

# Study of Elliptical Core Step Index Fiber Using Marcatili's Method

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**Abstract**—This paper is interested to study the propagation characteristics of the elliptical core step index fiber by the marcatili's method. With the help of this method, we can analyze the elliptical core fiber and extract its birefringence and chromatic dispersion behaviors at different wavelengths. Also the study of optical and geometrical structural parameters influences on the birefringence, dispersion and polarization mode dispersion (PMD) is done.

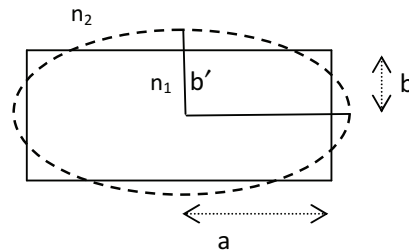


Fig. 1. Solid line shows cross-section of the core rectangular waveguide while dashed lines shows the core cross-section of elliptical fiber

**Keywords:** Elliptical core fiber, marcatili method, polarization mode dispersin

## I. INTRODUCTION

Elliptical core fibers have many applications such as in coherent optical communication systems [1] fiber optic sensors [2], and acousto-optic frequency shifters [3]. There are numerical, approximate and experimental methods for analysis of elliptical core fiber. Propagation characteristics of elliptical core optical fibers with any ellipticity can be obtained exactly using a series of Mathieu Functions [4], and using simple approximate methods like effective-index method [5], Marcatili's method [6] and experimental method like white-light spectral interferometry [7,8]. Because propagation characteristics of elliptical core fibers are nearly to those of a rectangular-core waveguide having the same core area and aspect ratio [9], in this letter we using marcatili's method.

## II. ANALYSIS

We consider a step – index elliptical core fiber with  $n_1$  and  $n_2$  as the core and the cladding refractive indices, respectively, and  $a'$  and  $b'$  as the semimajor and semiminor axes of the core ellipse. Now we approximate elliptical-core fiber with a rectangular dielectric waveguide as illustrated in Fig.1.

The rectangular core dimensions are chosen such that it and the elliptical core have the same area and aspect ratio (i.e.  $a/a' = b/b' = (\pi)^{1/2} / 2$ ) [9]. The key assumption made in marcatili's analysis is that the mode is well guided [6], i.e., most of its guided power is contained within the core region on the other hand, far from the cutoff. Most of the power travels in core region and a small part travels in cladding region. Hence, Maxwell's equations can be solved by assuming relatively simple sinusoidal and exponential field distributions. Rectangular dielectric waveguide is found to support a discrete number of guided modes that can be grouped in two families,  $E_{pq}^x$  and  $E_{pq}^y$  [10]. The sub-index  $p$  and  $q$  indicate the number of extreme of the electric or magnetic field in the  $x$  and  $y$  directions, respectively. The components of the  $E_{pq}^x$  mode are  $E_x, E_z, H_x, H_y$ , and  $H_z$ , with  $E_y=0$ . The components of the  $E_{pq}^y$  mode are  $E_y, E_x, E_z, H_x, H_z$ , and  $H_y$ , with  $H_x=0$ . The  $E_{pq}^x$  mode has  $E_x$  and  $H_y$  as principal field components, and the  $E_{pq}^y$  mode has  $E_y$  and  $H_x$  as principal field components.  $E_{11}^x$  and  $E_{11}^y$  are the fundamental modes.

We consider only the case  $E_{pq}^x$  mode, as  $E_{pq}^y$  mode can be treated similarly. According to Marcatili's method the propagation constant ( $\beta$ ) of these modes are determined from:

$$\beta^2 = k^2 n_1^2 - k_x^2 - k_y^2 \quad (1)$$

Where  $k$  is wave number in the vacuum and defined by  $(2\pi)/\lambda$ ,  $\lambda$  is the wavelength,  $k_x$  and  $k_y$  are the transverse propagation constants along the  $x$  and  $y$  directions respectively, they are the solutions of the following transcendental equations:

$$k_x a = p\pi - (2) \times \tan^{-1} \left( \frac{k_x}{(k^2 (n_1^2 - n_2^2) - k_x^2)^{0.5}} \right) \dots\dots\dots(2)$$

$$k_y b = q\pi - (2) \times \tan^{-1} \left( \frac{n_2^2 k_y}{(n_1^2 (k^2 (n_1^2 - n_2^2) - k_y^2)^{0.5})} \right) \dots\dots\dots(3)$$

The propagation constant  $\beta_{pq}^x$  (or  $\beta_{pq}^y$ ) of the  $E_{pq}^x$  mode (or  $E_{pq}^y$  mode) of the given elliptical core fiber can now be obtained by the above relation.

The effective refractive index is given by:

$$n_{eff} = \frac{\beta}{k} \dots\dots\dots(4)$$

Also for easy handling of the problem and identifying the proposed fiber, following optical parameters are defined.

$$b = \frac{n_1^2 - n_2^2}{n_1^2 - n_2^2}, \dots\dots\dots(5)$$

$$\zeta = \frac{(a - b)}{(a + b)}, \dots\dots\dots(6)$$

$$\Delta = \frac{n_1^2 - n_2^2}{2n_2^2} \approx \frac{n_1 - n_2}{n_2} \dots\dots\dots(7)$$

**III. RESULT and DISCUSSION**

It is well known that fibers with large birefringence are used to preserve the polarization state of the incident light over large distance. Large birefringence can be achieved by asymmetric waveguide geometry such as an elliptical core fiber. Modes in an elliptical core fiber are nondegenerate. Elliptical core is used to introduce birefringence in order to maintain the polarization thus elliptical optical fibers are one type of the polarization maintain fibers.

Birefringence is defined by :  $\Delta\beta = (\beta_x - \beta_y) / k$

Where  $\beta_x$  and  $\beta_y$  are the propagation constants of  $E_{pq}^x$  and  $E_{pq}^y$  modes, respectively.

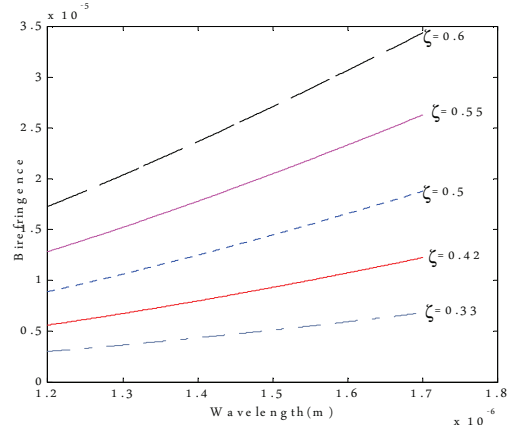


Fig. 2. Variation of birefringence versus wavelength for different ellipticity

At first, the influence of  $\zeta$  (called Ellipticity) on the birefringence is extracted and demonstrated in Fig. 2. From these curves, it is clear that birefringence of elliptical core fiber is increased by increasing the ellipticity of the core and birefringence increases slightly with the increase in wavelength. In other words, the more ellipticity, the better polarization maintenance.

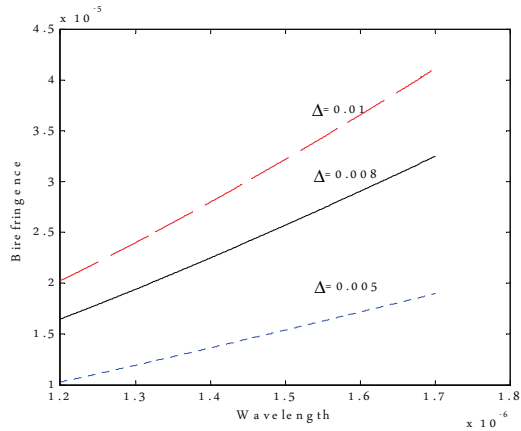


Fig. 3. Variation of birefringence versus wavelength for different  $\Delta$

Fig. 3 illustrates the variation of the birefringence as a function of wavelength for different refractive index differences. It is evident that the birefringence of such elliptical core fibers is grown up proportional to  $\Delta$ , that high index difference necessary to give a sufficiently large birefringence.

The propagation constant varies with wavelength, thus the group velocity depends on wavelength too and, consequently, different spectral components of the signal will travel with different group velocities. This

phenomenon is called chromatic dispersion and since chromatic dispersion limits the performance of fibers, we have evaluated this parameter in the elliptical core fiber. From Fig. 4 which shows dispersion curves versus wavelength for different ellipticities, it is clear that the dispersion decreases with ellipticity increasing and zero-dispersion wavelength shifts to a higher value of wavelength.

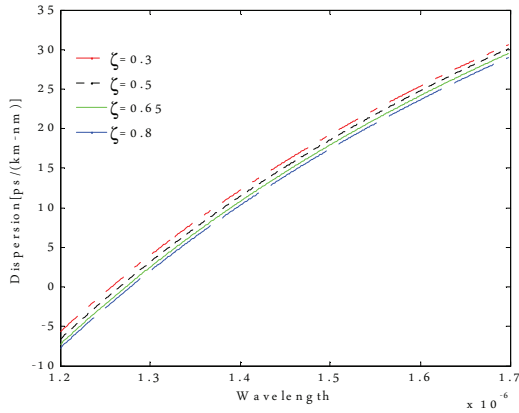


Fig. 4. Dispersion curves of fundamental mode for different ellipticities

Fig. 5 shows dispersion curves at different  $\Delta$ . It is evident that by  $\Delta$  increasing, the dispersion value is increased to and zero-dispersion wavelength shifts to a lower value of wavelength. Meanwhile, there is no appreciable change in the dispersion slope regime.

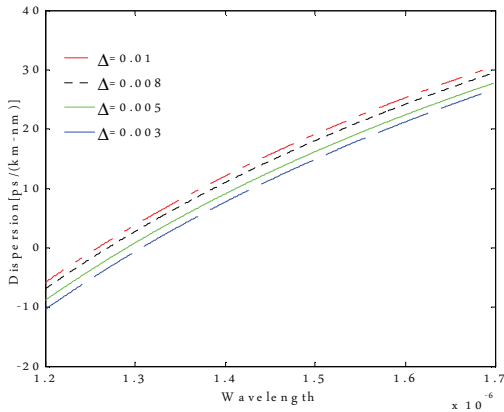


Fig. 5. Dispersion curves of fundamental mode for different  $\Delta$

Polarization mode dispersion ( $\Delta\tau$ ) between  $E_{pq}^x$  and  $E_{pq}^y$  modes per unit length of the fiber can be obtained by the following relation:

$$\Delta\tau = \frac{1}{c} \left( \frac{dB_x}{dk} - \frac{dB_y}{dk} \right) = \frac{1}{c} (n_x^x - n_x^y) \quad (8)$$

Elliptical core fiber are engineered in such a way that the two orthogonally polarized modes are forced to travel at different propagation constant, group velocities between the fundamental modes is zero but where there is a large enough difference in the phase velocities to preserve polarization. Fig. 6 illustrates variation of the Polarization mode dispersion versus wavelength for different  $\Delta$ . It is observed that the elliptical core fiber with large index difference hold polarization well.

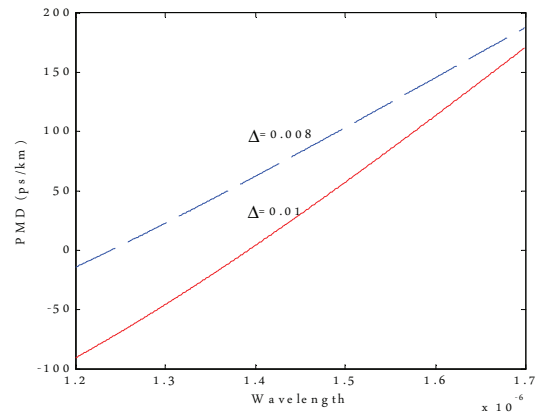


Fig. 6. PMD curves of fundamental mode for different  $\Delta$

Fig. 7 illustrates variation of the Polarization mode dispersion versus wavelength for different ellipticities. The results prove the strong dependence of the Polarization mode dispersion on the fiber core ellipticity i.e. Polarization mode dispersion increases strongly with core ellipticity. It is clear from this figures that Polarization mode dispersion becomes zero with careful design, the optical and geometrical parameters of elliptical core fiber. i.e. elliptical core fiber reduces mode coupling between orthogonal modes.

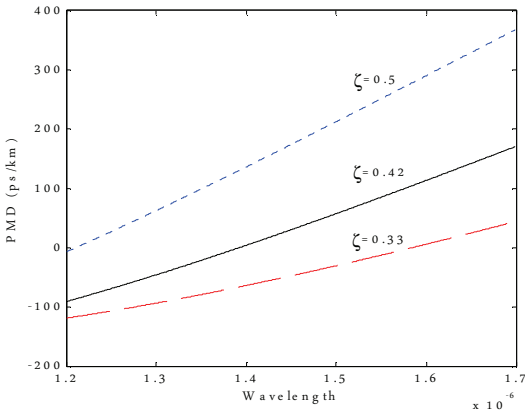


Fig. 7. PMD curves of fundamental mode for different ellipticities

#### IV. CONCLUSION

Using marcatili's method we have obtained birefringence, dispersion and polarization mode dispersion at elliptical core optical fiber. we show that birefringence of elliptical core fibers can be increased by increasing the ellipticity of the core or the index difference  $\Delta$  between the core and cladding and high value of birefringence is usually desired to maintain the polarization. We show that at elliptical core optical fiber, birefringence can be increased and zero Polarization mode dispersion can be obtained.

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