No. 14

DETAILS OF MACHINE TOOL DESIGN

SECOND EDITION

CONTENTS

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CHAPTER I

ELEMENTARY PRINCIPLES OF CONE PULLEYS AND BELTS*

Everyone knows that cone pulleys are usually made with regular steps; that is, if it is one inch from one step to the next, it is also one inch from the second to the third, etc., the reason being that when the centers of the shafts on which the cones run are a fair distance apart, the belt will pass very nearly half way around that part of each cone on which it is running, and the length of the belt will consequently be approximately equal to twice the distance between the shafts, added to half the circumference of the grade of one of the cones on which it is running, and half the circumference of the grade of the other cone on which it is running. As the steps are even, the half circumference of any two grades of each cone will, when added together, produce the same result. For example, if we had two cones, the diameters of the several grades of which were 6, 8, 10 and 12 inches, it is evident that the sum of half the diameters taken anywhere along the cones, as they would be set up for work, would in every case be the same. If the diameters are the same, it follows that the circumference must also be the same, and, of course, that half the circumference must be the same, so that when the centers of the shafts are a fair distance apart, and the difference between the largest and smallest step of the cone not too great, the same belt will run equally well anywhere on the cone, because it runs so near half way around each grade of the two cones on which it is running, that the slight difference is within the practical limit of the stretch of the belt.

But when the shafts are near together, and when the difference between the largest and smallest step of the cone is considerable, the belt is not elastic enough to make up this difference. Fig. 1 shows a three-step cone, the grades being 4, 18, and 32 inches diameter, respectively, there being a difference of 14 inches on the diameter for each successive grade, and the step being therefore 7 inches in each case. Of course, it is not likely that such a cone as this would be made for practical use, but it is well to go to extremes when looking for a principle. Now, it is evident that two cones, even if like the one shown in the cut, were set up far enough apart, they would still allow the belt to run very nearly half way around each grade of the two cones, the angularity of the belt would be slight, and the length of belt would therefore still be as mentioned above.

But (again taking an extreme case) by reference to Fig. 2, which is intended to represent a belt running from the largest grade of one cone to the smallest grade of the other cone, we see that the belt runs three quarters of the way around the large pulley, and only one quarter

* MACHINERY, April, 1895.
of the way around the small one, the distance between the shafts in this case being 19\%\% inches.

The length of this belt will evidently be equal to three quarters of the distance around the large pulley, plus one quarter the distance around the small pulley, plus the distances A and B, which we find to be each 14 inches. The circumference of a 32-inch diameter pulley is 100\% inches, and the circumference of a 4-inch diameter pulley is 12\%\% inches (near enough for our present purpose); three quarters of 100\% is 75\%, and one quarter of 12\%\% is 3\%\%; the length of a belt, then, to go around a 4-inch pulley and a 32-inch pulley, running at a distance of 19\%\% inches apart, is 75\% plus 3\% plus 14 plus 14; total, 106\%\% inches.

Now, let us take the middle cone, when the belt is running on two pulleys, both 18 inches diameter (see Fig. 3), and, of course, the same distance apart as before. The circumference of an 18-inch pulley is 56\%\% inches, and half the circumference of two 18-inch pulleys is evidently the same as the whole circumference of one 18-inch pulley; the length of belt in this case will then evidently be 56\%\% plus 19\%\% plus 19\%\%; total, 96 inches. It is therefore evident that a belt long enough to run on a 4- and 32-inch pulley, 19\%\% inches apart, is 10\%\% inches too long to run on two 18-inch pulleys 19\%\% inches apart, and, of course, it is therefore 10\%\% inches too long to run on the middle grades of such a cone as we have under consideration.

The thing to do, then, is to make the middle grades of these cones (or the two 18-inch pulleys) enough larger than 18 inches diameter to just take up this 10\%\% inches of belt, and if this were the only case we had to deal with, it would be very easy to settle it by saying that as half the circumference of two 18-inch pulleys is the same as the whole circumference of one 18-inch pulley, we should make the two 18-inch pulleys enough larger in diameter to make an additional circumference of 10\%\% inches; and as 3\% inches is nearly the diameter of
a 10½-inch circumference pulley, by making the middle of both cones 18 plus 3¾ inches diameter (that is, 21¾ inches diameter) our trouble would be ended in this particular case. It is easy enough to see, by looking at Fig. 2, that the belt being obliged to go three quarters of the way around the large pulley, is what makes it so much too long to go around the two middle pulleys, where, of course, it goes but half way around each. But, of course, what we want is some way of calculating the diameters to turn any pair of cones, running at any distance apart.

If we were to draw these same 32- and 4-inch pulleys twice 19¾ inches apart, and then three times 19¾ inches apart, and so on, until we got them far enough apart so that the belt would practically run half way around each, and should calculate the diameter of the middle grade of the cone to fit each distance, we would probably formulate a rule that would work for any distance apart, with this particular cone; but as it is evident that the further apart the cones are to run, the nearer to the nominal diameter of 18 inches must the middle of

the cones be turned, so also must it be evident that the less difference between the largest and smallest diameter of the cone, the less must also be the excess over nominal diameter of the middle of the cones.

Any method, then, of calculating such problems must take both of these things into consideration. The nominal diameter of the middle of any cone will be equal to half the sum of the diameters of the largest and smallest part respectively. This is almost self-evident, and no proof of it is necessary in this connection. What we want, then, is some way to find out how much larger than the nominal diameter to turn any one cone or cones to fit the conditions under which they are to run. The following formula is the result of a thorough investigation of this subject by Prof. Rankine, and has proved itself to be practically correct in the shop, as well as satisfactory to those mathematicians who are competent to criticise it. This formula is:

\[ R = \frac{R_1 + R_2}{2} + \frac{(R_1 - R_2)^2}{2\pi C}. \]

This formula translated into plain English means that the radius of the center of a cone will be equal to the radius of the smallest part, added to the radius of the largest part, and this sum divided by
2, and added to this the difference in radii between the largest and smallest part squared, and then divided by twice the center distance between the cones multiplied by 3.1416. That is, the first half of the formula gives the radius at the center of a cone, when the largest and smallest radii are known, and, of course, if the middle radius is equal to the smallest radius added to the largest radius and the sum divided by 2, it follows that the middle diameter is equal to half the sum of the diameters of the largest and smallest part, respectively, as mentioned before. The second part of the formula allows us to calculate how much larger than this nominal diameter to make the middle of a cone, no matter what the size or center distance.

Applying this formula to the case of the cones shown in Figs. 1, 2 and 3, we find the radius of the middle of the cone to be 10 6/10 inches, or, what is the same thing, the diameter to be 21 2/10 inches, which, in view of the extreme case under consideration, is very near the first result obtained (21 3/4), and shows that the formula is perfectly safe in any case likely to occur in practice.

When this formula is reduced so as to express the numerical value of diameters instead of radii, it takes the following form:

\[
\text{Diameter at center of cone} = \frac{D + d}{2} + \frac{(D - d)^2}{12\frac{1}{2}C},
\]

the 12\(\frac{1}{2}\) being the nearest value in plain and easy figures to which the quantity containing \(\pi\) in the original formula can be reduced.

Applying this simplified formula to the cone which we have been considering, it will be found that the middle diameter is 21 2/10, the same as by the original Rankine formula.

If a cone has five steps instead of three, it will be practically correct to add half as much to the nominal diameters of the second and fourth grades as was added to the middle grade, or, if it has four grades, add two-thirds of what is found by the calculation to the second and third grades (as there is evidently no middle grade). If more than four or five grades, add to each grade according to the same principle.

We have so far been considering two similar cones, but it often happens that one cone is larger than the other. In such case the problem becomes a little longer to work, and the length of belt necessary to go around each pair of steps of the cones must be used to find the diameters; that is, starting with one end of the cone, find the length of belt, and then calculate how much larger or smaller (as the case may be) than the nominal diameter it is necessary to make each grade, in order to make the same length of belt run properly.

Prof. Rankine has worked out a formula for the length of belt also, which, reduced to diameters, is as follows:

\[
\text{Length of belt} = 2C + \frac{11D + 11d}{7} + \frac{(D - d)^2}{4C}.
\]

That is, the length of a belt to pass around any two pulleys (and, of course, a cone is simply a set of pulleys) is the sum of the following quantities: First, twice the center distance of the shafts; second, 11 times the diameter of the larger pulley, plus 11 times the diameter
of the smaller pulley, and this sum divided by 7. This gives the nominal length of belt, or what would be practically correct if the center distance was fairly great; for the excess, the last part of the formula must be used, which is the difference between the diameters of the larger and smaller pulleys squared, and this result divided by 4 times the center distance.

Having found the length of belt to run on one end of the cones, and keeping this for a starter, we can easily find how much to add to, or take from, the nominal diameter of any other part of the cone to make the same belt run, as explained before. If, for instance, we find that the nominal diameters of the next grades that we try to bring the length of belt one-half inch shorter than the first calculation, we add enough to one or both diameters to make up one-half inch of circumference, which would be about 5/32 of diameter, and this could all be added to one pulley, or half of it could be added to each pulley, as convenient, and this would be practically correct.

CHAPTER II

CON€ PULLEY RADII*

In the present chapter a method presented by Dr. L. Burmester in his "Lehrbuch der Kinematik," for the solution of the cone pulley problem, has been extensively treated. Dr. Burmester's method is entirely graphical, and is exceedingly simple in application. While it is not theoretically exact, it is, as will be shown later, much more accurate than practice requires.

In order to bring out more clearly the points which will come up in the case of open belts, let us first consider the simple case of crossed belts. It is a well-known fact that in this case the only calculation necessary in order to find the radii of the various steps is to make the sum of the radii of any two corresponding steps a constant. This may be shown in the following manner:

\[ a = \text{radius of step, driving cone.} \]
\[ A = \text{length of belt from contact on driving cone to contact on driven cone.} \]
\[ b = \text{radius of step, driven cone.} \]
\[ E = \text{distance between centers of cones.} \]
\[ K = \text{the constant sum of the radii of two corresponding steps.} \]
\[ \theta = \text{angle shown, Figs. 4 and 5.} \]

Then

\[
\sin \theta, = \frac{a + b}{E} = \frac{K}{E}
\]
\[
\sin \theta, = \frac{a + b}{E} = \frac{K}{E}
\]

---

* MACHINERY, September, 1905.
Therefore $\theta_1 = \theta_2 = \theta = \text{a constant.}$

Therefore the arc of contact on each pulley = $180^\circ + 2\theta = \text{a constant.}$

Also $\cot \theta = \frac{A_1}{a_1 + b_1} = \frac{A_1}{K} = \frac{A_2}{a_2 + b_2} = \frac{A_2}{K}$

$A_1 : A_2 = K \cot \theta = \text{a constant.}$

But length of belt $= 2 A + \frac{180^\circ + 2\theta}{360^\circ} \times (2\pi a_1 + 2\pi b_1)$

$= 2 A + \frac{180^\circ + 2\theta}{360^\circ} \times 2\pi \times (a_1 + b_1)$

in which, as has been shown above, all the terms are constants, therefore length of belt is constant.

Note: The subscript applied to the letters denotes that the letters are used for the corresponding quantities in a special case; thus $a_1$, in Fig. 4, refers to $a$.

The radii of the various steps may be determined graphically by the following diagram (Fig. 6):

Draw a horizontal line from $A$, and also draw $AC$ making an angle of 45 degrees with it. On this line lay off $AS$ equal to the distance between the cone centers, using any scale most convenient, bearing in mind, however, that the scale adopted now must be used consistently throughout the diagram. At $S$ erect the perpendicular $TST'$ to the line $ASC$. From some convenient point on $AC$, as $D$, drop a vertical equal to some known radius of the cone $a$, as $DE$, and then
from $E$ measure back on this vertical the radius of the corresponding step on cone $b$, as $EF$, and from these points $E$ and $F$ draw lines parallel to $ASC$. From the point $G$, where the line $FG$ intersects the line $TST'$ drop a vertical. This will intersect the line $EH$ in $H$. Through $H$ draw the horizontal $MN$, $O$ being the point where this line intersects the line $TST'$. Then, distances on the line $MO$ may be taken to represent radii on cone $a$; and to find the corresponding radii on cone $b$ erect perpendiculars at the extremities of these radii, producing them until they intersect the line $TST'$. These perpendiculars then represent the desired radii. It may be shown as follows that the sum of the two corresponding radii, as obtained from this diagram, is always a constant, and the diagram therefore satisfies the conditions for crossed belts.

Let $MJ$ represent any radius on cone $a$, then $JI$ represents the corresponding radius on cone $b$.

The $\angle JIO = \angle JOI = 45$ degrees.
Therefore $JI = JO$.
Therefore $MJ + JI = MJ + JO = MO = a$ constant.

Dr. Burmester's diagram for open belts is a modification of the diagram just shown, the only difference being that the line $TST'$ is replaced by a curve. This curve was determined by plotting a series of points, and after several pages of exceedingly intricate mathematics he arrives at the astonishing result that this curve can be replaced by a simple circular arc without any appreciable error.

The diagram is shown in Fig. 7, and may be drawn as follows: Proceed as in Fig. 6 until the line $TST'$ is drawn, then lay off distance $SK$ equal to $\frac{1}{2} AS$. Next, with the center at $A$, and a radius equal to $AK$, describe the arc $XY$, and the diagram is ready for use.

In order to give an idea of the extreme accuracy of the diagram, let us observe the values obtained by Dr. Burmester in his calculations.
Let $R = AK$ (Fig. 7).

When $\theta = 0$, $R = E \times 1.11815172$.
$\theta = 15^\circ$, $R = E \times 1.11806542$.
$\theta = 30^\circ$, $R = E \times 1.11798671$.
$\theta = 45^\circ$, $R = E \times 1.11803397$.

The value used for $R$ in the diagram is $E \times 1.11803397$; so the maximum error of $R$ occurs when $\theta = 0$, and is equal to $E \times 0.00011775$.

which is much more accurate than the work of the most careful draftsman. Dr. Burmester gives values of $R$ up to $\theta = 90^\circ$, but as it is evident from Fig. 8 that it would be practically impossible to have a value of $\theta$ greater than $45^\circ$, the writer has omitted the other values.

In order to make the use of the diagram perfectly clear, let us solve the following problems:

**Problem 1.** Fig. 9

**Given:**

Distance between centers of cones $= 3' 4''$. 

![Fig. 7](image-url)

![Fig. 8](image-url)
CONE PULLEY RADII

Diameters of driving cone, 4", 8", 14", 20".
Diameters of driven cone, X, X, 14", X.

Required:
All diameters of driven cone.

Lay out the diagram and determine the point M as previously directed. Now the radii of driving cone may be laid off as abscissas or ordinates, whichever happens to be the more convenient, as the results obtained will be exactly the same in either case. In this particular problem it is evidently more convenient to lay them off as abscissas. Then the ordinates erected at the ends of these abscissas will represent the corresponding radii of the driven cone. The problem is solved in Fig. 9 and the following results obtained:

Results:
Diameters of driven cone, 221/2", 193/8", 14", and 71/4".

This problem does not bring out all of the fine points of the diagram, so let us solve a more complicated one, in which the different steps of
the cone are to transmit given velocities.

Problem 2. Fig. 10

Given:
Distance between centers of cones = 3' 4".
Maximum velocity of belt (assumed) 30 feet per second.
R. P. M. of driving cone = 240.

Required:
Driven cone to make 100, 240, 400 and 580 R. P. M.
The maximum belt speed will be attained when the belt is on the largest step of the driving cone.

Therefore

\[ \frac{2 \pi a_1 \times 240}{12 \times 60} = 30; \quad 2a_1 = 28\frac{5}{8}" \]

But \[ \frac{2b_1}{28\frac{5}{8}} = \frac{240}{580}; \quad 2b_1 = 11\frac{7}{8}". \]

Now having obtained a value for \( a_1 \) and \( b_1 \), the point \( M \) on the diagram may be found. Next draw a line from \( M \) as \( MO \), inclined so that any horizontal projection, as \( MN \), will be to the corresponding vertical projection, \( NO \), as the R. P. M. of the driver are to the R. P. M. of the driven; thus,

\[ \frac{MN}{NO} = \frac{R. \ P. \ M. \ of \ driving \ cone}{R. \ P. \ M. \ of \ driven \ cone}. \]

Also from similar triangles

\[ \frac{MN}{NO} = \frac{MN'}{N'O'}. \]

But we know that

\[ \frac{R. \ P. \ M. \ of \ driving \ cone}{R. \ P. \ M. \ of \ driven \ cone} = \frac{\text{rad. of driven cone}}{\text{rad. of driving cone}}. \]

Therefore \( MN' \) equals radius of driven cone, while \( N'O' \) equals radius of driving cone, thus making, for this case, radii of driving cone vertical and of driven cone horizontal. The problem is solved in Fig. 10 and the following results obtained:

Results:

Dia. or driving cone, 28\( \frac{5}{8} \)", 25\( \frac{5}{8} \)", 20\( \frac{7}{8} \)", 11\( \frac{7}{8} \)".
Dia. of driven cone, 11\( \frac{7}{8} \)", 15\( \frac{5}{8} \)", 20\( \frac{7}{8} \)", 28\( \frac{7}{8} \)".

We have seen that the Burmester diagram is under all conditions much more exact than is required in practice; and a more compact, simpler, or quicker method of finding cone pulley radii could not be desired. An experienced draftsman should be able to solve a problem like No. 2 above in less than 10 minutes, while to obtain the same results by an analytical method would require as many hours. Results of sufficient accuracy can usually be obtained by making the diagram to half scale, although there is no reason for reducing the scale, unless the distance between centers is inconveniently large, and in that case
the results do not need to be so accurate, as the belt will stand more stretching.

Another graphical method for laying out a pair of cone pulleys is as follows: First draw straight line \( AA \), Fig. 11, supposed to connect the centers of the cones to be laid out; then set off the centers of the cones \( B \) and \( C \) on line \( AA \) (full size is best); then bisect the distance between the centers of the cones and draw perpendicular line \( DE \). Now assume the size of the two cones—say the largest is 25 inches and the smallest 3 inches diameter. Then draw a line tangent to the circles, or the line representing the inside of the belt \( G \), which will intersect the line \( DE \) at \( E \), and taking the point \( E \) for a center scribe the circle \( F \). Then divide the circle \( F \), commencing at the line of the belt \( G \), into as many parts as needed, of a length to suit the required speeds.

Fig. 10. Solution of Cone Pulley Problem when Velocity Ratios, Maximum Belt Speed, Center Distance, and R. P. M. of Driver are Known

\[
\frac{MN}{NO} = \frac{240}{100} \\
\frac{MN}{NP} = \frac{240}{240} \\
\frac{MN}{NQ} = \frac{240}{400} \\
\frac{MN}{NR} = \frac{240}{580}
\]
Draw the other radiating belt lines through the point $E$ and the divisions on the circle $F$, extending them toward the cone $B$, and they will be the inside of the other belt lines. Draw circles tangent to these lines. We now have all the diameters of the rest of the steps of the cone to match the first, and the belts will correctly fit all the steps. This is, of course, only an approximation rule. This method was contributed to the June, 1905, issue of MACHINERY by John Swanberg.
CHAPTER III

STRENGTH OF COUNTERSHAFTS*

There is scarcely a shop in existence which has not had a more or less serious accident from a countershaft some time in its history. It may have been caused by a heavy pulley running very much out of balance, or the shaft may have been bent in the beginning. Possibly the shaft was too light, or too long between hangers. The latter is responsible for most of the trouble, and is the one with which this discussion is principally concerned.

There are two methods in vogue for turning cones and pulleys; one is to set the rough casting to run true on the inside, and the other on the outside. This latter method makes a cheaper and an easier job, but when turned, it requires an enormous amount of metal to balance it. And here is the source of considerable trouble. We may balance a large cone perfectly on straight edges, but that is a standing balance only; and when the cone is put in place and speeded up to several hundred revolutions per minute, it shakes, and shows that it is decidedly out of balance. The trouble is that we have not placed the balance weights directly opposite, or in the plane of the heavy portion of the cone. The result is that neither weight, when rotating, has its counterbalance pulling in the same line, and, of course, the pulley is sure to be out of balance. All cones and all other pulleys which have a wide face should be set to run true on the inside before turning.

A certain countershaft failed because it had been welded near the center. The weld twisted and bent open, and some one was badly injured by the fall. A weld in machine steel is so very uncertain that it should never be trusted for such a purpose. The extra expense of a new shaft would not warrant the hazard of such a risk.

In the calculations which follow, the spring of the shaft is limited to 0.06 of an inch. There are plenty of countershafts which have been running for years with about this much spring. Now, from the general formula for the deflection of a simple beam, we have:

$$ \frac{W L^3}{48 E I} $$

in which $W =$ the load at the center in pounds.
$L =$ the length between center of hangers in inches.
$E =$ the coefficient of elasticity $= 29,000,000$.
$I =$ the moment of inertia of the cross-section of the shaft.

For a round shaft,

$$ I = \frac{\pi d^4}{64} $$

(1)

*Machinery, April, 1903.
in which \( d \) = the diameter of the shaft in inches. We then have:

\[
\frac{W L^3}{48 E I} = 0.06 \tag{2}
\]

From (1) and (2), we have

\[
\frac{WL^3}{64 EI} = 0.06, \quad \text{and} \quad 48 E \pi d^4
\]

\[
L = \sqrt{\frac{0.06 \times 48 \pi d^4}{64 W}} = \sqrt{\frac{0.06 \times 48 \times 29,000,000 \times \pi}{64 W}} \times \frac{d^4}{W}
\]

\[
= \sqrt{\frac{4,100,000}{W}} \tag{3}
\]

Fig. 12 shows a countershaft which is in actual service, and which is known to be all right. \( A \) and \( B \) are keyed to the shaft. \( C \) and \( D \) are loose pulleys arranged for open and cross belts.

\( A \) weighs 30 pounds, and \( B, C \) and \( D \) weigh 110 pounds. The belts run as shown in the figure. If \( A \) weighs 30 pounds, and the centers of the hangers are 54 inches apart, then by taking the left-hand hanger as the center of moment, we have

\[
30 \times 12 = x \times 27, \quad \text{when} \quad x \text{ is the weight at the center.}
\]

Solving we find

\[
x = \frac{30 \times 12}{27} = 13
\]

In the same way, by taking the right-hand hanger for the center moment, we find that

\[
x_2 = \frac{110 \times 18}{27} = 73
\]

As to the belt pull, it is possible for a single belt to run up to 70 pounds per inch of width of belt, and a double belt can be taken at 100 pounds. As a double belt is used in this case, and as the slack side of the belt is very loose when the tight side is pulling its maximum, we will take the pull at the pulley \( A = 6 \times 100 = 600 \) pounds, and getting this in terms of a load at the center, we have

\[
x_3 = \frac{600 \times 12}{27} = 266
\]
and the downward pull is $13 + 73 + 266 = 352$.

The pull at the pulley $B$ will be $6 \times 100 = 600$, and by transferring this to the center we have

$$\frac{600 \times 18}{27} = 400$$

The resultant of these two forces will be the diagonal of the force diagram, and is equal to 530 pounds, which is equal to $W$ in the formula. Introducing these terms in equation (3) we have

$$L = \sqrt[3]{\frac{4,100,000 \times (2.44)^4}{530}}$$

and by solving we find $L = 65$, which means that for this condition of loading the countershaft would be safe even with the hangers 65 inches apart.

Introducing this value of $W$ in equation (3) we have,

$$L = \sqrt[3]{\frac{4,100,000 \times (1.75)^4}{189}} = 59$$

This is considerably less than the distance between the hangers, and it shows that it is not safe to place the hangers in this way. If the
belts ran as shown at the right-hand side of Fig. 13, we would then have:

- Weight due to pulleys (as before) = 148
- Pull on 30-inch pulley = 250
- Pull on 15-inch pulley = 208
- Total downward pull = 606
- Horizontal pull on 14-inch pulley = 417

From these two forces we find a resultant of \( W = 736 \). Substituting this in (3) and solving as before, we find \( L = 38 \), which is the greatest safe distance between hangers for this condition of loading.

There are cases where one must have an extra long shaft in order to work in the pulleys, cones, etc., as shown in Fig. 14. Here the downward loads amount to 820 pounds, and the pull at right angles amounts to 360 pounds. The resultant 895 pounds = \( W \).

![Diagram showing belt setup and calculations](image)

Introducing in the formula we have

\[
L = \sqrt[3]{\frac{4,100,000 \cdot (2.44)^4}{895}} = 55
\]

This means that for this condition of loading, the center distance should not exceed 55 inches, and since in this case it could not be made as small as this, the pulleys should be arranged for a third hanger.

In every case, therefore, where the centers are so far apart as formula (3) would indicate to be unsafe, a third hanger should be used. If all the flimsy countershafts had a third hanger added to them there is no doubt but that the number of accidents would be greatly diminished. In the above calculation the weight of the countershaft has not been considered, as it is usually very small. If the belts run at any other angle than that shown, the construction is made in exactly the same way, using the required angle instead of a right angle, the resultant of the two forces being used as \( W \) in the formula.
CHAPTER IV

TUMBLER GEAR DESIGN*

Of the different mechanisms that have been used in the machine tools of the past, one—the tumbler gear—could be found in some form or other in almost every machine. Its office, in most cases, was to reverse the direction of the feed. Fig. 15 shows the usual form in which it is found when used for this purpose. The gears $A$ and $B$ are to be connected so that motion may be transmitted from one, which runs constantly in one direction, to the other, which it is desired to run in either direction. Suppose that $A$ is the driver and runs as shown by the arrow. As connected, $A$ drives $B$ through the intermediate gears $D$ and $C$, $B$ rotating in an opposite direction to $A$, as shown by the arrows.

This mechanism is termed the tumbler gear, because the gears $D$ and $C$ are supported in a frame which swings about the axis of either the driving or the driven gear. In the case in hand, the intermediate gears are carried in the frame $E$, which rotates about the axis of the gear $B$. Some means, not shown, must be provided by which the rocker frame may be changed from one position to the other, and locked. Fig. 16 shows the mechanism shifted so that the motions of $A$ and $B$ are in the same direction.

The tumbler gear has been used as a reversing gear ever since present forms of machine tools were first invented. While it has always

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given considerable trouble, it has shown up to disadvantage mostly when applied to the modern machine with positive gear feed, where great power has to be transmitted by it. It is the purpose here to show where this gear may be used to advantage, and also to explain the theory on which the principles of its design are based.

All of us have met with this mechanism in some form or other, and may have formed an unfavorable opinion. The prejudice thus created keeps us from fully appreciating the tumbler gear, even when properly designed, and when used in the right place. It has been placed by many along with the worm drive and the spiral gear as undesirable, and to be avoided unless it is absolutely impossible to get along without it. This opinion has been responsible for the adoption of many combinations used for purposes that rightly belong in the field of the tumbler gear, and many times, in order to avoid using this mechanism, much unnecessary complication has resulted.

What are the faults of the tumbler reversing gear? That one on So-and-So's lathe used to kick furiously when one tried to throw it over. Then, the one used on the milling machine used to go into mesh easily enough, but when any amount of strain was put onto it, the teeth used to crack and growl, showing that the tendency was to drag the gear farther into mesh, causing the teeth to bind on one another and sometimes break. Let us look into the case represented in Fig. 15. Fig. 17 shows the gear \( D \) just entering into mesh with \( A \). An examination of this figure shows that the tendency is for the teeth of gear \( A \), when they strike those of gear \( D \), to cause the latter to rotate about the axis of the rocker frame, should the gear \( B \) be locked against turning. This tendency opposes the motion in the opposite direction necessary to bring the gears wholly into mesh. In practice, \( B \) is not locked, but it is necessary to overcome a certain amount of resistance in order that it may be set in motion, and the presence of this resistance has the same effect as if the gear were locked. The greater this resistance is, the greater is the effort necessary to bring the gears into working position.

Examining the conditions in the case of Fig. 16, we see that the effect would be just the opposite, that is, the gears would come into mesh of their own accord as soon as a contact is produced between the teeth of \( A \) and \( C \). Practically no effort is necessary to bring the gears into mesh, but, in order to withdraw the gear \( C \) from \( A \), considerable effort would be required. When the gears \( C \) and \( A \) are in mesh and transmit power, the tendency for gear \( C \) is to crowd farther into mesh with \( A \), which has the effect of binding the teeth. Should the pressure of contact be sufficient, the binding tendency would cause the motion to cease, or would break the teeth. This is one of the points on which many have based their verdict against the tumbler gear, and when designed so that such results are obtained, it is not to be wondered at.

Direction of Tooth Pressure in Ordinary Cut Gears

The first consideration in the design of tumbler gears in any form is that of tooth pressure and its line of application. As all cut gears
used in machine tools are made to the 14½-degree involute system, we will confine ourselves to that system. In this, the force tending to revolve the driven gear is not a tangential force, applied as a tangent to the pitch circle, but is a force applied at an angle of 14½ degrees to the tangent of the pitch circle, this 14½-degree line being termed the line of pressure. In case that there may be some confusion as to the above statement regarding the tangential force and the line of pressure on the teeth, the case is graphically shown in Fig. 18. The tangential force is equal to the twisting moment divided by the radius of the pitch circle. This force is equivalent to that which transmits motion between two disks by friction alone, the diameters of the disks being equal to the pitch circles of the gears. This force is, in the case of a gear, resolved into two component forces. One component acts perpendicular to the tangential force and tends to force the gears apart; the other acts in the direction of the line of tooth pressure shown in Fig. 18. The tooth pressure thus is somewhat less than the total twisting force, and equals the twisting or tangential force multiplied by the cosine of 14½ degrees.

Influence of Direction of Tooth Pressure on Tumbler Gear Design
To show what effect the line of pressure has upon the layout of the tumbler gear, we will use the simple case shown in Fig. 19. In this figure, A is the driving gear and B is the driven gear. These gears are connected by means of the intermediate gear C, which is carried in the swing frame E, which, in turn, swings about the axis of A. This mechanism is a simple case of tumbler gear, and while it is little used, it is useful as a means for disconnecting a train of gears when it is desired to stop the motion of the driven section. If we consider gear B locked in the position shown, and exerting a turning effort on the gear A in the direction indicated by the arrow, this effort is transmitted by the teeth of A and C, and a pressure is produced...
between the teeth of B and C, two of which are shown in the cut. The direction in which this force is applied is shown by the line of pressure HK, and is is exerted in the direction of H. Since every force is opposed by an equal and opposite force when in a state of equilibrium, we have in this instance a force or reaction opposing the force along the line of pressure referred to. It is this reaction that causes our troubles. In the mechanism shown in Fig. 19, the gear C and the link E are free to rotate about the axis of A, and since the line of pressure does not go through the center of gear A, the force acting along this line tends to rotate the arm E about the axis of A, the direction of rotation being dependent on which side of the center of A the line falls. Thus in Fig. 19, the line falls in a position that produces a tendency for the arm to force the gear C further into mesh with B. The twisting moment thus set up is equal to the tooth pressure multiplied by the normal distance from the axis of A, or GL.

Fig. 19. Objectionable Tumbler Gear Design

If now, instead of trying to turn the gear A in the direction of the arrow, we exert a torque in the other direction, the opposite sides of the teeth would come into contact, and the line of pressure would be located as shown by the dotted line H'K'. The normal distance of this line from the axis of A is much greater than in the former case; consequently the twisting moment tending to rotate the arm E about the axis is also increased, but the direction in which the torque is applied has changed the direction in which the reacting force along the line of pressure acts, and, since this line falls on the same side of the axis, the tendency of the arm is to rotate in the opposite direction, and to separate the gears C and B. Had the line of pressure gone directly through the axis of the gear A, where E is pivoted, the effect of any force acting along it would have had no rotating influence upon the tumbler gear arm. That this would be the ideal case needs not to be mentioned, and it should be the aim of the designer to approach that condition as nearly as possible.
The tendency for the tumbler to crowd the gears into mesh might be of some advantage were it desirable to throw them into mesh while the gears are in motion; but in cases where any considerable amount of power is being transmitted, a very stiff and rigid design will be necessary for the tumbler frame and the locking device. It is also well in such cases, when setting the locking device, to have the gears mesh with plenty of play or backlash, so that, if there be any spring in the frame, the gears will not be likely to bind and cramp. Should B be the driver and run in the direction of the arrow, the line of pressure would be \( H'K' \), and the pressure would be in the direction of \( H' \). The arm would then tend to carry the gear C out of mesh with B.

**Fig. 20. Correct Design of Tumbler Gear to run in Both Directions**

Should the direction be reversed, \( HK \) would be the line of pressure, and the tendency would be to crowd the gear in.

The layout in Fig. 19 has two bad features. In the first place, the gears have a tendency to crowd farther into mesh, which limits the amount of power that can be transmitted, and increases the liability of breakage of the gear teeth and of the tumbler frame, should an overload be imposed upon the mechanism. Inaccuracy in the shape and spacing of the teeth aggravate the above conditions. In the second place, the mechanism should be used to transmit motion in but one direction.

In most cases the throwing in or out of the tumbler is a secondary matter, as it is done either when the gears are not in motion, or while not under load, if running. In such cases it should be the aim of the designer to overcome the objection of the crowding of the teeth into
mesh by having the line of pressure properly located, so that the tendency is in the opposite direction. When it is desired to provide a tumbler gear that can be run in either direction, the layout in Fig. 20 is recommended. The object in this case is to have the twisting moment equal in either direction, and such that the gears have no crowding tendency. The arrangement in Fig. 20 is laid out as follows: Draw the pitch circles of the gears B and C and connect their centers by the line DF. Through the pitch point O draw a line GH normal to DF. Then locate the gear A at some point on GH so that its pitch circle will be tangent with that of C.

The Single Tumbler Gear

The single tumbler gear is the basis of many of our modern rapid change speed and feed mechanisms, and the principles treated above apply to this as well as to the regular tumbler gear. Take the simple case shown in Fig. 21, which shows the pitch circles of a four-gear cone and the driver A and tumbler gear C. It is evident that only one position of the gear C can be such that the ideal condition prevails, that is, only when in mesh with one gear of the cone can the line of pressure pass through the axis of the tumbler frame. Fig. 21 shows this to be the case when C is in mesh with the gear B'. Each subsequent shifting of the tumbler along the cone brings the line of pressure eccentric to the axis, until the position of extreme eccentricity is reached when C is in mesh with B'''. In mechanisms of this kind, it should always be the aim of the designer to have the line of pressure pass as close to the center of rotation of the tumbler frame as is possible, because the locking devices used with this type of tumbler gear are necessarily of such a design as to be quick in action, and in consequence are not very stiff or rigid. The line of pressure should always be made to fall on that side of the axis where the tendency is to separate the gears rather than to bring them closer together. When the gear C is supported in a swinging frame which does not
slide in a lateral direction, but the changes are made by shifting \( C \) along an intermediate shaft, the supporting member should be located at the end where the line of pressure has the greatest eccentricity, as the greatest strain comes at that end. Thus, in Fig. 21 the support should be at the same end as \( B''' \). The diameter of the intermediate gear \( C \) has an important effect on the location of the line of pressure. It will be found that it should in most cases be as large as \( B''' \), in order that the line of pressure may come right. However, no exact rule can be given by which the diameter of \( C \) can be calculated, as it depends greatly on the difference in the diameters of \( B' \) and \( B''' \), and also on the diameter of \( A \).

**Rules for the Design of Tumbler Gears**

What direct rule can be given that may be used as a guide in laying out the tumbler gear? Referring to Fig. 19, we see that the gear \( C \) is revolving in a direction away from the axis of the tumbler at the point of tangency of the pitch circles of \( C \) and \( B \), and that the reacting force tends to crowd the gears farther into mesh. Had this line of pressure fallen on the other side of the axis of the tumbler frame, the tendency would have been opposite in effect. When the gear \( C \) is revolving so that a point on the pitch circle travels away from the pivot of the tumbler, and the line of pressure falls somewhere between the pivot point and the axis of the driven gear \( B \), the tendency will be to crowd. From this we therefore may formulate the following rules:

**Rule I.** When the gear about which the tumbler gear swings is the driver, and the line of pressure falls between the axis of that gear and that of the driven gear, the motion of a point on the pitch circle of the tumbler or intermediate gear, when near the contact point, must be toward the axis of the tumbler frame. Should the direction of a point on the pitch circle be opposite, the line of pressure must fall outside of that area included between the axis of the pivot gear and the driven gear.

Referring again to Fig. 19, it is seen that should the driving gear be \( B \), the above rule does not apply, but may be altered to read thus:

**Rule II.** When the gear about which the tumbler gear swings is the driven gear, and the line of pressure falls in the space between the axis of this gear and that of the driving gear, the motion of a point on the pitch circle of the intermediate gear at the contact point must be away from the axis of the pivot gear; when the line of pressure falls outside of this space, this motion must be reversed.

By following these two rules, more as a precaution than as a compulsory condition, much better success may be expected in the results obtained.
CHAPTER V

FAULTS OF IRON CASTINGS*

POINTS FOR THE MACHINE TOOL DESIGNER

The most useful and indispensable of all the materials with which the designer has to do, is cast iron. Of all the metals used in the construction of machinery, it is the cheapest. It is the one to which we can the most readily give the form and proportions which we desire. It is, of all the common materials, the most easy to machine. While it is lacking in strength and ductility, its cheapness makes it possible to use it in such ample quantity as to overcome these disadvantages, and in many constructions it may be so shaped and proportioned, or so reinforced by other materials, as to make this lack but a slight detriment. It is therefore a matter of interest to the designer to learn of the various faults to which this valuable material is subject, and the best ways in which they can be avoided or minimized.

Causes of Blow-holes

Probably the one fault which spoils more castings than any other, is the result of an outrush of gas from the materials of the cores or the mold, into the molten iron, at the instant of solidification. If the solidification of the iron has proceeded so far that the outrushing gas or steam cannot bubble through it, and escape through the vents which should be provided for the purpose, it will be imprisoned in the substance of the casting, forming one or more holes, according to the special shape of the casting, and the quantity of the escaping gas. These holes, which are known as blow-holes, may not be apparent on the outside, and quite often occur in such a location that they do no particular harm, but it is more often the case that they occur at some point where they become apparent when the metal is being cleaned, or where their presence weakens the casting greatly.

Steam from Moisture in Sand

The gases which cause blow-holes may come from three sources. They may be, and generally are, caused by the generation of quantities of steam from the moisture contained in the molding sand, by the heat of the iron. In the case of dry sand and loam castings, the quantity of steam so generated is so insignificant, if the molds have been properly heated, that it gives no trouble whatever. In the case of green sand castings, however, the moisture present, and therefore the steam generated, is quite large in amount, and special precautions have to be taken to prevent blow-holes.

When the molten iron pours into a green sand mold, all the moisture in the layer of sand immediately in contact with the iron will at once be transformed into steam. The depth of the sand layer so affected

* MACHINERY, October and November, 1907.
depends on the thickness and extent of the fiery mass to which it is adjacent. The steam so formed must either force its way through the molten iron in the form of a mass of bubbles, or else it must escape through the sand. To facilitate its escape, the mold is vented. That is, after the damp sand has been packed around the wooden pattern by ramming it closely into place, a wire is thrust repeatedly into the mold, making numerous passages for the escape of the steam and gas.

It is obviously impossible that one of these vent-holes should extend to every point in the layer of sand adjacent to the casting, so it is necessary that most of the steam and gas should force its way for some small distance through the sand, before it can reach a vent-hole. This it can only do when the sand is somewhat porous. If the sand is too tightly rammed, it will lack the necessary porosity, and even though it be unusually dry, and the venting carefully done, the casting will be full of blow-holes. Cases have occurred where molds have been rammed so hard that the castings were nothing better than shells, the whole interior being a mass of blow-holes.

Decomposition of Binder in Cores, and Entrapping of Air

The second cause of blow-holes in iron castings is the decomposition of the material, generally flour or molasses, used as a binder in preparing the cores, and its escape in the form of gas, into the iron, at the instant of pouring. It is impossible to prepare and bake a core in such a way that it will not give off large quantities of gas when the iron is poured, and so means must be provided for the escape of this gas. In order to do this, the cores are prepared with wax strips running through them. When the core is baked, the wax melts, leaving passages, known as core vents, for the escape of these gases. If the core is of such form, and so set in the mold, that the gases can escape from these vents in an upward or sidewise direction, and leave the mold without forcing their way through the molten iron, no blow-holes will result.

A third source of blow-holes is the entrapping of air in certain parts of the mold, and its mixing, on expansion, with the iron. This trouble is due to insufficient venting of the mold, and is not a fault to which the designer need pay any particular attention.

Dry Sand or Loam Advisable for Large Complicated Castings

In the case of large and complicated castings, it is generally advisable to make dry sand or loam castings, in order to avoid, as far as possible the chance of blow-holes. When the mold is very large, it is difficult to vent it thoroughly, and when the work on it extends over a period of three or four weeks, it is impossible to keep the vents from filling up; hence the general use of dry sand work for large castings. Often, however, for the sake of economy, fairly large and complicated pieces must be undertaken in green sand, and it becomes a matter of importance that they be so designed that the molder will not be compelled to invite disaster by keeping his sand too wet, or ramming it too hard, and that there be no part of the mold which may not be thoroughly vented.
Elements of Green Sand Molding

In order that we may understand thoroughly the effect of the design of a casting on the probability of blow-holes, it is necessary that we review, in a brief way, the elements of green sand molding. The sand is sprinkled with water, and thoroughly mixed and sifted, preparatory to packing or "ramming" it around the pattern. The object of wetting the sand is of course to cause it to stick when it is packed. Up to a certain point, the wetter it is, the better it will stick, but the molder should not wet it any more than is necessary. In the same way, the more tightly the sand is rammed, the better its particles will cohere, and the more easily will the mold be handled, and the pattern drawn. However, tight ramming and wet sand, while they make a solid and easily handled mold, invariably produce blow-holes, and are therefore to be avoided.

It will be apparent then, that if a pattern be of complicated form, or hard to draw, or if when it is drawn it leaves the sand in such a form that the mold will easily fall together at a little jarring, the molder will be compelled to wet the sand more and to ram it harder than usual. Small, deep openings, sharp fillets, and thin walls and partitions of sand, are especially troublesome. Not only do they make it difficult to draw the pattern, and handle the mold, and so make excessive wetting and hard ramming imperative, but they cause spots in the mold which the venting wire is unlikely to reach. For these reasons, they are to be avoided when possible, in any class of molding, whether it be green sand, dry sand, or loam work, and on no account should such work be permitted in the case of large green sand castings.

When designing a casting to be made in green sand, the designer ought to know the position which it will occupy in the mold, when it is poured. In general, the parts of a casting which lie in the bottom of the mold will be the soundest, and those parts which must be machined, or which require the greatest strength, should therefore occupy the bottom of the mold, if possible, when the casting is poured. Having decided which side will be down, the designer needs generally to pay no particular attention to the configuration of the lower part of the mold, provided only that all of the pattern can be drawn, and that there are no sand partitions which overhang, or whose extent is large in proportion to their thickness. To insure a sound casting, the sand in the lower parts of the mold must be comparatively dry, and loosely rammed. This condition of affairs is not generally hard to attain, since all the work on the sand is done with the pattern in place, and that part of the mold is not generally moved or handled after the support of the pattern has been withdrawn. In the lower part of the mold, the sand is generally supported at all points in a very thorough manner by the sand lying under it, and so hard ramming or wet sand is unnecessary. If, however, the pattern must be made with loose pieces, or with sharp fillets, or must leave thin walls or tongues of sand when it is withdrawn, the case is changed. Then hard ramming and wet sand are almost compulsory, and the molder
is not to be blamed if he does not produce sound green sand castings. The fault is with the designer.

The upper part of the mold must of necessity be rammed harder than the lower part, since the sand is not supported from beneath, but hangs from above. This is not as great a disadvantage as it might seem to be at first sight, since the escaping gases do not have to make their way through the iron, as they would if they were given off by the sand in the lower part of the mold. The venting, however, must be just as thorough, and it is desirable that the sand should be as dry as possible. The whole arrangement of the upper part of the casting should be such that the sand may be well supported from above. Generously rounded fillets and corners, simple surfaces, plenty of "draft," and an absence of depending walls and masses of sand, make the mold easy to handle, and therefore promote freedom from blow-holes.

When Green Sand and Dry Sand Both May Be Used

It often occurs that the larger part of a casting is of simple form, and easy to mold. A certain part of it, however, may be of a form exceedingly difficult to mold, and therefore likely to give a great deal of trouble. It is not necessary that the whole casting should be made in a dry sand mold, but a core-box may be made to take care of the difficult part of the work, even though the work would ordinarily be done without a core. It is just as easy, and often just as desirable to cast the external face of a casting against a core, as the internal face. While it may not pay to do this if only one casting is wanted, if a great many are wanted it is often the cheapest possible way of making them, and reduces to a minimum both the work of the molder and the chance of a spoiled casting. Often forms may be cast in this way which could not be attempted in any other. If it is desirable to use this method of working, the designer has it in his power to make the construction of the core-box much simpler and cheaper than it might otherwise be, by giving the matter a little thought.

In arranging the coring of a mold, it is always better, if possible, to support the cores at the top. The gases formed in the core, being light, tend to rise, and if the core is supported at the bottom only, they tend to escape into the iron, and to bubble through it. If they can escape at the top, they will pass off without coming in contact with the iron. When it is impossible to support the cores at the top, they should be so arranged that the gases can pass off at the sides, and escape from the mold without coming in contact with the iron.

Sponginess

A second fault to which iron castings are subject is that of sponginess. Sponginess is due to the formation of gas bubbles in iron, at the instant of solidification. In all ordinary cases this is due to an improper mixture of iron. However, if the casting is very thick at one place, and thin at most others, it will be impossible to obtain a mixture which will have satisfactory properties for general work, and not be spongy at points of extraordinary thickness. It is an excellent
rule to allow no part of a casting to be at a greater distance from a sand surface than 2 1/2 inches. In case this rule is strictly adhered to, and the castings are of fairly uniform thickness no trouble will be experienced from sponginess, except from the use of poor iron. When, however, we are obliged to concentrate a considerable quantity of metal at one place, and give it a greater thickness than 5 or 6 inches, either we must take care that it will be at some point where the sponginess will do no harm, or else we must make provision to do away with it.

The only practical method for doing this is to place a riser immediately over the heavy spot. When the metal is poured, and the riser is full, a rod of wrought iron is inserted and worked up and down until the metal has almost solidified. By so doing, the bubbles have a better chance to escape, and the iron is left perfectly solid. Of course, it is not possible to use a riser effectively in this manner, unless it can be placed directly over the heavy spot. A riser at a point a few inches distant is useless. The use of a riser in this way, and for this purpose, is unnecessary when the part of the casting in which the heavy spot occurs is subject to no particular stress, or is not required to be tight under steam, air, or hydraulic pressure, but nevertheless, a spongy spot is a defect in a casting, which should, if possible, be avoided.

Shrink-holes

A third fault to which iron castings are subject is that of shrink-holes. A shrink-hole is a cavity in a casting caused by the shrinking away of the metal in cooling. Like sponginess, this defect is especially likely to occur in those parts of a casting which are excessively thick. To avoid this fault, it is best to avoid sudden changes in the thickness of a section. If the part of a casting which is unusually thick does not have to be machined, the difficulty may be overcome by placing in the mold at that point a piece of iron, so that the casting will be caused to solidify at that point first, on account of the chilling effect of the cold iron. If, however, the heavy spot in the casting has to be machined, or if it is subject to heavy stress, this method of preventing shrink-holes is to be avoided, since the chilling of the iron makes it so hard as to be impossible to cut with a tool, and at the same time creates stresses within the metal which weaken it. In such a case, shrink-holes are best prevented in the same manner as has already been described for the prevention of sponginess, namely, the use of a heavy riser, and the working of the iron with a rod when it is cooling.

The designer must therefore avoid heavy spots in castings, whenever possible, for the reason that they are likely to produce two serious faults, sponginess and shrink-holes. He must avoid them especially in those parts of castings which are to be machined or which are subject to heavy stresses. If they cannot be avoided entirely, in such a case, they should be so arranged that risers may be placed immediately over them, so that a rod may be inserted into the riser, and into the heart of the spot where the metal is thickest.
FAULTS OF IRON CASTINGS

Scabbiness

A fourth fault often encountered in iron castings is that of scabbiness. Although iron in the molten condition does not permeate the sand of the mold, as water would if it were poured in, nevertheless, on account of the great weight of the fluid, it has a considerable erosive action on the materials of the mold. If, as it flows into the mold, the iron eats away fillets or partitions, or scour away patches of sand, it is obvious that the casting will not be of the proper form, but will have its angles partly filled up, and unsightly protuberances upon its surfaces. Such imperfections as these are known as scabs. The sand so washed from its proper place may float on the iron, and rise to the top of the mold, where it forms a dirty mixture, which, when cleaned away, leaves a rough depression in the surface of the casting, also known as a scab.

The remedy for this trouble is to avoid as far as possible sharp fillets, and thin tongues of sand, projecting into the mold, and to so gate the casting that the current of iron, as it enters the mold, will spread itself out, and not concentrate itself in any particular direction, for if it does, it will eat away the part against which it flows, just as quickly and surely as would a current of water. In general, proper gating is a matter which must be attended to by the molder, but if the designer has arranged things so that proper gating is inconvenient or impossible, the castings will almost surely be scabby.

Sand-holes

The fifth fault to which iron castings are subject, namely sand-holes, is one which is almost invariably associated with that of scabbiness. If the sand which has been eroded by the entering current of iron does not rise immediately to the surface, the iron may partially solidify before it will float to the top. As a result, it will rise till it strikes a part of the iron which has so solidified, and will remain there, imprisoned within the body of the casting. Sand-holes generally occur in that part of a casting which lies near the top of the mold, and just a little ways under the skin. They may occasionally form large cavities which seriously impair the strength of the casting, but more often they form very small holes, which, being full of sand, destroy the edge of any tool which may be used for the purpose of machining the casting, and leave the finished surface pitted and unsightly.

From this description of their origin, it must be apparent that the cure for sand-holes, as far as the designer's work is concerned, is the same as that for scabbiness. The fault may also be avoided by the use of a riser, so arranged that the current of iron will sweep the loose sand out of the mold and into the riser, where it will do no harm; but while this remedy eliminates sand-holes, it does nothing to remedy scabbiness, which is generally the cause of sand-holes.

Floating Cores

A sixth difficulty often encountered in the production of sound iron castings, is caused by floating cores. The buoyant effect of the molten
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Iron on a core is equal to about three times the weight of the core, if it is solid, and very much more than that in case the core is hollow. Large cores are generally built-up about cast-iron skeletons known as core frames. These core frames are roughly of the same shape as the core, and serve to support it, and to bind it together. Were it not for these frames, heavy cores would fall to pieces by their own weight, and would be broken up in the process of casting, owing to the buoyant effect of the iron. A projecting piece of core having a volume of four cubic feet, for instance, will weigh approximately 500 pounds, and have a buoyant force of about 1,500 pounds thrusting it upward when the mold is poured. If the core frame is not amply strong and stiff, this force will bend the projection upward, or even break it off entirely. Hence the necessity of making large cored cavities of such form that the cores may be rigidly anchored and thoroughly secured. Nor is it sufficient that provision be made for securing the cores, but they must be of such shape, and so reinforced by the frames, that they will be stiff and strong, and not bend appreciably under the tremendous forces which will act on them when they are surrounded by the molten iron.

One of the most difficult things to cast properly is a long iron pipe having a small diameter and thin walls. If such a pipe be cast in the usual position, that is, lying horizontally, there will be an upward thrust along the whole length of the core, tending to bend it. On account of its slenderness, the central portions will be deflected upward, making the walls of the pipe thinner on one side, and thicker on the other. Often the deflection proves sufficient to thrust the core against the side of the mold, if it is long, or, in case the thickness of the wall is great in proportion to the length of the core, to break it off entirely. On this account, pipes and hollow columns of cast iron are often cast on end, thus avoiding any deflection of the core. The same principle may be applied to many other pieces, by taking care to so design them that long and slender cores shall have a vertical position when the mold is poured. If they have such a vertical position, and are supported at both ends, they will have no tendency to deflect one way or another, and this source of trouble may be completely avoided.

Cold Shuts

The seventh fault is known as a cold shut. A cold shut is caused by the imperfect uniting of two or more streams of molten iron, flowing together, which are too cold to coalesce. Such a fault often occurs on the upper side of a thin cylinder cast horizontally, when the iron is not sufficiently hot at the instant of pouring. It there appears as a seam in the side of the cylinder, where it is very apparent that the metal has united imperfectly. It is not only a weak spot in the casting, along which it will readily split if called upon to sustain any great stress, but it is a spot which will surely leak under pressure, and which it is impossible to calk. The cause of the imperfection is generally improper gating, or else too great thinness of metal. If the iron is obliged to flow in thin streams for long distances, it will be
cooled very much, and probably the advancing face will be partially solidified. Consequently, when it meets a similar advancing face of metal, which has been similarly cooled, there is small likelihood of their uniting properly.

The remedy is obviously to so design the casting that the metal will not have to flow in thin streams for long distances. The arrangement of gates and risers is often of great importance in minimizing cold shuts, and if the casting is large, and at the same time has thin walls, the designer must see that the gates may be so arranged that the iron may quickly fill up the mold. While the arrangement of the gating generally depends on the molder’s fancy, he may often be limited by the shape of the casting, and obliged to place the gate at some point where the iron, in flowing in, must spread itself into a thin sheet, or pass for a considerable distance through a narrow passage. Under such circumstances, a cold shut is hardly to be avoided.

**Shrinkage Strains**

The eighth and last fault is that of shrinkage strain. If we have two pieces of iron fastened end to end, as shown in Fig. 22, one piece being notably thinner than the other, the thinner piece will solidify first in the mold, and cool some hundreds of degrees below its freezing point, before the thicker part solidifies. As a result, the thicker part, when cooled to air temperature, will have, or rather tend to have, a less length than the thinner part, the reason being that at the instant of solidification of the thicker part, both pieces had the same length, although the thinner part was much the cooler. The thin part will then be in compression, while the thick part is in tension, and severe stresses will exist within the piece, which make it weaker than it would otherwise be in most cases.

Sometimes, however, we are enabled to utilize the shrinkage stresses to advantage. For instance, when cast iron was the standard material for the manufacture of ordnance, guns were cast with cores through which water was circulated, so as to cool the surface of the bore before the outer parts solidified. When a gun is fired, it is known that the inner layers of metal are stretched more than the outer ones. By cooling the inner layers of metal first, shrinkage strains are produced in the walls of the gun, causing the outer layers of metal to compress the inner ones. The combined effect of the shrinkage stresses and the stresses produced by the explosion is to produce a uniform stress throughout the walls of the guns, and so reduce the chance of rupture.

It is not often, however, that we are able to take advantage of shrinkage strains in this way. More often they are troublesome, caus-
ing work to warp in the process of machining, or causing mysterious cracks to develop without apparent cause. Since these strains are due to unequal rates of cooling in the different parts of the casting, the best way to eliminate them is to so arrange the thickness of the various parts, that the entire casting shall solidify at the same time. The second best way is to so arrange the parts of the casting that the unequal contraction shall not produce dangerous stresses at any point. In order that the entire casting shall cool at a uniform rate, it is necessary that all parts of it shall be of approximately uniform thickness, and that there shall be no sudden changes of section. In order that unequal contraction shall not produce dangerous stresses in the metal, it is necessary that there shall be no sharp corners, and that the various parts shall be free to expand when necessary. For instance, a wheel or pulley with a solid rim is likely to have severe stresses set up within the arms by unequal cooling, but if the hub be divided as shown in Fig. 23, by means of a thin core, and then bolted subsequently, no shrinkage strains will occur, since the arms are free to expand or contract, independently of the rim.

Shrinkage strains often become so serious that it becomes necessary to make pieces in two or more parts, which it would be perfectly possible to make, at much less expense, in one piece. Large jacketed cylinders, for steam and gas engines, are good examples of this. When cast in one piece, the shrinkage stresses, together with the stresses set up by the varying temperatures incident to services, are often sufficient to crack them. Were the piece shown in Fig. 22 made in two parts, as shown in Fig. 24, there would be no shrinkage strains in either part, although the cost of machining the surfaces which are
fitted together, and of putting in the bolts, would not always warrant the construction.

Relative Economy of Simple and Complicated Castings

In conclusion, it may be well to state that most of the faults enumerated will be more likely to occur in a part of a complicated casting, than in a similar part of a simpler casting. For instance, the cylinder of a gas engine will be more likely to have some imperfection if it is cast integral with the frame, than if it is cast separately.

In the same way, the frame will be more likely to have an imperfection of some kind, than if it were cast separately. Assuming that ten per cent of the cylinders or frames would be lost if they were cast separately, it is more than likely that fifteen per cent of the castings, having cylinder and frame cast together, would be rejected for faults in the frame, and fifteen per cent of the remainder would be rejected for faults in the cylinder. In other words, twenty-eight per cent of these castings would be rejected, against ten per cent of the simpler forms. If more than eighteen per cent of the cost of the castings is saved in machining, or in other ways, by casting cylinder and frame together, it is well to do so, but if the saving is not more than sufficient to balance the loss, it is well to make several simple forms, instead of one complicated one.
CHAPTER VI

PROPORTIONS OF MACHINES BUILT IN A SERIES OF SIZES*

The problem of cost reduction forces itself, with increasing vividness upon the mind of every person who has to do with the manufacture of machinery. To the “small shop” people, and to those whose product is unsystematized and whose ideas of methods to pursue are, as yet, vague, this chapter may prove of some assistance.

* There are three important means by which the shop product may be systematized: By the use of formulas; by the use of tables; and by the use of charts. As the two latter may be considered as the tabulated, or graphic results of the former, we will deal only with the formulas. In determining sizes, weights, and costs, these formulas are generally most efficient time-savers. For convenience, formulas in this chapter will be divided into two classes: The class used to produce the first of a type of machine we will call fundamental; and the class used to produce several sizes of this type of machine, empirical. Upon seeking fundamental formulas in text books and in mechanical engineers’ pocket-books we are confronted by a diversity of opinions and tabulated results that are, at least to a noyice, a bit confusing. These formulas, it is always to be remembered, have their application in the special case under consideration, and are to be used only as guide posts in our journey of design. It is evident to most designers that some kind of a tentative method must, sooner or later, be resorted to in the type design, for in nearly all machines the governing conditions soon become so numerous or indefinite as to render a subdivision of the problem a necessity. A certain amount of judgment is absolutely essential in the use of most fundamental formulas, and discrimination is always necessary.

Graphically, fundamental formulas can be represented by curves, and will be correct for all sizes under identical conditions, while empirical formulas rest on no such basis and hold true for but a certain series within certain limits. This constitutes the vital difference between fundamental and empirical formulas. A fundamental formula is one found through mathematical reasoning, while an empirical formula is made up by means of trial methods.

Suppose that we have built two or three sizes of a certain type of engine and that they are successful; we desire to put on the market an entire line. Our sizes of this type of engine will run from 10-inch cylinder diameter in the smallest to 30-inch cylinder diameter in the largest. We have built a 12-inch and a 24-inch engine and perhaps an 18-inch. These engines were, as was imperative, tentatively designed. In seeking the derivation of the empirical formula for the

* MACHINERY, November, 1902.
length of the cross-head shoe, we find that on our 12-inch engine we have given it an area of \(55\frac{1}{2}\) square inches, and on our 24-inch engine its area is \(190\frac{1}{2}\) square inches. In each case the length of the shoe was nearly twice its width, so we decide to make it so in our line of engines; solving for the width, we have in the 24-inch engine

\[2x^2 = 190.125; \quad x = \sqrt{95.0625} = 9.75.\]

making our shoe length for the 24-inch engine 19\(\frac{1}{2}\) inches, and for the 12-inch engine 10\(\frac{1}{2}\) inches.

To any scale in Fig. 25, perpendicular to the line NL, lay off these shoe lengths PB and P'B'—10\(\frac{1}{2}\) inches, and 19\(\frac{1}{2}\) inches, respectively—making the distance BB' equal to 12 inches, the difference between our sizes 12 inches and 24 inches. Through points PP' draw line SA intersecting NL at A. At B''—for our 18-inch size—erect a perpendicular B''P''. Draw PF intersecting B''P' at F. Using the notation given in the figure, we get the simple equations

\[
e : y = c : d; \quad ed = cy; \quad e = \frac{c}{d}; \quad x = \frac{c}{d} + c.
\]

In this last formula many will recognize an old acquaintance—the equation for a straight line. Let us now analyze this equation. From the figures it is seen that \(x = \) the desired dimension and that \(\frac{c}{d} = \) the rate of increase in the slope of the line. If now we measure the distances and substitute their values for \(c\) and \(d\) we may determine the ratio \(\frac{c}{d}\), which we will call \(f\).

Then \(f = \frac{10.5}{14} = \frac{3}{4}\), and

\[x = fy + c, \text{ or } x = \frac{3}{4}y + c.
\]

In interpreting our empirical formula \(x = fy + c\), we have \(y = \) a common unit to which all other sizes are to be referred, \(x = \) desired dimension,
\[ f = \text{a factor of } y, \]
\[ c = \text{a constant increment to be added in each case.} \]

The unit of value \( y \), as generally selected, is a bolt or cylinder diameter, or the capacity of the machine. Obviously, in our line of engines, we select the cylinder diameter \( D \) as our value of \( y \), and our unit formula then becomes \( \frac{3}{4}D + c \). The value of \( c \) is now determined by direct substitution in the following manner: \( x \) being the shoe length, we substitute for it 19\( \frac{1}{2} \) inches (its length on the 24-inch cross-head); then

\[ \frac{3}{4} \times 24 + c = 19\frac{1}{2}; \quad c = 19\frac{1}{2} - 18 = 1\frac{1}{2}. \]

Note, that while we have assigned to \( y \) and \( c \) other values, we have not altered the relations; our formula for this particular cross-head dimension now becomes \( \frac{3}{4}D + 1\frac{1}{2} \) inches.

For convenience in charting these sizes, some point is determined upon as a pole about which these lines (represented by our formulas) are drawn as vectors, the ordinate length for a particular size giving the desired dimension. If now in the determination of other formulas it be found, as is likely to be the case, that these lines do not all pass through a common point, it becomes necessary to select one. In well-designed machines the intersection of these lines with the base line will come close together, and an average of these intersections is selected as a pole. Figs. 26 and 27 will serve to illustrate the purport of this paragraph.

Experienced designers are well aware that the final test of any dimension in a design is that of satisfying all fundamental calculable
conditions; nevertheless, the instances where our empirical\textsuperscript{6} formulas prove incorrect are very few indeed. With the design for our line of engines thus systematized, let us consider what are to be the advantages that will naturally result from it. In the first place, the weights of any particular parts, or details of any size in our line of engines may be determined prior to its design or manufacture. In the determination of weights, cubic contents, and similar processes, the use of "differences" as applied to higher mathematics, will not only prove an efficient time-saver, but relieve much of the drudgery attendant upon such operations.

A brief explanation of the use of "differences" is as follows: When we have a series of numbers connected by a regular, though not obvious law, the nature of that law may be discovered by forming a new series of differences between each two terms of the original series, and then treating the new series (which we may call the series of first differences) in the same way, until we reach a series of differences, the law of which is obvious. In the table above will be found both the arithmetic and algebraic solutions of problems by "differences."

In column 1 of the table is given a series of numbers, which we suspect follows some definite, though not obvious law, and which we desire to discover. We here take the differences between each two terms in column 1 and put them down in column 2. Having proceeded with the two orders of differences, the law becomes apparent early in the process of determining the values in column 3. Referring again to the table, it is evident that the next term of column 3 must be 11, which gives 68 (57 + 11) as the next term of column 2 (the series of first differences) and 312 (244 + 68) for the original series. Note that this series can thus be obtained indefinitely, and that ultimately,
in any regular series, some one series of differences will become a constant. It is on the principle of differences that calculating machines are constructed to compute logarithmic tables, etc.

In the algebraic solution of such problems as involve the determination of weights and volumes, it will be necessary to calculate these weights or volumes for the 1st, 2d, 3d, and 4th terms of our given series. By substituting in the formulas in column 5 the numerical value of \( W \), which is the first term in our given series, we may equate these expressions and our calculated values for the 2d, 3d, and 4th terms, and determine, by simple algebraic processes, the values of \( c, x, y \), and ultimately, those values which we are requiring in the original series, column 5.

The computations concerning the cost of materials logically follow the determination of volumes and weights and are made with comparative ease. However, our next problem concerning the determination of the cost of labor is a more difficult one to solve. Formulas should express this cost in so many cents per pound of product, including all shop charges, and be established partially by experience and partially by methods suggested in this chapter.

In many instances it will be found both desirable and convenient to have this cost formula embody the unit dimension. When this is the case, the formula is, as are most cost formulas, established by the tentative methods to which we have just alluded. As the methods employed in the deduction of these formulas render them purely empirical, one or another form of expression may have to be adopted. However, formulas of this class usually assume the form of, or at least may be solved into, the familiar type form

\[
ax^2 - bx + c,
\]

where \( a \) and \( b \) are factors of the unit dimension, and \( c \) is a constant.

For the purposes of illustration we will assume that the formula for the cost of labor which we have established is

\[
\frac{3}{2}D^2 - 15D + 314.
\]

In this form the formula gives the total cost of labor in dollars for the size desired. The cost of labor \( C \) for our 18-inch engine would then be

\[
C = \frac{3}{2} (18)^2 - 15 \times 18 + 314 = 530.
\]

The computation of a final cost formula, embodying the unit dimension, is the last process in our development of shop formulas; this formula is derived directly from those relative to the costs of material and labor.
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