PERISTALTIC TRANSPORT OF A MICROPOLAR FLUID IN AN INCLINED CHANNEL WITH PERMEABLE WALLS

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Abstract - The peristaltic transport of a micro polar fluid in an inclined channel of half width 'a' with permeable walls is discussed . The effect of various parameters on the pumping characteristics is discussed through graphs

Keywords: Peristalsis, Micropolar Fluid , Permeable Walls

I. INTRODUCTION

Many researchers studied the peristaltic transport of fluids by considering Newtonian and non-Newtonian models. Classical continuum theory cannot explain the mechanical behavior of rheologically complex fluids, such as liquid crystals, colloidal fluids and blood. Due to this fact a new approach was necessitated. There are several approaches to the formulation of micro continuum theories of fluids such as simple deformable directed fluids, dipolar fluids, polar fluids, simple micro-fluids, micropolar fluids etc. All these consider the existence of couple stress and body couples.

Eringen [1,2] reported the theory of micropolar fluids in which the fluid micro elements undergo rotations without stretching. Micropolar fluids are superior to the Navier-stokes fluids and they can sustain stresses and body couples. Here the micro particles in the volume rotate with an angular velocity about the centre of gravity of the volume in an average sense and are described by the micro rotation vector $\overline{\Omega}$ .The micropolar fluids can support stress and body couples and find their applications in a special case of fluid in which micro rotational motions are important. Arimon and Cakmak [3] discussed three basic viscous flows of micropolar fluids. They are couette and poiseuille flows between two parallel plates and the problem of a rotating fluid with a free surface. Srinivasacharya et al. [4] made a study on the peristaltic pumping of a micropolar fluid in a tube. In this paper, we considered the peristaltic transport of a micropolar fluid in an inclined channel with permeable walls. The effect of various parameter on the pumping characteristics is discussed through graphs. Kh. S. Mekheimer [5] discussed the micropolar fluid model for blood flow through a tapered artery with a stenosis. Anuar Ishak at al [6] studied on MHD boundary-layer flow of a micropolar fluid past a wedge with constant wall heat flux. Krishna Kumari et al. [7] worked on Peristaltic Pumping of a Jeffrey Fluid under the Effect of Magnetic Field in an Inclined Channel. Y.V.K.Ravi Kumar et al [8] made a study on Peristaltic transport of a power-law fluid in an asymmetric channel bounded by permeable walls. P. Muthu [9] discussed on the influence of wall properties in the peristaltic motion of micropolar fluid

II. MATHEMATICAL FORMULATION

Consider the peristaltic pumping of a micropolar fluid in an inclined channel of half-width $'a'$. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity we restrict our discussion to the half-width of the channel as shown in figure (1).

The wall deformation is given by

$$H(X,t) = a + b\sin\frac{2\pi}{\lambda}(X - ct) \tag{1}$$

where b is the amplitude, λ is the wavelength and c is the wave speed.

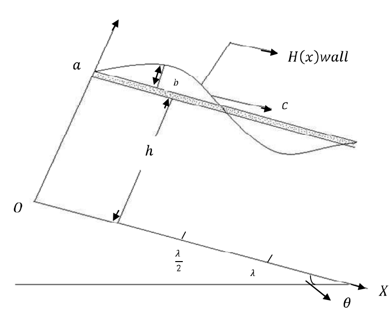

Figure. 1 Physical Model

III. EQUATIONS OF MOTION

Under the assumption that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x,y) moving with velocity c away from the fixed (laboratory) frame (X,Y). The transformation between these two frames is given by

$$x = X - ct : y = Y : u(x,y) = U(X-ct,Y) - c : v(x,y) = V(X-ct,Y) \tag{2}$$

where U and V are velocity components in the laboratory frame and u, v are velocity components in the wave frame. In many physiological situations it is proved experimentally that the Reynolds number of the flow is very small. So, we assume that the flow is inertia-free. Further, we assume that the wavelength is infinite.

Using the non-dimensional quantities.

$$\bar{u} = \frac{u}{c}; \bar{x} = \frac{x}{\lambda}; \bar{y} = \frac{y}{a}; \bar{p} = \frac{p a^2}{\lambda c \mu}; \bar{\Omega} = \frac{\Omega a}{c}; h = \frac{H}{a}$$

The non-dimensional form of equations governing the motion (dropping the bars) is

$$\frac{\partial^2 u}{\partial y^2} + N\frac{\partial \Omega}{\partial y} - (1-N)\frac{\partial p}{\partial x} + \eta \sin\theta = 0 \tag{3}$$

$$\frac{2-N}{m^2}\frac{\partial^2 \Omega}{\partial y^2} - \frac{\partial u}{\partial y} - 2\Omega = 0 \tag{4}$$

where $N = \dfrac{k}{\mu + k}$ coupling number, Ω is the micro rotation velocity, u is the velocity, μ is the viscosity of the fluid, k is the micropolar viscosity

m is the micropolar parameter, p is the fluid pressure.

θ is the inclination angle to the horizontal.

α is dimensionless Beavers - Joseph constant which depends on the nature of the porous medium but not the fluid viscosity.

where 'Da' is the Darcy number given by

Da = $\frac{k}{a^2}$

The corresponding non-dimensional boundary conditions are

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0 \tag{5}$$

$$\frac{\partial \Omega}{\partial y} = 0 \quad \text{at} \quad y = 0 \tag{6}$$

$$u = -\frac{\sqrt{Da}}{\alpha}\frac{\partial u}{\partial y} \quad \text{at} \quad y = h(x) \tag{7}$$

$$\Omega = -1 - \frac{\sqrt{Da}}{\alpha}\frac{\partial \Omega}{\partial y} \quad \text{at} \quad y = h(x) \tag{8}$$

IV. SOLUTION OF THE PROBLEM

The general solution of (3) and (4) is given by

$$u = L_8\left(y + h - L_4 - \frac{\sin L_3 y}{L_3}\right) + \frac{L_5}{L_3}\left(\frac{L_6 \cos L_3 h}{L_3} - L_4 \sin L_3 h - \frac{\cos L_3 h}{L_3}\right) - 1 + L_5 L_7 L_8 + 2h L_4 L_5 \tag{9}$$

$$\Omega = -\frac{L_5}{L_3}\cos(L_3 h)\sin(L_3 h) + L_5 h \cos(L_3 h) + \frac{L_5 L_6}{2 L_3} - \frac{L_5 L_6 h}{2} \tag{10}$$

Where $L_1 = (1-N)\frac{\partial p}{\partial x} - \eta \sin\theta$,

$L_2 = \frac{2-N}{m^2}, L_3 = \frac{N-2}{L_2} = -m^3,$

$$L_4 = -\frac{\sqrt{Da}}{\alpha}$$

$$L_5 = 2 + L_2 L_3^2, L_6 = L_3 L_4 \, Sin\, L_3 h - Cos L_3 h$$

$$L_7 = \frac{L_5}{L_3}(-hL_3 - L_4 L_3 + L_3 L_4 Cos\, L_3 h + Sin L_3 h)$$

$$L_8 = 1 - \frac{L_1 L_5 L_6}{L_3^4} + \frac{L_1}{L_3^2}(2hL_4 - h^2)$$

The volume flux q through each cross section in the wave frame is given by

$$q = \int_0^h u\,dy \quad (11)$$

The pressure gradient is obtained from equation (11)

$$\frac{\partial p}{\partial x} = \frac{1}{1-N}\left\{\frac{s_1}{s_2(s_3-s_4)} + \eta\sin\theta\right\} - s_5(s_6-s_7) + \frac{h^3 m^3}{3} \quad (12)$$

where

$$s_1 = (q+h)m^5\left\{1 - \frac{1}{N}[2\cosh(mh) - 2mL_4\sinh(mh)]\right\}$$
$$s_2 = (-sinhmh + mL_4coshmh - mL_4 + mh)$$
$$s_3 = 2coshmh - m^2h^2 - 2hmsinhmh + \frac{2hmL_2L_4}{N}coshmh$$
$$s_4 = \frac{4hm^2L_4coshmh}{N} - 2h^2m^2 - 2L_4hm^2 - 2$$
$$s_5 = 1 - \frac{1}{N}(2coshmh - 2mL_4sinhmh)$$
$$s_6 = Nsinhmh + m^4L_2L_4sinhmh - 2L_4m^2sinhmh$$
$$s_7 = Nmcoshmh + 2m^5h^2 + 4m^5hL_4$$

The time averaged flow rate is

$$\overline{Q} = q + 1 \quad (13)$$

V.PUMPING CHARACTERISTICS

Integrating the equation (12) with respect to over one wave length, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x}dx \quad (14)$$

The dimensionless frictional force F at the wall across one wavelength in the inclined channel is given by

$$F = \int_0^1 h\left(-\frac{\partial p}{\partial x}\right)dx \quad (15)$$

VI. RESULTS AND DISCUSSIONS

The variation of pressure rise Δp with time averaged flow rate for different values of α is shown in figure (2). It is observed that for a given $\overline{Q}$, Δp increases as the slip parameter α increases in the pumping and free pumping regions. The opposite behavior is observed in co-pumping region. And also for a given Δp the flux $\overline{Q}$, increases with increasing α .

The effect of the inclination angle θ on pumping characteristics is shown in figure (3). It is observed that for a given $\overline{Q}$, Δp increases as the angle of inclination θ increases.

The variation of pressure rise with time averaged flow rate for different values of micropolar parameter m is shown in figure (4). It is observed that for a given $\overline{Q}$, Δp decreases for a decreasing m in pumping and free pumping regions. For a given Δp the flux $\overline{Q}$, depends on m and it increases with increasing$m$.The effect of coupling parameter N on the pumping characteristics is shown in figure (5).We observed that the large the coupling number N the pressure rise against which the pumping works. For a given $\overline{Q}$, the pressure difference increases with increases N. The effect of amplitude ratio on pumping characteristics is shown in figure (6).
It is observed that the large the amplitude ratio,
the greater the pressure rise against which the pump works. For a given Δp , the flux $\overline{Q}$, depends o n ϕ and it increases with increasing ϕ .

Figures (7) to (11) are drawn to study the effect of various parameters on the microrotation velocity. From figure (7) it is observed that an increase in the angle of inclination decreases the microrotation velocity. From figure (8) it is noticed that decrease in the darcy number decreases the microrotation velocity. It is also observed that the velocity profiles are parabolic and the velocity attains the maximum at the central line of the channel. From figure (9) it is observed that decrease in M decreases the microrotation velocity. The effect of micropolar parameter on the microrotation velocity is shown in figure (10). It can be seen that the decrease in m decreases the microrotation velocity.

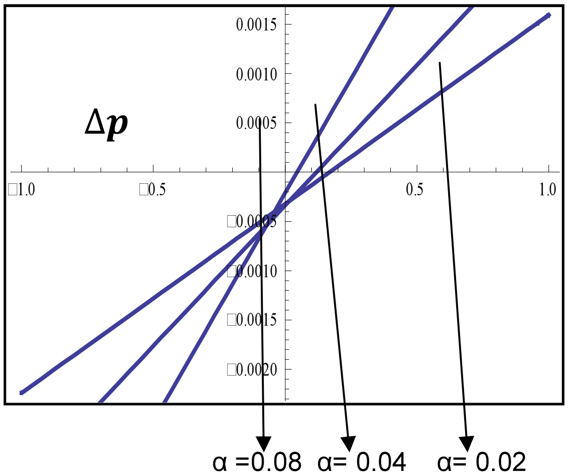

Fig.2. Variation of with $\overline{Q}$ for different values of slip parameter α ϕ .

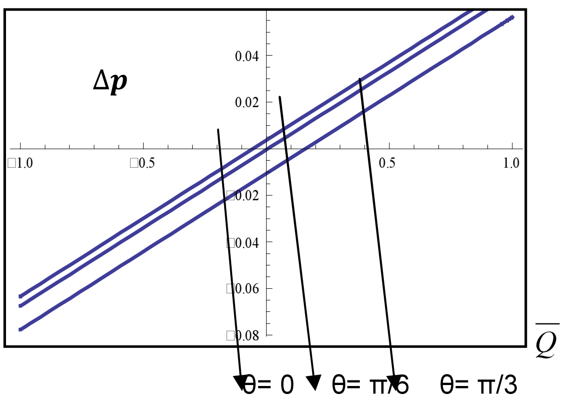

Fig 3. Variation of Δp with $\overline{Q}$ for different values of θ

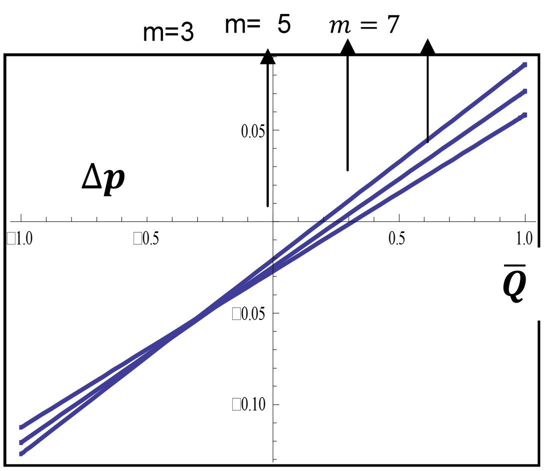

Fig.4 Variation of Δp with $\overline{Q}$ for different values of m .

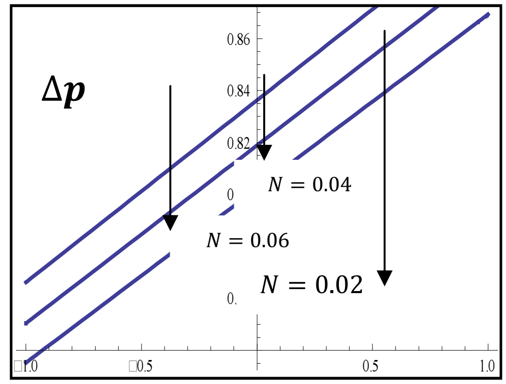

Fig. 5 Variation of Δp with $\overline{Q}$ for different values of N .

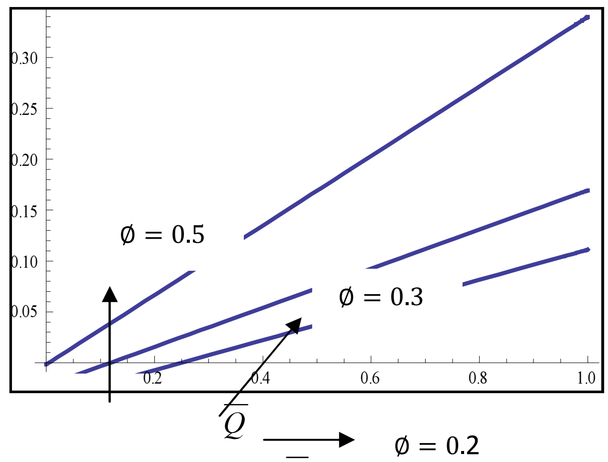

Fig. 6 Variation of Δp with $\overline{Q}$ for different values of ϕ .

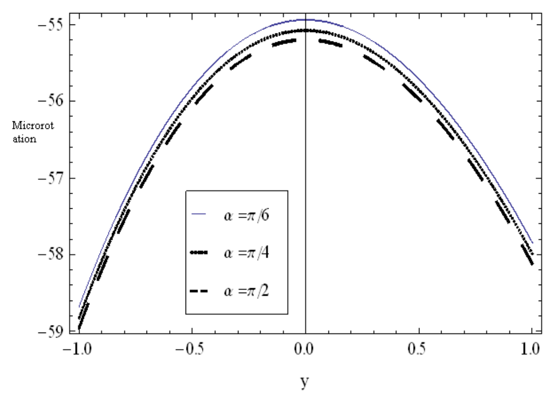

Fig.7 Variation of micro rotation velocity with y for different values of α

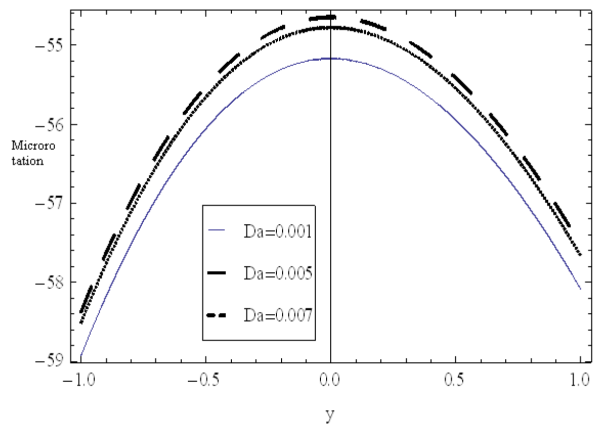

Fig. 8 Variation of microrotation velocity with y for different values of Da.

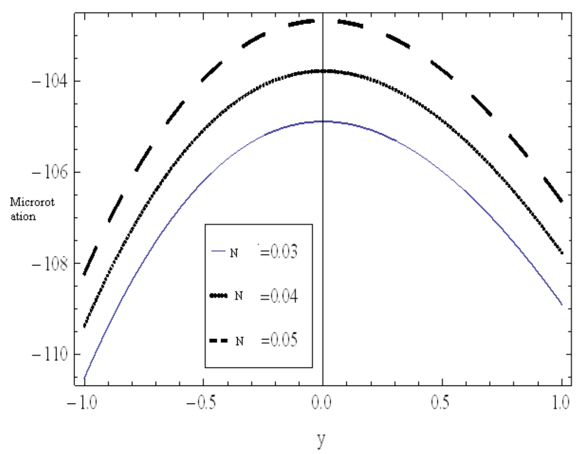

Fig.9 Variation of microrotation velocity with y for different values of N

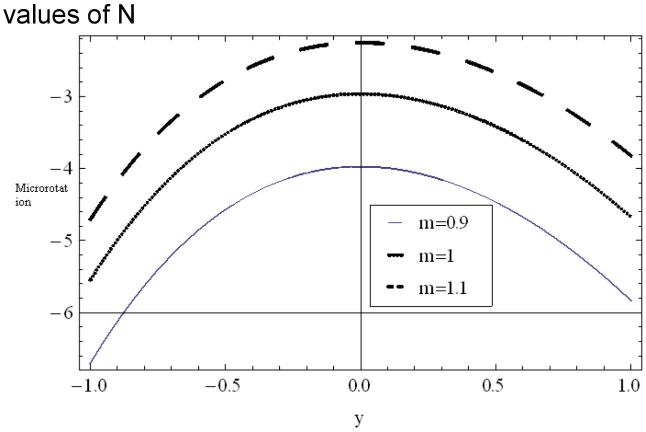

Fig.10 Variation of microrotation velocity with y for different values of m .

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