Peristaltic transport of Bingham fluid in a Channel with permeable walls

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Abstract: Peristaltic transport of a Bingham fluid in a channel with permeable walls is studied under long wavelength and low Reynolds number assumptions. This model can be applied to the blood flow in the sense that erythrocytes region and the plasma regions may be described as plug flow and non-plug flow regions. The effect of yields stress, Darcy number and slip parameter on the pumping characteristics are discussed through graphs.

Keywords: Peristaltic transport, Bingham fluid, permeable walls.

I. INTRODUCTION

Peristaltic transport is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible channel/tube containing the fluid. Peristalsis is used by a living body to propel or mix the contents of the tube such as, transport of urine from the kidney through the ureter to the bladder, food through the digestive tract, bile from the gall bladder into the duodenum, movement of ovum in the fallopian tube etc. It is accepted that most of the physiological fluids behave like non-Newtonian fluids. This approach provides a satisfactory understanding of the peristaltic mechanism involved in small blood vessels, lymphatic vessels, intestine, ductus efferentus of the male reproductive tract and in transport of spermatozoa in the cervical canal.

Nicoll and Webb (1946) and Nicoll reported that peristalsis plays an important role in blood circulation. Some theoretical and experimental investigations have been made on the peristaltic motion of blood considering blood as a non-Newtonian fluid. Latham (1966) first made an experiment to study the fluid mechanics of peristaltic transport. Base on this experiment work. Shapiro et al. (1971) made a study on peristaltic pumping a tube and a channel under the assumptions of low Reynolds number and long wave lengths.

Peristaltic transport of chyme modeled by a nonNewtonian fluid or power-law fluid in small intestine and oesophagus has been studied by Srivastava and Srivastava (1985)

El Shehawey et al. (1999) investigated the peristaltic flow of a Newtonian fluid through a porous medium. Mekheimer and Al-Arabi (2003) have discussed the peristaltic flow of a Newtonian fluid through a porous medium in a channel under the effect of magnetic field. Elshahed and Haroun (2005) discussed peristaltic flow of a Johnson Segalman fluid under the effect of a magnetic field. Ravi Kumar et al. (2010) studied the peristaltic pumping in a finite lengths tube with permeable wall. Most of the researchers considered various non-Newtonian fluid models under peristaltic transport. Ravi Kumar et al. (2011) considered power-law fluid in an asymmetric channel with permeable walls under peristalsis in their studies. Krishna Kumari et al. (2011) studied the peristaltic pumping of a Casson fluid in an inclined channel under the effect of a magnetic field.

In view of this, peristaltic pumping of a Bingham fluid in a channel with permeable walls is studied under long wavelength and low Reynolds number assumptions. The effect of various parameters on the pumping characteristics is discussed through graphs.

II. MATHEMATICAL FORMULATION

Consider the peristaltic pumping of a Bingham fluid in a channel with permeable walls, under long wave length and low Reynolds number assumptions (Figure.1). The flow in a channel is governed by Navier - Stokes equations whereas the flow in the permeable wall is described by Darcy's law. The channel is of half width 'a '.

The region between $y=0$ and $y=y_0$ is called plug flow. In this region $|\tau_{yx}|\le\tau_0$. In the region between $y=y_0$ and $y=h$, we have $|\tau_{yx}|>\tau_0$.

The wall deformation is given by

$$H(X,t)=a+b\sin\frac{2\pi}{\lambda}(X-ct) \quad (1)$$

where b is the amplitude, λ is the wave length 'c' is the wave speed.

Under the assumptions that the channel length is an integral multiple of the wave length λ and the pressure difference across the ends of the tube is a constant, the flow becomes steady in the wave frame(x,y)moving with velocity 'c' away from the fixed frame (X,Y) The transformation between these two frames is given by

x=X-ct, y=Y, u(x,y)=U(X-ct,Y)-c, v(x,y)=V(X-ct,Y)

where (U, V) are velocity components in the laboratory frame and (u, v) are velocity components in the wave frame.

We introduce the following non-dimensional quantities:

$$\bar{u}=\frac{u}{c};\bar{x}=\frac{x}{\lambda};\bar{y}=\frac{y}{a};\bar{h}=\frac{h}{a};\bar{t}=\frac{ct}{\lambda};\bar{\tau_0}=\frac{a\tau_0}{\mu c}$$

$$\bar{\psi}=\frac{\psi}{ac};\bar{p}=\frac{pa^2}{\lambda\mu c};\bar{q}=\frac{q}{ac};\boldsymbol{\phi}=\frac{\boldsymbol{b}}{\boldsymbol{a}};\bar{F}=\frac{\boldsymbol{Fa}}{\boldsymbol{\lambda\mu c}}$$

After non-dimensionalisation (after dropping bars), the governing equations and boundary conditions become

$$\frac{\partial}{\partial y}(\tau_0-\psi_{yy})=\frac{-\partial p}{\partial x} \quad (2)$$

$$0=\frac{\partial p}{\partial y} \quad (3)$$

$$\psi=0;\psi_{yy}=\tau_0 \text{ at } y=0 \quad (4)$$

$$u=\psi_y=-1-\frac{\sqrt{Da}}{\alpha}\frac{\partial u}{\partial y} \text{ at } y=h \quad (5)$$

where ψ is the stream function, Da=$\frac{K}{a^2}$,

α=Slip parameter, and τ_0 is the yield stress.

SOLUTION OF THE PROBLEM

Solving equation (2) and (3) subject to the boundary conditions (4) and (5) we obtain the velocity as

$$u=\frac{Ph^2}{2}[1-\frac{y^2}{h^2}-\frac{2\tau_0}{Ph}(1-\frac{y}{h})]-1+\frac{\sqrt{Da}}{\alpha}(Ph-\tau_0) \quad (6)$$

We find the upper limit of plug flow region using the boundary condition that

ψ_{yy}=0 at $y=y_0$ so we have $y_0=\frac{\tau_0}{P}$

Also by using the condition $\tau_{yx}=\tau_h$ at $y=h$

we obtain $P=\frac{\tau_h}{h}$

Hence, $\frac{y_0}{h}=\frac{\tau_0}{\tau_h}=\tau, 0<\tau<1$ (7)

Taking $y=y_0$ in equation (6) and using the relation (7) we get the velocity in plug flow region as

$$u_P=\frac{Ph^2}{2}[(1-\frac{y_0}{h})^2+\frac{\sqrt{Da}}{\alpha}(Ph-\tau_0)]-1, 0\le y\le y_0 \quad (8)$$

Integrating the equations (6) and (8) and using the conditions $\psi_P = 0$ at $y=0$, and $\psi = \psi_p$ at $y = y_0$

we get stream function as

$$\psi = -y + \frac{Ph^2}{2}[y - \frac{1}{3h^2}(y^3 - y_0^3) - \frac{2\tau_0}{h}(y - \frac{y^2}{2h})] + \frac{\sqrt{Da}}{\alpha}(Ph - \tau_0)y, 0 \le y \le y_0 \quad (9)$$

The volume flux q through each cross – section on the wave frame is give by

$$q = \int_0^{y_0} u_P dy + \int_{y_0}^{y} u dy$$

$$= \frac{Ph^3}{3}[1 - \frac{3}{2}(\frac{y_0}{h}) + \frac{1}{2}(\frac{y_0}{h})^3] - h + \frac{\sqrt{Da}}{\alpha}h^2(1-\tau) \quad (10)$$

The instantaneous volume flow rate Q(x, t) in the laboratory frame between the centre line and the permeable wall is

$$Q(X,t) = \int_0^h U(X,Y,t)dy$$

$$= \frac{Ph^3}{3}[1 - \frac{3}{2}(\frac{y_0}{h}) + \frac{1}{2}(\frac{y_0}{h})^3] + h^2\frac{\sqrt{Da}}{\alpha}(1 - \frac{y_0}{h}) \quad (11)$$

From equation (10) we have

$$\frac{dP}{dx} = \frac{-3(q+h)}{h^3[1 - \frac{3}{2}(\frac{y_0}{h}) + \frac{1}{2}(\frac{y_0}{h})^3 + 3h^2\frac{\sqrt{Da}}{\alpha}(1 - \frac{y_0}{h})}$$

$$= \frac{-3(q+h)}{h^3[1 - \frac{3}{2}\tau + \frac{1}{2}\tau^3] + 3h^2\frac{\sqrt{Da}}{\alpha}(1-\tau)} \quad (12)$$

Averaging equation (2.11) over one period yields the time mean flow $\overline{Q}$ as

$$\overline{Q} = \frac{1}{T}\int_0^T Q dt = q + 1 \quad (13)$$

Integrating the equation (12) with respect to x over one wave length, we get the pressure rise over one cycle of the wave as

$$\Delta P = -\int_0^1 \frac{3(q+h)}{h^3[1 - \frac{3}{2}\tau + \frac{1}{2}\tau^3] + 3h^2\frac{\sqrt{Da}}{\alpha}(1-\tau)}dx \quad (14)$$

where $h(x) = 1 + \phi \sin 2\pi x$

The time average flux at zero pressure rise is denoted by $\overline{Q_0}$ and the pressure rise required to produce zero average flow rate is denoted by ΔP_0 so we have

$$\Delta P_0 = -\int_0^1 \frac{(3h-3)}{h^3[1 - \frac{3}{2}\tau + \frac{1}{2}\tau^3] + 3h^2\frac{\sqrt{Da}}{\alpha}(1-\tau)}dx \quad (15)$$

It is observed that when $\tau \to 0$ and $\mathrm{Da} \to 0$ equation (6), (10) and (14) reduce to the corresponding results of Jaffrin and Shapario (1971) for the peristaltic transport of a Newtonian fluid in a channel.

The dimensionless friction force F at the wall across one wave length is given by

$$F = \int_0^1 h(-\frac{dP}{dx})dx$$

$$= \int_0^1 \frac{3(q+h)}{h^2[1 - \frac{3}{2}\tau + \frac{1}{2}\tau^3] + 3h\frac{\sqrt{Da}}{\alpha}(1-\tau)}dx \quad (16)$$

III. DISCUSSION OF THE RESULT

From equation (14), we have calculated the pressure difference as a function of $\overline{Q}$ for different values of τ, the ratio of yield stress to the wall shearing stress for a fixed amplitude ratio (ϕ), Darcy number (Da), and slip parameter (α) is shown in figure (2). It is observed that for a Bingham fluid, the peristaltic wave passing over the channel wall pumps against more pressure rise (ΔP) compared to Newtonian fluid. This type of behaviour may be due to the presence of plug flow in Bingham fluid. Further, we observe that there is no difference in flux for Bingham fluid and Newtonian fluids for free pumping case (ΔP=0). For a given ΔP the flux $\overline{Q}$ for a Bingham fluid depends on yield stress and it increases with increasing yield stress.

The variation of pressure rise with time averaged flow rate is calculated from equation (14) for different amplitude ratios (ϕ) and is shown in figure (3) for fixed τ=0.2, Da=0.001, α=0.1. We observe that the higher the amplitude ratio, the greater the pressure rise against which the pump works. For a given ΔP, the flux $\overline{Q}$ for Bingham fluid depends on yield stress and it increases with increasing ϕ.

From equation (14), we have calculated the pressure difference as a function of $\overline{Q}$ for different values of Darcy number Da, for fixed ϕ=0.6, τ=0.2, α=0.1 and is shown in figure (4). We observe that for a given ΔP, the flux $\overline{Q}$ depends on Darcy number and it decreases with increasing Darcy number Da. For free pumping the flux $\overline{Q}$ is constant and it is independent of Da.

The variation of pressure rise with time averaged flow rate is calculated from (14) for different values of α and is shown in figure (5) for fixed Da=0.001, τ=0.2 and ϕ=0.6. We observe that the longer the slip parameter, the greater the pressure rise against which the pump works. For a given ΔP, the flux $\overline{Q}$ increases with increasing 'α'. For a given flux $\overline{Q}$, the pressure difference ΔP increases with increasing α.

From equation (16), we have calculated the frictional force as a function of $\overline{Q}$ for fixed ϕ=0.6, Da=0.001, α=0.1 and for different values of τ and depicted of figure (6). As τ increases the flux $\overline{Q}$ decreases for a given frictional force. Further the frictional force for different amplitude ratios, different Darcy numbers , and different slip parameters is shown in the figures(7)-(9) and it is observed that the frictional force F has the opposite behavior compared to pressure rise (ΔP).

We have calculated the pressure rise from equation (15) required to produce zero average flow rate ΔP_0 as a function of the amplitude ratio ϕ, Darcy number Da for different values of τ and is shown in the figure (10). The value of ΔP_0 is larger for Bingham fluid when compared to Newtonian fluid ($\tau = 0$). As $\phi \longrightarrow$1, ΔP_0 becomes indefinitely large for a given value of τ.

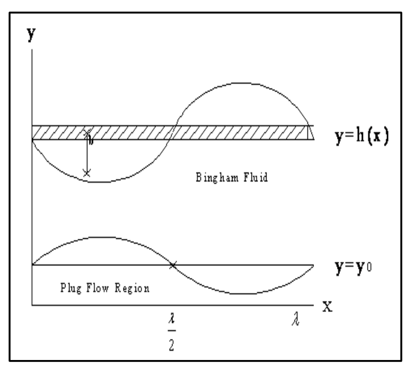

Fig 1: Physical Model

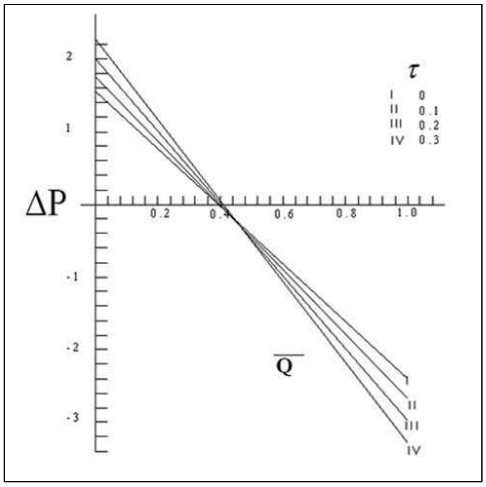

Fig.2: The variation of ΔP with $\overline{Q}$ for different τ with ϕ =0.6, Da=0.001, α=0.1

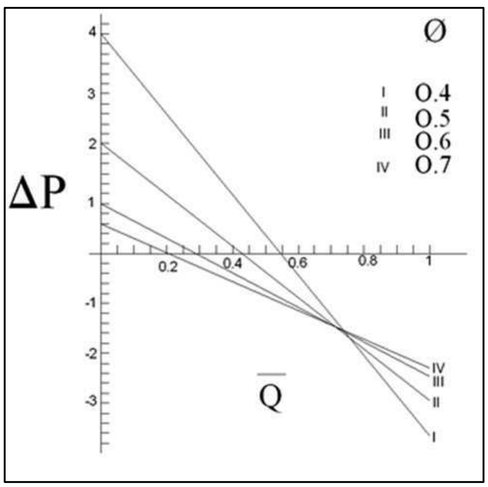

Fig.3: The variation of ΔP with $\overline{Q}$ for different ϕ with τ=0.2, Da=0.001, α=0.1

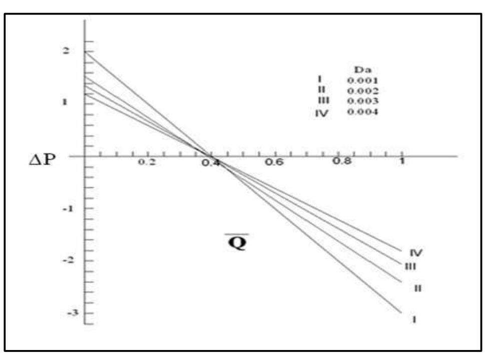

Fig.4: The variation of ΔP with $\overline{Q}$ for different Da with ϕ =0.6, τ=0.2, α=0.1

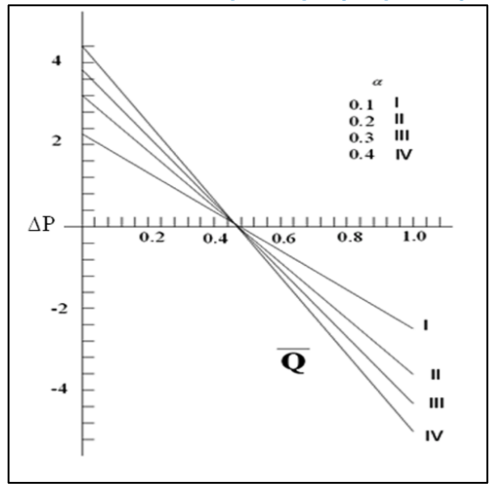

Fig.5: The variation of ΔP with $\overline{Q}$ for different α with ϕ =0.6, τ=0.2, Da=0.001,

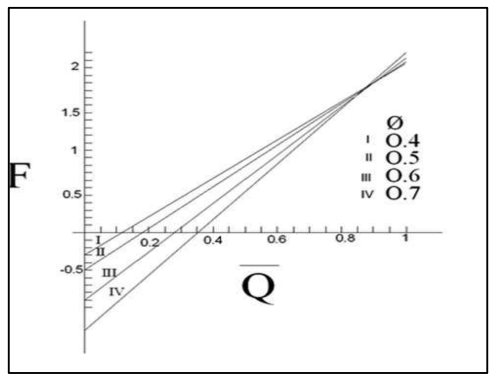

Fig.6: The variation of F with $\overline{Q}$ for different τ with ϕ =0.6, Da=0.001, α=0.1

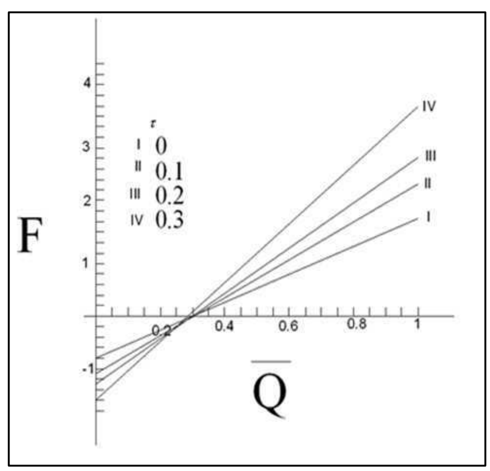

Fig.7: The variation of F with Q for different ϕ with τ=0.2, Da=0.001, α=0.1

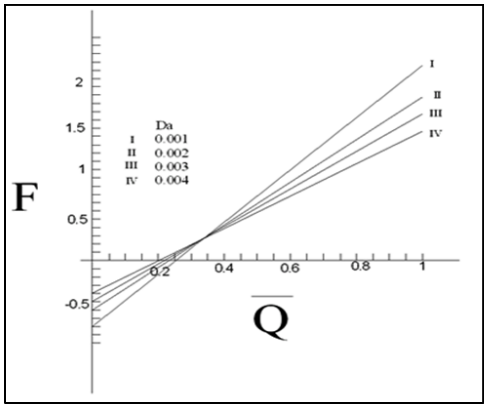

Fig.8: **The variation of F with Q for different Da with ϕ =0.6, τ=0.2, α=0.1**

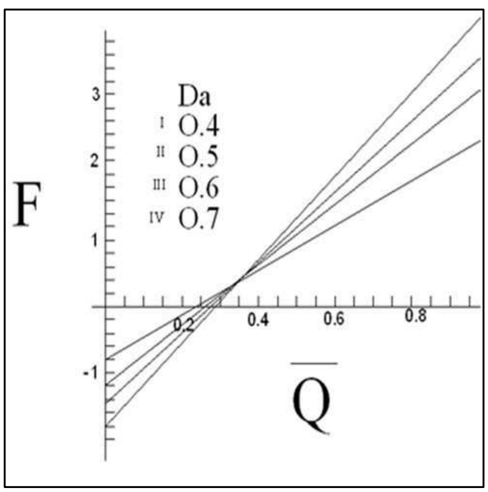

Fig.9: **The variation of F with Q for different Da with ϕ =0.6, τ=0.2, Da=0.001**

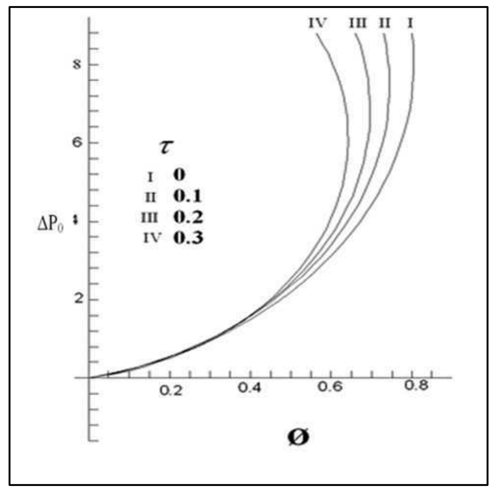

Fig.10: **The variation of ΔP_0 with ϕ for different τ with Da = 0.001, α = 0.1**

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