum analyzer, Fig. 8 shows the spectrum of the shear loads on a fighter-type airplane wing under buffeting conditions. The continuous curve was obtained from a magnetic tape record played through the analyzer and the circled points were obtained by reading the time-history record and numerically calculating the spectrum.

The results from the two methods differ by a maximum of about 10 per cent at the low-frequency end of the spectrum. This difference is due, in part at least, to a large, undesired, very-low-frequency component in the original data sample which had to be attenuated before the analysis. The electrical high-pass filter used to attenuate this component was not identical to the equivalent numerical high-pass filter used for the same purpose; therefore, some differences in the spectrum at the low-frequency end were to be expected.

Some difference between the analyzer and the numerical values also result from the fact that the shape and bandwidth of the analyzer's scanning filter were not identical to the equivalent filter resulting from the numerical process.

Actually, differences of 10 per cent are not partic-

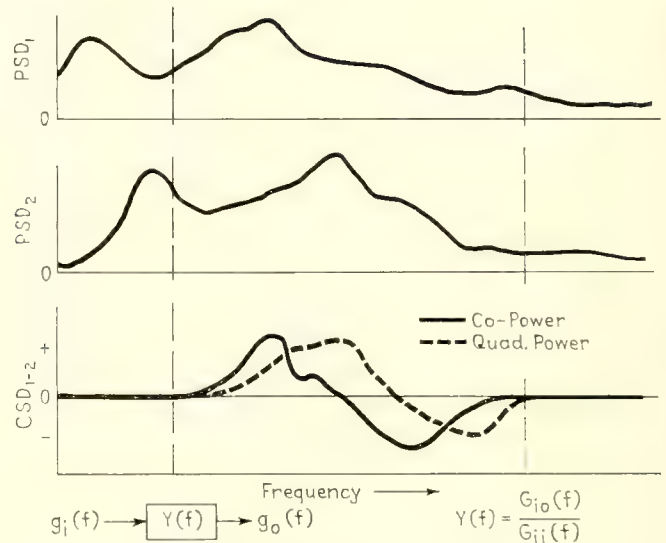


Fig. 9 Cross spectral density

ularly alarming. We are dealing with statistical processes such that, even with errorless data processing schemes, the results obtained from repeating the same test might easily vary as much as plus or minus 25 to 30 per cent. More will be said about this subject later.

Here, to read the record and determine the spectrum using an automatic digital computer required about 10 man-hours; the time required for setting up and running the electronic analyzer was less than 30 min.

Cross Spectral Density

It has been shown that the power spectral density furnishes information regarding the frequency content of fluctuating quantities which is not provided by the probability distribution. However, where the phenomenon being investigated involves study of two or more related fluctuating quantities, which is usually the case, some knowledge of the time or phase correlation between the two quantities might also be required. This information is not provided by the power spectra of the individual functions but may be obtained from another statistical process called cross spectra analysis.

The cross spectrum between two time functions is a vector quantity and two spectra are required to furnish the complete cross spectral density. These are illustrated in Fig. 9. The real part is called the "co-power spectrum" and indicates the product of the in-phase frequency components in the two functions. The imaginary part is called the "quadrature spectrum" and indicates the product of the 90° out-of-phase frequency components in the two functions. The absolute value and phase angle of the cross spectrum are determined by vectorially combining the in-phase and quadrature spectra.

To gain an understanding of the physical significance of cross spectra, consider two fluctuating time functions

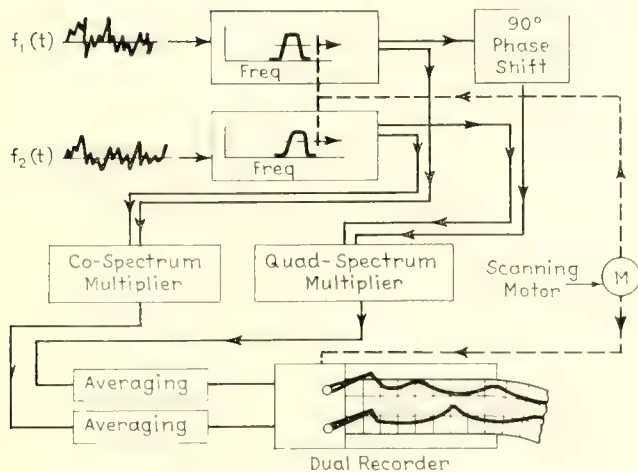


Fig. 10 Cross spectral density analyzer

whose power spectra are represented on Fig. 9 as PSD_1 and PSD_2 . The cross spectrum between the two functions will indicate only those frequencies in PSD_1 which are also contained in PSD_2 and which bear a specific, nonrandom phase relationship to the frequencies in PSD_1 . For example, PSD_1 and PSD_2 may include power in the same frequency band, but, if the corresponding frequency components are entirely independent of one another, the phase between them would be random and the cross spectral density would be zero throughout the band. However, if some of the frequency components in PSD_1 bear a definite phase or time relationship to the corresponding components in PSD_2 , this relationship will be indicated in the cross spectrum.

One useful application of the cross spectrum is in the determination of the phase response of a linear system subjected to a random-type input. Recall that the power spectra of the input and output made possible calculation of the absolute amplitude of a system's transfer function but did not furnish any knowledge of the phase response. By using cross spectra, both the amplitude and the phase response may be calculated. As shown by Lee [9] the transfer function $Y(f)$ equals the input-output cross spectrum $G_{io}(f)$ divided by the power spectrum of the input $G_{ii}(f)$.

An interesting feature of this relationship is the fact that the equation holds true even though other independent random noises are present in the output. This is true since the input-output cross spectrum will ignore the presence of uncorrelated random fluctuations in the output.

The cross spectra has other useful, practical applications to systems with multiple inputs and outputs, but these are too involved to discuss here. A good illustration of this usage, however, is included in Summers' paper on atmospheric turbulence measurements [6].

The numerical process for determining cross spectra is

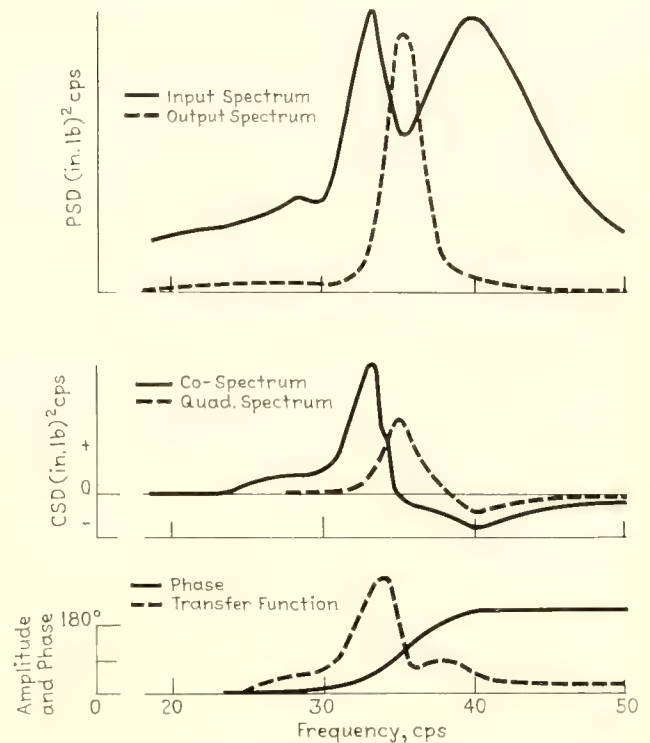


Fig. 11 Cross spectra of wing bending moment

even more lengthy and expensive than that required for determining power spectra. However, the numerical results can be duplicated by the analog process illustrated in Fig. 10. The two fluctuating data samples recorded on a continuous loop of dual-channel tape are simultaneously applied to two synchronized filters of the same type used with the power spectral density analyzer. However, instead of squaring and averaging the outputs of the filters as would be done to determine power spectra, the two outputs are fed into a multiplier whose output is averaged and automatically plotted against frequency to furnish the co-power spectral density. At the same time, one of the filter outputs is run through a 90° phase shifter and again multiplied by the output of the other filter. The product is then averaged and plotted to give the quadrature power spectral density.

To illustrate the use of the cross spectrum analyzer, Fig. 11 shows some data secured from a buffeting airfoil in a supersonic wind tunnel. The double-peaked spectrum is the power spectral density of the fluctuating aerodynamic forces existing at a particular spanwise station and the single-peaked spectrum is the power spectral density of the fluctuating bending moment existing at the root of the wing. The cross spectral densities between these two quantities as determined by the analyzer are shown in the center of the figure. The calculated amplitude and phase response shown at the bottom of the figure indicate that the wing is sim-

ilar to a spring-mass system with low damping. (This simple example does not provide aerodynamic data of any particular value but was chosen to demonstrate the use of the analyzer and the type of problem to which it is applicable.)

To set up and run these four spectra on the electronic analyzers required about 1½ hours. To read the time-history records and run the same four analyses digitally would require an estimated 40 to 50 man-hours—which explains why no numerically calculated check points are shown in the figure.

Accuracy and Reliability Considerations

Electronic analog analyzers might introduce errors in the order of plus or minus 10 per cent where a digital computer can process data to almost any degree of precision that might practically be desired. Consequently, a given data sample can be statistically analyzed by a digital computer more precisely than by an analog machine.

However, it must be remembered that we are dealing with statistical quantities. Consequently, if an experiment or test is repeated several times, sizeable variations in experimental results occur even if the instruments and analyzing processes introduce no error at all. For example, consider again the wing loads power spectrum, previously shown in Fig. 8, which is fairly typical of the type of problems encountered. The statistical reliability in this case was such that only one out of three tests would be expected to yield power estimates within plus or minus 10 per cent of the true value. If the length of the data sample could be increased by a factor of 20, however, 19 out of 20, instead of 1 out of 3, of the tests would be expected to yield results within plus or minus 10 per cent of the true value.

In analysis of actual data samples, then, consideration of the statistical aspects of the problem usually shows that improved reliability might be obtained, not by improving computational accuracy, but by analyzing longer data samples.

Longer samples are not always possible, especially in airplane or missile flights, where steady flight conditions often cannot be maintained for longer than a few seconds. Where longer samples can be secured, however, the electronic analyzers can economically handle samples of such length that analysis by any other process would be quite impractical.

In conclusion, the various statistical analyses required in aeronautical research and development can be secured rapidly and economically by the use of electronic analog analyzers. The accuracy of the analog

analyzers in processing a specific data sample is somewhat inferior to that available from digital computers. This factor is normally outweighed, however, by the analog equipment's capability for economically handling longer data samples and providing improved statistical reliability.

It appears that more extensive use of the analog analysis techniques might facilitate further advancement in the application of statistical methods to aerodynamic problems.

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